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On the stabilized asymptotic response of a system of solids in contact with wear

Sur la réponse asymptotique stabilisée d'un système de solides en contact avec usure

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ABSTRACT

This Note deals with the modelling of the response in wear of a system of two solids maintained in contact and subject to mechanical cyclic loading. The aim is to reproduce wear contact mechanisms commonly observed under cyclic loading, numerically and experimentally, for bearing journals. The present analysis is devoted to the determination of the system behaviour, the stabilization of its response and more particularly to the asymptotic state in wear under cyclic loading. On the one hand, the proposed modelling also enables to give useful analytical expressions inherent to the asymptotic response, such as the cumulated wear per cycle. On the other hand, it offers the possibility to perform direct sensitivity analyses, for instance to the wear coefficient effect, as studied in the last part of this paper.

RÉSUMÉ

On s'intéresse dans cette Note à la modélisation de la réponse en usure d'un système de solides en contact sous cycles de chargement afin de reproduire certains mécanismes de contact avec usure communément observés numériquement et expérimentalement dans les paliers à arbre tournant. L'analyse porte sur la détermination du comportement du système, sur la stabilisation de la réponse de celui-ci et plus particulièrement sur l'état asymptotique en usure sous chargement cyclique. La modélisation proposée permet aussi, d'une part, de donner des expressions analytiques utiles de grandeurs caractéristiques de la réponse asymptotique, telles que l'usure cumulée par cycle et offre, d'autre part, la possibilité d'effectuer des analyses paramétriques rapides, par exemple au coefficient d'usure, comme étudié en dernière partie.

1. Introduction

Despite the highly non-linear and time-dependent character of wear mechanisms, many advances have been made in this field and it is now possible to simulate numerically the behaviour in wear for some mechanical systems with good accuracy. Most of the encountered wear models remain mainly based on the use of Archard's law (e.g. [1–5]), as well as Coulomb's law of friction is essentially addressed in Contact Mechanics. The originality of those wear models comes from

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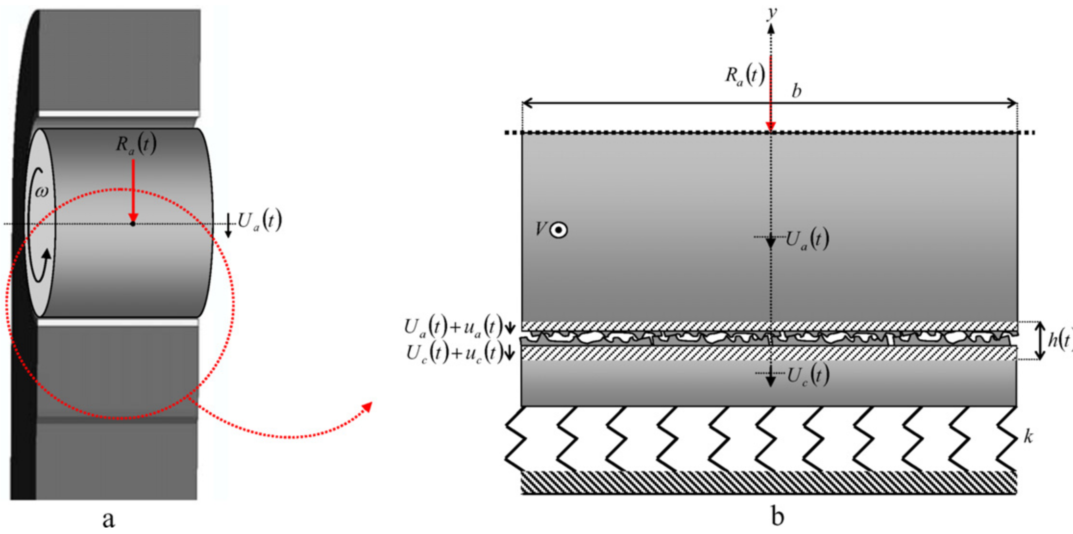


Fig. 1. Real system (b) and associated contact model involving wear (a).

thermodynamical [5–7], micromechanical [8], third body [9,10] approaches and from the type of adopted coupling with contact and friction laws, and with evolution laws of surface textures characteristics, as considered in the present Note.

The response in wear of bearing journals under mechanical cyclic loadings due to a rotating loaded pin, such as conrod systems of internal combustion engines, is investigated (Fig. 1a). As a first approach, a system composed of two rigid solids maintained in dry contact with wear is considered (Fig. 1b). The upper solid represents the pin, subject to both rotational and vertical translational motions, and the lower one, supported by elastic springs, represents the bearing fitted in its vertically deformable housing. In addition, both solids exhibit a time-dependent surface roughness, in presence of wear. Fluid lubrication is not considered in the present study. For the sake of simplicity, the contact state is assumed sufficiently uniform along the housing circumference to neglect the dependence of physical quantities in angular position and thermal effects are not taken into account.

The aim of the present study is not to provide predictive wear simulations for bearing journals but to propose an analytical modelling leading to a better mechanical understanding of wear mechanisms often observed, experimentally and numerically, for such complex systems. The mechanical analysis is first devoted to the determination of the model response, then to its stabilization, and more particularly, to the asymptotic behaviour in wear under cyclic loading. Since finite element simulations of the considered problem can require long-term and costly computations, the adopted modelling offers several interests. First, the asymptotic response is derived to give some useful analytical expressions of quantities inherent to the stabilized behaviour, such as the increment in wear per cycle. In a second part, a sensitivity analysis to the wear coefficient is performed, as a direct application of the studied model.

2. Modelling assumptions and governing equations

2.1. Kinematics

Let $U_a(t)$ and $U_c(t)$ be the proper translations of the rigid solids (pin and bearing resp.). Let $u_a(t)$ (resp. $u_c(t)$) be the vertical displacement of the pin (resp. of the bearing) mean roughness surface, only due to wear. Let h_0 be the initial closure of the mean contacting surfaces. The closure $h(t)$ between mean contacting surfaces at time $t \geqslant 0$ is thus given by:

$$h(t) = h_0 + \left[u_c(t) + U_c(t)\right] - \left[u_a(t) + U_a(t)\right] \tag{1}$$

2.2. Equilibrium

Let $R_a(t)$ and $p(t)$ be the pin reaction force and the contact pressure on the pin/bearing interface assumed axially uniform, respectively. The equations governing the equilibrium of the pin and the bearing supported by a uniform distribution of elastic springs with stiffness $k > 0$ are written as:

$$p(t)bL - R_a(t) = 0, \qquad kU_c(t) - p(t) = 0 \tag{2}$$

2.3. Contact law

Let $\sigma_a(t)$ (resp. $\sigma_c(t)$) be the pin (resp. bearing) surface roughness. The usual equivalent roughness (so-called Root Mean Square, R.M.S.) of the contact system is given for all times $t \geqslant 0$ by (e.g. [11]):

$$\bar{\sigma}(t) = \sqrt{\sigma_a(t)^2 + \sigma_c(t)^2} \tag{3}$$

Let us consider here a regularized contact law, based on normal compliance power-laws (e.g. [12]), to represent the normal stiffness and deformation of contacting asperities. Not only does the following formulation enable to define a coupling between the contact pressure, closure and roughnesses, but it also allows a regular saturation in contact pressure when the closure h tends towards zero, corresponding to the plastic deformation of contacting asperities. The contact pressure is non-zero as soon as $0 \leqslant h(t) \leqslant 3\bar{\sigma}(t)$ and is defined as:

$$p(t) = \frac{1}{2} p_{\max}\left[1 + \tanh\left(\frac{4\bar{\sigma}(t) - 2h(t)}{\bar{\sigma}(t)}\right)\right] \tag{4}$$

where $p_{\max}$ is the maximum in admissible contact pressure (for instance the yield stress of the softest constitutive material in presence). For $h(t) > 3\bar{\sigma}(t)$, contact is not active, i.e. $p(t) = 0$.

2.4. Wear law

The pin is assumed subject to a time-constant rotation speed magnitude ω. Since the housing is fixed, the relative slip rate on the contact interface is thus equal to the pin one, i.e. $V = R\omega$. It is also assumed that wear mechanisms are not participating to the creation of a third body on the contact interface. In other words, the detached material is totally ejected from the contact zone.

The standard Archard's law of wear (e.g. [1–5]) is here adopted, but coupled with a time-evolution in surface roughness, to take the influence of wear on the surface textures into account. Let H_a and K_a (resp. H_c and K_c) be the hardness and the wear coefficient of the surface pin (resp. bearing). The evolution laws are written as:

$$\begin{cases} \dfrac{\mathrm{d}u_a}{\mathrm{d}t} = K_a \dfrac{p(t)V}{3H_a} \\ \dfrac{\mathrm{d}\sigma_a}{\mathrm{d}t} = f_{aa}\dfrac{\mathrm{d}u_a}{\mathrm{d}t} + f_{ac}\dfrac{\mathrm{d}u_c}{\mathrm{d}t} \end{cases} \qquad \begin{cases} \dfrac{\mathrm{d}u_c}{\mathrm{d}t} = K_c \dfrac{p(t)V}{3H_c} \\ \dfrac{\mathrm{d}\sigma_c}{\mathrm{d}t} = f_{cc}\dfrac{\mathrm{d}u_c}{\mathrm{d}t} + f_{ca}\dfrac{\mathrm{d}u_a}{\mathrm{d}t} \end{cases} \tag{5}$$

The given quantity f_{ij} $(i, j = a, c)$ represents the variation in surface roughness of solid (i) due to the variation in wear of solid (j), that is $f_{ij} = \partial\sigma_i/\partial u_j$.

It is assumed here that the wear evolution of one solid has only an influence on its own surface roughness evolution. Therefore, the coupling coefficients f_{ij} $(i \neq j)$ are assumed identically zero. We will simply denote, in the following, $f_{ii} = f_i$ $(i = a, c)$.

Note that $u_a(t)$ and $u_c(t)$ are positive and time-increasing functions whereas $\sigma_a(t)$ and $\sigma_c(t)$ are positive functions that can either decrease (if $f_i < 0$) or increase (if $f_i > 0$) with time t.

2.5. Initial conditions

It is assumed that the initial state is a contact state without wear. Let us denote as σ_a^0 (resp. σ_c^0) the initial surface roughness of the pin (resp. bearing). The initial equivalent roughness is thus $\bar{\sigma}(0) = \sqrt{(\sigma_a^0)^2 + (\sigma_c^0)^2}$. It implies that the initial closure h_0 is such that $0 \leqslant h_0 \leqslant 3\bar{\sigma}(0)$. Therefore, the initial conditions are:

$$h(0) = h_0, \qquad u_c(0) = 0, \qquad u_a(0) = 0, \qquad \sigma_a(0) = \sigma_a^0, \qquad \sigma_c(0) = \sigma_c^0 \tag{6}$$

2.6. Time-evolution problem

The system of equations governing the behaviour of the studied system of solids in contact with wear for all times $t \geqslant 0$ is thus the following:

$$
\begin{aligned}
&\text{(a)}\quad h(t) = h_0 + \big[u_c(t) + U_c(t)\big] - \big[u_a(t) + U_a(t)\big] \\
&\text{(b)}\quad p(t)bL = R_a(t) \\
&\text{(c)}\quad U_c(t) = \frac{p(t)}{k} \\
&\text{(d)}\quad \bar{\sigma}(t) = \sqrt{\sigma_a(t)^2 + \sigma_c(t)^2} \\
&\text{(e)}\quad p(t) = \frac{1}{2} p_{\max}\left[1 + \tanh\left(\frac{4\bar{\sigma}(t) - 2h(t)}{\bar{\sigma}(t)}\right)\right] \\
&\text{(f)}\quad \begin{cases} \dfrac{\mathrm{d}u_a}{\mathrm{d}t} = K_a \dfrac{p(t)V}{3H_a} \\ \dfrac{\mathrm{d}\sigma_a}{\mathrm{d}t} = f_a \dfrac{\mathrm{d}u_a}{\mathrm{d}t} \end{cases} \qquad \begin{cases} \dfrac{\mathrm{d}u_c}{\mathrm{d}t} = K_c \dfrac{p(t)V}{3H_c} \\ \dfrac{\mathrm{d}\sigma_c}{\mathrm{d}t} = f_c \dfrac{\mathrm{d}u_c}{\mathrm{d}t} \end{cases} \\
&\text{(g)}\quad h(0) = h_0, \qquad u_c(0) = 0, \qquad u_a(0) = 0, \qquad \sigma_a(0) = \sigma_a^0, \qquad \sigma_c(0) = \sigma_c^0
\end{aligned}
\tag{7}
$$

Two types of time-evolution problem of the system can be distinguished:

(i) For a given applied load on the pin, the non-linear time-evolution problem to be solved is:

Being given $R_a(t)$, find $(U_a(t), U_c(t), h(t), u_a(t), u_c(t), \sigma_a(t), \sigma_c(t), \bar{\sigma}(t), p(t))$ for all times $t \geqslant 0$ such that Eqs. (7)(a)–7(g) are satisfied.

(ii) For a prescribed displacement of the pin, the non-linear time-evolution problem to be solved is:

Being given $U_a(t)$, find $(R_a(t), U_c(t), h(t), u_a(t), u_c(t), \sigma_a(t), \sigma_c(t), \bar{\sigma}(t), p(t))$ for all times $t \geqslant 0$ such that Eqs. (7)(a)–7(g) are satisfied.

3. Solving and discussions

Problem (i) is not of interests here since driving the pin in applied force amounts to prescribing the contact pressure on the interface, according to (7)(b). The behaviour in wear is thus directly given in this case by Eq. (7)(f) and initial conditions (7)(g).

On the other hand, problem (ii) leaves the contact pressure free of evolution and the mechanical coupling contact-wear-equilibrium-kinematics remains. This is the relevant mechanical problem we shall consider here.

Taking Eqs. (7) into account and after some algebra, the primal variables become the wear ones and the condensed problem to be solved is rewritten as:

(iii) Being given $U_a(t)$, find $(u_a(t), u_c(t))$ for all times $t \geqslant 0$ such that:

$$
\begin{cases}
\dfrac{3H_c}{K_c V}\dfrac{\mathrm{d}u_c}{\mathrm{d}t} = \dfrac{1}{2}p_{\max}\Bigg[1 \\
\qquad + \tanh\left(\dfrac{4\sqrt{[\sigma_a^0 + f_a u_a(t)]^2 + [\sigma_c^0 + f_c u_c(t)]^2} - 2\big[h_0 + \big[u_c(t) + \frac{3H_c}{kK_cV}\frac{\mathrm{d}u_c}{\mathrm{d}t}\big] - [u_a(t) + U_a(t)]\big]}{\sqrt{[\sigma_a^0 + f_a u_a(t)]^2 + [\sigma_c^0 + f_c u_c(t)]^2}}\right)\Bigg] \\
u_a(t) = \dfrac{K_a}{K_c}\dfrac{H_c}{H_a} u_c(t) \\
u_c(0) = 0
\end{cases}
\tag{8}
$$

It is clear that when the solution of problem (iii) is obtained, that is when $(u_a(t), u_c(t))$ are determined, problem (ii) is completely solved. The variables $(\sigma_a(t), \sigma_c(t))$ and $p(t)$ result from Eqs. (7)(e)–(f)–(g) respectively. Then, Eqs. (7)(c) and (7)(d) lead to $U_c(t)$ and $\bar{\sigma}(t)$ respectively. The closure $h(t)$ finally results from Eq. (7)(a) and $R_a(t)$ from Eq. (7)(b).

It is worth noting that problem (iii) can be rewritten in the following general form:

$$
\begin{cases}
\dfrac{\mathrm{d}u_c}{\mathrm{d}t}(t) = Q\left(u_c(t), \dfrac{\mathrm{d}u_c}{\mathrm{d}t}(t), t\right) \\
u_c(0) = 0
\end{cases}
\tag{9}
$$

where Q is a non-linear function that can be easily derived from Eq. (8).

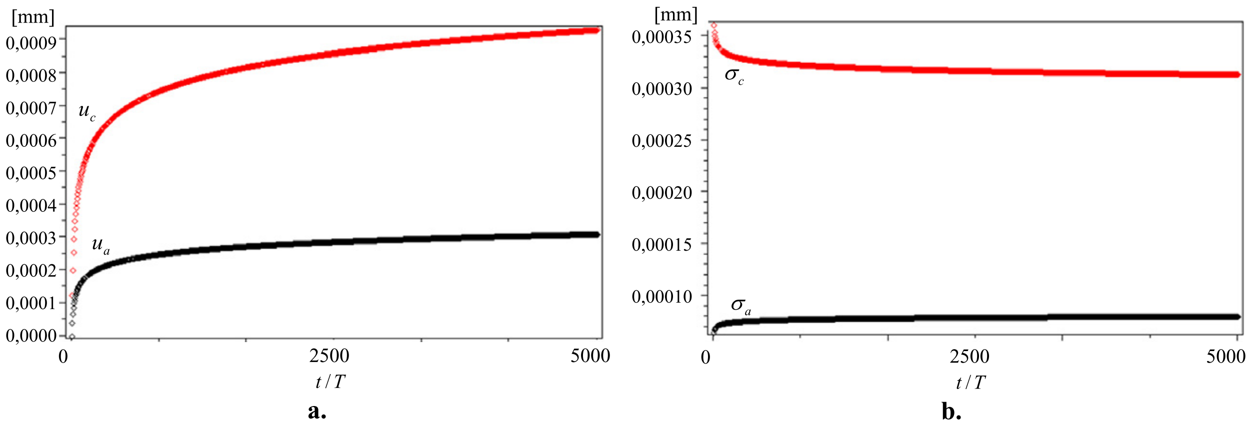


Fig. 2. Time-evolutions in wear (a) and surface roughness (b) of the bearing and pin.

Due to the highly non-linear character of problem (9), an adequate numerical solving method is required. For the sake of simplicity, let us assume in the following that the stiffness k of the support is infinitely high. The previous problem can be then reduced to the more common following form with $\tilde{Q} = \lim_{k\to\infty} Q$:

$$\begin{cases} \dfrac{\mathrm{d}u_c}{\mathrm{d}t}(t) = \tilde{Q}\big(u_c(t), t\big) \\ u_c(0) = 0 \end{cases} \tag{10}$$

In this case, the proper vertical displacement U_c of the bearing is identically zero.

As numerical solution example, let us consider a prescribed pin displacement of then form $U_a(t) = U\sin(2\pi t/T)$ where T is the solicitation period. The applied loading is thus cyclic.

The following numerical values are used, corresponding to a tribological couple made of a steel pin against an aluminium alloy bearing:

$$R = 22\ \text{mm}, \quad n = 2000, \quad \omega = n\pi/30, \quad U = 0.0005\ \text{mm}, \quad T = 120/n, \quad p_{\max} = 300\ \text{MPa}$$
$$f_a = 0.05, \quad f_c = -0.05, \quad H_a = 600\ \text{MPa}, \quad H_c = 200\ \text{MPa}, \quad \sigma_a^0 = 0.000065\ \text{mm}, \quad \sigma_c^0 = 0.00036\ \text{mm}$$
$$K_a = 10^{-6}, \quad K_c = 10^{-6}, \quad h_0 = 3\bar{\sigma}(0) = 0.0011\ \text{mm}$$

The numerical solution of Eq. (10) has been plotted in Fig. 2a, depicting the time-evolution in wear of the pin and bearing during applied cycles. The time-evolution in surface roughness of the pin and bearing during applied cycles has been also depicted in Fig. 2b.

The obtained response in wear is completely similar to the experimental observations [2] and numerical results by finite element computations. It can be decomposed into a first transient evolution, so-called lapping stage, followed by a stabilized evolution, studied here after. In reality, a final unstable evolution is often observed, corresponding to seizing wear by a quick deterioration of contacting surfaces, but cannot be represented by the present model.

4. Asymptotic response and stabilization

4.1. Theoretical background and definitions

Some interesting works and analyses can be found in the literature on the asymptotic response of mechanical systems under cyclic loading. Standard plasticity (asymptotic states are well known as accommodation, shakedown and ratchetting, cf. [13]) and, more recently, frictional contact problems (cyclic slip, slip-shakedown and cumulative slip, cf. [14]) can be cited as investigated fields in non-linear Mechanics. Let us give the following general definition:

Definition 1. A system Ω admits an asymptotic response if for all its fields G, there exists a limit G_∞ for $t \to +\infty$, that is:

$$\lim_{t\to+\infty}\big[G(x,t) - G_\infty(x,t)\big] = 0 \quad \forall x \in \Omega \tag{11}$$

The notion of stabilization is above all useful in the framework of cyclic loadings. Under a periodical loading, the rate response of the system tends to become periodic after a certain number of cycles. This property has been established in standard plasticity (e.g. [13]) and friction [14], but there is no proof in the general case considered here. We can however postulate the following formulation of the stabilized asymptotic response:

Definition 2. Under a periodical loading of period T, the asymptotic response of a system Ω is stabilized if, for all its fields G, there exists an asymptotic response in rate $\dot{G}_\infty$ periodical of period T, that is:

$$\begin{cases} \forall t \geqslant 0 \quad \dot{G}_\infty(x,t) - \dot{G}_\infty(x,t+T) = 0 \\ \lim_{t\to+\infty}\big[G(x,t) - G_\infty(x,t)\big] = 0 \end{cases} \quad \forall x \in \Omega. \tag{12}$$

The number of cycles to stabilization, i.e. the time for which Eq. (12) is satisfied, can be very high. Particular cases of stabilized asymptotic responses for all points $x \in \Omega$ can be given:

$$\text{(a)} \quad \int_t^{t+T} \dot{G}_\infty\big(x,t'\big)\,\mathrm{d}t' = 0, \qquad \text{(b)} \quad \int_t^{t+T} \dot{G}_\infty\big(x,t'\big)\,\mathrm{d}t' \neq 0, \qquad \text{(c)} \quad \dot{G}_\infty(x,t) = 0 \quad \forall t \geqslant 0$$

Case (a) corresponds to a cyclic response in field G whereas case (b) corresponds to an accumulation in field G for each loading cycle. The case for which field G becomes asymptotically constant is given by (c).

4.2. Determination of the asymptotic response

Let us reconsider the studied model and determine its stabilized asymptotic response. For the sake of simplicity in the analytical derivations, the case of a perfect surface texture without any roughness is assumed for the pin. In addition, only bearing wear is taken into account. The condensed problem to be solved is rewritten from (8) as:

(iv) Being given $U_a(t)$ periodical of period T, find $u_c(t)$ for all times $t \geqslant 0$ such that:

$$\begin{cases} \dfrac{3H_c}{K_c V}\dfrac{\mathrm{d}u_c}{\mathrm{d}t}(t) = \dfrac{1}{2}p_{\max}\left[1 + \tanh\left(\dfrac{4[\sigma_c^0 + f_c u_c(t)] - 2[h_0 + u_c(t) - U_a(t)]}{[\sigma_c^0 + f_c u_c(t)]}\right)\right] \\ u_c(0) = 0 \end{cases} \tag{13}$$

Let us assume the existence of a stabilized asymptotic response in wear. According to Definition 2, the response in wear rate is periodical of period T for $t \to \infty$, that is:

$$\lim_{t\to+\infty}\left[\frac{\mathrm{d}u_c}{\mathrm{d}t}(t+T) - \frac{\mathrm{d}u_c}{\mathrm{d}t}(t)\right] = 0 \tag{14}$$

Let us then introduce an asymptotic cumulative wear per cycle $u_{cum} = \lim_{t\to+\infty}[u_c(t+T) - u_c(t)]$ and assume $u_{cum} \neq 0$. Eq. (13) being valid at times $t \geqslant 0$ and $t+T$, it results from the passage to the limit $t \to \infty$:

$$\begin{cases} \lim\limits_{t\to+\infty}\dfrac{3H_c}{K_c V}\dfrac{\mathrm{d}u_c}{\mathrm{d}t}(t) = \lim\limits_{t\to+\infty}\dfrac{1}{2}p_{\max}\left[1 + \tanh\left(\dfrac{4[\sigma_c^0 + f_c u_c(t)] - 2[h_0 + u_c(t) - U_a(t)]}{[\sigma_c^0 + f_c u_c(t)]}\right)\right] \\ \lim\limits_{t\to+\infty}\left[\dfrac{4[\sigma_c^0 + f_c u_c(t+T)] - 2[h_0 + u_c(t+T) - U_a(t+T)]}{[\sigma_c^0 + f_c u_c(t+T)]} - \dfrac{4[\sigma_c^0 + f_c u_c(t)] - 2[h_0 + u_c(t) - U_a(t)]}{[\sigma_c^0 + f_c u_c(t)]}\right] = 0 \end{cases} \tag{15}$$

Using the relation $\lim_{t\to+\infty} u_c(t+T) = u_{cum} + \lim_{t\to+\infty} u_c(t)$ on the one hand and taking the periodicity of $U_a(t)$ into account on the other hand, operations on limits result in:

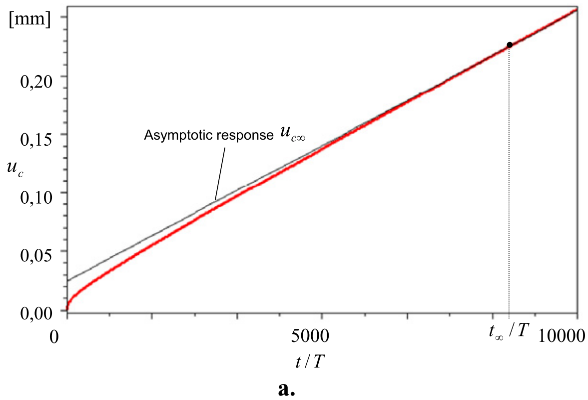
$$\begin{cases} \lim\limits_{t\to+\infty}\dfrac{3H_c}{K_c V}\dfrac{\mathrm{d}u_c}{\mathrm{d}t}(t) = \lim\limits_{t\to+\infty}\dfrac{1}{2}p_{\max}\left[1 + \tanh\left(\dfrac{4[\sigma_c^0 + f_c u_c(t)] - 2[h_0 + u_c(t) - U_a(t)]}{[\sigma_c^0 + f_c u_c(t)]}\right)\right] \\ u_{cum}\lim\limits_{t\to+\infty}\big[(4f_c - 2)\big[\sigma_c^0 + f_c u_c(t)\big] - f_c\big[4\big[\sigma_c^0 + f_c u_c(t)\big] - 2\big[h_0 + u_c(t) - U_a(t)\big]\big]\big] = 0 \end{cases} \tag{16}$$

Since $u_{cum} \neq 0$, it follows that:

$$\begin{cases} \lim\limits_{t\to+\infty}\dfrac{3H_c}{K_c V}\dfrac{\mathrm{d}u_c}{\mathrm{d}t}(t) = \lim\limits_{t\to+\infty}\dfrac{1}{2}p_{\max}\left[1 + \tanh\left(\dfrac{4[\sigma_c^0 + f_c u_c(t)] - 2[h_0 + u_c(t) - U_a(t)]}{[\sigma_c^0 + f_c u_c(t)]}\right)\right] \\ \lim\limits_{t\to+\infty}\left[\dfrac{4[\sigma_c^0 + f_c u_c(t)] - 2[h_0 + u_c(t) - U_a(t)]}{[\sigma_c^0 + f_c u_c(t)]}\right] = \left(4 - \dfrac{2}{f_c}\right) \end{cases} \tag{17}$$

That is:

$$\lim_{t\to+\infty}\frac{\mathrm{d}u_c}{\mathrm{d}t}(t) = \frac{K_c V}{6H_c}p_{\max}\left[1 + \tanh\left(4 - \frac{2}{f_c}\right)\right] \tag{18}$$



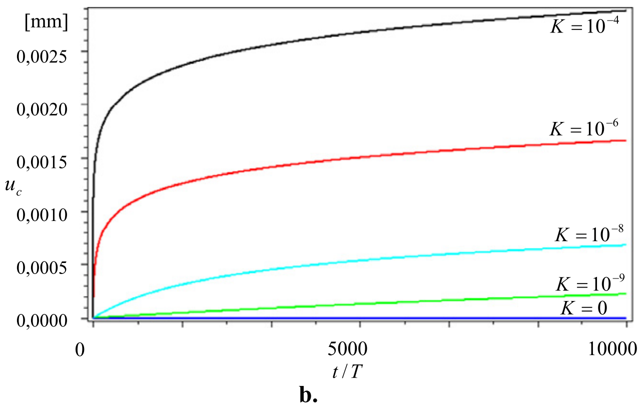


Fig. 3. Time-evolution in bearing wear and associated stabilized asymptotic response (a). Time-evolution in bearing wear and sensitivity to the wear coefficient K (b).

The stabilized asymptotic response in wear $u_{c\infty}(t)$ is thus of the form:

$$u_{c\infty}(t) = \frac{K_c V}{6H_c} p_{\max}\left[1 + \tanh\left(4 - \frac{2}{f_c}\right)\right](t - t_\infty) + u_{c\infty}(t_\infty) \tag{19}$$

The quantity t_∞/T denotes the number of cycles to stabilization. The asymptotic cumulated wear per cycle is defined as:

$$u_{cum} = \int_t^{t+T} \frac{\mathrm{d}u_{c\infty}}{\mathrm{d}t}(t')\,\mathrm{d}t' = u_{c\infty}(t+T) - u_{c\infty}(t) \tag{20}$$

That is finally:

$$u_{cum} = \frac{K_c V T}{6H_c} p_{\max}\left[1 + \tanh\left(4 - \frac{2}{f_c}\right)\right] \tag{21}$$

Also note that the variation in asymptotic cumulated wear per cycle is affine as a function of the wear coefficient K_c and non-linear as a function of the part of wear f_c, removed or added to the surface roughness. Eq. (18) also leads to the stabilized asymptotic response in contact pressure p_∞:

$$p_\infty = \frac{1}{2} p_{\max}\left[1 + \tanh\left(4 - \frac{2}{f_c}\right)\right] \tag{22}$$

It is worth noting that result (22) is only valid if $p_\infty > 0$, i.e. if $\tanh(4 - 2/f_c) > -1$, that is if $f_c > 1/8$. It thus implies that $u_{cum} > 0$.

As a numerical application, let us consider the following numerical values:

$$R = 22\ \text{mm},\quad n = 2000,\quad \omega = n\pi/30,\quad U = 0.0005\ \text{mm},\quad T = 120/n,\quad p_{\max} = 300\ \text{MPa}$$
$$f_c = 0.25,\quad H_c = 200\ \text{MPa},\quad \sigma_c^0 = 0.00036\ \text{mm},\quad K_c = 10^{-6},\quad h_0 = 3\bar{\sigma}(0) = 0.00108\ \text{mm}$$

The time-evolution in wear bearing through the cycles and the associated stabilized asymptotic response are depicted in Fig. 3a. The approximate number of cycles to stabilization is given by the value $t_\infty/T = 9000$ if adopting the following convergence criterion $|\frac{\mathrm{d}u_c}{\mathrm{d}t}(t_\infty) - \frac{u_{cum}}{T}|/\frac{u_{cum}}{T} < 10^{-6}$.

As a verification, the asymptotic cumulated wear per cycle can be computed, on the one hand, from the solution $u_c(t)$ of Eq. (13) and using next the approximate relation $u_{cum} \approx u_c(t_\infty + T) - u_c(t_\infty)$. By this way, we roughly obtain $u_{cum} \approx 2.34 \times 10^{-5}$ mm.

On the other hand, the value directly given by Eq. (21) is $u_{cum} = 2.35 \times 10^{-5}$ mm. It shows well the efficiency of the obtained analytical expression.

5. Example of sensitivity analysis: effect of the wear coefficient

Let us analyze the impact of the wear coefficient K (for the sake of convenience, it is assumed that $K = K_a = K_c$) on the behaviour in wear of the system. The applied loading is cyclic and the considered numerical values are the following:

$$R = 22\ \text{mm},\quad n = 2000,\quad \omega = n\pi/30,\quad U = 0.0005\ \text{mm},\quad T = 120/n,\quad p_{\max} = 300\ \text{MPa}$$
$$f_a = 0.05,\quad f_c = 0.05,\quad H_a = 600\ \text{MPa},\quad H_c = 200\ \text{MPa}$$
$$\sigma_a^0 = 0.000065\ \text{mm},\quad \sigma_c^0 = 0.00036\ \text{mm},\quad h_0 = 3\bar{\sigma}(0) = 0.0011\ \text{mm}$$

The time-evolution in bearing wear through the cycles has been plotted in Fig. 3b for several values of wear coefficient K. This graph shows well that the lapping transient stage is all the more quicker, i.e. the number of cycles to stabilization is all the more lower, as the wear coefficient is low enough.

6. Conclusion

The present modelling has enabled to reproduce, in a first order approach, the time-response in wear, commonly observed experimentally and numerically by FEM for bearing journals. An original coupling between contact and wear laws has been formulated and the solution in wear has been numerically derived.

From the obtained solutions and general theoretical notions of the stabilization of systems under cyclic loading, the study of the stabilized asymptotic response has lead to the determination of useful analytical expressions of the cumulated wear per cycle and asymptotic contact pressure.

The consistent model obtained can be also used to perform quick parametric studies to have a better understanding of some numerical and experimental results, complex to be analyzed in presence of high non-linearities, such as wear mechanisms.

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References

[1] J.F. Archard, Contact and rubbing of flat surfaces, J. Appl. Phys. 24 (1953) 981–988.
[2] J.-L. Ligier, Avaries en Lubrification : Application aux Paliers, Publ. de l'IFP, Editions Technip, 2004.
[3] E. Rabinowicz, Friction and Wear of Materials, 2nd edition, John Wiley and Sons, 1995.
[4] J.M. Georges, Frottement, Usure et Lubrification, Editions Eyrolles, 2000.
[5] N. Stromberg, L. Johansson, A. Klarbring, Derivation and analysis of a generalized standard model for contact, friction and wear, Internat. J. Solids Structures 33 (13) (1996) 1817–1836.
[6] A. Zmitrowicz, A thermodynamical model of contact, friction and wear: I. Governing equations, Wear 114 (2) (1987) 135–168.
[7] H.D. Bui, M. Dragon-Louiset, C. Stolz, A thermodynamic analysis of wear, in: Handbook of Materials Behavior Models, 2001, pp. 768–776.
[8] M. Dragon-Louiset, On a predictive macroscopic contact-sliding wear model based on micromechanical considerations, Internat. J. Solids Structures 38 (9) (2001) 1625–1639.
[9] M. Godet, The third body approach: a mechanical view of wear, Wear 100 (1984) 437–452.
[10] N. Fillot, I. Iordanoff, Y. Berthier, Wear modeling and the third body concept, Wear 262 (2007) 949–957.
[11] S. Belghith, S. Mezlini, H. Belhadjsalah, J.-L. Ligier, Modeling of contact between rough surfaces using homogenisation technique, C. R. Mecanique 338 (1) (2010) 48–61.
[12] N. Kikuchi, J.T. Oden, Contact Problems in Elasticity: A Study of Variational Inequalities and Finite Element Methods, SIAM, 1988, pp. 307–329.
[13] M. Peigney, C. Stolz, An optimal control approach to the analysis of inelastic structures under cyclic loading, J. Mech. Phys. Solids (2003) 575–605.
[14] N. Antoni, Q.-S. Nguyen, Shakedown theorems in contact mechanics, C. R. Mecanique 336 (2008) 341–346.