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Comment

Comment on: "Influence of inter-granular void ratio on monotonic and cyclic undrained shear response of sandy soils" by M. Belkhatir, A. Arab, H. Missoum, T. Schanz [C. R. Mecanique 338 (2010) 290–303]

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A R T I C L E I N F O

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A B S T R A C T

The authors should be commended for their interesting experimental work on sandy soils presented in Belkhatir et al. (2010) [1]. They used the inter-granular void ratio, e_s, to interpret the experimental results and developed some useful correlations with e_s. However, the Note fails to address the further development of e_s over a decade to a more generalized form of equivalent granular void ratio, e^*. This comment aims at adding missing literature on e^* and presents a re-interpretation of the experimental data based on e^*. The advantages of using e^* over e_s are significant and explained in subsequent sections.

1. Equivalent granular void ratio, *e**

The authors rightly mentioned that the inter-granular void ratio, e_s as in Eq. (1) of [1] was first presented by Mitchell [2] and then Thevanayagam and Mohan [3]. The under laying assumption of e_s is that fines particles may simply be occupying void in the sand particles and therefore, the measured behaviour is controlled by the force chain in sand particles. Thus, by considering fines particles as void one may achieve Eq. (1) for e_s. However, Zlatovic and Ishihara [4] reported that fines particles started to come in between sand particles contacts from 5% fines content and sand particles contacts vanished completely at 30% fines content. Many others reported the same observation, i.e. some fines particles are active in between sand particles contacts at higher fines content (Kuerbis et al. [5], Pitman et al. [6]) and they should be consider in 'equivalent' void ratio formulation. Then in 2002, Thevanayagam et al. [7] presented a more general form of 'equivalent' void ratio, called equivalent granular void ratio, e^* by introducing a parameter b which represents the fraction of fines that actively take part in the force structure of sand particles. Thus the e^* can be presented as following

$$e^* = \frac{e + (1 - b)Fc/100}{1 - (1 - b)Fc/100} \tag{1}$$

where e^* is equivalent granular void ratio and b is the fraction of fines particle that are active in between sand particles contact. The b ranges from 0 to 1. At low fines content $b = 0$ which degenerates Eq. (1) to Eq. (1) of Ref. [1] and $b \neq 0$ for higher fines contents. The main challenge of using e^* was to obtain a physically reasonable b for a dataset. Its value was often assumed/back analysed in the most of the literature (Ni et al. [8], Thevanayagam et al. [7], Yang et al. [9]). However, a number of literature reported correlation between soil grading properties and back analyzed b values (Kanagalingam and Thevanayagam [10], Ni et al. [8]). Recently, Rahman et al. [11,12], by re-analyzing the experimental data of McGeary [13] on

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Table 1
Grading properties of sand and fines used to convert e^* from e.

Types	D_{10}	d_{50}	χ	r	F_{thre}
Sand	0.158	–	2.77	0.361	33%
Fines	–	0.057			

binary packing studies, concluded that b is a function of both Fc and $\chi = D/d$, where D is the size of sand and d is the size of fines. The functional relationship, $b = f(Fc, \chi)$, can be presented by the following equation:

$$b = \left[1 - \exp\left(-0.3\frac{(Fc/F_{thre})}{k}\right)\right] \times \left(r\frac{Fc}{F_{thre}}\right)^r \tag{2}$$

where r = particle size ratio, d/D, $k = 1 - r^{0.25}$, F_{thre} is the threshold fines content. The fines content that defines the reversal in the movement of the location of steady state data points in e–$\log(p')$ space is called F_{thre} (Yang et al. [14]), where e is void ratio and p' is mean effective stress. It is noted that the concept of b is only applicable for $Fc < F_{thre}$. Since sand and fines are generally not single-size materials, D/d was generalized to D_{10}/d_{50} based on the argument in Ni et al. [8], where the subscripts denote fractile passing. The F_{thre} is an input parameter for Eq. (2) and it can be obtained from experimental procedure as outline in Yang et al. [14]. However, it can be approximated as an average value of 30% or can be estimated by an empirical equation (3) as presented in Rahman and Lo [15,16] where enough information on steady state behaviour are not available.

$$F_{thre} = 40\left(\frac{1}{1 + e^{\alpha - \beta\chi}} + \frac{1}{\chi}\right) \tag{3}$$

where $\alpha = 0.50$ and $\beta = 0.13$. Thus, Eqs. (3) and (2) can be used to determine b without the need of back analyzing a substantial triaxial datasets covering a range of Fc. Eq. (1) can be then used to calculate e^* from e. To the best of the author's knowledge, this is the only currently available approach that can be used to predict e^* from e. It will be referred to as 'prediction approach' here after. However, the performance of the prediction approach and back analysis based other correlations are discussed in elsewhere (Rahman et al. [17], Rahman and Lo [18]) and concluded that the prediction approach yields an overall better correlation for large datasets and almost as good as back analysis. Thus the prediction approach is used here to re-interpretation of the experimental data presented in the Belkhatir et al. [1].

2. Results and discussions

2.1. F_{thre} and soil parameters

The F_{thre} was defined by the movement of steady state (SS) data points in e–$\log(p')$ space (Yang et al. [14]). With increasing fines content SS strength decreases but when fines content becomes sufficiently large and controls mixed soil behaviour, the SS strength increases again. A similar trend for steady state and quasi-steady state (near phase transition) was observed for Toyoura sand with fines by Zlatovic and Ishihara [4]. The fines content at the reversal point is F_{thre}. Fig. 11 in [1] showed a reversal on shear strength at phase transition in between 30% to 40% fines contents. This indicates that the interpretation of these data up to 50% fines contents in terms of e_s or e^* is not reasonable as presented in [1]. They should be better interpreted with e^* provided $Fc < F_{thre}$. The F_{thre} for Chlef sand with fines is 33%, consistent with Fig. 11 in [1], as obtained from Eq. (3). The grading properties for sand and fines for converting e^* from e are presented in Table 1.

2.2. Undrained shear strength in terms of e^*

Fig. 13 in [1] shows three correlations based on relative densities for normalized shear strengths at phase transition versus e_s. However, the normalized shear strengths at phase transition come to a single correlation up to 30% fines content when they are plotted against e^* as shown in Fig. 1. This single correlation can be presented by the following equation:

$$S_{US}/\sigma'_c = 0.293 - 0.152e^* \tag{4}$$

where S_{US} is shear strength at phase transition and σ'_c is initial effective stresses. It is important to note that $e^* = e$ for clean sand ($Fc = 0\%$), i.e. Eq. (4) for clean sand and clean sand with up to 33% fines contents is the same. This means that the coefficients in Eq. (4) can be obtained from clean sand or clean sand with any single fines content and this equation can be used to predict S_{US} for sand with any other unknown fines contents, say 15% and 25% fines contents in this particular case. The details of these prediction technique can be found in Rahman and Lo [15,19].

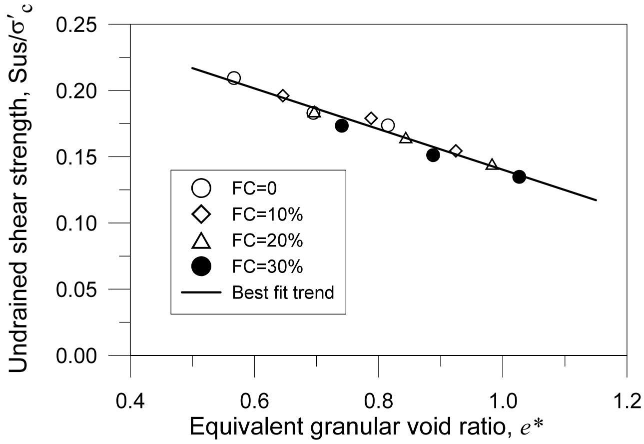

Fig. 1. Undrained shear strength at phase transition point versus equivalent granular void ratio, e^* ($\sigma_3' = 100$ kPa).

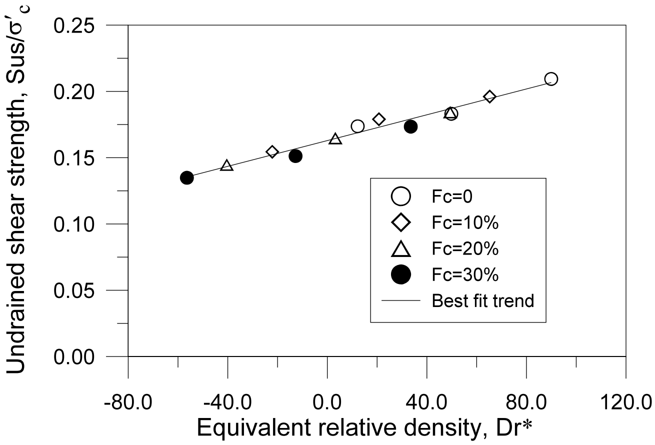

Fig. 2. Undrained shear strength at phase transition point versus equivalent relative density, Dr^* ($\sigma_3' = 100$ kPa).

2.3. Undrained shear strength in terms of equivalent relative density, Dr*

Relative density, Dr, is very commonly used comparison basis in soil mechanics. However, the concept of equivalent relative density can also be proposed for sandy soils. Along the line with e^*, equivalent relative density is defined as:

$$Dr^* = \frac{(e_{\max,cs} - e^*)}{(e_{\max,cs} - e_{\min,cs})} \tag{5}$$

where Dr^* is equivalent relative density, $e_{\max,cs}$ is maximum void ratio of clean sand and $e_{\min,cs}$ is minimum void ratio of clean sand. This relative index compares equivalent granular state of sandy soils with two extreme states of clean sand. Again, the normalized shear strengths at phase transition come to a single correlation up to 30% fines content when they are plotted against Dr^* as shown in Fig. 2. This correlation can be presented by Eq. (6) and can be used to predict S_{US} for sandy soils.

$$S_{US}/\sigma_c' = 0.163 + 0.000486\,Dr^* \tag{6}$$

2.4. Cyclic tests results

A large number of cyclic tests require to evaluate the effect of void ratio on cyclic stress ratio, CSR. The CSR at the same initial effective stress normalized by an equivalent number of cycles from an earthquake (say 15 cycles) varies with void ratio. Three data points for CSR normalized at 15 cycles, CSR_{15} at three e_s for 5% fines content can be obtained from Fig. 19a of [1]. Two other data points for 0% and 10% fines content can be obtained from their Fig. 18a. These data points have a significant influence of fines content on CSR_{15} when they are plotted in CSR_{15}–e space as shown in Fig. 3a. However, they

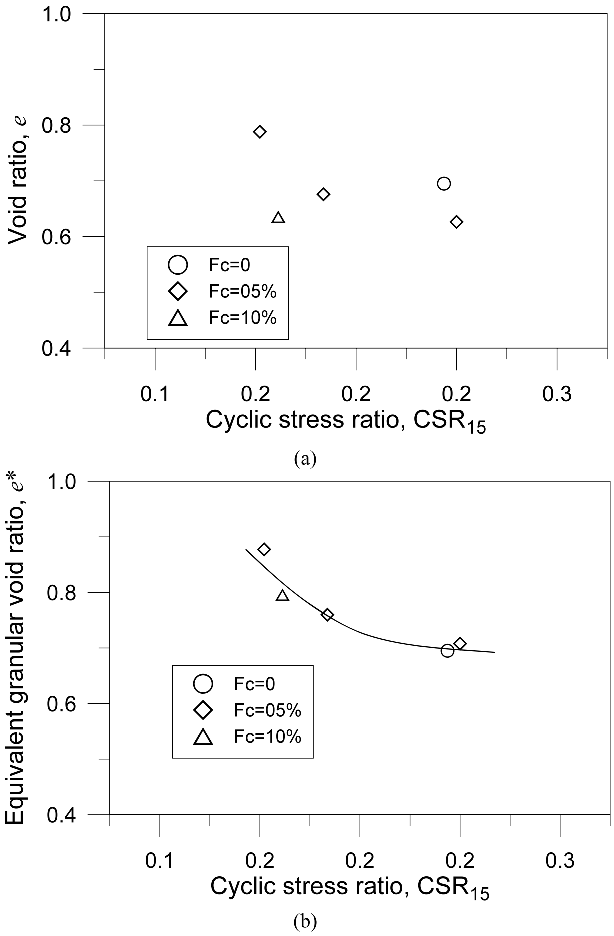

Fig. 3. Effect of fines contents on normalized cyclic stress ratio, CSR_{15} ($\sigma_3' = 100$ kPa): (a) CSR_{15}–e, (b) CSR_{15}–e^*.

come to a single trend when they are plotted in CSR_{15}–e^* space as shown in Fig. 3b. This is consistent with many other sands with fines as presented in Rahman et al. [11].

The concept of e^* is a generalization over e_s and more appropriate for higher fines contents. The re-interpretation of the data presented in Belkhatir et al. [1] supports this, i.e. e^* exhibits better correlation than e_s for Chlef sand with fines.

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