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A priori evaluation of the Pantano and Sarkar model in compressible homogeneous shear flows

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ABSTRACT

In this study, a Reynolds stress closure, including the Pantano and Sarkar model of the mean part of the pressure–strain correlation is used for the computation of compressible homogeneous at high-speed shear flow. Several studies concerning the compressible homogeneous shear flow show that the changes of the turbulence structures are principally due to the structural compressibility effects which significantly affect the pressure field and then the pressure–strain correlation. Eventually, this term appears as the main term responsible for the changes in the magnitude of the Reynolds stress anisotropies. The structure of the gradient Mach number is similar to that of turbulence, therefore this parameter may be appropriate to study the changes in turbulence structures that arise from structural compressibility effects. Thus, the incompressible model of the pressure strain correlation and its corrected form by using the turbulent Mach turbulent only, fail to correctly evaluate the compressibility effects at high shear flow. An extension of the widely used incompressible Launder, Reece and Rodi model on compressible homogeneous shear flow is the major aim of the present work. From this extension, the standard coefficients C_i become a function of the extra compressibility parameters (the turbulent Mach number M_t and the gradient Mach number M_g) through the Pantano and Sarkar model. Application of the model on compressible homogeneous shear flow by considering various initial conditions shows reasonable agreement with the DNS results of Simone et al. and Sarkar. The observed trend of the dramatic increase in the normal Reynolds stress anisotropies, the significant decrease in the Reynolds shear stress anisotropy and the increase of the turbulent kinetic energy amplification rate with increasing the gradient Mach number are well predicted by the model. The ability of the model to predict the equilibrium states for the flow in cases A_1 to A_4 from DNS results of Sarkar is examined, the results appear to be very encouraging. Thus, both parameters M_t and M_g should be used to model significant structural compressibility effects at high-speed shear flow.

1. Introduction

The extension of the standard models to compressible flows represents a research topic of great scientific and industrial interest. A major challenge related to this extension is to involve compressibility effects that appear in most of high-speed turbulent flows, such as supersonic and hypersonic of aerodynamic importance. In this context, extensive research initiatives in compressible modeling have been conducted during the last few years. It is well known that the standard models fail to predict high compressible flows. In recent studies, Spezial et al. [1] employed the Favre Reynolds-stress closure with the

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addition of the compressible dissipation and pressure–dilatation correlation models [2,3] for the prediction of compressible homogeneous shear flow.

The poor predictions for the turbulent kinetic energy and the changes in the magnitude of the Reynolds-stress anisotropies show that the dilatational terms effects are much smaller than one believes. According to the DNS results of Blaisdell and Sarkar [4], the dilatational terms represent nearly 9 to 12% of the turbulent kinetic energy production, which do not provide support for the assumption that the level of compressibility is controlled by these terms. In contrast to dilatational effects, structural compressibility effects may cause significant changes on turbulence structures. Accordingly, Sarkar [5] and Simone et al. [6] find in their compressible homogeneous shear flows DNS results that the structural compressibility effects strongly affect the pressure field and then the pressure strain correlation which recognized as the main responsible for the changes in magnitude of Reynolds stress anisotropies. It appears from DNS results of Simone et al. [6] that there is an amplification of the turbulent kinetic energy with increasing the gradient Mach number at low St ($St < 5$), for $St > 5$, the trend change and compressibility tend to stabilize turbulent shear flows. To study these effects, Sarkar [5] and Simone et al. [6] suggested the use of the gradient Mach number which is defined by: $M_g = Sl/a$ where $S = (\overline{U}_{i,j}\overline{U}_{i,j})^{0.5}$ and l is an integral length scale. From the DNS results (cited above), one can see that after an initial slight increase with St ($St < 5$), M_g shows a trend to become asymptotically constant, contrary to the turbulent Mach number M_t ($M_t = \sqrt{2k}/\bar{a}$, where k is the turbulent kinetic energy and $\bar{a}$ is the mean speed sound) which grow constantly with St. Obviously, the structure of this parameter is similar to that of turbulence. As a consequence of this, M_g seems to be an appropriate parameter to study the structural compressibility effects and it may be useful to establish compressible turbulence models that are indispensable for a precise simulation of high shear flows. More recently, a M_t-corrected form of the Launder, Reece and Rodi model for the pressure–strain correlation has been proposed by Marzougui, Khlifi and Lili [7]. Applications of this model on compressible homogeneous shear flow have shown a qualitative agreement with the DNS results of Sarkar [5] for cases A_1, A_2, and A_3 that correspond to moderate mean shear. Contrary, in case A_4, the predictions model [7] are in disagreement with the DNS results [5]. Pantano and Sarkar [8] use the gradient Mach number M_g in addition to the turbulent Mach number M_t to express the pressure strain correlation. An extended form of the incompressible Launder, Reece and Rodi model through the Pantano and Sarkar model [8] is proposed. The aim of this work is an a priori evaluation of the proposed model for different simulation cases of compressible homogeneous shear flow. Thus, the predicted results by the proposal model are compared with the DNS results and those obtained by the standard Launder, Reece and Rodi model [9] and its corrected form [7]. The results obtained for equilibrium states of major flow characteristics are also used for comparisons.

2. Governing equations

The general equations governing the motion of a compressible fluid are the Navier–Stokes equations. They can be written as follows for mass, momentum and energy conservation:

$$\frac{\partial}{\partial t}\rho + \frac{\partial}{\partial x_i}\rho u_i = 0 \tag{1}$$

$$\frac{\partial}{\partial t}\rho u_i + \frac{\partial}{\partial x_j}\rho u_i u_j = \frac{\partial}{\partial x_j}\sigma_{ij} \tag{2}$$

$$\frac{\partial}{\partial t}\rho e + \frac{\partial}{\partial x_j}\rho e u_j = \frac{\partial}{\partial x_j}\sigma_{ij}u_i - \frac{\partial}{\partial x_j}q_j \tag{3}$$

where $e = c_v T$, $\sigma_{ij} = -p\delta_{ij} + \tau_{ij}$, $\tau_{ij} = 2\mu S_{ij}$, $S_{ij} = (u_{i,j} + u_{j,i})/2$ and $q_i = -\lambda T_{,i}$.

For an ideal gas, the relation between pressure, density and temperature can be written as follows:

$$p = \rho RT \tag{4}$$

2.1. Averaged equations

In compressible flows, two averaging techniques can be used to split a physical quantity f, into an averaged and a fluctuating term. Such techniques are known as the ensemble and the mass-weighted averages, which are often referred to as the Reynolds and the Favre averages respectively. For the Reynolds average technique, f is divided into a mean part, $\bar{F}$, and a fluctuating part, f', as

$$f = \bar{F} + f', \quad \overline{f'} = 0$$

While in the Favre average, except density and pressure, the quantity f is written in the following form

$$f = \widetilde{F} + f''$$

where the Favre mean is defined as

$$\widetilde{f} = \overline{\rho f}/\bar{\rho}$$

and the Favre fluctuation f'' satisfies the following relationships:

$$\overline{\rho f''} = 0, \quad \overline{f''} = -\overline{\rho' f'}/\bar{\rho}$$

2.2. Basic equations of the Favre second-order closure in compressible homogeneous turbulent shear flow

For compressible homogeneous shear flow, the mean velocity gradient is given by:

$$\widetilde{U}_{i,j} = S\delta_{i1}\delta_{j2} \tag{5}$$

where S is the constant mean shear rate. These considerations lead to:

$$\widetilde{U}_{i,i} = 0 \tag{6}$$

$$\bar{\rho} = cte \tag{7}$$

At high Reynolds numbers, when assuming the hypothesis of isotropy dissipative structures of the turbulence, the Favre averaged Reynolds stress should be a solution of the transport equation:

$$\bar{\rho}\frac{\mathrm{d}}{\mathrm{d}t}\big(\widetilde{u''_i u''_j}\big) = -\big(\bar{\rho}\widetilde{u''_i u''_m}\widetilde{U}_{j,m} + \bar{\rho}\widetilde{u''_j u''_m}\widetilde{U}_{i,m}\big) + P^*_{ij} - \frac{2}{3}\bar{\rho}\epsilon\delta_{ij} + \frac{2}{3}\overline{p'u'_{i,i}}\delta_{ij} \tag{8}$$

where P^*_{ij} is the deviatoric part of pressure–strain correlation:

$$P^*_{ij} = 2\overline{p's'_{ij}} - \frac{2}{3}\overline{p'u'_{i,i}}\delta_{ij}, \quad s'_{ij} = \big(u'_{i,j} + u'_{j,i}\big)/2$$

The governing equation of turbulent kinetic energy, $k = \widetilde{u''_i u''_i}/2$, is:

$$\bar{\rho}\frac{\mathrm{d}k}{\mathrm{d}t} = -\bar{\rho}\widetilde{u''_i u''_j}\widetilde{U}_{i,j} + \overline{p'u'_{i,i}} - \bar{\rho}\epsilon \tag{9}$$

Classically, the second-order closure requires a transport equation of the turbulent dissipation rate. The new concept of dissipation in compressible turbulence was proposed by Sarkar et al. [3], Zeman [2] and Ristorcelli [10] and can be written as follows:

$$\epsilon = \epsilon_s + \epsilon_c \tag{10}$$

where for homogeneous turbulence $\bar{\rho}\epsilon_s = \overline{\mu\omega_i\omega'_i}$, ω'_i is the fluctuating vorticity, and $\bar{\rho}\epsilon_c = \frac{4}{3}\overline{\mu(u'_{i,i})^2}$ represent the solenoidal and compressible parts of ϵ respectively. Sarkar et al. [3] have mentioned that for moderate Mach numbers, ϵ_s is insensitive to the compressibility changes. This yields, for ϵ_s, a model transport equation, similar to what it was obtained in the incompressible case. Such a model equation is written as in [1], namely:

$$\bar{\rho}\frac{\mathrm{d}\epsilon_s}{\mathrm{d}t} = -C_{\epsilon 1}\bar{\rho}\frac{\epsilon_s}{k}\widetilde{u''_i u''_j}\widetilde{U}_{i,j} - C_{\epsilon 2}\bar{\rho}\frac{\epsilon_s^2}{k} \tag{11}$$

where $C_{\epsilon 1}$ and C_{ϵ_2} are respectively the model constants, $C_{\epsilon 1} = 1.35$ and $C_{\epsilon_2} = 1.85$.

We should remind that ϵ_c is generally taken to be proportional to ϵ_s through the following algebraic equation:

$$\epsilon_c = F(M_t)\epsilon_s \tag{12}$$

$F(M_t)$ is a function of the turbulent Mach number.

As it is suggested in model [3], one can write:

$$F(M_t) = \alpha M_t^2 \tag{13}$$

where α is constant model, $\alpha = 0.5$ in homogeneous turbulence.

Sarkar et al. [3] have also proposed a model for the pressure–dilatation correlation in term of the turbulent Mach number as follows

$$\overline{p'd'} = \alpha_1 M_t\bar{\rho}\left(R_{ij} - \frac{2}{3}K\delta_{ij}\right)\widetilde{U}_{i,j} + \alpha_2\bar{\rho}{M_t}^2\varepsilon_s + \alpha_3{M_t}^2\bar{\rho}k\widetilde{U}_{i,i} \tag{14}$$

The model constants, α_1, α_2 and α_3 take the values: $\alpha_1 = 0.15$, $\alpha_2 = 0.2$ and $\alpha_3 = 0$.

The turbulent Mach number is described in [1] by the transport equation as follows

$$\frac{\mathrm{d}M_t}{\mathrm{d}t} = -\frac{M_t}{2k}\widetilde{u''_i u''_j}\widetilde{U}_{i,j} + \frac{M_t}{2\bar{\rho}k}\left(1 + \frac{1}{2}\gamma(\gamma-1){M_t}^2\right)\big(\overline{p'd'} - \bar{\rho}\varepsilon\big) \tag{15}$$

3. The compressibility models to be tested

Many DNS and experiment results have been performed for compressible turbulent flows, most of which show that the compressibility has significant effects on the pressure–strain correlation via the pressure fields. Such effects induce reduction in the magnitude of the anisotropy of the Reynolds shear stress and increase in the magnitude of the normal stress anisotropy. Consequently, the pressure strain correlation requires a careful modeling in the Reynolds stress turbulence model. With respect to the incompressible case, many compressible models have been developed for the pressure–strain correlation. Hereafter, most of all these models are generated from a simple extension of its incompressible counter-part; they perform in simulating of important turbulent flows evolving with moderate compressibility. Adumitroaie et al. [11] modeled the pressure–strain correlation using the turbulent Mach number. Fujihiro et al. [12] related the correlation to the rate of change of the normalized pressure variance. Recently, Marzougui, Khlifi and Lili [7] used the concept of growth rate of turbulent kinetic energy to introduce correction on the Launder, Reece and Rodi model coefficients [9] which become a function of the turbulent Mach number. Using the wave equation of the pressure propagating, Pantano and Sarkar [8] pointed out that for compressible homogeneous turbulence evolving with high mean shear rate, compressibility effects are closely linked with the turbulent Mach number and the gradient Mach number. They used these two parameters to extent the incompressible models for the pressure–strain correlation to compressible homogeneous shear flow. The detailed models read as below:

- **Launder, Reece and Rodi (LRR) model [9]**

$$P_{ij}^{I*} = -C_1^I \bar{\rho}\epsilon_S b_{ij} + C_2^I \bar{\rho} k\left(\widetilde{S}_{ij} - \frac{1}{3}\widetilde{S}_{ll}\delta_{ij}\right) + C_3^I \bar{\rho} k\left(b_{ik}\widetilde{S}_{jk} + a_{jk}\widetilde{S}_{ik} - \frac{2}{3} b_{ml}\widetilde{S}_{ml}\delta_{ij}\right) + C_4^I \bar{\rho} k\left(b_{ik}\widetilde{\Omega}_{jk} + b_{jk}\widetilde{\Omega}_{ik}\right) \quad (16)$$

where $C_1^I = 3$, $C_2^I = 0.8$, $C_3^I = 1.75$, $C_4^I = 1.31$, $\widetilde{S}_{i,j} = 0.5(\widetilde{U}_{i,j} + \widetilde{U}_{j,i})$ and $\widetilde{\Omega}_{i,j} = 0.5(\widetilde{U}_{i,j} - \widetilde{U}_{j,i})$.

- **Marzougui, Khlifi and Lili (MKL) model [7]**

$$P_{ij}^{*} = -C_1 \bar{\rho}\epsilon_S b_{ij} + C_2 \bar{\rho} k(\widetilde{S}_{ij} - \frac{1}{3}\widetilde{S}_{ll}\delta_{ij}) + C_3 \bar{\rho} k\left(b_{ik}\widetilde{S}_{jk} + a_{jk}\widetilde{S}_{ik} - \frac{2}{3} b_{ml}\widetilde{S}_{ml}\delta_{ij}\right) + C_4 \bar{\rho} k\left(b_{ik}\widetilde{\Omega}_{jk} + b_{jk}\widetilde{\Omega}_{ik}\right) \quad (17)$$

where

$$C_1 = \frac{C_1^I}{(1+0.5M_t^2}\left(1 - 0.44M_t^2\right)$$
$$C_2 = C_2^I = 0.8$$
$$C_3 = C_3^I\left(1 - 1.5M_t^2\right)$$
$$C_4 = C_4^I(1 - 0.5M_t)$$

- **Pantano and Sarkar (PS) model [8]**

$$P_{ij}^{*} = \left(1 - f(M_t, M_g)\right)P_{ij}^{I*}$$
$$f(M_t, M_g) = \alpha_1 M_t^2 + \alpha_2 M_g^2 + \alpha_3 M_t M_g \quad (18)$$

In this study, we have chosen to extent the incompressible model of Launder, Reece and Rodi [9]. The calibration of the coefficients α_1, α_2 and α_3 by using the DNS results of Sarkar [5] for homogeneous shear flow gives for C_1, C_2 and C_3 the following expressions

$$C_1 = C_1^I\left(1 - 0.9M_t^2\right)$$
$$C_2 = C_2^I\left(1 + 0.4M_t^2\right)$$
$$C_3 = C_3^I\left(1 - 1.4M_t^2 - 0.012M_g^2\right)$$
$$C_4 = C_4^I\left(1 - 0.8M_t^2 - 0.005M_g^2\right)$$

4. Results and discussion

The transport equations (8), (9), (11) and (15) on which the second-order closure for compressible homogeneous shear flow is based, are solved using the fourth-order accurate Runge–Kutta numerical scheme.

The ability of the proposed form of the Pantano and Sarkar model [8] to predict the anisotropy of the compressible homogeneous turbulent shear flow will now be considered. The model predictions will be compared with DNS results conducted by Sarkar [5] for cases: A_1, A_2, A_3 and A_4 and by Simone et al. [6] for cases B_0 and B_1. These cases correspond to different initial conditions for which the initial values of the gradient Mach number M_{g0} change by changing the initial values of $(SK/\varepsilon)_0$ and taking M_{t0} constant as it is listed in Tables 1 and 2.

Table 1
Initial conditions for DNS [5] of homogeneous turbulent shear flow.

Case	M_{g0}	M_{t0}	$(Sk/\varepsilon)_0$	b_{11}	b_{22}	b_{12}
A_1	0.22	0.4	1.8	0	0	0
A_2	0.44	0.4	3.6	0	0	0
A_3	0.66	0.4	5.4	0	0	0
A_4	1.32	0.4	10.8	0	0	0

Table 2
Initial conditions for DNS [6] of homogeneous turbulent shear flow.

Case	M_{g0}	M_{t0}	$(Sk/\varepsilon)_0$	b_{11}	b_{22}	b_{12}
B_0	0.4	0.25	5.35	0	0	0
B_1	0.6	0.25	8	0	0	0

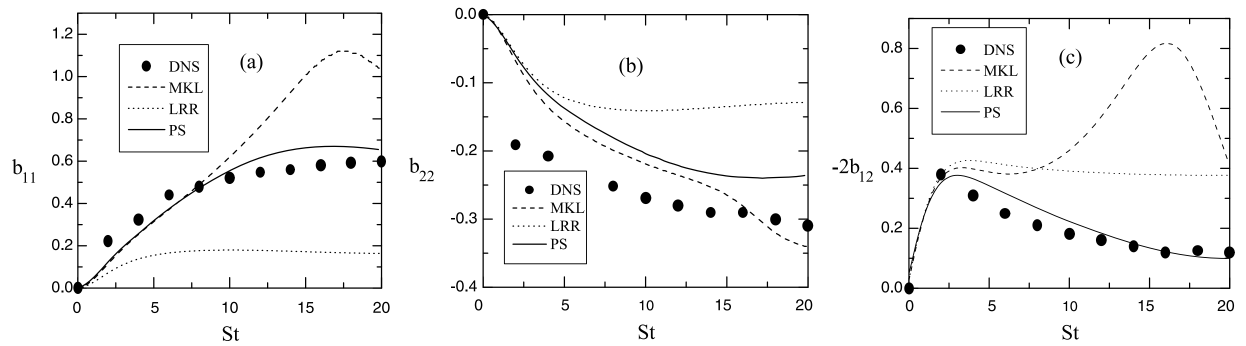

Fig. 1. Time evolution of the Reynolds-stress anisotropy: (a) b_{11}, (b) b_{22}, and (c) Pb_{12}. Case A_4.

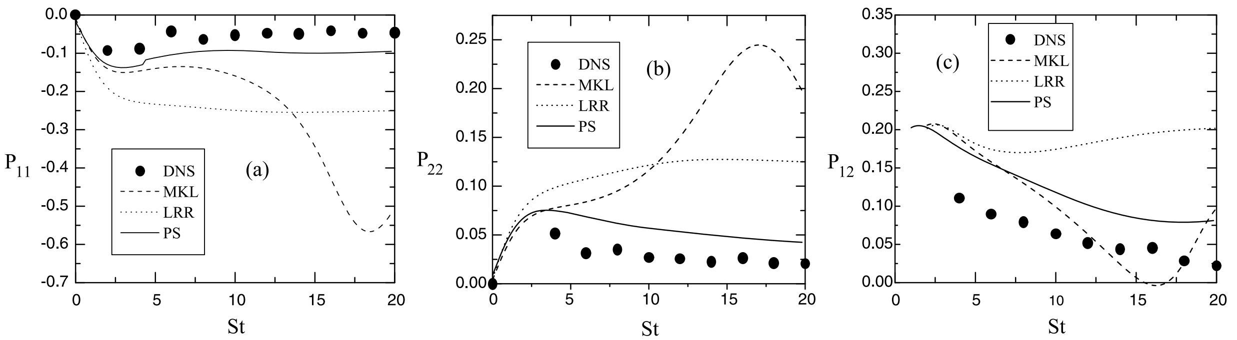

Fig. 2. Time evolution of the pressure–strain correlation: (a) P_{11}, (b) P_{22}, and (c) P_{12} in case A_4.

From Figs. 1, 2 and 3, it is clear that the incompressible Launder, Reece and Rodi (LRR) model [9] is still unable to predict the dramatic changes in the magnitude of the Reynolds-stress anisotropy that arise from compressibility. Because the compressibility correction model proposed by Marzougui, Khlifi and Lili contains the turbulent Mach number M_t only, the predicted values for case A_4 are in disagreement with the DNS results [5]. The proposed form of the Pantano and Sarkar model provides an acceptable performance in reproducing the DNS results for A_4. This model explains the importance of the involving of M_g with the commonly used parameter M_t in modeling the high compressible turbulent flows. Figs. 1(a–c) show that the Pantano and Sarkar model [8] appears to be able to predict correctly the significant decrease in the magnitude of the normalized production term $-2b_{12}$ and the increase in the magnitude of the diagonal components of Reynolds-stress tensor b_{11} and b_{22} in case A_4.

Figs. 2(a–c) show the time history of the components of the pressure–strain correlation P_{11}, P_{22} and P_{12} for case A_4. The Pantano and Sarkar model [8] yields reasonably acceptable results that are in good qualitative agreement with the DNS results [5].

Fig. 3 presents the behavior of (ϵ_S/Sk) for case A_4. It can be seen there is a decrease in ϵ_S/Sk ($\epsilon_S/Sk = -2b_{12}\frac{\epsilon_S}{P}$) when M_{g0} increases, since the compressibility effects cause significant reduction on the Reynolds turbulent shear stress b_{12} from numerical simulation case A_1 to A_4 of DNS results [5] (Table 1). It is clear that the proposed form of the Pantano and Sarkar model likes the DNS results.

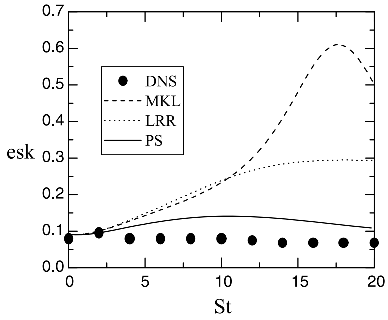

Fig. 3. Time evolution of $esk = \frac{\epsilon_s}{Sk}$ in case A_4.

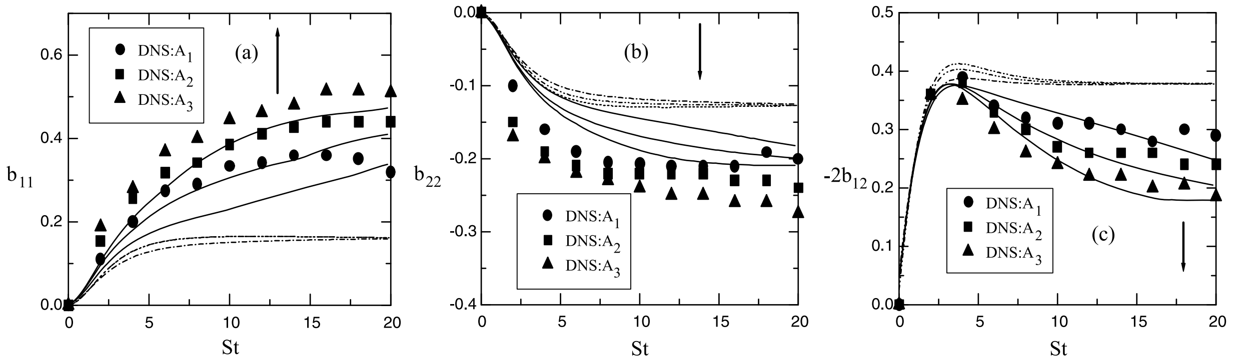

Fig. 4. Time evolution of the Reynolds-stress anisotropy: (a) b_{11}, (b) b_{22}, and (c) b_{12} in cases A_1, A_2 and A_3. PS model (line), LRR model (dash dot: A_1, dash dot dot: A_2, short dash: A_3), arrow shows the trend with increasing M_g.

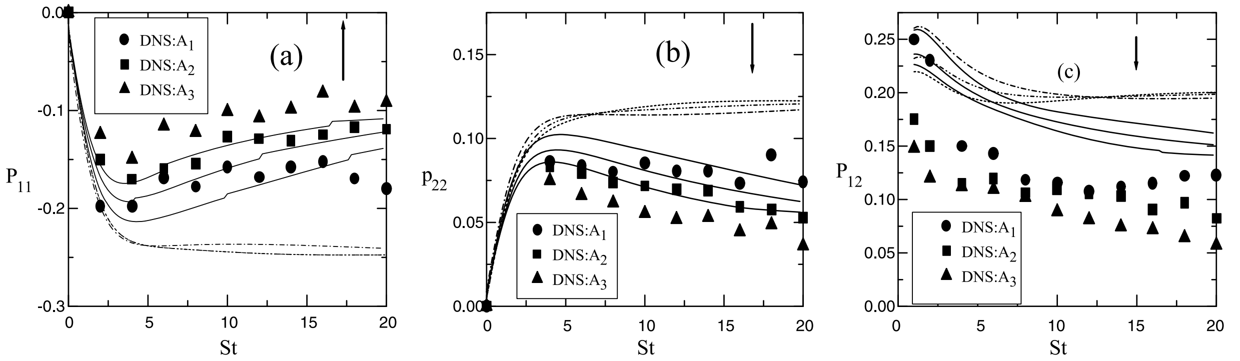

Fig. 5. Time evolution of the pressure–strain correlation: (a) P_{11}, (b) P_{22}, and (c) P_{12} in cases A_1, A_2 and A_3. PS model (line), LRR model (dash dot: A_1, dash dot dot: A_2, short dash: A_3), arrow shows the trend with increasing M_g.

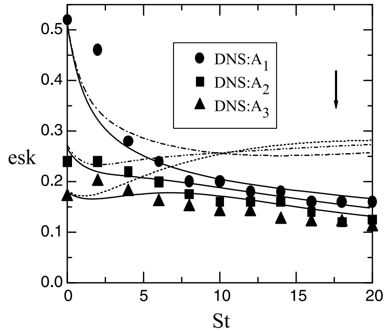

Fig. 6. Time evolution of $esk = \frac{\epsilon_s}{Sk}$ in cases A_1, A_2 and A_3. PS model (line), LRR model (dash dot: A_1, dash dot dot: A_2, short dash: A_3), arrow shows the trend with increasing M_g.

Table 3
Comparison of the present model predictions for the long-time values of the anisotropy tensor for cases A_1 to A_4 and with the DNS results of Sarkar [5] and the formula of Stefan [13].

Case	LRR model			Pantano and Sarkar model			DNS results of Sarkar			Formula of Stefan		
	b_{11}	b_{22}	b_{12}	b_{11}	b_{22}	b_{12}	b_{11}	b_{22}	b_{12}	b_{11}	b_{22}	b_{12}
A_1	0.159	−0.125	−0.1895	0.338	−0.169	−0.125	0.32	−0.2	−0.145	0.372	−0.207	−0.136
A_2	0.161	−0.127	−0.1895	0.425	−0.182	−0.1025	0.44	−0.24	−0.12	0.444	−0.238	−0.113
A_3	0.162	−0.127	−0.189	0.473	−0.19	−0.0895	0.051	−0.275	−0.092	0.48	−0.254	−0.099
A_4	0.164	−0.128	−0.1885	0.65	−0.236	−0.05	0.6	−0.31	−0.06	0.563	−0.289	−0.066

Table 4
Comparison of the present model predictions for the long-time values of ε_s/SK for cases A_1 to A_4 and with the DNS results of Sarkar [5] and the formula of Stefan [13].

Case	LRR model	Pantano and Sarkar model	DNS results of Sarkar	Formula of Stefan
A_1	0.257	0.167	0.15	0.16
A_2	0.273	0.148	0.115	0.133
A_3	0.282	0.132	0.10	0.116
A_4	0.294	0.1	0.075	0.078

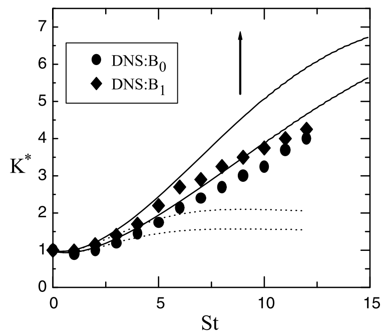

Fig. 7. Time evolution of the turbulent kinetic energy in cases B_0 and B_1. PS model (line), LRR model (dot), arrow shows the trend with increasing M_g.

The ability of the Pantano and Sarkar model [8] in the cases A_1, A_2 and A_3 that correspond to initial moderate mean shear rates and gradient Mach numbers M_{g0} is examined. From Figs. 4, 5, and 6 it can be seen that the Pantano and Sarkar model [8] shows the same tendency as the DNS results [5] in cases A_1, A_2 and A_3. From the DNS results of Sarkar [5], one can remark that for $St = 20$ the turbulence seems to evolve to equilibrium states. This can be seen more clearly in Tables 3 and 4 that show a systematic comparison between the models predictions for the long-time values of b_{ij} and (ϵ_S/Sk), the DNS results of Sarkar [5] and those given by the formula of Stefan [13], namely

$$b_{11} = \frac{2}{3} - 0.4e^{-(0.3M_g)}$$
$$b_{12} = -0.17e^{-(0.2M_g)}$$
$$b_{22} = -\frac{1}{3} + 0.17e^{-(0.3M_g)}$$
$$\varepsilon/SK = 0.2e^{-(0.2M_g)}$$

According to Tables 3 and 4, it seems that the proposed form of the Pantano and Sarkar model appears to be able to predict accurately the equilibrium values of b_{ij} and ε_s/SK for compressible shear flow. According to the DNS results of Simone et al. [6], the monotonic increase of the turbulent kinetic energy amplification rate with the initial M_{g0} at low St ($St < 5$), is the one of the important compressibility effects on the turbulent homogeneous shear flow. The principal reason of this amplification is the reduced level of b_{12} and then the production in the turbulent kinetic energy equation which seems to be due to the structural compressibility effects on the pressure–strain correlation via the pressure field. Fig. 7 compares the turbulent kinetic energy predictions with the DNS results [6] in cases: B_0 and B_1 (Table 2). It is clearly seen that the proposed form of the Pantano and Sarkar model [8] predictions is in acceptable agreement with the DNS results. From the previous results, one can conclude that the Reynolds-stress closure involving the parameter M_g appears to be suited to study the structural compressibility effects on compressible homogeneous turbulence at high speed shear flows.

5. Conclusion

In this study, the commonly second-order closures have been used for the prediction of homogeneous compressible turbulent shear flow. The standard Reynolds-stress turbulence closure with the addition of the pressure–dilatation and compressible dissipation models gives very poor predictions of the changes in the Reynolds-stress anisotropy magnitude. The deficiency of this closure is due to the use of the incompressible models of the pressure–strain correlation. Such models are based on the incompressible Poisson equation for the pressure. Thus, it does not appear to be useful for compressible flows. This same report was revealed by Spezial et al. [14,1], Vreman et al. [15] and Pantano and Sarkar [8]. The authors concluded that new representations incorporating some compressible physics are needed for the pressure–strain correlation. This is what has been confirmed by the DNS performed by Blaisdell et al. [4] and Sarkar [5].

A comparison of different turbulence models [7–9] of the pressure–strain correlation is made with the several DNS results of Sarkar [5], Simone et al. [6] in order to evaluate its ability in prediction of the time evolving fields and equilibrium states for different cases of compressible homogeneous shear flow, particularly in case A_4 that corresponds to high mean shear. It is found that none of the used incompressible models or then its corrected forms with the turbulent Mach number M_t only, can yield to acceptable results for high compressibility. With the help of the Pantano and Sarkar model for the pressure–strain correlation, an extended form of the Launder, Reece and Rodi model involving the gradient Mach number M_g with the commonly used M_t has been proposed. This model appears to be able to predict accurately the structural compressibility effects. The significant decrease in the magnitude of the Reynolds shear stress, the increase in magnitude of the diagonal components of the Reynolds stress anisotropies, the increase of the turbulent kinetic energy amplification rate with increasing initial values of the gradient Mach number and the asymptotic states of the flow are well predicted by the proposed model. Therefore, the gradient Mach number M_g is concluded to be an important parameter in addition to M_t in the modeling of the pressure–strain correlation for high compressible homogeneous turbulence.

References

[1] C.G. Spezial, R. Abid, N. Mansour, Evaluation of Reynolds-stress turbulence closures in compressible homogeneous shear flow, ZAMPS 17 (1985).
[2] O. Zeman, Dilatation dissipation, the concept and application in modeling compressible mixing layers, Phys. Fluids A 2 (1990) 178.
[3] S. Sarkar, G. Erlebacher, M.Y. Hussaini, H.O. Kreiss, The analysis and modeling of dilatational terms in compressible turbulence, J. Fluid Mech. 227 (1991) 473.
[4] G.A. Blaisdell, S. Sarkar, Investigation of the pressure strain correlation in compressible homogeneous shear flow, Transitional and turbulent compressible flow, ASME 151 (1993) 133–138.
[5] S. Sarkar, The stabilizing effects of compressibility in turbulent shear flow, J. Fluid Mech. 282 (1995) 163.
[6] A. Simone, G.N. Coleman, C. Cambon, The effect of compressibility on turbulent shear flow: a rapid distortion—theory and direct numerical simulation study, J. Fluid Mech. 330 (1997) 307–338.
[7] H. Marzougui, H. Khlifi, T. Lili, Extension of the Launder, Reece and Rodi on compressible homogeneous shear flow, Eur. Phys. J. B 45 (2005) 147–154.
[8] C. Pantano, S. Sarkar, A study of compressibility effects in the high-speed turbulent shear layer using direct simulation, J. Fluid Mech. 451 (2002) 329–371.
[9] B.E. Launder, G.J. Reece, W. Rodi, Progress in the development of a Reynolds-stress turbulence closure, J. Fluid Mech. 68 (1975) 537.
[10] J.R. Ristorcelli, A pseudo-sound constitutive relationship for the dilatational covariances in compressible turbulence: An analytical theory, J. Fluid Mech. 347 (1997) 37.
[11] V. Adumitroaie, J.R. Ristorcelli, D.B. Taulbee, Progress in Favre Reynolds stress closures for compressible flows, Phys. Fluids A 9 (1999) 2696.
[12] F. Hamba, Effects of pressure fluctuations on turbulence growth compressible homogeneous shear flow, Phys. Fluids A 6 (1999) 1625.
[13] H. Stefan, Statistical Mechanics of Turbulent Flows, Springer-Verlag, Berlin, 2003.
[14] C.G. Spezial, S. Sarkar, Second-order closure models for supersonic turbulent flows, ICASE Report 91-9, NASA Langley Research Center, Hampton, 1991.
[15] A.W. Vreman, N.D. Sandham, K.H. Luo, Compressible mixing layer growth rate and turbulence characteristics, J. Fluid Mech. 320 (1996) 325.