C. R. Mecanique 340 (2012) 536–542

Contents lists available at SciVerse ScienceDirect

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Analytical and innovative solutions for heat transfer problems involving phase change and interfaces

Analytical computation of transient heat transfer and macro-constriction resistance applied to thermal spraying processes

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ARTICLE INFO

Article history:
Available online 15 May 2012

Keywords:
Transient heat transfer
Thermal macro-constriction resistance

Mots-clés :
Transferts thermiques
Résistance thermique de macro-constriction

ABSTRACT

An analytical solution of the thermal macro-constriction resistance is derived by using the Hankel finite transform and the Duhamel theorem leading to a simple expression of the solution as a serial expansion with fast convergence. The application concerns the thermal macro-constriction resistance estimate during the spreading and coated formation involved in thermal spraying process.

In such a process, the heat flux reaches a few hundreds of W/mm^2 while the spreading duration is extremely short (about µs). The phenomena of thermal macro-constriction deposit/substrate interface play are of primary importance because they control the coating cooling and the thermo-mechanical behavior of the deposited layer, as well.

The effect of the spreading velocity on the thermal macro-constriction resistance has been studied. Results show the existence of a critical threshold of spreading velocity for which the transient problem has to be considered. On the other side (below the threshold) the study state regime should be sufficient for the macro-constriction estimate.

RÉSUMÉ

Nous présentons dans cette note une approche analytique destinée au calcul de la résistance thermique de macro-constriction basée sur la transformée de Hankel et le théorème de Duhamel aboutissant à un développement d'une solution sous forme d'une série à convergence rapide.

L'application physique concerne la projection thermique pour laquelle la formation de dépôts et les densités de flux mises en jeu lors du procédé culminent à quelques centaines de W/mm^2. La durée de l'étalement des gouttes est extrêmement faible, de l'ordre de la µs. Les phénomènes interfaciaux de micro-constriction thermique dépôt/substrat jouent un rôle extrêmement important puisqu'ils contrôlent le refroidissement du dépôt par le substrat et conditionnent de ce fait le comportement thermomécanique du dépôt. La résistance thermique de macro-constriction conditionne les niveaux de température à l'échelle de la goutte et permet de mettre en évidence le gradient thermique dans les directions radiale et axiale. C'est ce dernier phénomène que nous étudions dans cette note. On s'intéresse au développement de la macro-constriction en fonction de la vitesse d'étalement.

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1631-0721/$ – see front matter © 2012 Published by Elsevier Masson SAS on behalf of Académie des sciences.
doi:10.1016/j.crme.2012.03.007

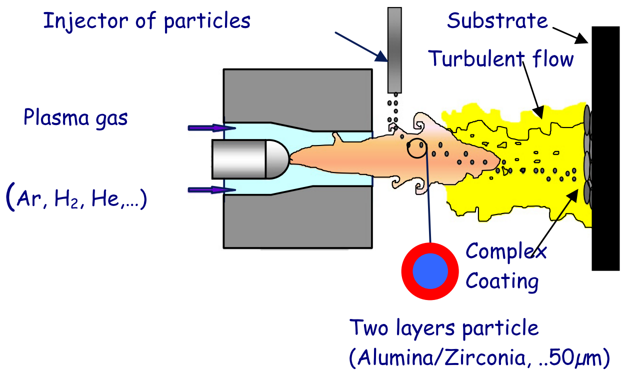

Fig. 1. Sketch of the thermal projection process.

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La réalisation de dépôts par projection thermique (Fig. 1) est largement utilisée dans l'industrie du traitement de surface. Le dispositif d'étude élémentaire de ce procédé étant une goutte liquide déposée sur un support solide (substrat). Ce dépôt passe par plusieurs phases (impact, étalement, solidification, ...) et met en jeu également la qualité du contact entre la goutte et le substrat [1–4]. Les échelles caractéristiques en temps et en espace diffèrent de plusieurs ordres de grandeurs et la compréhension demeure incomplète à présent. Le caractère évolutif de l'aire de contact et les phénomènes physiques qui s'opèrent durant le dépôt, tels que les variations de contraintes et le changement de phase sont autant de facteurs qui rendent difficile l'interprétation et la modélisation, fine, de ce processus. Des études thermiques portant sur la mesure de la résistance de contact durant l'étalement d'une goutte sur un substrat ont été développées pour des vitesses d'étalement modérées [5,6]. Le problème de la constriction thermique avec une aire évolutive est posé pour d'autres applications, notamment celles du procédé de mise en forme des matériaux [7,8]. Il s'agit en effet des procédés de forgeage à chaud, du laminage, du formage à froid, ...

Nous proposons dans cet article une approche analytique pour étudier l'évolution transitoire de la température et de la résistance thermique de macro-constriction durant l'étalement d'une goutte sur un substrat. La géométrie étudiée est celle d'un cylindre semi-infini (le substrat) recevant sur l'une de ses faces planes une source de chaleur circulaire, centrée par rapport à son axe, et dont le rayon est variable. La solution proposée est très simple d'utilisation. L'évolution de la résistance thermique de macro-constriction est étudiée en fonction de la vitesse de variation de l'aire de contact et des paramètres physiques du problème.

Cette solution est pratique et a permis de mettre en évidence le phénomène d'établissement de la macro-constriction thermique et de déduire un seuil critique de vitesse d'étalement dans le cas particulier de croissance linéaire du rayon de la goutte. On peut étudier à l'aide de ce modèle d'autres lois d'évolution du rayon de la goutte qui ne sont pas forcément monotones. Cela peut être le cas par exemple de décollement après étalement (le rayon de l'aire de contact goutte–substrat augmente puis diminue suite au décollement au niveau des bords). Le modèle peut être étendu sans difficultés à d'autres morphologies de gouttes (aire de type prismatique, ou en forme de doigts).

1. Introduction

The design of multilayered materials by thermal spraying process is largely used in the surface treatment industry (Fig. 1). The elementary phenomenon of this process being a liquid drop deposited on a solid support (substrate).

This coated matter passes by several phases (impact, spreading, solidification) [1–4] and the quality of the contact between the drop and the substrate has to be considered. The characteristic scales in time and space differ from several orders of magnitude and the understanding of involved phenomena still under progress and attracts numerous studies.

The time dependent character of the problem in its various stages, such as the variations of constraints and the phase change are as many factors which make difficult the complete interpretation and need sophisticated models. Thermal studies related to the measurement of the contact resistance were developed for moderate spreading velocities [5,6]. The problems of thermal constriction with an evolutionary surface are also encountered in other applications [7]. The model will be developed in such a way to be of mutual use.

The concept of thermal constriction plays an important part in many processes [8,9]. The physical parameter introduced to characterize this phenomenon is the thermal constriction resistance R_c. Many works were devoted to its analytical evaluation according to the geometry of the contact area, the boundary conditions nature and the heat flow regime stationary or transitory.

In Refs. [10,11] the authors studied the random distribution of the real contact area effect on the behavior of R_c. The authors show that the distribution of the contacts is disordered, for higher R_c. This phenomenon is accentuated when the contacts sizes are very different. In other works [12,13] the authors were interested in the geometry of the contacts

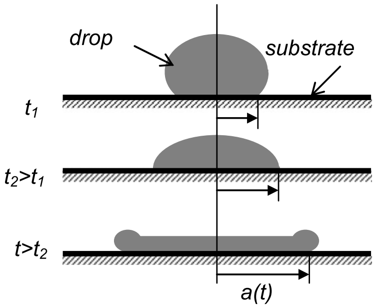

Fig. 2. Simplified problem of a radius growth drop spreading.

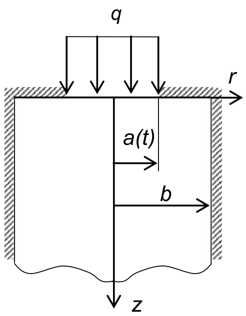

Fig. 3. Sketch of the model.

influence and their mobility. The authors give analytical expressions of R_c according to the geometrical parameters and the relative speed of the movement of solids.

Other work concerned the influence of a surface coating on the behavior of R_c in statics and dynamics [14]. Few works are devoted to the evolution of R_c in transient regimes. A recent study permits to establish an exact analytical expression for R_c corresponding to a single contact dissipating a uniform or non-uniform heat flux [15].

We propose in this article an analytical approach to study the time evolution of the temperature and the thermal macro-constriction resistance during the drop spreading on a substrate. The studied geometry is that of a semi-infinite cylinder (the substrate) receiving on one of its plane faces a heat source, circular centered compared to its axis, and whose radius is time dependent. The solution suggested is very simple of use. It avoids complex numerical resolution.

2. Model description

The problem is essentially characterized by rapid phenomena, namely the spreading during spraying (Fig. 2) the substrate is assumed semi-infinite medium. This choice is not a limitation for the proposed model. The considered medium receives a surface heat flux on a circular area of $a(t)$ radius varying in time (Fig. 3). The substrate is a cylinder $r = b$ with zero flux at its lateral surface (adiabatic).

The surface occupied by the sprayed is represented by a heat source q, uniform and constant (without inducing any limitation of the model). Thermophysical properties are supposed constant. Zero is the reference temperature. According to the notations in Fig. 3, the problem formulation writes:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{\partial^2 T}{\partial z^2}=\frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{1}$$

$$T(r,z,0)=0 \tag{2}$$

$$\frac{\partial T(0,z,t)}{\partial r}=0; \qquad \frac{\partial T(b,z,t)}{\partial r}=0 \tag{3}$$

$$-\lambda\frac{\partial T(r,0,t)}{\partial z}=\begin{cases} q & (r\leqslant a(t)) \\ 0 & (a(t)\leqslant r\leqslant b)\end{cases}; \qquad T(r,\infty,t)=0 \tag{4}$$

3. Analytical solution

The problem described by Eqs. (1)–(4) being linear, and can be solved analytical. Indeed, taking into account the cylindrical geometry and the boundary conditions (3), it is judicious to use the Hankel integral transform as:

$$\tilde{T} = \int_0^b rT\,J_0(\beta_n r)\,\mathrm{d}r \tag{5}$$

where β_n are the positives roots of the transcendental equation given by (6) as:

$$J_1(\beta_n b) = 0 \tag{6}$$

In Eqs. (5) and (6) J_0 and J_1 represent the Bessel functions of order 0 and 1 respectively.

The first value $\beta_0 = 0$ is one solution of (6). It corresponds to the average term. The other values of β_n can be determined by using the McMahon relation ([16], p. 371). This permits to avoid computing the zeros of Eq. (6):

$$\beta_n b = \sigma_n - \frac{p-1}{8\sigma_n} - \frac{4(p-1)(7p-31)}{3(8\sigma_n)^3} - \frac{32(p-1)(83p^2-982p+3779)}{15(8\sigma_n)^5} - \frac{64(p-1)(6949p^3-153\,855p^2+1\,585\,743p-6\,277\,237)}{105(8\sigma_n)^7} - \cdots \tag{7}$$

where $\sigma_n = (n + \nu/2 - 1/4)\pi$, $p = 4\nu^2$, $\nu = 1$.

The new system in the Hankel domain leads to 1D unsteady heat problem of unknown $\tilde{T}(z,t)$:

$$\frac{\partial^2 \tilde{T}}{\partial z^2} - \beta_n^2 \tilde{T} = \frac{1}{\alpha}\frac{\partial \tilde{T}}{\partial t} \tag{8}$$

$$\tilde{T}_{z,0} = 0 \tag{9}$$

$$-\lambda\left(\frac{\partial \tilde{T}}{\partial z}\right)_{0,t} = q\frac{a(t)J_1(\beta_n a(t))}{\beta_n}; \quad (\tilde{T})_{\infty,t} = 0 \tag{10}$$

A variable change as: $\tilde{T}(z,t) = \widetilde{W}(z,t)e^{-\alpha\beta_n^2 t}$ leads to writing Eqs. (8)–(10) as follows:

$$\frac{\partial^2 \widetilde{W}}{\partial z^2} = \frac{1}{\alpha}\frac{\partial \widetilde{W}}{\partial t} \tag{8a}$$

$$\widetilde{W}_{z,0} = 0 \tag{9a}$$

$$-\lambda\left(\frac{\partial \widetilde{W}}{\partial z}\right)_{0,t} = q\frac{a(t)J_1(\beta_n a(t))}{\beta_n}e^{\alpha\beta_n^2 t}; \qquad \widetilde{W}_{\infty,t} = 0 \tag{10a}$$

The problem described by (8a) to (10a), for the unknown $\widetilde{W}(z,t)$ corresponds to the 1D diffusion in a semi-infinite, medium initially at zero temperature and submitted to a time dependent heat flux at $z = 0$. It can be solved by using the Duhamel Theorem [17]. We obtain $\tilde{T}(z,t)$ as:

$$\tilde{T} = \frac{q\sqrt{\alpha}}{kb\beta_n\sqrt{\pi}}\int_0^t a(t-\tau)J_1\big[\beta_n a(t-\tau)\big]e^{-(\alpha\beta_n^2\tau + z^2/4\alpha\tau)}\frac{\mathrm{d}\tau}{\sqrt{\tau}} \tag{11}$$

The inverse finite Hankel transform gives:

$$T(r,z,t) = \frac{2}{b^2}\sum_{n=0}^{\infty}\tilde{T}(z,t)\frac{J_0(\beta_n r)}{J_0^2(\beta_n b)} \tag{12}$$

Leading to the final expression of $T(r,z,t)$:

$$T(r,z,t) = \frac{q\sqrt{\alpha}}{kb^2\sqrt{\pi}}\left\{\int_0^t \frac{a^2(t-\tau)e^{-z^2/4\alpha\tau}\mathrm{d}\tau}{\sqrt{\tau}} + \sum_{n=1}^{\infty}\frac{2J_0(\beta_n r)}{\beta_n J_0^2(\beta_n b)}\int_0^t a(t-\tau)J_1\big[\beta_n a(t-\tau)\big] \times e^{-\alpha\beta_n^2\tau - z^2/4\alpha\tau}\frac{\mathrm{d}\tau}{\sqrt{\tau}}\right\} \tag{13}$$

The first term of (13) is the average temperature for which $\beta_0 = 0$. It gives the medium temperature of the section z. Then, for a fixed radius, $r = a$, the expression (13) reduces to $T_{z=0,t} = 2q(a^2/b^2)\sqrt{\alpha t/\pi}/k$, which corresponds to the solution of the semi-infinite 1D medium submitted to a uniform heat flux, here: $q(a^2/b^2)$.

The thermal constriction resistance is defined by:

$$R_c = \frac{T_c - T_a}{q\pi a^2(t)} \tag{14}$$

In Eq. (14), T_c and T_a are the average temperatures of the real contact area and the apparent one respectively. They are determined by integrating Eq. (13) as:

$$T_c = \frac{1}{\pi a^2(t)} \int_{r=0}^{a(t)} T(r,0,t) 2\pi r \, \mathrm{d}r \tag{15}$$

$$T_a = \frac{1}{\pi b^2} \int_{r=0}^{b} T(r,0,t) 2\pi r \, \mathrm{d}r \tag{16}$$

(15) and (16) in (14) permit to explicit the constriction resistance R_c as:

$$R_c = \frac{4b^2\sqrt{\alpha}}{k\pi^{3/2}a^3(t)} \left\{ \sum_{n=1}^{\infty} \frac{J_1(\beta_n a(t))}{\beta_n^2 J_0^2(\beta_n b)} \int_0^t a(t-\tau) J_1\big[\beta_n a(t-\tau)\big] e^{-\alpha\beta_n^2\tau} \frac{\mathrm{d}\tau}{\sqrt{\tau}} \right\} \tag{17}$$

In dimensionless form we have: $\psi = R_c \lambda \sqrt{A_c}$, where $A_c = \pi a^2$ is the real contact area.

A solution of the particular case $a = Const.$, can be deduced from (17) in the form:

$$\psi(a = Cste, t) = \frac{4b}{\sqrt{\pi} a} \sum_{n=1}^{\infty} \frac{J_1^2(\beta_n a)}{(\beta_n b)^3 J_0^2(\beta_n b)} erf(\beta_n \sqrt{\alpha t}) \tag{18}$$

where erf is the error function defined by: $erf(u) = \frac{2}{\sqrt{\pi}} \int_0^u e^{-\xi^2} \, \mathrm{d}\xi$.

For $t \to \infty$, classical expression gives the permanent regime:

$$\psi(a = Cste, t \to \infty) = \frac{4b}{\sqrt{\pi} a} \sum_{n=1}^{\infty} \frac{J_1^2(\beta_n a)}{(\beta_n b)^3 J_0^2(\beta_n b)} \tag{19}$$

4. Thermal constriction resistance estimate

The expression of the thermal constriction resistance, given by (17) can be easily solved numerically. The solution is expressed as an infinite convergent series. The convergence is reached by a number of terms depending on the ratio $\varepsilon(t) = a(t)/b$ (smallest ratio needs a higher number of terms). Since spreading, the value of the parameter ε increases with time. To reduce the computing time, we adopted a number of terms of the series decreasing with time. It starts with 2000 terms (at beginning of spreading) and reaches 20 terms (at the end). We underline that the analytical solution is extremely fast converging, even when it is expressed by 2000 terms.

To implement this solution, we consider the case of a linear evolution of the drop radius according to time, that is to say:

$$a(t) = a_0 + Vt \tag{20}$$

a_0 is the initial radius, V is the spreading velocity. The choice of the linear evolution of $a(t)$ is arbitrary done, then the model can be adapted to another evolution law of the radius.

Two following dimensionless parameters are introduced: $\varepsilon = \sqrt{A_c/A_a} = a/b$ ($A_a = \pi b^2$ is the apparent contact area) and $V^* = Vb/\alpha$ (is a Peclet number).

Fig. 4 presents the evolution of the dimensionless thermal macro-constriction resistance ψ versus dimensionless time, $t^* = t\alpha/b^2$ for various V^*. These shapes show the decrease of ψ, with the time or the drop radius. This decrease is slow for small velocities. All the curves start of the same value $\psi(t = 0) = 0.4789$, which corresponds to the thermal resistance of established macro-constriction for a single spot such as $\varepsilon \to 0$. The final value $\psi = 0$ corresponds for each curve to $\varepsilon(t^*) = 1$, i.e. $a(t^*) = b$. The time $a(t^*)$ to reach b can be deduced from Eq. (20) as $t = (b - a_0)/V$. Knowing that $a_0 \ll b$, $\psi = 0$ for $t = b/V$, i.e. $t^* = 1/V^*$.

Fig. 5 shows ψ variation versus the geometric parameter ε (instantaneous drop radius) for different velocities. A value of ε corresponds to one drop radius which is reached quickly with V^*. The curves show that in the case of linear evolution ($V^* \approx 0.1$) corresponds to a critical value under which the macro-constriction is fully developed. This result is interesting

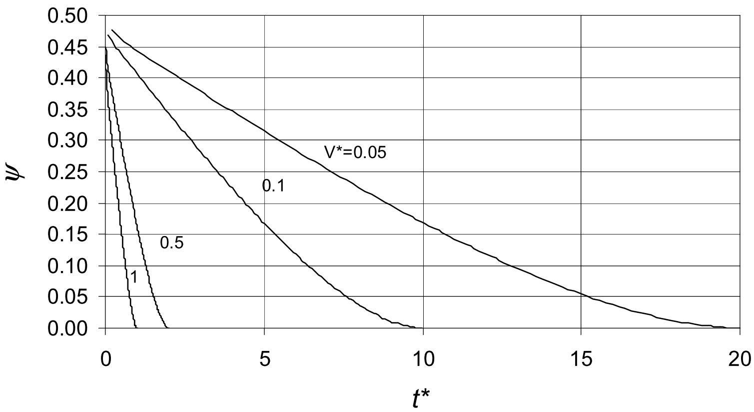

Fig. 4. Dimensionless thermal macro-constriction resistance vs. time for various spreading velocities.

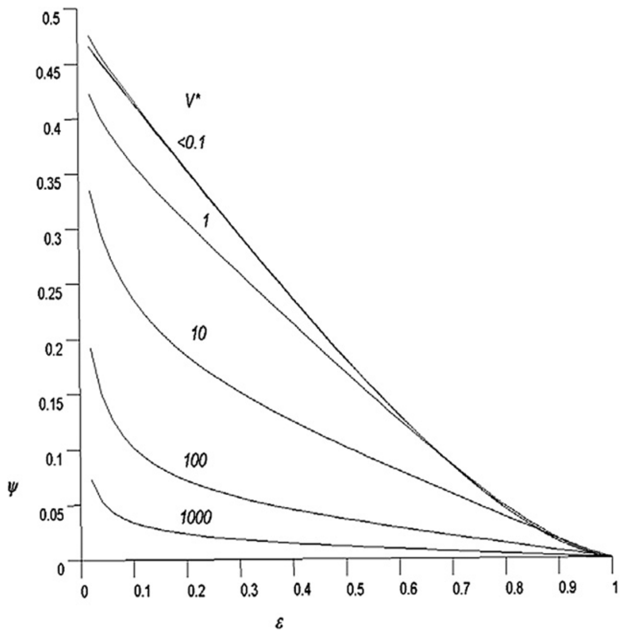

Fig. 5. Dimensionless thermal macro-constriction resistance vs. $\varepsilon = a(t)/b$ for various spreading velocities.

because it shows that for $V^* \leqslant 0.1$, transient calculation is not necessary, steady solution is enough giving ψ versus ε. A currently used correlation for steady regime writes:

$$\psi(\varepsilon) = \psi_0(1-\varepsilon)^{1.5} \tag{21}$$

with: $\psi_0 = \frac{8}{3\pi\sqrt{\pi}} \cong 0.4789$.

5. Conclusion

We presented an analytical solution to compute the temperature field and the thermal macro-constriction resistance for a drop in the course of spreading out on a substrate. This solution is simple for use. It enabled us to highlight the phenomenon of establishment of thermal macro-constriction and to deduce a threshold speed of spreading out in the particular case of linear growth of the drop radius. One can study by using this model other laws of evolution of the drop radius which are not inevitably monotonous. That can be the case for example of separation after spreading (drop breaking). The radius of the drop increases then decreases following separation on the level of the edges. The model can be used without difficulties with other morphologies of drops spreading (surface of the prismatic type, or in the shape of fingers).

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