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Closed form solution for the finite anti-plane shear field for a class of hyperelastic incompressible brittle solids

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ABSTRACT

The equilibrium solution of a damaged zone in finite elasticity is given for a class of hyperelastic materials which does not suffer tension when a critical stretching value is reached. The study is made for a crack in anti-plane shear loading condition. The prescribed loading is that of linearized elastostatics conditions at infinity. The geometry of the damaged zone is found and the stationary propagation is discussed when the inertia terms can be neglected.

1. Introduction

In classical fracture mechanics, the solution of equilibrium for linearized elastostatics proposes an outer expansion of the displacement with respect to the distance from the crack tip. Some investigations must be made aimed to take into account the non-linear effects for determination of the inner expansion in finite elastostatics or in elastoplasticity. The finite antiplane shear field near the tip of a crack in an incompressible elastic solids has been studied [1–3]. The local behaviour presents singularities or discontinuities of deformation gradient, depending on the shape of the stress strain curve. The stress is defined for any amount of shear γ, which is not bounded $0 \leqslant \gamma < \infty$.

For hyperelastic materials such as polymers or elastomers, rupture may occur when a maximal stretch of the polymeric chains is reached. For any material direction, the stretch λ in a hyperelastic body is bounded by a critical value λ_c. When this value is reached, the material is broken and cannot support further tension; then, a damaged zone develops inside the body. The purpose of this Note is to study the evolution of the damaged zone in the case of anti-plane shear.

This approach is connected with classical approach of fracture mechanics. The damaged zone takes the shape of a quasi-crack. This shape is determined considering a stationary propagation of the damaged zones, when the applied loading at infinity corresponds to the linearized elastostatics field for a classical crack. Such a problem has been solved for linearized elastodynamics [4] and for elastoplasticity [5] in case of antiplane-shear. Connections with previous results [6] are also discussed.

2. Finite anti-plane shear

Suppose an isotropic homogeneous incompressible elastic body occupies a domain Ω. The anti-plane shear is the transformation

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$$\underline{x}(X) = \underline{X} + w(x_1, x_2)\underline{e}_3 \tag{1}$$

The gradient of this transformation and its inverse are

$$\mathbf{F} = \mathbf{I} + w_{,i}\underline{e}_3 \otimes \underline{e}_i, \qquad \mathbf{F}^{-1} = \mathbf{I} - w_{,i}\underline{e}_3 \otimes \underline{e}_i \tag{2}$$

Then we can deduce the left Cauchy–Green deformation tensor

$$\mathbf{B} = \mathbf{I} + w_{,i}(\underline{e}_3 \otimes \underline{e}_i + \underline{e}_i \otimes \underline{e}_3) + \left(w^2_{,x_1} + w^2_{,x_2}\right)\underline{e}_3 \otimes \underline{e}_3 \tag{3}$$

which has the fundamental scalar invariants

$$I_1 = \operatorname{tr}\mathbf{B}, \qquad I_2 = \frac{1}{2}\left(I_1^2 - \operatorname{tr}\left(\mathbf{B}^2\right)\right), \qquad I_3 = 1 \tag{4}$$

The two invariants are equal $I_1 = I_2 = 3 + R^2$, where R is the norm of ∇w. As the two invariants are equal, the free energy W is replaced by its value Ψ for the antiplane-shear transformation:

$$\Psi(I_1) = W(I_1, I_2) \tag{5}$$

2.1. A special class of hyperelastic brittle material

The local stress-strain curve is given by the combined power law

$$\begin{aligned} &\tau = \mu_o R, \quad R \leqslant R_o \\ &\tau = \mu_o R_o \left(\frac{R}{R_o}\right)^{\alpha} = \hat{\mu}_o R^{\alpha}, \quad R_o \leqslant R \leqslant R_m \\ &\tau = 0, \quad R > R_m \end{aligned} \tag{6}$$

For $\alpha = 1$ and $R_m = \infty$ the material is neo-Hookean. For $\alpha = -0.5$ and $R_m = \infty$ the special material studied by Abeyaratne [7] is recovered. For $-1 < \alpha < 0$ and $R_m = \infty$ the field near the crack tip presents a discontinuity of the deformation gradient along a curve. This fact is due to the loss of the ellipticity condition for the differential equation of motion [3]. The local behaviour is reversible for $R < R_m$. When $R = R_m$, for unloading ($\dot{R} < 0$) then the behaviour is reversible, otherwise the material is totally broken.

The goal of the present study is to determine the influence of a finite critical stretch (R_m) on the local solution.

2.2. The equilibrium equation

For the energy Ψ, the constitutive behaviour is written as:

$$\boldsymbol{\Theta} = 2\Psi'(I_1)\mathbf{F}^T - \eta\mathbf{F}^{-1}, \qquad \boldsymbol{\sigma} = 2\Psi'(I_1)\mathbf{B} - \eta\mathbf{I} \tag{7}$$

where $\boldsymbol{\Theta}$ is the first Piola–Kirchhof stress and $\boldsymbol{\sigma}$ the Cauchy stresses. The Lagrange multiplier η associated with the condition of incompressibility can be eliminated assuming that in the direction $\underline{e}_1, \underline{e}_2$ the stress vectors are zero, then $\eta = 2\Psi'(I_1)$ and the differential equation of equilibrium is reduced to:

$$\operatorname{Div}\boldsymbol{\Theta}^T = \left(2\Psi'(I_1)w_{,\beta}\right)_{,\beta} = 0 \tag{8}$$

where $\beta = 1, 2$ and the summation is implicit.

2.3. Property of the differential equation

As the constitutive law satisfies the general relation

$$\boldsymbol{\Theta} = 2\Psi'\left(\mathbf{F}^T - \mathbf{F}^{-1}\right)$$

the condition of ellipticity of the differential equation of equilibrium is reduced to

$$2\Psi' + 4k^2\Psi'' > 0$$

which corresponds to the fact that the local response $\tau(R) = 2R\Psi'(I_1) = R\mu(R)$ is an increasing function of R [2,3]. The differential equation of equilibrium is elliptic if $\alpha > 0$.

3. The hodograph method

To determine the mechanical fields around the propagating damaged zone, the hodograph transformation is useful as pointed out by Knowles and Sternberg [1] and Rice [8]. Other techniques can be used [6]. In this transformation, the components of the gradient ∇w become the new independent variables.

$$(x_1, x_2) \longrightarrow (\xi_1, \xi_2), \qquad \xi_\alpha = w_{,\alpha}(x_1, x_2) \tag{9}$$

The mapping of the physical plane $\Re^*$ is denoted by $\Re^{\mathcal{H}}$ in the hodograph plane where the Cartesian coordinates are ξ_α. The displacement w appears as a potential ($\mathrm{d}w = \xi_\alpha\, \mathrm{d}x_\alpha$). The mapping is considered as inversible provided the Jacobian $H = w''_{,11} w''_{,22} - (w''_{,12})^2$ does not vanish on $\Re^*$. Denote by U the Legendre transformation of w with respect to ξ_i

$$U(\xi_1, \xi_2) = x_\alpha w_{,\alpha}(x_1, x_2) - w(x_1, x_2), \quad \forall (x_1, x_2) \in \Re^* \tag{10}$$

By differentiating U with respect to ξ_α and by using (9), we obtain the conjugate relations:

$$x_\alpha = \frac{\partial U}{\partial \xi_\alpha}(\xi_1, \xi_2), \qquad w = \xi_\alpha \frac{\partial U}{\partial \xi_\alpha}(\xi_1, \xi_2) - U(\xi_1, \xi_2), \quad \text{over } \Re^{\mathcal{H}} \tag{11}$$

In the hodograph plane, polar coordinates are used $\xi_1 = w_{,1} = R\cos\Phi$, $\xi_2 = w_{,2} = R\sin\Phi$ and the mapping is given by

$$x_1 = \cos\Phi \frac{\partial U}{\partial R} - \frac{\sin\Phi}{R}\frac{\partial U}{\partial \Phi}, \qquad x_2 = \sin\Phi \frac{\partial U}{\partial R} + \frac{\cos\Phi}{R}\frac{\partial U}{\partial \Phi} \tag{12}$$

4. Anti-plane shear motion

The conservation of the momentum in a stationnary motion ($w(x_1, x_2, t) = w(x_1 - Vt, x_2)$) with speed V in the $\underline{e}_1$ direction is given by ($\mu(R) = 2\Psi'$):

$$(\mu w_{,\alpha})_{,\alpha} = \rho_o V^2 w''_{,11} \tag{13}$$

The equations of motion are written in the hodograph plane (R, Φ) using the mapping (12). The notation $()'$ indicates a differentiation with respect to R. $\nabla R, \nabla \Phi$ are functions of the new coordinates R, Φ using the potential $U(R, \Phi)$. By differentiation of the mapping, the differential equation in statics is:

$$Eq(U) = \frac{\partial}{\partial R}\left(\mu R \frac{\partial U}{\partial R}\right) + \frac{(\mu R)'}{R}\frac{\partial^2 U}{\partial \Phi^2} = 0 \tag{14}$$

The equation of motion in the hodograph plane is linear and homogenous of degree one in U, [3]. The displacement is finite, this implies a condition on U. Taking account of the relation between w and U:

$$w = R^2 \frac{\partial}{\partial R}\left(\frac{U}{R}\right), \quad \text{is bounded when } R \longrightarrow \infty,\ 0 \leqslant \Phi \leqslant \pi \tag{15}$$

4.1. Matching conditions

The points far from the origin in the plane $\Re^*$ are transformed in points close to the origin in $\Re^{\mathcal{H}}$. The map of the crack lips ($\theta = \pm\pi$) in $\Re^{\mathcal{H}}$ is on the ξ_1 axis, i.e. ($\Phi = 0; \Phi = \pi$). In addition to the free surface condition, at infinity the displacement tends to the solution of an equilibrated crack in a medium under linearized elastostatics conditions. At a large r, $x_1 = r\cos\theta$, $x_2 = r\sin\theta$, the displacement takes the form

$$w \to \frac{2K}{\mu_o}\sqrt{\frac{r}{2\pi}}\sin(\theta/2) \tag{16}$$

The matching conditions at infinity are then:

$$r \longrightarrow \infty, \quad -\pi \leqslant \theta < \pi, \qquad R = \frac{K}{\mu_o\sqrt{2\pi r}}, \qquad \Phi = \frac{1}{2}(\pi + \theta) \tag{17}$$

Therefore, in the hodograph plane, a solution of the differential equation must satisfy the corresponding boundary conditions:

$$\begin{cases} \dfrac{\partial U}{\partial \Phi} = 0, \quad \text{for } \Phi = 0 \text{ and } \Phi = \pi,\ \forall R > 0 \\ U = -\dfrac{K^2}{\pi \mu_o^2 R}\cos\Phi, \quad \text{when } R \to 0,\ 0 \leqslant \Phi \leqslant \pi \end{cases} \tag{18}$$

The existence of one solution in the hodograph plane ensures the existence of a solution in the physical plane taking the mapping between the two geometries into account. For ellipticity of the differential equation of equilibrium, the solution exists and is unique.

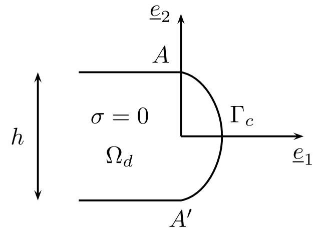

Fig. 1. The damaged zone Ω_d.

5. The solution for a crack in statics

An explicit solution of the differential equation of equilibrium is given by:

$$U(R,\Phi) = -ARI_0(R)\cos\Phi \quad \forall R > 0,\ 0 \leqslant \Phi \leqslant \pi \tag{19}$$

where:

$$I_0(R) = \int_R^\infty \frac{\mathrm{d}t}{\mu(t)t^3}\,\mathrm{d}t \quad \forall R > 0 \tag{20}$$

This potential determines a solution of the equilibrium of a crack. In the same manner a solution can be obtained when the differential equation of equilibrium is no longer elliptic [2,3].

For neo-Hookean material, the Cauchy stress and the displacement posses the same singularity as in linearized elasticity. For this case, the modulus μ is constant and the potential is given by

$$U(R) = -\frac{A}{\mu_o R}\cos\Phi, \quad w = \frac{2A}{\mu_o R}\cos\Phi \tag{21}$$

where the constant $A = \frac{K^2}{\pi\mu}$ due to matching condition at infinity.

6. The case of a quasi-crack

In outer point of view, a crack is a straight line. The direction of the line is $\underline{e}_x$. The applied loading is that of a crack in classical linear elasticity or neo-Hookean solid (21). In inner point of view the crack is modelling by a propagating damaged zone, shape of which is defined by a specific constitutive law. This inner solution is studied assuming that after a critical stretch λ_c the material is destroyed and does not support any further tension. The damaged zone is bounded by a curve along which the maximal stretch is less or equal to R_m. Under stationary condition the damaged zone propagates in the $\underline{e}_1$ direction with a constant speed V conserving its shape. This shape is given by two horizontal lines of direction $\underline{e}_1$ separated by a band of thickness h and closed by a symmetric curve Γ_c in the direction of the propagation, as depicted in Fig. 1.

Inside the damaged zone the stress is identically null. Then the boundary of the damaged zone is stress free. On the front of damage Γ_c the value of the gradient of the displacement is equal to the critical value R_m. The local behaviour is defined by the combined power law (6). The modulus $\mu(R)$ depends on the strain R and the general solution in the hodograph plane is written ($\tau(t) = \mu(t)t$):

$$U(R,\Phi) = AR\cos\Phi \int_R^{R_o} \frac{\mathrm{d}t}{\tau(t)t^2} + U_p(R,\Phi), \quad 0 \leqslant \Phi \leqslant \pi \tag{22}$$

where $U_p(R,\Phi)$ is a solution of the differential equation of equilibrium which is regular and compatible with the boundary conditions ($R = R_m$). The matching condition (18) imposes the constant A for $R \to 0$.

6.1. Particular solution

When the modulus $\mu(R)$ is a power law of R, $\tau(R) = \mu_o R^\alpha$, peculiar solutions are found

$$R\cos\Phi, \qquad R\left(\frac{2\alpha}{\alpha+1}\cos\Phi\ln R - \Phi\sin\Phi\right) \tag{23}$$

For the considered behaviour (6), the solution is obtained as: In the linear part $R \leqslant R_o$,

$$U_2 = A_2 R\cos\phi \int_{R_o}^{R} \frac{\mathrm{d}t}{\mu_o t^3} + B_2 R(\ln R\cos\phi - \phi\sin\phi) + C_2 R\cos\phi \tag{24}$$

in the power part: $R_o \leqslant R \leqslant R_m$

$$U_1 = A_1 R\cos\phi \int_{R}^{R_o} \frac{dt}{\hat{\mu}_o t^{\alpha+2}} + B_1 R\left(\frac{2\alpha}{\alpha+1}\ln R\cos\phi - \phi\sin\phi\right) + CR\cos\phi \tag{25}$$

The continuity of the potential at $R = R_o$ $(U_1(R_o,\phi) = U_2(R_o,\phi))$ implies that $B_1 = B_2 = B$ and the relation

$$BR_o \ln R_o \frac{2\alpha}{\alpha+1} + CR_o = BR_o \ln R_o + C_2 R_o \tag{26}$$

C_2 is considered to be zero, that determines the position of the front in the $\underline{e}_1$ direction.

The displacement $w_i = R^2 \frac{\partial}{\partial R}(\frac{U_i}{R})$ must be continuous for $R = R_o$ then

$$w_2 = A_2\cos\phi\frac{1}{\mu_o R} + BR\cos\phi, \qquad w_1 = A_1\cos\phi\frac{1}{\hat{\mu}_o R^{\alpha}} + \frac{2B\alpha}{\alpha+1}R\cos\phi \tag{27}$$

so the relation of continuity and the matching condition give

$$-A_1 + \mu_o R_o^2 B\frac{\alpha-1}{\alpha+1} = A_2, \qquad A_2 = \frac{K^2}{\pi\mu_o} \tag{28}$$

The mapping $(R,\phi) \to (x_1,x_2)$ is given for $R \geqslant R_o$ by

$$x_1 = A_1\int_{R}^{R_o}\frac{\mathrm{d}t}{\hat{\mu}_o t^{\alpha+2}} - \frac{A_1}{\hat{\mu}_o R^{\alpha+1}}\cos^2\phi + \frac{2B\alpha}{\alpha+1}\ln\frac{R}{R_o} + B\left(\frac{2\alpha}{\alpha+1}\cos^2\phi + \sin^2\phi\right) + B\ln R_o \tag{29}$$

$$x_2 = -\frac{A_1}{\hat{\mu}_o R^{\alpha+1}}\cos\phi\sin\phi - B\phi + B\frac{\alpha-1}{\alpha+1}\cos\phi\sin\phi \tag{30}$$

Along Γ_c the amount of shear is constant $R = R_m$ and the stress vector is null, that implies the condition

$$\tau_1\,\mathrm{d}x_2 - \tau_2\,\mathrm{d}x_1 = \cos\phi\left(-\frac{A_1}{\hat{\mu}_o R_m^{\alpha+1}} - 2\frac{B}{\alpha+1}\right) = 0 \tag{31}$$

For the considered class of hyperelastic brittle material, the curve Γ_c is a cycloïd. The thickness h of the damaged zone is given by $h = \pi B$ and taking account of conditions (28), the thickness h is defined by the loading condition ($\tau_o = \mu_o R_o$)

$$K^2 = 2h\tau_o^2\frac{1}{\alpha+1}\left(\left(\frac{R_m}{R_o}\right)^{\alpha+1} + \frac{\alpha-1}{2}\right) \tag{32}$$

This relation is nothing else than an account of energy lost when the front propagates conserving the shape explained in macroscopic form as a moving crack (D_M) and in a local form as a moving surface (D_m). For a macroscopic point of view the dissipation is $D_M = VK^2/\mu_o$ and locally the loss of energy along the curve Γ_c is $D_m = W(R_m)2hV$ where

$$W(R_m) = \int_{o}^{R_o}\mu_o t\,\mathrm{d}t + \int_{R_o}^{R_m}\hat{\mu}_o t^{\alpha}\,\mathrm{d}t = \frac{1}{\mu_o}\left(\frac{1}{2}\tau_o^2 + \frac{\tau_o^2}{\alpha+1}\left(\left(\frac{R_m}{R_o}\right)^{\alpha+1} - 1\right)\right) \tag{33}$$

For $\alpha > 0$ Neuber [6] shows that along notch of cycloïd shape, the stress τ is constant. Moreover, these results generalizes the results of Bui and Ehrlacher [4] for a special class of hyperelastic material. The interpretation in terms of a critical value defining a propagating damaged zone is explicitly taken into account and the relation (32) between thickness and loading at infinity generalizes the results obtained for elastic brittle material in small strain.

For $\alpha < 0$, the curve Γ_c is also a cycloïd. In the domain ($R_o < R < R_m$) the equations of motion are hyperbolic. When R_m tends to infinity, the thickness h tends to zero; for $\alpha = 0.5$ the potential U given by Abeyaratne is recovered.

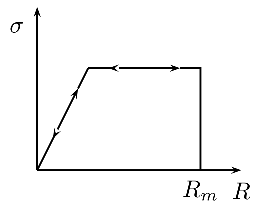

Fig. 2. The bilinear-brittle material.

7. Particular constitutive laws

- For the linear hyperelastic brittle material, $\alpha = 1$ and $R_o = R_m$. The mapping from the hodograph plane to the physical one is

$$x_1 = \frac{A}{2\mu R^2}\cos 2\Phi + \frac{A}{R_m^2} - B\ln R, \qquad x_2 = \frac{A}{2\mu R^2}\sin 2\Phi + B\Phi \tag{34}$$

  The relations (28) are reduced to:

$$B - \frac{A}{\mu R_m^2} = 0, \quad h = \pi B = \left(\frac{K}{\mu R_m}\right)^2 \tag{35}$$

  This solution generalizes the solution of Bui and Ehrlacher [4] obtained in linearized elasticity. When R_m tends to infinity, the constant B tends to zero, the thickness of the damaged zone vanishes and the solution is that of classical crack in a neo-Hookean material.
- For a bilinear brittle solid, with $\alpha = 0$, see Fig. 2.
  The previous framework can be applied for each domain. For R less than R_o the differential equation of equilibrium is elliptic. For R between R_o and R_m the differential equation is parabolic.
  - For $R < R_o$ the solution is given by the potential U

$$U_e(R,\Phi) = AR\cos\Phi \int_R^{R_o} \frac{\mathrm{d}t}{\sigma(t)t^2} + B(R\ln R - R)\cos\Phi - BR\Phi\sin\Phi \tag{36}$$

  - For $R_o \leqslant R \leqslant R_m$ then the solution is written as

$$U_p(R,\Phi) = AR\cos\Phi \int_R^{R_o} \frac{\mathrm{d}t}{\sigma_o t^2} + RB(\ln R_o\cos\Phi - \Phi\sin\Phi) - BR_o\cos\Phi \tag{37}$$

    in this domain the stress is uniform ($\sigma_o = 2\mu_o R_o$, $R > R_o$) and $\tau_x = \sigma_o\cos\Phi$, $\tau_y = \sigma_o\sin\Phi$.

This potential ensures the continuity of the displacement along the curve $R = R_o$

$$u_e(R_o,\Phi) = u_p(R_o,\Phi) = \left(\frac{A}{\mu_o R_o} + BR_o\right)\cos\Phi \tag{38}$$

The condition of stress free surface, where $R = R_m$, and the matching conditions give

$$\frac{A}{2\mu_o R_o R_m} + \frac{B}{2}\left(1 - \frac{R_o}{R_m}\right) + B = 0, \qquad A = -\frac{K^2}{\pi\mu_o} \tag{39}$$

The thickness of the band $h = 2\pi B$ is found as a function of the loading parameter K and the characteristics of the constitutive law $W(R_m)$:

$$2hW(R_m) = h\sigma_o(2\mu R_m - \sigma_o) = \frac{K^2}{\mu_o} \tag{40}$$

Taking account of this value, the damaged front is a cycloïd given by

$$x_1 = \frac{h}{2\pi}(\cos 2\Phi - 1), \qquad x_2 = \frac{h}{2\pi}(\sin 2\Phi - 2\Phi). \tag{41}$$

The solution has a similar form to the solution that Bui [5] obtained for an elastoplastic-brittle material. However here, the solution is not only a static solution, but also a dynamic solution for a stationnary motion at sufficient low speed such that we can neglect the inertia terms.

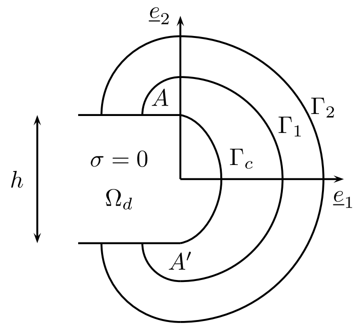

Fig. 3. On J integrals.

If R_m tends to infinity the curve R_o is a circle and there is no dissipation in the process. In the circle, the stresses are bounded and there are no singularities, the curve Γ_c is reduced to a point. This solution cannot be a solution for a translation of the crack, the tangent modulus being null, the inertia effects cannot be neglected. The analysis must be reformulated taking account of inertia effects.

8. Comments

We have performed closed form solutions for the propagation of damaged zone in finite strain at low speed and make connection with linear fracture mechanics. The solutions are obtained for a special class of hyperelastic brittle materials in anti-plane shear loading conditions. This point of view completes the discussion of the finite anti-plane shear field near a crack tip. The structure of the local fields depends strongly on the non-linear behaviour at large strain. Here, the non-linearity contains a criterion of rupture.

It is well known, [9], that for a homogeneous body, the closed loop integral

$$\int_{\mathcal{C}} W \underline{n}.\underline{e}_1 - \underline{n}.\boldsymbol{\Theta}.\mathbf{F}.\underline{e}_1 \,\mathrm{d}s = 0 \tag{42}$$

Because the stress vector vanishes along the boundary of the quasi-crack, this induces that for any curve with extremities on the horizontal lines the integral

$$\int_{\Gamma} W \underline{n}.\underline{e}_1 - \underline{n}.\boldsymbol{\Theta}.\mathbf{F}.\underline{e}_1 \,\mathrm{d}s = J_\Gamma = \mathcal{G} \tag{43}$$

is independent of the chosen curve Γ.

Taking a curve at infinity, $J_\infty = K^2/(2\mu)$ and taking the curve along the boundary of the damaged zone, $J_c = W_m h$. So we recognize the loss of strain energy during the propagation of the quasi-crack, the equality of the local dissipation and the global one gives the depth of the damaged zone. The J integral plays a fundamental role to study the propagation of a crack (Fig. 3), and therefore

$$\mathcal{G} = \lim_{\Gamma \to \Gamma_c} J_\Gamma \tag{44}$$

and the dissipation is

$$D_m = \mathcal{G}\dot{a}. \tag{45}$$

This is an illustration of the connection between the driving force due to the discontinuity of the gradient of displacement along Γ_c [7] and the global dissipation.

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