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Convection of a binary mixture under high-frequency vibrations

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ABSTRACT

The spatiotemporal evolution of steady and oscillatory convective roll patterns in a plane horizontal layer subject to steady gravity and high-frequency vibrations is investigated numerically, considering a laterally periodic convective cell with rigid, impermeable horizontal boundaries. Simulations are performed for parameters adapted to laboratory experiments with an ethanol/water mixture with negative Soret coupling between temperature and concentration fields. The characteristics of the patterns are investigated in relation to heat intensity. Bifurcation diagrams are presented in the "Rayleigh number–Gershuni number" plane.

1. introduction

Thermal vibrational convection is a set of phenomena associated with the appearance of regular flows in an inhomogeneous fluid under the action of vibrations. An instability mechanism for such motion manifests itself even in microgravity [1–3]. The wide variety of vibrational phenomena in convection is connected with the influence of vibrational effects on mass and heat transfer in the Earth's gravitational field [2]. Both stabilization and destabilization of the convective flows is possible, depending on the characteristics of vibration. In general, the effect of vibration on liquid flows is characterized by amplitude, frequency, and the direction of the axis of vibration. The theoretical analysis and experimental investigations of the thermal vibrational convection are considered in detail [2]. At high-frequency forcing, the period of vibrations is smaller than the system characteristic time, and therefore it is reasonable to use the procedure of averaging for derivation of the governing equations of convection [4]. The amplitude and frequency of vibrations are combined into a non-dimensional parameter referred to as the vibrational Rayleigh number, which is also called the Gershuni number [3]. The convective instability of a horizontal binary-mixture layer under transversal vibrations was first studied in [5], but without consideration of the effect of thermal diffusion. The interrelation of convective flow and thermal diffusion in binary mixtures should be controllable in different technological processes such as, for example, the separation of isotopes or fractions in petrochemical industry, and others. The theory of convection in binary mixtures employs the Boussinesq approximation with allowance made for dissipative processes of diffusion and thermal diffusion [6]. The stability of a horizontal, binary-mixture layer, which is bounded by rigid impermeable plates, possesses thermal diffusion and is subjected to longitudinal vibrations, has been considered in [7,8].

The present paper focuses on the vibrational convection of an incompressible binary mixture with thermal diffusion, which fills a horizontal layer subjected to high-frequency harmonic vibrations, whose axis is angled to layer boundaries.

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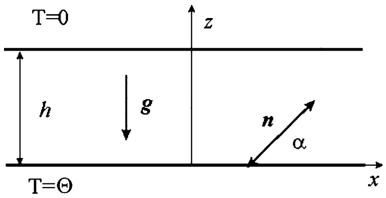

Fig. 1. The geometry of the problem.

2. Formulation of the problem

Consider an infinite plane horizontal layer bounded by two parallel rigid plates $z = 0; h$ (Fig. 1). The x-axis of the Cartesian coordinate system is directed along the layer. The plates are maintained at different constant temperatures: $T(z = 0) = \Theta$, $T(z = h) = 0$. The layer is filled with a binary mixture of non-reacting constituents, and its boundaries are impermeable to these mixture components. There is no external difference in the concentration of a mixture, but due to the effect of thermal diffusion, a concentration gradient appears in an originally homogeneous mixture.

The equation of state of the mixture can be written in the form:

$$\rho = \bar{\rho}(1 - \beta_T T - \beta_C C)$$

where $\bar{\rho}$ is the density of the mixture at the certain mean values of temperature and concentration, T and C are the deviations of temperature and concentration from their mean values, and β_T and β_C are the thermal solutal expansion coefficients. Assuming that C is the concentration of the light component, we obtain $\beta_C > 0$.

The layer is located in the static gravity field $\boldsymbol{g} = -g\boldsymbol{\gamma}$ ($\boldsymbol{\gamma}$ is an upward unit vector), and subjected to harmonic vibrations with the axis directed at an angle α to the layer. Ω is the angular frequency, and b is the amplitude of vibrations. We consider the limit of high-frequency (but below acoustic) vibrations with the period T_v, which is much less than the hydrodynamic, thermal and concentration characteristic times of the system:

$$T_v \ll \min\left[\frac{h^2}{\nu}, \frac{h^2}{\chi}, \frac{h^2}{D}\right]$$

Here, ν and χ designate the coefficients of kinematic viscosity and thermal diffusivity of the fluid, respectively, and D is the diffusion coefficient.

Equations for the mean parts of velocity $\boldsymbol{v}$, temperature, concentration and additional variable $\boldsymbol{w}$, which changes slowly with time, are obtained by the standard averaging procedure [4] applied to the equations of convection in the Boussinesq approximation.

The following scales are used: h for distance, h^2/χ for time, χ/h for velocity, Θ for temperature, $\beta_T\Theta/\beta_C$ for concentration, and $\bar{\rho}\nu\chi/h^2$ for pressure.

After normalization, the averaged system of equations can be written as:

$$\begin{aligned}
&\frac{1}{Pr}\frac{\partial \boldsymbol{v}}{\partial t} + \frac{1}{Pr}(\boldsymbol{v}\nabla)\boldsymbol{v} = -\nabla p + \Delta\boldsymbol{v} + Ra(T + C)\boldsymbol{\gamma} + Gs(\boldsymbol{w}\nabla)\big[(T + C)\boldsymbol{n} - \boldsymbol{w}\big] \\
&\frac{\partial T}{\partial t} + (\boldsymbol{v}\nabla)T = \Delta T, \qquad \operatorname{div}\boldsymbol{v} = 0, \qquad \operatorname{div}\boldsymbol{w} = 0, \qquad \operatorname{rot}\boldsymbol{w} = \nabla(T + C)\times\boldsymbol{n} = 0 \\
&\frac{\partial C}{\partial t} + (\boldsymbol{v}\nabla)C = Le\Delta(C - \psi T)
\end{aligned} \tag{1}$$

where p is the pressure, $Ra = g\beta_T\Theta h^3/\nu\chi$ is the Rayleigh number, $Gs = (b\Omega\beta_T\Theta h)^2/2\nu\chi$ is the Gershuni number, $Pr = \nu/\chi$ is the Prandtl number, $Le = D/\chi$ is the Lewis number, and $\psi = -\alpha\beta_C/\beta_T$ is the separation ratio.

The equation of motion of the mixture in (1) includes the additional vibrational force that is dependent on the slow variable $\boldsymbol{w}$ and on temperature and concentration inhomogeneities. It is properly accounted by the diffusion equation that temperature inhomogeneities imply an increase in the inhomogeneity of concentration, and the thermal diffusion effect, the intensity of which is characterized by the Soret parameter ψ, is responsible for the process. The sign of this parameter points to the direction of the flux of matter under thermal diffusion. We consider the case $\psi < 0$ (so-called anomalous thermal diffusion, or negative Soret coupling), when the light component migrates in the direction opposite to the temperature gradient.

The problem specified by the system of equations (1) has a quasi-equilibrium solution describing the linear fields of $\boldsymbol{w}$, mean temperature and concentration:

$$T_0 = 1 - z, \qquad \frac{\mathrm{d}C_0}{\mathrm{d}z} = -\psi, \qquad \boldsymbol{w}_0 = (w_{0x}, 0, 0), \qquad w_{0x} = -(1 + \psi)(z - 1/2)\cos\alpha \tag{2}$$

When $\alpha = \pi/2$ the system may stay in the rest state and $\boldsymbol{w}_0 = 0$ [8].

To solve numerically the problem of thermovibrational convection in the form of y-axial rolls, two stream functions Ψ and F and the vorticity function φ are introduced, which are related to velocity as follows:

$$v_x = \frac{\partial \Psi}{\partial z}, \qquad v_z = -\frac{\partial \Psi}{\partial x}, \qquad w_x = \frac{\partial F}{\partial z}, \qquad w_z = -\frac{\partial F}{\partial x} \tag{3}$$

The system of governing equations takes the form:

$$\begin{aligned}
&\frac{1}{Pr}\frac{\partial \varphi}{\partial t} + \frac{1}{Pr}\left\{\frac{\partial \Psi}{\partial z}\frac{\partial \varphi}{\partial x} - \frac{\partial \Psi}{\partial x}\frac{\partial \varphi}{\partial z}\right\} \\
&\quad = \Delta\varphi + Ra\frac{\partial (T+C)}{\partial x} + Gs\left\{\cos\alpha\left[\frac{\partial (T+C)}{\partial z}\frac{\partial^2 F}{\partial x \partial z} - \frac{\partial^2 F}{\partial z^2}\frac{\partial (T+C)}{\partial x}\right]\right. \\
&\qquad \left. + \sin\alpha\left[\frac{\partial (T+C)}{\partial x}\frac{\partial^2 F}{\partial x \partial z} - \frac{\partial^2 F}{\partial x^2}\frac{\partial (T+C)}{\partial z}\right]\right\} \\
&\frac{\partial T}{\partial t} + \frac{\partial \Psi}{\partial z}\frac{\partial T}{\partial x} - \frac{\partial \Psi}{\partial x}\frac{\partial T}{\partial z} = \Delta T \\
&\frac{\partial C}{\partial t} + \left\{\frac{\partial \Psi}{\partial z}\frac{\partial C}{\partial x} - \frac{\partial \Psi}{\partial x}\frac{\partial C}{\partial z}\right\} = Le\Delta(C - \psi T) \\
&\varphi = -\Delta\Psi \\
&\Delta F = -\frac{\partial (T+C)}{\partial z}\cos\alpha + \frac{\partial (T+C)}{\partial x}\sin\alpha
\end{aligned} \tag{4}$$

These equations are subject to boundary conditions:

$$\begin{aligned}
&z = 0: \quad \Psi = 0, \quad \frac{\partial \Psi}{\partial z} = 0, \quad F = 0, \quad T = 1, \quad \frac{\partial C}{\partial z} - \psi\frac{\partial T}{\partial z} = 0 \\
&z = 1: \quad \Psi = 0, \quad \frac{\partial \Psi}{\partial z} = 0, \quad F = 0, \quad T = 0, \quad \frac{\partial C}{\partial z} - \psi\frac{\partial T}{\partial z} = 0
\end{aligned} \tag{5}$$

corresponding to no-slip, impermeable, and isothermal horizontal plates, while lateral boundaries of the computational domain are treated as periodic. So all the variables $f \equiv \{\Psi, \varphi, F, T, C\}$ in x-direction are set to periodic:

$$f(x, z, t) = f(x + \lambda, z, t) \tag{6}$$

with $\lambda = 2$.

To obtain approximate solutions of the initial boundary value problem for the system of PDEs (4), the finite-difference technique is applied. The parabolic equations are solved using an alternating-direction implicit scheme of the second order with central differences for spatial derivatives and one-sided right differences for time derivatives. The elliptic equations for the stream functions were worked out by means of an iterative method of successive over-relaxation at each time step. Typically, a regime of steady state, finite-amplitude convective oscillations obtained at a particular set of parameters was used as initial condition for a run with a different set of parameters. All calculations were executed on a grid of 42×21 nodes. Further refining the mesh did not have any significant effect on characteristics of the oscillations. To compare the numerical results presented in this paper with the well-known experimental, analytical, or numerical ones of other authors, the reduced Rayleigh number $r = Ra/Ra_0$ is used (as in [9], for example), i.e. Ra is scaled by Ra_0, where $Ra_0 = 1686$ is the critical Rayleigh number for the onset of convection in an homogeneous liquid obtained by means of our numerical code.

3. Numerical results

Consider the nonlinear evolution of convective patterns in a binary mixture layer heated from below. In calculations, we employed the typical for molecular liquid mixture parameters: the Lewis number $Le = 0.01$, and the Prandtl number $Pr = 10$, and the separation ratio $\psi = -0.25$. This set of parameters is typical for ethanol/water mixtures [9].

We are concerned with the properties of the convective solutions at different angles of the vibration axis inclination. The bifurcation diagram of various numerically obtained convective regimes in the case of longitudinal vibration is given in Fig. 2. It illustrates the maximum vertical velocity v_z (a) and the frequency of oscillations ω in the travelling wave (TW) regime (b) versus the reduced Rayleigh number r at given values of the Gershuni number $Gs = 1000$.

When the heating intensity increases quasi-statically, the onset of convection occurs via a Hopf bifurcation at the value of reduced Rayleigh number r_{osc} and is characterized by the Hopf frequency ω_H. Arisen 2D flow pattern bifurcates backwards from the conductive state at r_{osc}, which is in a good agreement with the predictions of the linear stability theory, and has a form of symmetry-degenerated travelling waves. The waves gain stability via a saddle-node bifurcation at the value of reduced Rayleigh number r^S_{TW}. Within the range $r < r^S_{TW}$ the binary-mixture convection decays and the system turns to the conductive state. The branch of stable TWs ends at the value of the reduced Rayleigh number r^*, and the pattern of

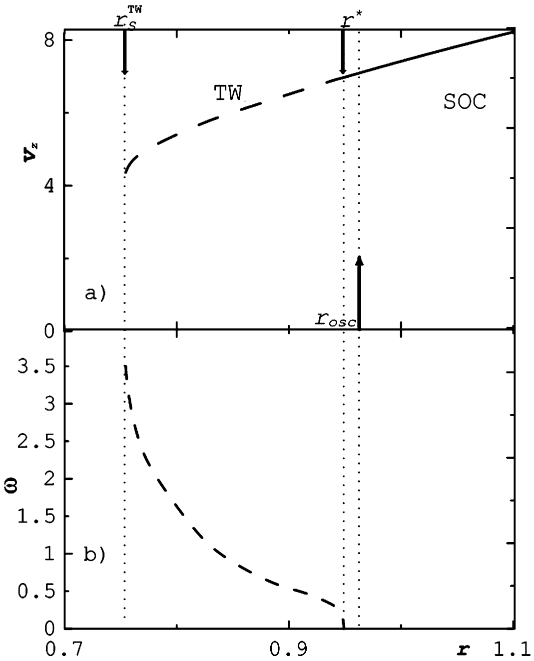

Fig. 2. The bifurcation map (a) and the frequency ω (b) of stable solution at the Gershuni number $Gs = 1000$.

Table 1
The dependencies of critical Rayleigh numbers on the intensity of longitudinal vibrations Gs.

Gs	r_{TW}^{S}	r_{osc}	r^*
0	1.176	1.318	1.36
500	0.965	1.144	1.121
1000	0.754	0.967	0.935
1500	0.546	0.776	0.747
2000	0.336	0.594	0.569

stable stationary overturning convection (SOC) is formed. Thus, the highly developed, nonlinear travelling wave convection is stable in the interval of reduced Rayleigh numbers $r_{TW}^{S} < r < r^*$. The TW (dashed) branch of curve in Fig. 2(a) fits the findings. The minimal value of the reduced Rayleigh numbers in Fig. 2(a) is $r = r_{TW}^{S} = 0.754$. In the Earth's gravitational field, it is possible to reach this value of the r parameter with reducing the temperature difference between the horizontal boundaries of the layer. Fig. 2(b) shows the dependency with $\omega(r)$. At the saddle point, r_{TW}^{S}, the frequency of stable TW is about 3.5. It decreases to zero at r^*, where the SOC regime appears.

Bifurcation diagrams constructed at different values of the Gershuni number Gs look similarly; however, as it can be seen from Table 1, the increase of the Gershuni number leads to significant lowering of the critical values of the reduced Rayleigh numbers r_{TW}^{S}, r_{osc}, r^*.

It is necessary to note that in the absence of vibration $r_{osc} < r^*$. This property is kept in the case of transversal vibration; however, the growth of the Gershuni number in the case of longitudinal vibration changes this ratio: $r_{osc} > r^*$.

Fig. 3 represents the dependencies of the reduced Rayleigh numbers r_{TW}^{S}, r_{osc}, r^* on the intensity of longitudinal vibrations (a) and of the axis inclination angle at $Gs = 1000$ (b). As shown in Fig. 3(b), the critical values of the control parameter increase linearly with angle α.

The stream function F contours are demonstrated in Fig. 4 for specified α values and the reduced Rayleigh number values pertinent to the central point of the corresponding stability interval $r_{TW}^{S} < r < r^*$. In the case of longitudinal vibrations (Fig. 4(a)) the field of the stream function F consists of two parts.

In the first part, or the basic flow, the stream function is described by square-law dependence on the z-coordinate:

$$F = -(1+\psi)\left(z^2/2 - z/2\right) \tag{7}$$

The second part of the field is formed by imposing vortex disturbances on the basic flow. The intensity of $\boldsymbol{w}$ field is approximately twenty times less than that of the averaged velocity $\boldsymbol{v}$ field, and $\Psi_{max}/F_{max} \cong 10$.

The growth of the angle between a horizontal line and the axis of vibration tends to reduce the intensity of the basic flow (Fig. 4(b)). In the case of transversal vibrations, the basic flow is not operative and only vortex disturbances specify the structure of the field of the stream function F (Fig. 4(c)). With that ratio, Ψ_{max}/F_{max} increases.

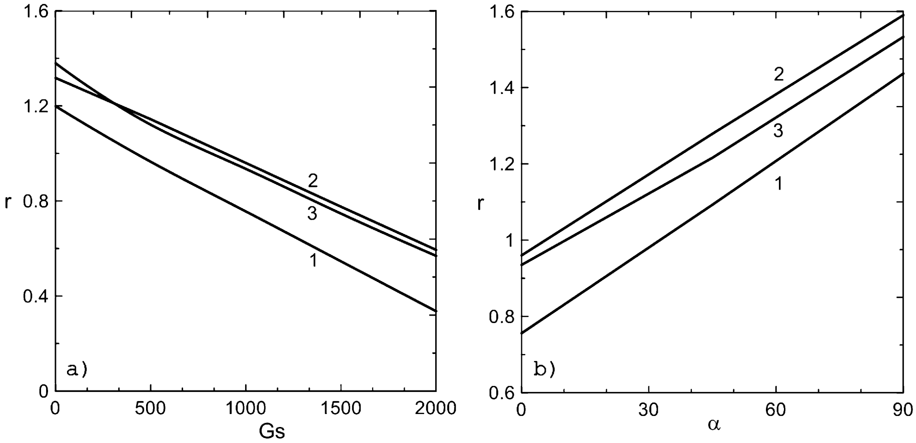

Fig. 3. The influence of the intensity of longitudinal vibrations (a) and axis inclination at $Gs = 1000$ (b) on critical control parameters: 1, r^S_{TW}; 2, r_{osc}; 3, r^*.

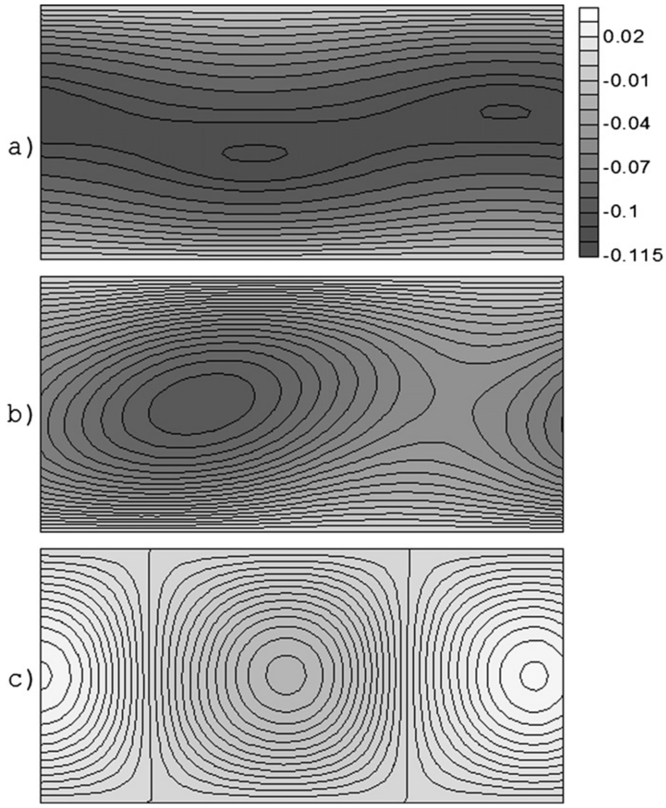

Fig. 4. The stream function F contour lines at the parameter set: (a) $\alpha = 0$, $r = 0.848$; (b) $\alpha = 45°$, $r = 1.097$; (c) $\alpha = 90°$, $r = 1.435$.

In the absence of vibration in a molecular binary mixture, both TW- and SOC-modes have the mirror-glide (MG) symmetry [9]:

$$f(x, z, t) = -f\left(x + \frac{\lambda}{2}, 1 - z, t\right) \tag{8}$$

This property is observed in experimental investigations [10].

Convection of binary mixture under high-frequency harmonic vibrations, whose axis is angled to layer boundaries, is described by the problem (4) containing the stream function F, defined by the oscillatory velocity component. In the case of vertical vibrations ($\alpha = \pi/2$), the important property (8) holds true for all fields, including F, which is demonstrated in Fig. 4(c). For the arbitrary inclination angle of vibration axis, the mirror-glide (MG) symmetry (8) is broken because of terms in Eqs. (4), which are proportional to $\cos\alpha$. The broken MG symmetry is seen for the stream function F in Figs. 4(a), (b).

4. Conclusion

The influence of high-frequency vibration with arbitrary orientation of the axis on the appearance and nonlinear evolution of a convective travelling wave regime in a horizontal layer heated from below and filled with a binary mixture of fluids with anomalous thermal diffusion is studied numerically. The effect of mirror-glide symmetry breaking is found in the case of non-vertical vibration. It is shown that in the case of longitudinal vibration, the Gershuni number growth leads to the lowering of critical control parameters. Growth of the inclination angle increases critical control parameters. Convective regimes are observed at larger values of the reduced Rayleigh number. Thus vibrations can be used in order to control convection in binary mixtures during terrestrial experiments.

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