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A gradient method for viscoelastic behaviour identification of damped sandwich structures

Une méthode de gradient pour l'identification du comportement viscoélastique des structures sandwich

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ABSTRACT

The damping properties estimation assumes a viscoelastic model calibrated from experiments and simulations. This Note presents a gradient method for viscoelastic behaviour identification of damped sandwich structures devoted to the passive control of mechanical vibration. The method combines experimental data, numerical simulations realized with a complex non-linear eigenvalue solver using the asymptotic numerical method, and optimal control for the identification of viscoelastic parameters. An automatic differentiation tool is used to get numerical derivatives exact up to the machine precision with minimal user effort. Results are presented for a sandwich beam with a frequency dependent viscoelastic core.

RÉSUMÉ

L'estimation des propriétés amortissantes suppose un modèle viscoélastique calibré par des expériences et des simulations. Cette Note présente une méthode de gradient pour l'identification du comportement viscoélastique de structures sandwich dédiées au contrôle passif des vibrations mécaniques. La méthode combine des données expérimentales, des simulations numériques réalisées avec un solveur de problèmes aux valeurs propres non-linéaires complexes, et du contrôle optimal pour l'identification de paramètres viscoélastiques. Un outil de différentiation automatique est utilisé pour obtenir des dérivées numériques exactes à la précision machine près avec un minimimum d'effort pour l'utilisateur. Des résultats sont présentés pour une poutre sandwich à cœur viscoélastique dépendant de la fréquence.

1. Introduction

Vibration control study is essential in the design of complex mechanical structures. In many fields (aerospace, automotive industry, electrical devices, civil engineering, ...), vibrations generate instabilities in the structure inducing failure and some discomfort for users. Such vibrations may be controlled using some passive damping treatments on the existing

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structure, viscoelastic materials used as layers sandwiched between elastic faces for instance, with the goal of increasing energy dissipation. The damping property modelling of viscoelastic sandwich structures assumes a viscoelastic model [1] whose parameters may be identified through a combination of experiments and simulations. As demonstrated in [2], the description of the actual behaviour of a damping material for a wide frequency band may be tackled considering a frequency and temperature dependent complex Young's modulus. Parametric representations of this modulus range from simple rheological models to higher order models such as the generalized Maxwell model.

An appropriate account of the frequency and temperature dependent complex Young's modulus is a major challenge in viscoelastic sandwich structures. This Note discusses an inverse method combining experiments [1], numerical simulations [2] and optimal control for the identification of the viscoelastic parameters to be used in damping modelling of sandwich structures. Gradients are obtained at a low development effort thanks to the Automatic Differentiation (AD) tool Tapenade [3] applied to the complex non-linear eigenvalue solver code [2] and the objective function measuring the discrepancies between computed and measured structural damping properties. Results are reported for a viscoelastic sandwich beam.

2. Problem statement

Within the finite element discretization, the linear free vibration problem leads to a complex non-linear eigenvalue problem [2]. This may be solved combining the asymptotic numerical method (ANM) and a homotopic deformation [2] which allow for a direct and accurate computation of the modal damping and resonance frequency for a large number of vibration modes. However, for some temperatures (see Table 2), numerical results do not fully agree with experiments when the Young's modulus identified for a pure polymer is used in the vibration modelling of a sandwich structure. A detailed discussion is presented in this section.

2.1. Damping property calculations

The complex non-linear eigenvalue problem is written [2] as

$$R(u,\omega) = \left[K(\omega) - \omega^2 M\right]u = 0 \tag{1}$$

where u is the complex eigenmode and ω is the complex eigenvalue. The mass matrix M is real. Assuming that the core and face materials of the sandwich structure are linear, homogeneous and isotropic, the complex stiffness matrix $K(\omega)$ may be written [2] as

$$K(\omega) = K(0) + E(p,\omega)K_c$$

where $K(0)$ is related to the delayed elasticity, K_c is a constant matrix, and $E(p,\omega)$ is a parametric frequency dependent Young's modulus defined by the set of parameters p. The complex eigenvalue ω is related to the resonant frequency Ω and the modal loss factor η_m following

$$\omega^2 = \Omega^2(1 + i\eta_m)$$

We denote by $\mathcal{R}$ the function that links ω to (Ω, η_m) through the solution of (1).

The complex non-linear eigenvalue problem (1) is solved combining the ANM and a homotopic deformation [2] involving functions S and T such that

$$R(u,\omega) = S(u,\omega) + T(u,\omega), \quad \text{where } S(u,\omega) = \left[K_0 - \omega^2 M\right]u \text{ and } T(u,\omega) = E(p,\omega)K_c u$$

For modes of interest, the homotopy (2) then enables to deform continuously the real eigenvalue problem $S(u,\omega)$, corresponding

$$R(u,\omega) = S(u,\omega) + aT(u,\omega) = 0, \quad \text{for } a \in [0,1] \tag{2}$$

to $a = 0$, into the complex eigenvalue problem (1). Under analyticity assumptions, eigensolutions (u,ω) are approximated as truncated Taylor series $(\sum_{n=0}^{N} u(a), \sum_{n=0}^{N} \omega(a))$ that, introduced in (2), yield a sequence of N linear systems involving the same tangent linear matrix and higher order differentiation recurrence formulas. Nowadays, these series may be evaluated using automatic differentiation [4,5].

2.2. Results with a generalized Maxwell model identified from the pure polymer

Numerical tests are realized on a cantilever sandwich beam (elastic/viscoelastic/elastic) (Fig. 1). The viscoelastic material is assumed to be linear, homogeneous and isotropic. In the frequency domain, the stress–strain law of the central layer is

$$\sigma = 2\mu\varepsilon + \lambda I_3 \operatorname{tr}(\varepsilon), \qquad \mu = \frac{E(\omega)}{2(1+\nu_c)}, \qquad \lambda = \frac{\nu_c E(\omega)}{(1+\nu_c)(1-2\nu_c)}$$

Table 1
Material properties.

	Elastic layers	Viscoelastic core
Young's modulus	$E_l = 2.1 \times 10^{11}$ Pa	$E_0 = 2.7216 \times 10^7$ Pa
Poisson's ratio	$\nu_l = 0.3$	$\nu_c = 0.44$
Density	$\rho_l = 7800\ \mathrm{kg\,m^{-3}}$	$\rho_c = 1200\ \mathrm{kg\,m^{-3}}$
Thickness	$h_l = 0.6$ mm	$h_c = 0.045$ mm

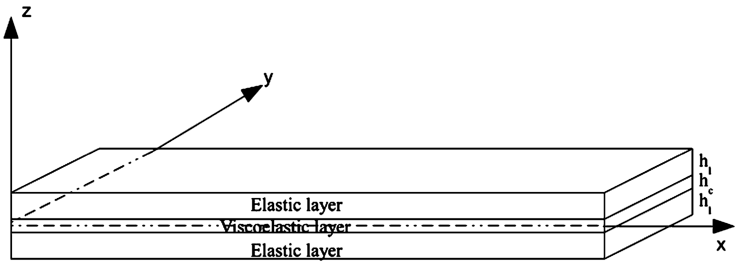

Fig. 1. Cantilever sandwich beam.

Table 2
Frequencies and loss factors. Data: (Ω^o, η_m^o), generalized Maxwell model: $(\Omega^{GM}, \eta_m^{GM})$ and relative errors, identified Young's modulus: (Ω^*, η_m^*) and relative errors.

	Data		Generalized Maxwell model				Identified Young's modulus			
	Ω^o	η_m^o	Ω^{GM}	η_m^{GM}	$\frac{\Omega^{GM}-\Omega^o}{\Omega^o}$	$\frac{\eta_m^{GM}-\eta_m^o}{\eta_m^o}$	Ω^*	η_m^*	$\frac{\Omega^*-\Omega^o}{\Omega^o}$	$\frac{\eta_m^*-\eta_m^o}{\eta_m^o}$
60 °C	34	2.75×10^{-2}	30.60	3.58×10^{-2}	1.00×10^{-1}	3.02×10^{-1}	30.34	2.75×10^{-2}	1.08×10^{-1}	7.93×10^{-5}
	147	5.03×10^{-2}	156.27	7.56×10^{-2}	6.31×10^{-2}	5.03×10^{-1}	153.81	4.98×10^{-2}	4.63×10^{-2}	9.25×10^{-3}
	357	6.68×10^{-2}	382.77	7.52×10^{-2}	7.22×10^{-2}	1.26×10^{-1}	382.75	6.68×10^{-2}	7.21×10^{-2}	4.72×10^{-5}
	636	6.77×10^{-2}					636.12	6.77×10^{-2}	1.86×10^{-4}	2.22×10^{-6}
70 °C	34	2.82×10^{-2}	30.25	2.92×10^{-2}	1.10×10^{-1}	3.55×10^{-2}	30.20	2.82×10^{-2}	1.12×10^{-1}	1.63×10^{-3}
	145	4.45×10^{-2}	152.49	8.60×10^{-2}	5.17×10^{-2}	9.33×10^{-1}	147.83	4.45×10^{-2}	1.95×10^{-2}	5.66×10^{-4}
	352	5.52×10^{-2}	376.11	7.88×10^{-2}	6.85×10^{-2}	4.26×10^{-1}	368.12	5.52×10^{-2}	4.58×10^{-2}	1.79×10^{-5}
	627	5.27×10^{-2}					634.47	5.26×10^{-2}	1.19×10^{-2}	1.05×10^{-3}

where I_3 is the identity matrix and Poisson's ratio ν_c is assumed to be constant. Material and geometrical characteristics of the beam are reported in Table 1, dimensions are $L \times l = 178$ mm $\times$ 10 mm. The complex Young's modulus is parameterized using the following generalized Maxwell model,

$$E_{GM}\big(\{\alpha_j, \beta_j\}_{j=0,\ldots,N_{\max}}, \omega\big) = \alpha_0 + i\beta_0\omega + \sum_{j=1}^{N_{\max}} \frac{i\omega}{\frac{i\omega}{\alpha_j} + \frac{1}{\beta_j}} \tag{3}$$

whose parameters were determined from experimental tests realized on the pure viscoelastic material [1] (provided by the steel company Usinor). This model involves a set $p = \{\alpha_j, \beta_j\}_{j=0,\ldots,N_{\max}}$ parameters, with $N_{\max} = 129$. The delayed elasticity is denoted by a_0. The structure is meshed using the sandwich finite element described in [6]. In this three node triangular element based on the discrete Kirchhoff theory, each node has eight degrees of freedom that are the longitudinal displacements of the elastic layers, the common deflection and three rotations. The chosen finite model, detailed in [6], has been validated by comparison to other models devoted to the finite modelling of viscoelastic sandwich structures. The vibration analysis of the sandwich beam is performed under clamped-free boundary conditions.

The truncature order of the ANM series is set to $N = 20$ and the parameter that controls the convergence of the ANM is equal to 10^{-6} as usual.

Table 2 compares measured and computed frequencies and loss factors for temperatures 60 °C and 70 °C. Relative errors are also provided. Each temperature row presents up to four modes. Data are denoted by (Ω^o, η_m^o). Values $(\Omega^{GM}, \eta_m^{GM})$ computed using (3) were reproduced from [2]. One observes that resonant frequencies are determined in a satisfactory manner (relative errors less than of 10%) because the structure stiffness is mainly due to elastic faces. On the contrary, misfits on the modal loss factor may be important. Such discrepancies may be attributed to either measurement errors on the data and/or the choice of the viscoelastic model. The generalized Maxwell model (3) identified from relaxation tests realized on a pure polymer at 20 °C and extended [1] to other temperatures by mean of the Williams–Landel–Ferry (WLF) law [7] is unadapted to the sandwich modelling we perform. The next section is devoted to the identification of a parametric complex Young's modulus. Related frequencies and loss factors are denoted by (Ω^*, η_m^*) in Table 2.

3. Optimization

The degree to which a particular Young's modulus formula is applicable to a given material strongly depends on the value of its parameters. Numerical methods proposed for the identification of viscoelastic parameters are usually based on gradient [8] or response surface [9] methods. We discuss a gradient method for which derivatives are obtained using AD techniques [10]. Such numerical derivatives are exact up to the machine precision with minimal user effort. Identification results are then presented discussing the last columns of Table 2.

3.1. Theoretical issues

Let $\mathcal{J}$ be an objective function that measures the discrepancies between simulated and observed structural damping properties, denoted by (Ω, η_m) and (Ω^o, η_m^o) respectively, taking into account the difference in the orders of magnitude,

$$\mathcal{J}(\Omega, \eta_m) = w_\Omega \frac{(\Omega - \Omega^o)^2}{(\Omega^o)^2} + w_\eta \frac{(\eta_m - \eta_m^o)^2}{(\eta_m^o)^2} \tag{4}$$

In Eq. (4), weights w_Ω and w_η are introduced to favour the minimization with respect to one data over the other. From a modelling point of view, the objective function $\mathcal{J}$ may be seen as a compound function

$$\mathcal{J}(\Omega, \eta_m) = \mathcal{J} \circ \mathcal{R} \circ E(p, \omega) \tag{5}$$

that links the viscoelastic parameters p of Young's modulus E to the discrepancies. Let $\mathcal{P}$ be a set of admissible viscoelastic parameters. Under continuity and differentiability assumptions, the minimization problem (6)

$$\text{Find } p^* \in \mathcal{P} \text{ such that } \min_{p \in \mathcal{P}} \mathcal{J} \circ \mathcal{R} \circ E(p, \omega) = \mathcal{J} \circ \mathcal{R} \circ E\big(p^*, \omega\big) \tag{6}$$

may be solved through a gradient method.

There exist several computational options for sensitivity analyses [11] and gradient computations [10,12]. On the one hand, gradients may be obtained differentiating the continuous (or discrete) equations of the model and evaluating the resulting equations in particular directions of perturbation δp. For instance the differentiation of (5) yields

$$\big[\nabla \mathcal{J}(\Omega, \eta_m)\big].\big[\nabla \mathcal{R}\big(E(p, \omega)\big)\big].\big[E(p, \omega)\big].\delta_p = \delta_J$$

which is evaluated at point p in the direction δp. On the other hand, AD techniques [10] may be used on the code implementing $\mathcal{J} \circ \mathcal{R} \circ E$. In a nutshell, AD views any computer code as a sequence of elementary operations and intrinsics functions, control statements and do-loops provided by the programming language. The differentiation is performed applying the chain rule to this sequence, statement by statement, operation by operation. This generic approach constitutes a reliable technique when dealing with large codes and/or large gradients [13], or with higher order differentiation [4,12]. The reader is referred to [14] and the references therein for an exhaustive discussion about AD and non-linear mechanics. In this Note, we do not discuss finite difference schemes because approximate gradients they produce may be of insufficient accuracy for optimization purposes.

Our Fortran code implements $\mathcal{J} \circ \mathcal{R} \circ E(p, \omega)$ following the ANM method presented in [2]. It contains about 5200 lines. Given this code, the tangent linear mode of differentiation allows to get a linear code that enables the computation of the gradient components. Among the available softwares, we choose Tapenade (freely available for research purposes) since it may be applied to general Fortran codes. The user provides the source code, the name of the top routine to be differentiated, here $\mathcal{J}$, a set of "independent" input variables (the viscoelastic parameters p) and the mode of differentiation. The resulting source code, generated in 16 seconds on an Intel Core2 Duo 2 GHz processor with 2 GB memory, evaluates both (4) and its partial derivative with respect to a prescribed direction δp. AD generally provides efficient tangent linear codes.

3.2. Numerical results

The viscoelastic model is written as

$$E(\omega) = E_0 + \big(E_R(\omega), E_I(\omega)\big) = \mathrm{Re}\big(E(\omega)\big)\big(1 + i\eta(\omega)\big) \tag{7}$$

where $E_0 \in \mathbb{R}$ is the delayed elasticity modulus, $E_R(\omega) = \mathrm{Re}(E(\omega))$ and $E_I(\omega) = \mathrm{Im}(E(\omega))$ are the real part and the imaginary part of the complex non-linear Young's modulus of the viscoelastic structure to be identified, respectively. The variable $\eta(\omega)$ is the material loss factor. For a given frequency ω, Eq. (7) may be turned into a constant complex modulus formulation containing two parameters. The independent variables are thus E_R and E_I at the AD stage. Weights (w_Ω, w_η) are set to $(10^{-6}, 1)$ in order to favour a better fit of the modal loss factor. The optimization process, including the L-BFGS-b routines [15], identifies optimal parameters p^* of Young's modulus (7), what enables to deduce related structural damping properties $(\Omega^*, \eta_m^*) = \mathcal{R} \circ E(p^*, \omega)$.

Results are reported in Table 2. As expected, error on the damping η_m is less than $\leqslant 10^{-3}$ when using the identified Young's modulus. Resonant frequencies (less than 10^{-2} for modes 2, 3 and 4) are well reproduced because the stiffness of the structure is mainly supported by elastic faces. Errors on the first mode are higher (about 10^{-1}) because the experimental device was not enough accurate in the measurement of damping properties for low frequencies [1]. Manufacturing error sources may be also invoked in [1] since the thickness and Young's modulus of the steel, and the complex modulus of the polymer are known with a precision of $\pm 10\%$. These identification results prove the efficiency of our optimization process: identified viscoelastic parameters enable an accurate computation of the modal loss factor.

4. Conclusion

This Note proposes an optimal control approach for the identification of viscoelastic parameters. The direct and accurate computation of the modal damping for a large number of vibration modes of the sandwich structure is realized by means of the complex non-linear eigenvalue solver described in [2]. The identification algorithm is provided with AD-based gradient computations. This optimization process is tested on a sandwich structure (steel/polymer/steel) and data acquired by the Usinor company (Arcelor group). The viscoelastic parameter identification is successful and enables to estimate the modal loss factor in an accurate manner. Future works are concerned with a data measurement campaign to provide reliable data, acquired on different materials and structures, to be used in this optimization process for instance. Parameters such as optimal thickness and shear modulus of the viscoelastic layer [16] may be identified in a similar manner considering another objective function and differentiating the resulting computer code with respect to the parameters of interest.

References

[1] J. Landier, Modélisation et étude experimentale des propriétés amortissantes des tôles sandwich, Ph.D. Thesis, University of Metz, 1993.
[2] E. Daya, M. Potier-Ferry, A numerical method for non-linear eigenvalue problems application to vibrations of viscoelastic structures, Computers & Structures 79 (5) (2001) 533–541.
[3] L. Hascoët, V. Pascual, Tapenade 2.1 user's guide, Technical Report 0300, INRIA, 2004, http://www.inria.fr/rrrt/rt-0300.html.
[4] I. Charpentier, M. Potier-Ferry, Différentiation automatique de la méthode asymptotique numérique typée: l'approche Diamant, Comptes Rendus Mecanique 336 (2008) 336–340.
[5] M. Bilasse, I. Charpentier, E. Daya, Y. Koutsawa, A generic approach for the solution of nonlinear residual equations. Part ii: Homotopy and complex nonlinear eigenvalue method, Computer Methods in Applied Mechanics and Engineering 198 (2009) 3999–4004, http://dx.doi.org/10.1016/j.cma.2009.09.015.
[6] E. Daya, M. Potier-Ferry, A shell finite element for viscoelastically damped sandwich structures, Revue Européenne des Eléments Finis 11 (2002) 39–56.
[7] M. Williams, R. Landel, J. Ferry, The temperature dependance of relaxation mechanisms in amorphous polymers and other glass forming liquids, Journal of the American Chemical Society 77 (1955) 3701–3707.
[8] A. Araújo, C.M. Soares, C.M. Soares, J. Herskovits, Optimal design and parameter estimation of frequency dependent viscoelastic laminated sandwich composite plates, Composite Structures 92 (2010) 2321–2327.
[9] E. Barkanov, E. Skukis, B. Petitjean, Characterization of viscoelastic layers in sandwich panels via an inverse method, Journal of Sound and Vibration 327 (2009) 402–412.
[10] A. Griewank, Evaluating Derivatives. Principles and Techniques of Algorithmic Differentiation, Frontiers in Applied Mathematics, vol. 19, SIAM, Philadelphia, 2000.
[11] F. van Keulen, R. Hafka, N. Kim, Review of options for structural design sensitivity analysis. Part 1: Linear systems, Computer Methods in Applied Mechanics and Engineering 194 (2005) 3213–3243.
[12] I. Charpentier, Sensitivity of solutions computed through the asymptotic numerical method, Comptes Rendus Mecanique 336 (2008) 788–793.
[13] I. Charpentier, Checkpointing schemes for adjoint codes: Application to the meteorological model Meso-NH, SIAM Journal on Scientific Computing 22 (2001) 2135–2151.
[14] I. Charpentier, On higher-order differentiation in non-linear mechanics, Optimization Methods & Software 27 (2012) 221–232.
[15] C. Zhu, R. Byrd, J. Nocedal, L-bfgs-b: Algorithm 778: L-bfgs-b, Fortran routines for large scale bound constrained optimization, ACM Transactions on Mathematical Software 23 (1997) 550–560.
[16] A. Baz, J. Ro, Optimum design and control of active constrained layer damping, Journal of Mechanical Engineering Design 117 (1995) 135–144.