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The idea of PGA stream computations for soil slope stability evaluation

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A R T I C L E I N F O

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A B S T R A C T

Designing and constructing of road embankments, deep excavations, landslide and snow avalanche predictions or profiling construction sites in slanting terrain need slope stability evaluations. Determination of a safety factor and the position of a potentially critical slip surface is one of the essential issues in classical and modern soil mechanics, which still remains a very important problem in engineering practice. Most of the stability evaluation methods, i.e. based on limit equilibrium assumptions, need optimization, which can be successfully realized with the assistance of a genetic algorithm. The authors propose a variational approach with a four-step technique to determination of the critical height of a slope, which can be treated as an alternative and variant method to the generally applied limit equilibrium and/or finite element methods. Some common obstacles encountered while adapting classical optimization procedures have been solved by application of a parallel genetic algorithm. Substantial acceleration of computations has been achieved by introducing SIMD stream technology, which generally relies on modern graphics processing units. Examples of the results of a slope stability analysis performed using the fast parallel computation technique are also presented.

1. Introduction

Slope stability evaluation is a regular element of a designing process in most civil engineering projects. Designing and constructing of road embankments, deep excavations, landslide and snow avalanche predictions or profiling construction sites in slanting terrain need evaluations of safety factors for slopes. Evaluation of a reliable safety factor and/or determination of the shape and position of a potentially critical slip surface are still among the most difficult issues in modern soil mechanics. They represent fundamental problems investigated by many researchers. There are many individual and complex formulations for the safety factor determination and approaches to slope designing, for example:

a) The limit equilibrium methods (LEM) based on slices discretization of soil mass above the assumed failure surface: with a circular slide line, like Fellenius [1], Bishop [2] and Spencer [3]; with a general slide line, like Janbu [4] and Morgenstern–Price [5]; with inclined slices, like Sarma [6,7], etc. Generally, equilibrium equations are not satisfied in these approaches, although they can be satisfied if one can make certain assumptions. The simplicity of their formulation and easy programming, as well as application, are their unquestionable advantages. In contrast, a huge number of computations in the optimization process including analysis of a large number of potentially critical slide lines and difficulty adjusting an arbitrarily chosen method to the real soil conditions remain an important weakness of these approaches. Conventional optimization procedures based on trial-and-error or gradient descent techniques usually lead to the minimum value of a safety factor defined as a relation between resisting and driving forces. However, applications of modern optimization techniques based on genetic management of computations are increasingly more common in search for the critical circular slide line

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[8–10], multiple-wedge analysis [11,12], adopting solutions with non-circular slide lines (Morgenstern–Price method [13], Janbu method [14] or Spencer method with spline curves [15]).

b) Numerical approaches based on continuum mechanics, e.g., using displacement-based finite element method (FEM). These methods need a constitutive model for the soil mass and enable one to calculate the progressive failure and safety factor defined as a relation between the real soil shear strength and the reduced shear strength by iterative approaches ("shear strength reduction" SSR or "phi-c reduction" techniques, [16–20]). The "black boxed" forms of these methods locked in commercial computer applications have been very successfully used by engineers (i.e. Plaxis), especially if a low-number-of-parameter forms of constitutive behavior are adopted (e.g., a five-parameter linear elastic-perfectly plastic model with Mohr–Coulomb criterion). Beside the LEM, numerical approaches of continuum mechanics are most often used by geotechnical practitioners. However, it should be emphasized that accurate simulation of real materials requires a well-fitted constitutive model and real mechanical parameters, while the SSR technique rests on changing material strength characteristics – consequently, the simulations are performed on fictitious materials, which are defined by an affected strength envelope (see [21]).

c) The limit analysis approaches based on lower and upper bound theorems of the classical plasticity, which are concerned with the development of efficient methods for computing the stability of soil slopes in a direct manner. In the lower-bound theorem, based on statically admissible stress field, the loads, determined by stress state that satisfies the equilibrium equations and the stress boundary conditions without violating the yield criterion, are not greater than the actual collapse loads. In the upper-bound theorem, based on kinematically admissible velocity field, the loads, determined by equating the external rate of work to the internal rate of dissipation in an assumed velocity field that satisfies the velocity boundary conditions and the strain/velocity compatibility conditions, are not less than the actual collapse load. The basic techniques and many numerical results have been summarized by Chen [22].

d) Variational method developed by Baker and Garber [23] which consists in transformation of the static equilibrium equations into the minimized functional using undefined Lagrange's multipliers. The analysis of the first variation using Euler's rule leads to a differential equation of a potentially critical slide line and the closed form of a safety factor defined like in the SSR technique. In a general case of the formulation presented by Chen and Liu [24], this approach can be classified as a mid-type method based on limit analysis theorems. It combines the upper- and lower-bound methods (assumed admissible velocity field satisfies the kinematic constrains, yield criterion and equilibrium equations). The method can be used with the variability of material or loading characteristics. The shape of the critical slip surface is represented by a log-spiral, but for layered soil it has a more complex form of a series of log-spiral sections with different initial inclinations which depend on the friction angles of the materials building a soil slope. Despite a very elegant mathematical notation, the method was criticized (see [25,26]) because of some ambiguities related to the actual possibility of attaining the minimal value of a safety factor. The guidelines proposed by Baker [27] are very helpful in a practical application of the method, although the intractable problem of the global solution identification still remains unsolved.

e) Combined methods based on probabilistic approaches to slope analysis and design with an aid of LEM and/or FEM (e.g. [28–30]), assisted by the artificial neural network (ANN, e.g. [31]), with built-in optimization techniques based on the self-optimizing genetic algorithms (GA) with adapted crossover and mutation probabilities [32] or two-stepped self-optimizing GA with structure evolution and parameter optimization [33].

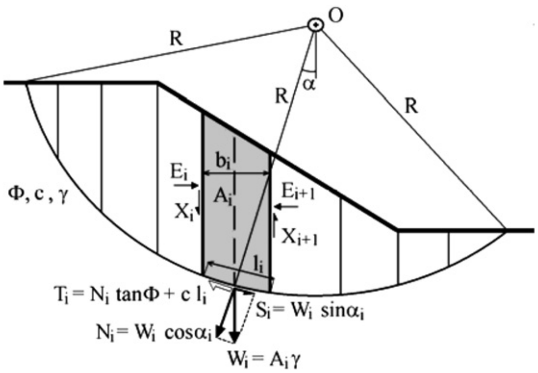
The results obtained with the above approaches are usually presented in the form of safety factors, which have very various definitions (ratio of: resisting and driving forces – (a), real and reduced strengths – (b) and (d), critical and real loads – (c) and (d), critical and real heights – (d), etc.), which hinders their direct comparison. In practice, however, doubts in slope stability analyses appear and verification of the performed computations using alternative approaches is highly recommended. It becomes even more important if the performed analyses indicate the position of a subtle equilibrium between safe and unsafe states. On the other hand, fundamental differences between approaches which are generally applied in practice can provide inconclusive results and an additional variant analysis is necessary. Only an overbalance of positive or negative valuations prepared by different methods can give some valuable information about the actual state of the problem. Because LEM and FEM techniques are commonly used by geotechnical practitioners, the authors tested a variational method as such an alternative of that type and a variant approach. The difficulties with global solution identification were defeated by application of a parallel-structured GA. Significant acceleration of the calculations was achieved by adopting the Single Instruction Multiple Data (SIMD) model of computational architecture.

The paper is organized as follows. In Section 2 the backgrounds of LEM-slices and variational calculus are presented. Section 3 contains some considerations on advantages of GA usage in variational investigations. Application of the SIMD model is discussed in Section 4. Examples of the results of slope stability analyses are compared in Section 5. Section 6 summarizes the authors' conclusions.

2. Background of LEM-slices and variational approaches

2.1. LEM-slices methods

Among LEM-slices methods, the simplest and most commonly used in engineering practice are Fellenius and Bishop approaches. The soil mass above a trial failure surface is divided into slices by vertical planes (Fig. 1). Each slice is taken as having a straight line base, implying mutual support between the slices. In Fellenius approach, the forces acting between



Fellenius: $F_s = \frac{\sum_i T_i}{\sum_i S_i}$

Bishop: $F_s = \frac{\sum_i [cb_i + (W_i + X_i - X_{i+1})\tan\Phi] \frac{\sec\alpha_i}{1+(\tan\alpha_i \tan\Phi)/F_S}}{\sum_i S_i}$

Simplified Bishop: $F_s = \frac{\sum_i [cb_i + W_i \tan\Phi] \frac{\sec\alpha_i}{1+(\tan\alpha_i \tan\Phi)/F_S}}{\sum_i S_i}$

Fig. 1. The idea of slices method. Definitions of safety factors.

each pair of slices (E, X) are neglected, so only the forces acting on the base of each slice $(W$, N and $S)$ are left for consideration. Taking into account the soil resistance (T) consequent to the friction rule, the safety factor can be defined as a relation between resisting and overturning moments regarding the center point of an assumed slip circle. The constant ray in moments equations can be reduced and a final formulation of the safety factor comprises only resisting and sliding forces (S, T).

In Bishop's approach, apart from the total moments equilibrium conditions, an additional equation of vertical forces equilibrium is taken into account individually for each of the slices. Such consideration leads to an implicit formulation of the safety factor which needs an iterative procedure. None of thus defined solutions satisfies the global equilibrium equations: $\sum H = 0$, $\sum V = 0$ and $\sum M = 0$.

2.2. Variational method

Contrary to the aforementioned methods, the initial establishment of the variational approach is based on equilibrium equations (Fig. 2), taking into considerations the following conditions:

- the critical height of the slope is studied, so the value of the safety factor is constant and assumed $F_s = 1.0$;
- the linear failure criterion of Coulomb is used: $\tau = \sigma \tan\Phi + c$, so only three constants define the material: internal friction angle Φ, cohesion c, weight γ by volume;
- for simplicity of the problem, the slope is homogeneous and self-weight loaded only.

The system of static equilibrium equations has the following form:

$$\begin{aligned} R_H &= \int H\,\mathrm{d}x = \int \left(-\tau + \sigma \frac{\mathrm{d}y}{\mathrm{d}x}\right)\mathrm{d}x = 0 \\ R_V &= \int V\,\mathrm{d}x = \int \left(-\tau \frac{\mathrm{d}y}{\mathrm{d}x} - \sigma + \gamma(y - \bar{y})\right)\mathrm{d}x = 0 \\ \int M\,\mathrm{d}x &= \int \left(\left(-\tau + \sigma \frac{\mathrm{d}y}{\mathrm{d}x}\right)y + \left(\tau \frac{\mathrm{d}y}{\mathrm{d}x} + \sigma\right)x - \gamma(y - \bar{y})x\right)\mathrm{d}x = 0 \end{aligned} \tag{1}$$

where $\bar{y}$ is a vertical coordinate of slope geometry (Fig. 2), τ, σ are tangent and normal components of stress distribution along the slide line. Combination of these conditions into one functional defines the variational functional:

$$T = \int Q\,\mathrm{d}x = \int (\lambda_1 H + \lambda_2 V + M)\,\mathrm{d}x = 0 \tag{2}$$

and

$$\operatorname*{Extr}_{y,\sigma} \int_{x_0}^{x_n} Q\,\mathrm{d}x = 0 \tag{3}$$

where λ_1, λ_2 are Lagrange's multipliers. The unknown functions $y(x)$ and $\sigma(x)$ should be established in the forms which would fulfill the condition (2).

The first variation of Q regarding to σ-unknown leads to a differential form of the slide line function:

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\tan\Phi(y - y_c) + x_c - x}{y - y_c - \tan\Phi(x_c - x)} \tag{4}$$

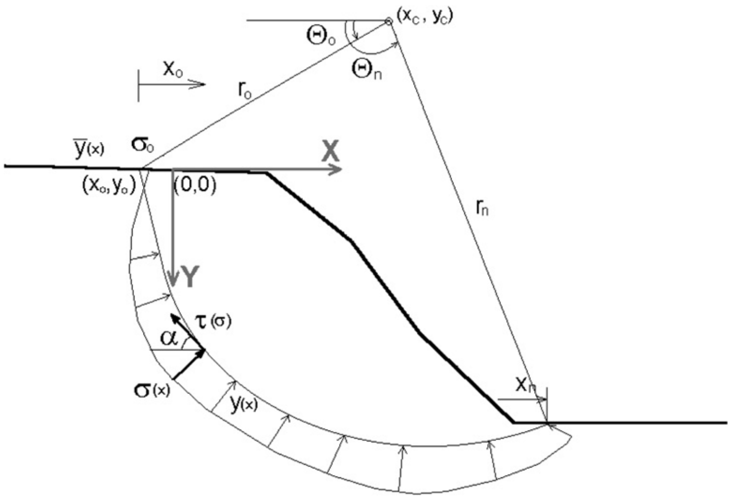


Fig. 2. Slip surface with normal stress distribution. Cartesian and polar coordinate systems [24,39].

where x_c and y_c are the coordinates of the pole center (Fig. 2); the shape of a potentially critical slip surface is log-spiral and has a simple form in polar coordinates:

$$r(\Theta) = r_0 e^{\tan\Phi(\Theta - \Theta_0)} \tag{5}$$

Constantly, the first variation of Q regarding y-unknown leads to a differential form of a normal stress distribution function which assumes the following explicit shape in polar coordinates:

$$\sigma(\Theta) = \gamma r_0 e^{\tan\Phi(\Theta-\Theta_0)} \frac{3\tan\Phi\cos\Theta + \sin\Theta}{9\tan^2\Phi + 1} - \frac{c}{\tan\Phi} + C_1 e^{-2\Theta\tan\Phi}$$
$$C_1 = e^{2\Theta_0\tan\Phi}\left(\sigma_0 - \gamma r_0 \frac{3\tan\Phi\cos\Theta_0 + \sin\Theta_0}{9\tan^2\Phi + 1} + \frac{c}{\tan\Phi}\right) \tag{6}$$

Requirement (2) is not sufficient for satisfaction of the whole system of static equilibrium equations, therefore an additional condition is introduced (resultant force, see Chen and Liu [24]):

$$R = \sqrt{R_H^2 + R_V^2} = 0 \tag{7}$$

The whole idea of slope stability evaluation consists in finding its minimal height H_{cr} defined by the exit point of a potentially critical slide line. The slide line is described univocally by three geometrical parameters x_0, x_c, y_c, although an additional unknown σ_0 is needed to guarantee the agreement of the general requirements:

$$\operatorname*{Min}_{x_0,x_c,y_c,\sigma_0}\big(|T|\big) \approx 0 \wedge \operatorname*{Min}_{x_0,x_c,y_c,\sigma_0}(R) \approx 0 \tag{8}$$

Determination of all the four variables which fulfill the requirements (7) does not guarantee achieving the global solution. There exists an infinite number of such four-coordinate (x_0–x_c–y_c–σ_0) points, although one of them provides the minimum height of the slope – the critical height which indicates the global solution:

$$\operatorname*{Min}_{x_0,x_c,y_c,\sigma_0}(H) \rightarrow H_{\mathrm{cr}} \tag{9}$$

The proposed strategy of computation consists of four basic operations:

a) determination of y_c (for assumed x_0, x_c and σ_0), fulfilling the requirement $|T| \approx 0$,
b) finding the minimum value of R by adjustment of σ_0,
c) fulfilling the requirement $R \approx 0$ by successive corrections of x_c and repetition of steps: a) and b),
d) determination of $H_{\min}$ by adjustment of x_0 and repetition of steps: a), b) and c).

The whole procedure consists of several multi-nested minimization algorithms – the quality of the solution strongly depends on assumed accuracy of fulfilling the requirements (8) and, due to finite precision of the computations, can lead to non-univocal results.

The theoretical background presented by Baker and Garber at the 9th International Conference on Soil Mechanics and Foundation Engineering in Tokyo became the starting point for many subsequent modifications of variational solutions (e.g.,

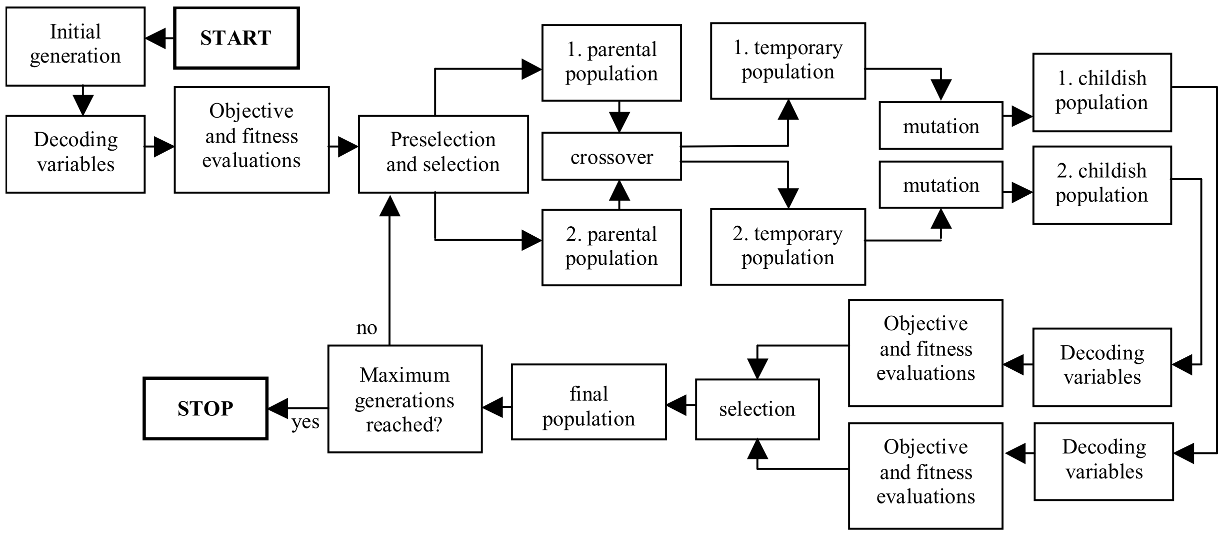


Fig. 3. A full cycle GA computation flow chart.

Baker, Chen and Liu [34,24] – application of nonlinear Coulomb's criterion; Leshchinsky and Reinschmidt [35], Lemonnier et al. [36,37] – analysis of reinforcement; Srokosz [38,39] – calculations optimized by genetic algorithms).

3. Parallel genetic algorithm

The computations can be highly accelerated by omitting the standard minimization procedures and entrusting the searching process to genetic management. Such modification makes the whole calculation process insensitive to local minima of T, R and H, which are usually created by the limited accuracy of computations.

The canonical form of a GA [40] is a sequential series of operations performed on a population of coded shapes of potential solutions (most often binary coding is used). Taking into consideration this simplified definition, it is easy to notice that the possibility of GA parallelization lies in the multi-element structure of a processed population. Thus, the simplest idea of acceleration of computations is based on parallel evaluation of a fitness function independently for each component of the population.

A genetically managed optimization process requires a definition of an objective function which evaluates the proximity of a currently analyzed set of variables to the optimal solution. The set of requirements (8) and (9) can be combined and transformed into objective function f in the following arbitrarily chosen way (other functions were also tested, e.g. see [39]):

$$f(w, Y) = w_T^{|T|} + w_R^R + w_H^H \tag{10}$$

where $w_{T,R,H}$ are weight coefficients (each less than 1.0).

The optimal set of x_0, x_c, y_c and σ_0 maximizes objective function f. The reader can notice that the specificity of T (2) and R (7) facilitates the definition of a minimized objective function (with its optimum value equal zero). However, in view of fitness maximization in the canonical form of genetically controlled calculations, the presented model of f is ready for direct implementation in GA computations.

The full cycle of GA computations consists of a few simple operations performed on the coded parameters being optimized (p_i, $i = 1..4$). The binary method of encoding is chosen to satisfy the rule of the minimum alphabet [40]. The binary representations of the parameters (the genes, b_i, $i = 1..4$) are linked creating a continuous chain of bits with a constant length, i.e. an individual or chromosome ($b_1 b_2 b_3 b_4$). The first collection of the individuals (the initial population) is created by a random number generator. The next ones evolve transforming, exchanging and transferring carried information, dependently on their fitness which is evaluated after decoding their individuals. The fitness is defined as a linearly scaled objective function f with an assumed scaling multiplier (see [40]). This operation helps to avoid premature convergence. The evolution of the information starts from the selection which rests on choosing the most hopeful chains using deterministic technique. As an effect of the selection, two subpopulations of parents are created (Fig. 3). The next stage – crossover – is the most essential mechanism of the transformation of carried information. The crossover procedure rests on copying of successive bits from the selected parental chain to the actually being created childish chain in accordance with randomly generated orders. While copying, randomly chosen bits are inverted, i.e. the mutation effect occurs. This process discovers new potential possibilities of the solution which are not known in the population in the current generation. Newly created children replace parental populations and then they are estimated by the fitness function. The next generation is finally created as a result of tournaments of the childish populations (Fig. 3).

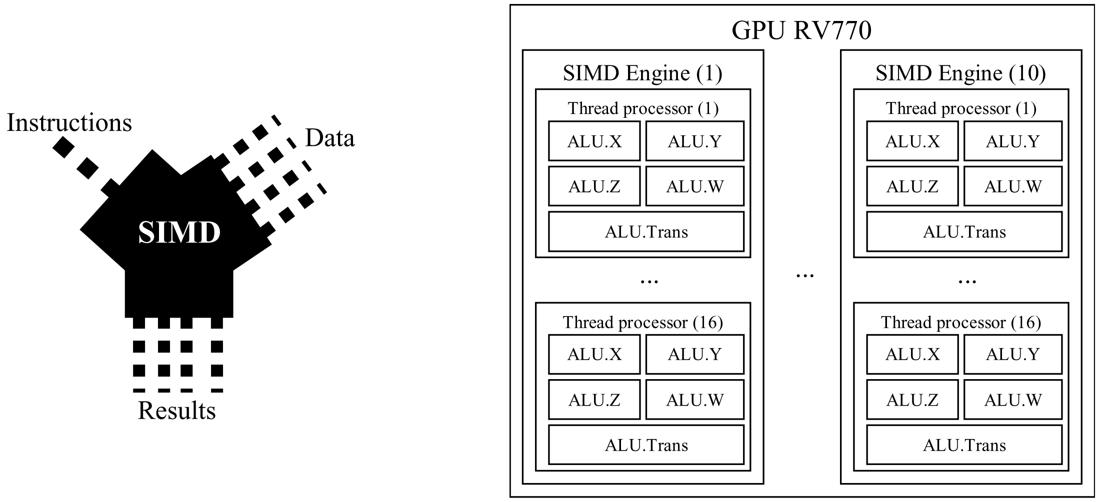


Fig. 4. SIMD processor & SIMD structure of GPU (ATI RV770) [41].

The fundamental concept of parallel GAs is based on their cooperation. The cooperation enables helping one another to reach the global solution via migration of partial solutions across the system of independently working GAs. There are many techniques of a selection of emigrants (e.g., alike in association for crossover), but the simplest one is the choice of the actually best individual (i.e., one per population). The number of migrants and the probability of migration are the controlling parameters specific for the parallel structures of GAs. Selected and gathered (copied) emigrants become immigrants during their incorporation. The incorporation of immigrants was realized by direct replacements, confronting migrants to chosen native individuals.

The schema of the migration depends on assumed structure of the PGA (in 2D structures: North-East-West-South interconnection stencil, NEWS; in 3D structures: North-East-West-South-Front-Back, NEWSFB). Summarizing, the simplest form of a parallel GA is configured by:

- definition of communication between a GA and the analyzed problem: objective function;
- seven global, numerical parameters: the size of a population, number of working GA, maximum number of generations, probability of crossover, mutation and migration, and a scaling coefficient;
- six parameters of behavior techniques: interconnection stencil, methods of coding, selection, crossover, mutation and incorporation;
- three local, numerical parameters (for coding): the number of variables, range of phenotypes (min and max) and number of bits coding each variable.

It is an obvious and well-known fact that the choice of proper configuring parameters totally decides about the efficiency of the optimization procedure, but many useful suggestions can be found in the literature of the problem.

4. SIMD application

Effective application of parallel GA requires utilization of a multi-processor machine, a set of personal computers interconnected via Ethernet or a personal computer equipped with a powerful many-core processor. The latest generation of graphics processing units (GPU) can execute non-graphics functions and work with non-graphics data, performing general purpose calculations in a many-core array structure. There are two technologies applied in general purpose computations: Stream Technology and Compute Unified Device Architecture (CUDA). Both technologies are based on the SIMD model of the GPU architecture (Fig. 4).

In the SIMD programming model, many sets of data (called streams) are processed independently by many thread processors which execute one sequence of instructions for each datum. This kind of parallel computations is known as a limited parallelism or data-parallelism.

Adopting the internal structure of a thread processors consisting of four general purpose arithmetic-logic units (ALU), a three-dimensional space of parallel data processing can be proposed (i.e. two-dimensional tables representing a stream and third dimension defining a short vector with four components: X, Y, Z and W, allocated in ALU.X, ALU.Y, ALU.Z and ALU.W respectively, see [41]).

The proposed structure of a parallel GA is shown in Fig. 5. The simplest interconnection stencil for 3D array of GAs thus created is a type of Linear-7 (Fig. 5).

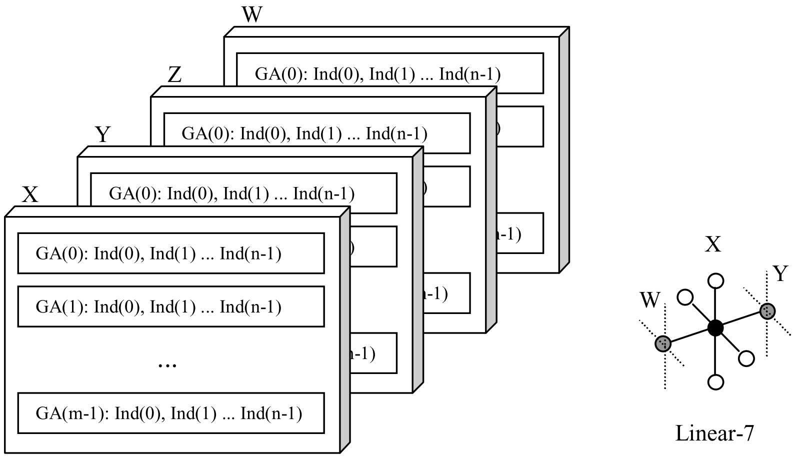


Fig. 5. 3D structure of parallel GA in Stream Technology concept. Interconnection stencil for PGA.

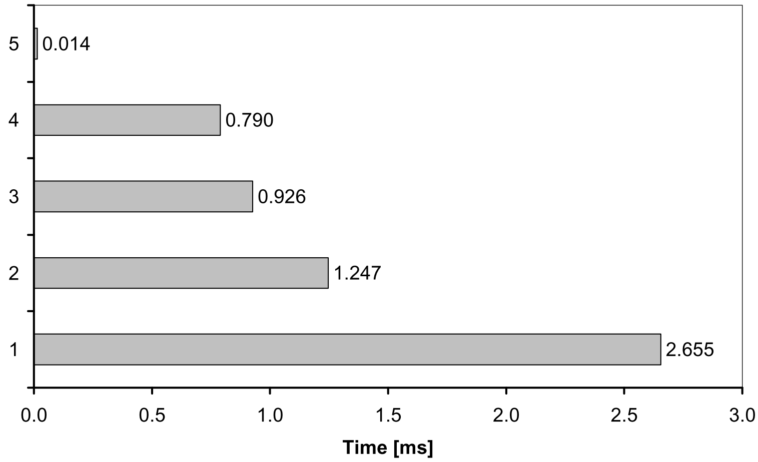


Fig. 6. A comparison of times consumed for one example of T and R evaluation (Eqs. (2) and (7)) performed on different CPUs and a CPU + GPU tandem, based on 2×10^5 multithread computations. 1 – Pentium 4 531 @3 GHz, 2 – Core 2 Duo E6750 @2.66 GHz, 3 – Core 2 Duo E8500 @3.16 GHz, 4 – Phenom X4 9550 @2.2 GHz, 5 – Core 2 Duo E6750 @2.66 GHz + GPU RV770.

Introduction of this communication structure needs a division of GA-vector into sections, therefore the length of a section is a special parameter forming the GA interconnection grid in 3D space. In the SIMD stream technology, the application of NEWSFB stencil creates a quadruple torus. The creation of spatial GA structures is limited only by the programmer's imagination, however, even the simplest structures can guarantee obtaining satisfactory results.

SIMD procedures regarding the Stream Computing concept can be easily coded using Brook+, a high-level C-like language compiler developed by Stanford University Graphics Laboratory [42]. The creation of the executable code is based on a two-step operation: transcoding Brook+ code (i.e. kernels for GPU) into a pure C-syntax file and then ordinary compilation with the linking of all components of the project into a heterogeneous (i.e. CPU/GPU) executable file.

Fig. 6 presents the average times consumed for a single evaluation of R and T functions (Eqs. (2), (7)) for the following data: $\Phi = 20°$, $c = 10$ kPa, $\gamma = 16$ kN/m^3, $x_0 = -2.39$ m, $x_c = 3.16$ m, $y_c = 3.68$ m, $\sigma_0 = -5.08$ kPa, $\beta = 70°$, using the simplest numerical integration technique – trapezoids rule with 10^4 intervals.

5. Examples of calculations

A simple case of a slope stability problem was chosen (see [24], Fig. 7) to illustrate application of the variational analysis.

The results of the performed calculations were collected in Figs. 7 and 8. The minimal value of H is assumed as the critical height of the slope H_{cr}. It can be noticed that small changes of H have considerable influence on the range of failure observed on the top of the slope. This inconvenience demands application of precision integration and minimization algorithms with special attention paid to satisfying the criteria (8) – the presented example was solved with an assumption of error tolerance $< 10^{-4}$% independently for each optimization and integration procedure. Such high-precision calculations usually involve numerical procedures with long, time-consuming operations (see Table 1, computations performed on Core 2

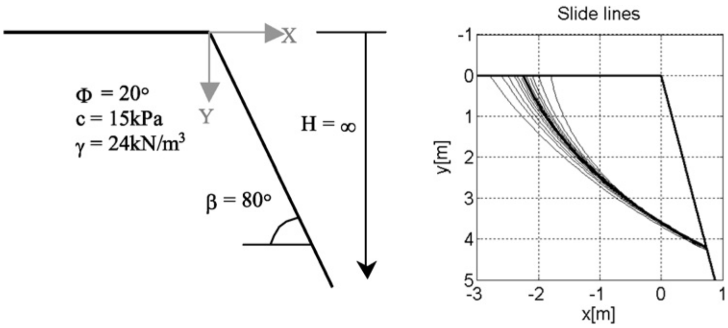


Fig. 7. A case of a slope. Examples of the shapes and positions of slide lines for $|T| < 10^{-8}$ and $R < 10^{-5}$, error tol. $< 10^{-4}$% (thick line – the global solution, i.e. the critical slide line).

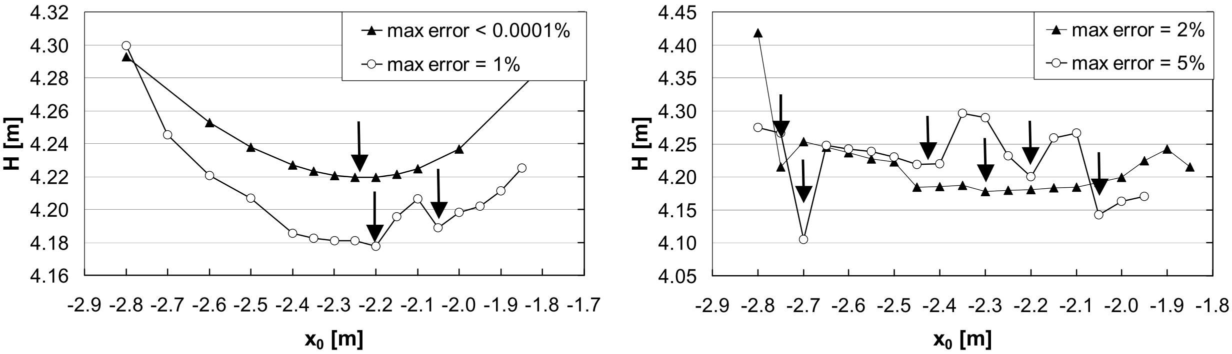


Fig. 8. Intractable identification of the global solution (a critical height of the slope H and a range of the most probable failure x_0) for different error tolerances.

Table 1
Time of computations (Matlab 7.1).

Error tol.	$< 10^{-4}$%	0.1%	1%	2%	5%
Time [s]	2158.4	217.6	86.3	72.9	21.8

Table 2
GA-configuring parameters.

Crossing probability	Mutation probability	Migration probability	Incorporation method	Scaling coefficient	w_T	w_R	w_H
1.0	0.005	1.0/pop.size	replacement	2.0	0.87	0.87	0.99

Table 3
Coding parameters (Gray reflected binary code).

Variable	Min. value	Max. value	Length [bits]	Resolution
x_c [m]	1.0	6.11	9	0.01
y_c [m]	−6.11	−1.0	9	0.01
x_0 [m]	−6.11	−1.0	9	0.01
σ_0 [kPa]	−12.23	−2.0	10	0.01

Duo E6750 @2.66 GHz considering stability problem defined in Cartesian coordinates with differential forms of $y(x)$ and $\sigma(x)$, using standard integration and optimization M-files built-in Matlab 7.1).

The example of a slope shown in Fig. 7 was re-analyzed taking into consideration the PGA application in the stream concept of SIMD computations. As a reference result, the solution from previous analysis is taken: $H_{\mathrm{cr}} = 4.22$ m, $x_0 = -2.23$ m. The GA-configuring parameters are collected in Tables 2 and 3. It should be emphasized that, except for weights coefficients, all GA-parameters were not optimized–adapted for this particular case of analysis.

A middle-size structure of PGA was taken into consideration: 8/64/256 (length of sector/number of GAs/population size), assuming execution of 1000 iterations with renewing the population every 125 generations (8 cycles). Examples of the results of trial computations performed on ATI Radeon HD4850 (RV770 GPU) with single-precision 32-bits floating-point

Table 4
Comparison of the results: the critical heights H_{cr} [m] for an assumed safety factor $F_s = 1.0$, $\gamma = 24.0$ [kN/m^3], $\Phi = 20$ [°], $c = 15$ [kPa].

β [°]	Chen and Liu*	PGA/GPU	Fellenius	Bishop (simpl.)	Plaxis
50	8.31	8.57	7.90	8.55	8.0
60	6.43	6.50	6.02	6.41	6.1
70	5.14	5.19	4.78	4.94	5.0
80	4.18	4.22	3.62	3.91	4.1

* [24], p. 450, Table 11.1 (recalculated).

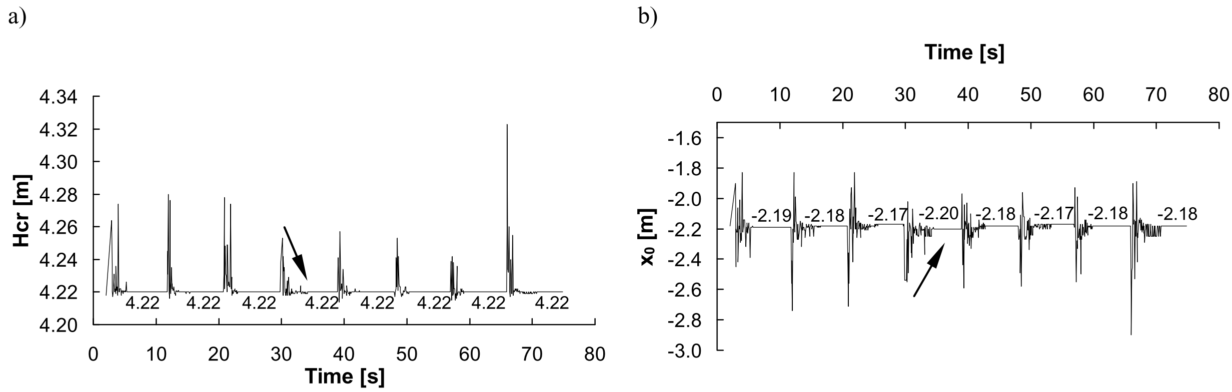


Fig. 9. Example of results: a) the critical height of the slope, b) the range of the failure. Stable solutions were marked with numbers. Evolution is indicated by decreasing of the changes of actually the best solution in a single cycle (examples marked with arrows).

Table 5
Comparison of the results: the safety factors F_s for assumed heights of the slopes H_{cr} [m], in brackets: $F_h = H_{cr}/H_{cr(PGA/GPU)}$; $\gamma = 24.0$ [kN/m^3], $\Phi = 20$ [°], $c = 15$ [kPa].

β [°]	H_{cr}	PGA/GPU	Fellenius	Bishop (simpl.)	Plaxis
50	8.57	1.0	0.96 (0.92)	1.00 (1.00)	0.96 (0.93)
60	6.50	1.0	0.97 (0.93)	1.00 (0.99)	0.96 (0.94)
70	5.19	1.0	0.98 (0.92)	0.99 (0.95)	0.97 (0.96)
80	4.22	1.0	0.99 (0.86)	1.00 (0.93)	0.97 (0.97)

functions were collected in Table 4 and in Fig. 9. It is noticeable that the rate of computations is outstanding (less than 5 seconds to the establishment of the final solution), although it should be emphasized that most of the time is consumed by procedures of slope stability evaluation (based on four-points Gauss quadrature with constant integration step $\Delta\Theta = 0.01$ [rad]). Table 4 collects comparative results obtained from Fellenius, Bishop and FEM methods. It can be noticed that the results of PGA/GPU computations are close to the results of the standard variational calculus for steep slopes, and in the case of smaller inclinations they are approaching the results generated by Bishop's method. On the other hand, LEM-slices evaluations of a safety factor for the analyzed inclinations but with an assumed critical height specified by PGA/GPU computations (see Table 5) do not give the values which should seem to be proportional to the ratio of the critical heights stated by LEM and PGA/GPU methods. The absence of such correlation is caused by a circular shape of the slide line in Fellenius and Bishop methods, which achieves the minimum ratio of resisting and sliding forces for an exit point situated above the foot of the slope (especially for steep slopes, over 70 degrees of inclination). Circles which embrace the full height of the slopes have lengths causing bigger slide-resistance because of stronger influence of the cohesion ($c*l$ component in shear resistance). The results collected in Table 5 confirm the basic difficulties in making comparisons of differently defined safety factors.

It is clearly visible that the determination of the critical height of a slope and identification of the range of failure x_0 are easy and rapid. However, the x_0-value (−2.18) proposed by PGA slightly differs from the solution obtained from the previously performed precise computations (−2.23). This discrepancy is related to the intentional low single-precision accuracy of R and T computations – the x_0-value proposed by PGA is placed very close to the global optimum. Additionally, taking into account the assumed wide range of admissible values of unknowns (Table 3), the distance between the obtained solutions can be treated as satisfactorily low. Verification of the critical height H_{cr} confirms the great potential of PGA optimization reinforced by SIMD technology. Prospective improvements of the accuracy of computations can be realized by usage of hardware with implemented double-precision mathematical functions in transcendent ALUs.

6. Conclusions

Development of slope stability evaluation methods is highly desirable in engineering practice. Alternative approaches can help to take right decisions regarding safety of civil engineering projects and products. Despite difficulties in obtaining univocal identification of the solution in the variational approach to slope stability evaluation, adaptation of PGAs assisted by the SIMD model of computation makes this analysis an alternative to the commonly used FE and slices methods, serving as an optional and variant solution. The consideration of a nonlinear law to describe a soil behavior including variability of material or loading characteristics in space creates this method as a very attractive one to civil engineers.

Application of a parallel GA in the SIMD stream technology in general purpose computations is crucial for substantially increasing the speed of many tasks which are dealt with by engineers in everyday work. The philosophy of stream programming in the frame of a virtual SIMD model initiates new approaches of coding apart from the above conventional numerical procedures. The presented example of PGA application in the General Purpose GPU (GPGPU) technology is the first part of a larger project including both Stream and CUDA technologies. Additionally, two improvements concerning the usage of the SIMD processor (Fig. 4) should be emphasized. Firstly, the data is understood to be processed in blocks, and a block containing thousands values can be loaded at once. Instead of a series of instructions saying "get one datum, now get the next datum", the SIMD processor has a single instruction that effectively says: "get lots of data at once". For a variety of reasons, this can take much less time than "getting" each datum individually as it is performed using traditional CPUs. Secondly, the data-parallel technique of an information processing is very important for landslide and snow avalanche predictions using wireless sensor networks (large scale network of sensors employed for observing various aspects of the physical world), our intent is to show the trends of the future work.

The sources of executable codes are available at: http://www.uwm.edu.pl/edu/piotrsrokosz/Gpgpu_en.htm.

References

[1] W. Fellenius, Calculations of the stability of earth dams, in: Transactions of the 2nd Congress on Large Dams, Washington, DC, vol. 4, 1936, p. 445.
[2] A.W. Bishop, The use of the slip circle in the stability analysis of slopes, Geotechnique 5 (1) (1955) 7–17.
[3] E. Spencer, A method of analysis of the stability of embankments assuming parallel inter-slice forces, Geotechnique 17 (1) (1967) 11–26.
[4] N. Janbu, Slope stability computations, in: R.C. Hirshfield, S.J. Poulos (Eds.), Embankment Dam Engineering. Cassagrande Volume, John Wiley and Sons, 1973, pp. 47–86.
[5] N.R. Morgenstern, V.E. Price, The analysis of the stability of general slip surface, Geotechnique 15 (4) (1965) 289–290.
[6] S.K. Sarma, Stability analysis of embankments and slopes, Geotechnique 23 (3) (1973) 423–433.
[7] S.K. Sarma, Stability analysis of embankments and slopes, Journal of the Geotechnical Engineering Division, ASCE 105 (12) (1979) 1511–1524.
[8] A.T.C. Goh, Search for critical slip circle using genetic algorithms, Civil Engineering and Environmental Systems 17 (2000) 181–211.
[9] P. McCombie, P. Wilkinson, The use of the simple genetic algorithm in finding the critical factor of safety in slope stability analysis, Computers and Geotechnics 29 (8) (2002) 699–714.
[10] Aniruddha Sengupta, Anup Upadhyay, Locating the critical failure surface in a slope stability analysis by genetic algorithm, Applied Soft Computing 9 (2009) 387–392.
[11] A.T.C. Goh, Genetic algorithm search for critical slip surface in multiple-wedge stability analysis, Canadian Geotechnical Journal 36 (2) (1999) 382–391.
[12] P. McCombie, Critical failure surface location using simple genetic algorithm and multiple wedge slope stability, in: 1st International Symposium on Computational Geomechanics, 2009.
[13] A.R. Zolfaghari, A.C. Heath, P.F. McCombie, Simple genetic algorithm search for critical non-circular failure surface in slope stability analysis, Computers and Geotechnics 32 (2005) 139–152.
[14] Wen Wan, Ping Cao, Tao Feng, Hai-Ping Yuan, Improved genetic algorithm freely searching for dangerous slip surface of slope, Journal of Central South University of Technology 12 (6) (2005) 749–752.
[15] Jianping Sun, Jiachun Li, Qingquan Liu, Search for critical slip surface in slope stability analysis by spline-based GA method, Journal of Geotechnical and Geoenvironmental Engineering 134 (2) (2008) 252–256.
[16] O.C. Zienkiewicz, C. Humpheson, R.W. Lewis, Associated and non-associated visco-plasticity and plasticity in soil mechanics, Geotechnique 25 (4) (1975) 671–689.
[17] D.V. Griffiths, P.A. Lane, Slope stability analysis by finite elements, Geotechnique 49 (3) (1999) 387–403.
[18] T. Matsui, K.C. San, Finite element slope stability analysis by shear strength reduction technique, Soils and Foundations 32 (1) (1992) 59–70.
[19] E.M. Dawson, W.H. Roth, A. Drescher, Slope stability analysis by strength reduction, Geotechnique 49 (6) (1999) 835–840.
[20] H. Maosong, J. Cang-Qin, Strength reduction FEM in stability analysis of soil slopes subjected to transient unsaturated seepage, Computers and Geotechnics 36 (2009) 93–101.
[21] R. Baker, A relation between safety factors with respect to strength and height of slopes, Computers and Geotechnics 33 (2006) 275–277.
[22] W.F. Chen, Limit Analysis of Soil Plasticity, Developments in Geotechnical Engineering, vol. 7, Elsevier Scientific Publishing Co., New York, 1975.
[23] R. Baker, M. Garber, Variational approach to slope stability, in: Proc. 9th International Conference on Soil Mechanics and Foundation Engineering, vol. 2, Tokyo, 1977, pp. 2–12.
[24] W.F. Chen, X.L. Liu, Limit Analysis in Soil Mechanics, Developments in Geotechnical Engineering, vol. 52, Elsevier, Amsterdam–Oxford–New York–Tokyo, 1990.
[25] E. Castillo, A. Luceno, A critical analysis of some variational methods in slope stability analysis, International Journal for Numerical and Analytical Methods in Geomechanics 6 (1982) 195–209.
[26] G. De Josselin De Jong, Variational fallacy, Geotechnique 31 (4) (1981) 289–290.
[27] R. Baker, Sufficient conditions for existence of physically significant solutions in limiting equilibrium slope stability analysis, International Journal of Solids and Structures 40 (2003) 3717–3735.
[28] H. El-Ramly, N.R. Morgenstern, D.M. Cruden, Probabilistic slope stability analysis for practice, Canadian Geotechnical Journal 39 (3) (2002) 665–683.
[29] K.S. Li, P. Lumb, Probabilistic design of slopes, Canadian Geotechnical Journal 24 (1987) 520–535.
[30] D.V. Griffiths, G.A. Fenton, Probabilistic slope stability analysis by finite elements, Journal of Geotechnical and Geoenvironmental Engineering 130 (5) (2004) 507–518.
[31] Sung Eun Cho, Probabilistic stability analyses of slopes using the ANN-based response surface, Computers and Geotechnics 36 (2009) 787–797.

[32] Guihua Xie, Jiasheng Zhang, Jixiang Li, Adapted genetic algorithm applied to slope reliability analysis, in: 4th International Conference on Natural Computation, vol. 1, 2008, pp. 520–524.
[33] C.X. Yang, L.G. Tham, X.T. Feng, Y.J. Wang, P.K.K. Lee, Two-stepped evolutionary algorithm and its application to stability analysis of slopes, Journal of Computing in Civil Engineering 18 (2) (2004) 145–153.
[34] R. Baker, Variational slope stability analysis of materials with nonlinear failure criterion, EJGE 2005-0514, 2005.
[35] D. Leshchinsky, A.J. Reinschmidt, Stability of membrane reinforced slopes, Journal of Geotechnical Engineering, ASCE 111 (11) (1985) 1285–1300.
[36] P. Lemonnier, A.H. Soubra, R. Kastner, Variational displacement method for geosynthetically reinforced slope stability analysis: I. Local stability, Geotextiles and Geomembranes 16 (1998) 1–25.
[37] P. Lemonnier, A.H. Soubra, R. Kastner, Variational displacement method for geosynthetically reinforced slope stability analysis: II. Global stability, Geotextiles and Geomembranes 16 (1998) 27–44.
[38] P. Srokosz, Slope stability analysis by variational method with genetic algorithm application. Part 2: Genetic algorithm with advanced techniques. Simple modeling of uncertainties, Archives of Civil Engineering 53 (2) (2007) 269–292.
[39] P. Srokosz, Slope stability analysis by variational method with genetic algorithm application. Part 4: Parallel genetic algorithms, Archives of Civil Engineering 55 (2) (2009) 229–256.
[40] D.E. Goldberg, Genetic Algorithms in Search, Optimization and Machine Learning, Addison–Wesley Publishing Company, 1989.
[41] ATI Stream Computing, User Guide, April 2009, rev. 1.4.0a, Advanced Micro Devices Inc., http://www.amd.com.
[42] BrookGPU – stream program language for modern graphics hardware, Stanford University Graphics Lab, http://graphics.stanford.edu/projects/brookgpu/.