



Analytical and innovative solutions for heat transfer problems involving phase change and interfaces

Foreword

Thermal science has always been a hot topic encompassing a broad list of problems such global warming, new energy sources, advanced materials performance, process optimization, etc. Development of modeling technologies, analytical solutions and numerical simulations and their interactions play special roles in this aspect. The well-known advection–diffusion equations are of parabolic type. For example, the Fourier law describes problems ranging from simple body heating [1] towards phase change [2] and moving boundary problems [3]. Exact and approximate analytical solutions of such problems are well developed and form a classic literature background for any scholar starting in modeling and simulation [1–3].

Commonly, analytical works focus on simplified or reduced models after scaling the governing equations. They attempt to establish the general relationships within the imposed constraints. In many cases they give a ground for refined numerical methods to attain accurate numerical solutions of complex problems. The numerical approaches based on analytical (and semi-analytical) developments made important advances in understanding physical phenomena and increasing engineering efficiency. Even though it is challenging to test the efficiencies of new numerical methods we have to take into account the deep physical background of the analytical approaches. In spite of the attraction of modern computational capabilities, we stress the attention on these classical fields. Analytical work is still strong for scholars and it leads to new approaches, efficient approximate solutions, as well, as it is exemplified in the papers of this collection.

This issue of the *Comptes Rendus Mecanique* follows two earlier special issues on the theme of ‘Microgravity and transfers’, published in 2004, and ‘Melting/solidification and interfaces’ in 2007. The first issue illustrated how the new techniques of reducing gravity, possible in experiments in planes in hyperbolic flight or in space vehicles, could advance the understanding of the movement of a liquid [4]. The theme of the second issue has been devoted to show how around different procedures, the use of experimentation and numerical simulation amply participate in academic progress and in actual techniques for the understanding and the control of the phase change by melting/solidification [5].

The present issue conveys strong, reliable, efficient, and promising developments, with articles on analytical problems on heat conduction with and without phase change. This collection, entirely devoted to heat transfer problems starts with an appraisal of the contribution of Joseph Stefan to the field of heat transfer [6], summarized by John Crepeau. This brief biography of the Austrian/Slovenian physicist is given, along with an outline of some of the most notable work he did at the University of Vienna including radiation heat transfer and solid–liquid phase change [7].

Phase change by internal heat generation has numerous applications, including geophysics and materials processing, but most work has been done in nuclear energy. The problem is developed in the work of Crepeau and Siahpush [8] where melting is driven by internal heat generation, in a cylindrical geometry. Simple transient models of reactor pins, successive approximation methods of flow in a reactor plug, numerical studies in a heat generating slab, and the enthalpy method to model the mushy zone have been studied. The comparison between the quasi-static analytical solutions for Stefan numbers less than one and numerical solutions show good agreement. Moreover, the scale analysis of the same problem shows four distinct regions of the melting process, each with an appropriate time scale associated with the process.

The Stefan problem is also analyzed and solved in an elegant manner by Sadoun et al. [9] where explicit numerical schemes obtained by using variable space grid (VSGM) and boundary immobilization (BIM) methods are considered for the solution of the transient heat conduction problem. Moreover, this article provides a brief review on the different approaches developed to track the phase change front with a particular interest to those tracking explicitly the moving boundary. The numerical results show that the use of the four-term backward finite difference approximation to evaluate the heat flux at the moving boundary refines the solution accuracy without affecting the central processing time.

The Stefan problem was efficiently solved by Feulvarch et al. [10] by an enriched finite element algorithm to simulate steady-state diffusion convection problems with isothermal phase changes. This technique is based on an enthalpic approach discretized by means of an enriched finite element approximation of the enthalpy in space. The phase changes interface is implicitly described without coupling with an interface-capturing technique. An example clearly shows the efficiency of the method developed. Such improvement can directly serve the use of this method on complex problems as the interaction between transitional flow and phase change [11].

Transient heat conduction without phase change is still a challenging problem for developing efficient approximate methods. Sahu and Behera [12] present an analysis of transient one-dimensional heat conduction in both Cartesian and cylindrical geometry by employing the polynomial approximation method (PAM). Four different models such as specified heat flux for both slab and tube and heat generation in both slab and tube has been analyzed. The transient temperature is found to depend on various model parameters, namely, Biot number, heat source parameter and time. It is demonstrated that PAM allows deriving a unified relation for the transient thermal behavior of solid (slab and tube) with both internal generation and boundary heat flux. The family of approximate solutions to transient 1-D heat conduction is enriched by new development of the classic heat-balance integral method of Goodman [13]. The first articles of Hristov [14] focus the attention on the correct calibration of a parabolic profile with unspecified exponent [15] by comparing three methods: calibration at the front surface, the Myers approach [16] and by the conceived similarity transformation. The latter avoids the ambiguities in application of the Myers method by neglecting the time-dependent terms in the minimization of the L_2 norm. These results clearly demonstrate that the optimal profile exponent should be time-dependent and cannot be accepted constant over the entire penetration depth, as it is constituted in the classical Goodman method [13]. The problem of the time-dependent exponent of the parabolic profile is consequently developed in the second article [17] where the concept of the self-adaptive exponent was conceived, analyzed and demonstrated by numerical examples.

An analytical solution of the thermal macro-constriction resistance is derived in [18], by using the Hankel finite transform and the Duhamel theorem, leading to a simple expression of the solution as a serial expansion with fast convergence. The application concern the thermal macro-constriction resistance estimate during the spreading and coated formation involved in complex thermal spraying process.

The entropy generation minimization has been a hot topic for years [19] and the work of Esfahani and Modirkhazeni [20] presents an elegant analytical solution of the classical Nusselt problem with forced convection. Numerical experiments were performed to demonstrate how the entropy generation number is affected by the Reynolds and Brinkman numbers and the geometrical characteristics of the pipe.

The numerical solutions are presented by two articles [21,22] on the Lattice-Boltzmann method (LBM). This method can be either regarded as an extension of the lattice gas automaton or as a special discrete form of the Boltzmann equation for kinetic theory. Unlike conventional numerical schemes based on discretization of partial differential equations describing macroscopic conservation laws, the LBM is based on solving the discrete-velocity Boltzmann equation from statistical physics. It describes the microscopic picture of particles movement in an extremely simplified way, while on the macroscopic level it gives a correct average description. In recent years, the lattice Boltzmann method has been developed into an alternative and promising numerical scheme for simulating fluid flows and solving various mathematical-physical problems. Esfahani and Vassel-Be-Hagh [21] shows the efficiency of LBM to the case of heat transfer in flow past a square unit of four isothermal cylinders as a good example of practical engineering application considering tubular heat exchangers.

The thermal interface impedance problem is developed. El Ganaoui et al. [22] solved transient heat conduction problems with cracks by application an extension of the Partial Bounce Back scheme (PBB) of the LBM. This scheme especially accounts the problem of the thermal contact resistance between the contacting surfaces. These examples demonstrate the contemporary interest in this old problem and the challenge in development of new analytical techniques.

Finally, this issue, which has international contributions (Algeria, Belgium, Bulgaria, France, India, Iran, Morocco, USA, ...), could provide a collection of solved problems serving as steps ahead to innovative schemes and elaborated solutions. We hope the articles will provide to the readers new ideas and may be a good source for further inspirations in the thermal science problems.

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