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Stability of Soret-induced flow in a vertical layer

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ABSTRACT

The stability of the Soret-induced convective flow of a binary mixture of non-reacting components to traveling thermosolutal perturbations is studied. Dependence of the threshold value of the Prandtl number, at which the viscous thermal mechanism of the conductive state crisis becomes most dangerous, on the separation ratio is obtained. For positive Soret effect, the range of Prandtl number values where the thermal waves are completely suppressed is discovered.

1. Introduction

Existence and specific manifestations of instability mechanisms of equilibrium and flows in various media depend on various factors, which can be divided into several groups. The first group comprises the properties of the medium. These are normal (or anomalous) thermal expansion, the presence of volumetric heat generation, the composition of the medium (homogeneous medium or medium with any inclusions (gas bubbles, solid particles, etc.)), a mixture of reacting and non-reacting components, the possibility of phase transformations, etc. The second group comprises various external factors. These are a configuration of the domain occupied by the medium, the orientation of the domain in a gravitational field, the different boundary conditions (including the conditions of heating), vibration action, etc. There are situations where these factors work together. An example of this can be a variety of phenomena related to the deformability of the free surface, where the dependence of surface tension on temperature manifests itself. As a result, scenario of the crisis can be extremely complicated.

A binary mixture of non-reacting components can serve as an example of the medium, whose properties make possible new mechanisms of crisis. Due to heterogeneity of the composition, a diffusion and thermosolutal instability mechanism arises.

The studies of convection in a binary mixture of non-reacting components go back to work by Shaposhnikov [1], in which the equations of buoyancy convection in a mixture were obtained in the Boussinesq approximation. The presence of diffusion, thermodiffusion and diffusive thermal conductivity changes the action of usual mechanisms of the crisis of equilibrium and flows and leads to the appearance of new instability modes.

Convective flow of a mixture in a vertical layer heated from the side was first considered by Hart [2]. A vertical gradient of lighter component concentration is created in the layer that forms a stable vertical stratification of the mixture. Unfortunately, the paper contained an error. More careful consideration of the stability of the mixture was performed by G.Z. Gershuni, E.M. Zhukhovitskii, and L.E. Sorokin in [3,4] (see, also [5] for a review). A rich set of instability mechanisms was discovered. It was shown that for all Prandtl number values there is a hydrodynamic instability mode related to the vortices at the boundary of towards flows. For large Prandtl number values, there is a thermosolutal wave instability mode.

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In the presence of thermodiffusion, the instability of the flow with respect to traveling disturbances changes considerably. In addition to lowering the instability threshold, for both positive and negative Soret effects, the decrease of the threshold value of the Prandtl number, at which the growing thermosolutal waves appear, is observed. The greatest decrease in the threshold of the Prandtl number is observed for a positive Soret effect. The thermodiffusion also leads to the appearance of a new instability mechanism associated with the development of cellular monotonic disturbances. For a negative Soret effect, the significant decrease in the stability of the flow is possible due to the long-wave thermosolutal instability mode.

In the present paper we consider the stability of the convective flow with respect to the traveling thermosolutal perturbations. The dependence of the threshold value of the Prandtl number at which the growing thermosolutal wave perturbation appears on the separation ratio is calculated.

2. Problem formulation

Let a binary mixture of non-reacting components be enclosed in an infinite vertical layer $x = \pm h$ (x is the horizontal coordinate in the direction orthogonal to the layer boundaries). Plane-parallel boundaries of the layer are of infinitely high thermal conductivity and are maintained at different temperatures $T = \pm\Theta$. The mass flux through the layer boundaries is absent.

We assume that the density of the mixture is a linear function of temperature and concentration C of lighter component:

$$\rho = \rho_0(1 - \beta_\mathrm{T} T - \beta_\mathrm{C} C)$$

Here ρ_0 is the density of the medium at the mean values of temperature and concentration, T and C are small deviations of the temperature and concentration from the mean values, β_T is the thermal expansion coefficient, $\beta_\mathrm{C} > 0$ is the concentration coefficient of the density. Below, analyzing the behavior of the mixture, we will use the Boussinesq approximation. The diffusive thermal conductivity is neglected.

In the situation under consideration, there exists a plane-parallel flow in a vertical direction with a cubic velocity profile and linear distributions of temperature and concentration [5]:

$$\mathrm{v}_0 = \frac{1+\varepsilon}{6}\left(x^3 - x\right), \qquad T_0 = -x, \qquad C_0 = -\varepsilon x \tag{1}$$

Here $\varepsilon = -\alpha\beta_\mathrm{C}/\beta_\mathrm{T}$ is the dimensionless parameter characterizing thermodiffusion (separation ratio), and α is the thermodiffusion coefficient. For a positive Soret effect, $\alpha < 0$, $\varepsilon > 0$ and for a negative Soret effect, $\alpha > 0$, $\varepsilon < 0$.

The problem of the linear stability of the flow (1) with respect to the normal plane perturbations is as follows:

$$-\lambda\Delta\varphi + \mathrm{v}_0 ikGr\Delta\varphi - \mathrm{v}_0'' ikGr\varphi = \Delta\Delta\varphi + \left(\vartheta' + \xi'\right)$$

$$-\lambda\vartheta + \mathrm{v}_0 ikGr\vartheta - T_0' ikGr\varphi = \frac{1}{Pr}\Delta\vartheta$$

$$-\lambda\xi + \mathrm{v}_0 ikGr\xi - C_0' ikGr\varphi = \frac{1}{Sc}\Delta(\xi - \varepsilon\vartheta), \quad \Delta = \frac{\mathrm{d}^2}{\mathrm{d}x^2} - k^2 \tag{2}$$

$$x = \pm 1:: \quad \varphi = \varphi' = \vartheta = 0, \quad \xi' - \varepsilon\vartheta' = 0 \tag{3}$$

Here λ is the complex decrement, k is the wave number of perturbations, φ is the amplitude of the stream function of the velocity perturbations, ϑ and ξ are the amplitudes of the normal perturbations of temperature and concentration, the prime denotes differentiation with respect to the coordinate x.

The problem (2)–(3) is written in dimensionless form. The following scales are chosen: for length h, for time h^2/ν, for velocity $g\beta_1\Theta h^2/\nu$, for temperature Θ, for concentration $\beta_1\Theta/\beta_2$.

The stability problem contains the following dimensionless parameters: Grashof number Gr, Prandtl number Pr, Schmidt number Sc and separation ratio ε:

$$Gr = \frac{g\beta_1\Theta h^3}{\nu^2}, \qquad Pr = \frac{\nu}{\chi}, \qquad Sc = \frac{\nu}{D}$$

where D is the diffusion coefficient.

The problem (2)–(3) was solved numerically using the method allowing us to construct the fundamental system of solutions. The set of complex ordinary differential equations obtained in this way was integrated by the Runge–Kutta–Merson method, with orthogonalization of the vectors of solutions by the method of Gram–Schmidt. Some results were obtained using the differential sweep method with matching at the intermediate point. The eigenvalues of the spectral boundary value problem were determined numerically by the two-dimensional secant method.

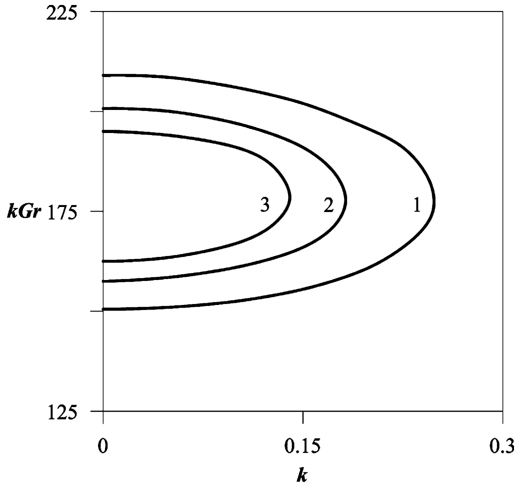

Fig. 1. Neutral curves $kGr(k)$ for thermal wave instability of convective flow at $\varepsilon = 0$, $Pr = 12, 11.8, 11.7$ (curves 1, 2, 3, respectively).

3. Numerical results

As known, for convective flow of a single-component fluid, which occurs in a plane vertical layer heated the side, for sufficiently large Prandtl number values $Pr > Pr^*$ there arises an instability mode associated with the growth of traveling thermal waves. The symmetry properties of the stationary flow (its oddness with respect to the transversal coordinate) and of the perturbations lead to the well-known degeneration: the critical Grashof number values are the same for the thermal waves traveling upward and downward (the waves localized in the ascending and descending flows). This is due to the fact that the lower real-valued thermal branches ν_0 and ν_1 of the decrement spectrum are responsible for the formation of wave disturbances; merging they form a complex conjugate pair of decrements. The corresponding perturbations realize the wave instability of stationary flow.

Such an instability mechanism can be dangerous for the flow of the binary mixture too, at least in the case of zero separation ratios, which corresponds to the single-component fluid. The results of the analysis of wave instability of a binary mixture flow are presented in [3–5], where the dependences of the minimal critical Grashof number on the Prandtl number are presented for traveling thermal waves. It is shown that taking into account thermodiffusion effect leads to a lowering of the threshold Prandtl number value Pr^* both for positive and negative Soret effects. Unfortunately, the relationships $Gr_{\mathrm{m}}(Pr)$ are obtained only for a few values of the separation ratio and with a quite large step in the Prandtl number. The threshold value Pr^* is obtained by extrapolation of the relationship $Gr_{\mathrm{m}}(Pr)$ to $Gr \to \infty$. As a result, the data of [3–5] demonstrate a monotonous decrease of the minimal critical Grashof number with increasing the Prandtl number. In addition, the effect of the threshold value of the Prandtl number on the separation ratio is not obtained there. Precise data on the value Pr^* can be obtained using the procedure described in [6].

As mentioned above, at $\varepsilon = 0$, we come to the case of a mixture the flow of which corresponds to the single-component fluid flow. Neutral curves of viscous thermal perturbations in the plane Gr–k have a looped shape, the axis Gr is a common asymptote of the upper and lower branches of neutral curves. When Pr^* approaches the large Prandtl number values, the stability of the flow with respect to the thermal waves increases, the neutral curves move upward. At $k \to 0$, the critical Grashof number on the upper and lower branches of neutral curves increases according to the k^{-1} law, as well as the phase velocity of neutral disturbances c. Thus, the product kGr remains finite all over the neutral curve. This makes it possible to describe the whole instability domain in the plane kGr–k.

The described features of the behavior of the neutral curves for thermal instability mode allow us to determine the value Pr^*. For that, one needs to follow the evolution of the neutral curves in the plane kGr–k. Eqs. (2) allow us to perform the calculations for any finite values of the wave number, including $k = 0$, since the Grashof number is presented in these equations only in the necessary combination with the wave number.

Fig. 1 presents the neutral curves $kGr(k)$ for $Pr = 12, 11.8, 11.7$ in the case when $\varepsilon = 0$ (curves 1–3, respectively).

At fixed value of the Prandtl number, the neutral curve of wave disturbances intersects the axis kGr in two points whose position depends on Pr. With the decrease of Pr, the instability domain decreases in size, the points of intersection of the curves with the axis kGr approach each other, at $Pr = Pr^*$ the neutral curve shrinks to a point on the axis kGr; this makes it possible to determine Pr^*.

Curve 1 in Fig. 2 shows the dependence of coordinates of points kGr_0 on the Prandtl number. The point furthest to the left on the dependence curve determines the value $Pr^* \approx 11.562$; its position is indicated by the dashed line. Curve 2 shows the corresponding dependence for the oscillation frequency of temperature perturbations.

The presence of thermodiffusion greatly affects the quantitative characteristics of flow stability with respect to traveling waves.

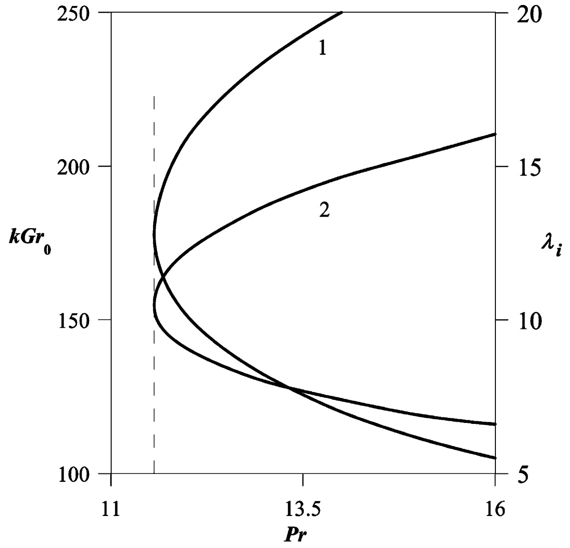

Fig. 2. Determination of Pr^*; curve 1, kGr_0; curve 2, frequency.

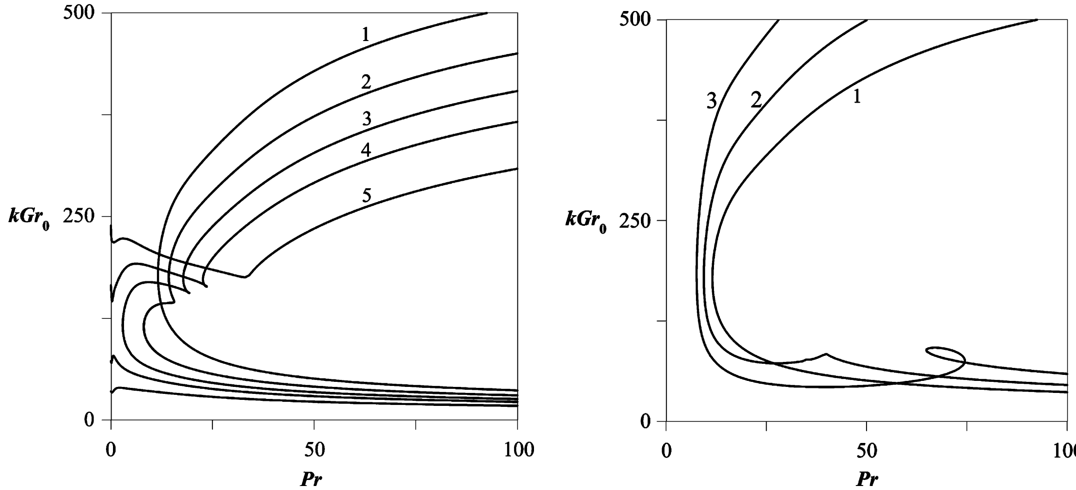

Fig. 3. Dependence $kGr_0(Pr)$, $Sc = 676.7$; (left) positive Soret effect, $\varepsilon = 0, 0.1, 0.2, 0.3, 0.5$, curves 1–5; (right) negative Soret effect, $\varepsilon = 0, -0.1, -0.2$, curves 1–3.

Fig. 3 (left) shows the dependences $kGr_0(Pr)$ for the case of a positive Soret effect at $\varepsilon = 0.1, 0.2, 0.3, 0.5$ (curves 2–5, respectively); Fig. 3 (right) displays the corresponding dependences for the case of a negative Soret effect at $\varepsilon = -0.1, -0.2$ (curves 2–3). Lines 1 correspond to the case $\varepsilon = 0$. The calculations were performed for a Schmidt number $Sc = 676.7$, which corresponds to the liquid mixtures. The flow is unstable with respect to thermal waves with $k = 0$ in the domains between the upper and lower branches of the dependences. The threshold value of the Prandtl number is determined by the point furthest to the left of the boundary of the instability domain.

First of all, it can be seen that the behavior of the boundary of the instability domain is different for positive and negative Soret effects. In the case $\varepsilon < 0$ (Fig. 3 (left), curves 2, 3), the threshold value of the Prandtl number decreases with decreasing the separation ratio; on the lower branch of the dependence $kGr_0(Pr)$ curve, at first the segment of non-monotonic behavior appears, then a loop is formed, which increases in size and is shifted toward the higher Prandtl number values with the decrease of ε.

For a positive Soret effect, the behavior of the instability domain is different. Increasing the separation ratio leads to the formation of the protrusion on the boundary of the instability domain and to a rapid decrease of the threshold value of the Prandtl number. For sufficiently large values of the separation ratio, the growing thermal waves are possible at any value of the Prandtl number.

In the parameter range where the dependences $kGr_0(Pr)$ are non-unique, there are two separate instability domains with respect to thermal waves, which correspond to the existence of two neutral curves for a fixed value of the Prandtl number. This is illustrated in Fig. 4, which shows the neutral curves $kGr(k)$ (left) and $Gr(k)$ (right) fort $Pr = 15$ (curve 1), $\varepsilon = 0.1$.

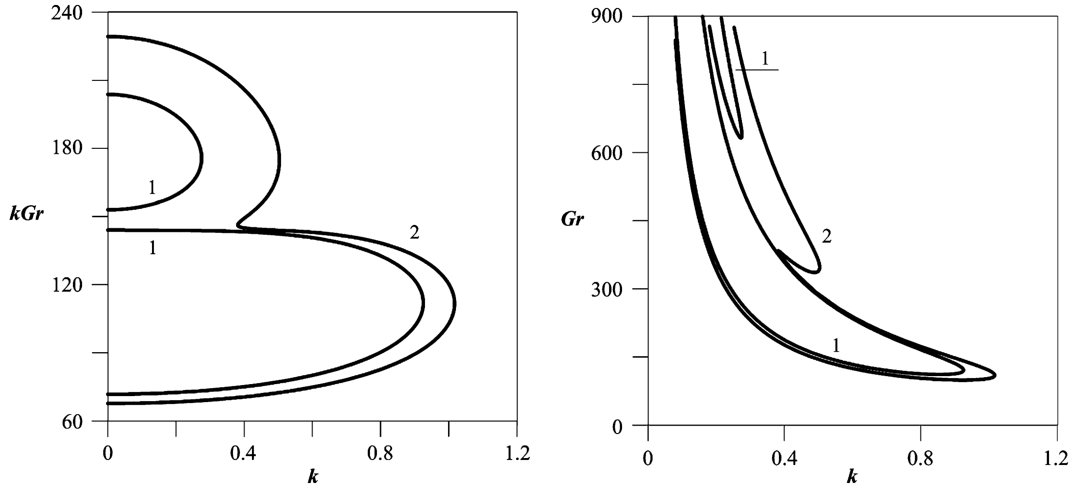

Fig. 4. Neutral curves $kGr(k)$ (left) and $Gr(k)$ (right) for $Pr = 15$ (lines 1) and $Pr = 17$ (lines 2).

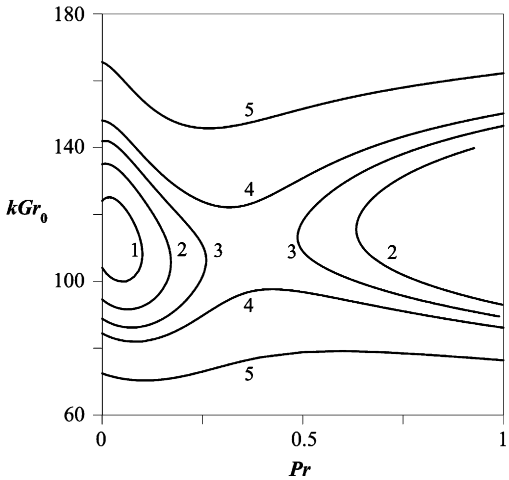

Fig. 5. Dependence $kGr_0(Pr)$; $\varepsilon = 0.265$ (1), 0.27 (2), 0.275 (3), 0.28 (4), 0.3 (5).

With the increase of the Prandtl number, the neutral curves converge and coalesce, forming a single neutral curve (curve 2 in Fig. 4, $Pr = 17$).

More detailed calculations have shown that an increase in the separation ratio leads to a more complex scenario of the changes in flow stability with respect to thermal waves than it might seem at the first glance.

In Fig. 5 the dependences $kGr_0(Pr)$ are plotted for such values of the separation ratio that the threshold value of the Prandtl number is low enough. At $\varepsilon < 0.2$, the viscous thermal mechanism of the instability is possible only if $Pr > Pr^*$. With the increase of the separation ratio, the growing thermal waves become possible at $Pr = 0$. This occurs at $\varepsilon \approx 0.2634$. At $\varepsilon > 0.2634$ the thermal wave mode exists for $0 \leqslant Pr \leqslant Pr_1^*$ and for $Pr > Pr_2^*$. The boundaries of the domains in Fig. 5 are described by curve 1 ($\varepsilon = 0.265$, only the left instability domain is shown), curves 2 ($\varepsilon = 0.27$), curves 3 ($\varepsilon = 0.275$).

With further growth of the separation ratio, the instability domains merge. At $\varepsilon > 0.278$, the thermal wave instability is possible for any values of the Prandtl number – see curves 4 and 5 ($\varepsilon = 0.28, 0.3$, respectively). On the lower boundary, the value of kGr_0 varies with the Prandtl number non-monotonically, which makes it possible to predict non-monotonic dependences of the minimal critical Grashof number on the Prandtl number as in the case of flow in a vertical layer with adiabatic boundaries [5].

In Fig. 6, the stability map of a thermosolutal flow with respect to the thermal waves is presented. The instability domains are located above the curves $Gr_{\mathrm{m}}(Pr)$. Curves 1 and 2 show the location of the instability boundaries at $\varepsilon = 0.27$; the position of Pr_1^* and Pr_2^* is shown by dashed lines 6 and 9. Curves 3–4 correspond to the case $\varepsilon = 0.275$; the position of Pr_1^* and Pr_2^* is shown by dashed lines 7 and 8. Curve 5 shows the location of the boundary of instability domain for $\varepsilon = 0.28$.

Finally, the dependence of the threshold value of the Prandtl number Pr^* on the separation ratio is presented in Fig. 7. The dependence consists of two intersecting lines. This is a consequence of the fact that the threshold value of the Prandtl

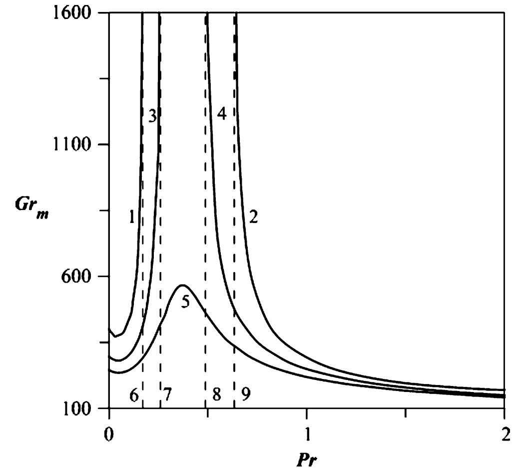

Fig. 6. Stability map for $\varepsilon = 0.27$ (1–2), 0.275 (3–4), 0.28 (5).

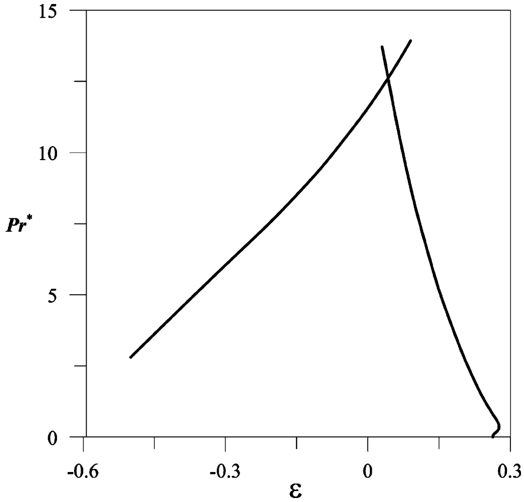

Fig. 7. Threshold value of the Prandtl number on the separation ratio.

number is formed in the cases of negative and positive Soret effects by two different parts of the dependence $kGr_0(Pr)$ (see Fig. 3).

4. Conclusions

The thermodiffusion modifies rather strongly the effect of the viscous mechanism of convective flow crisis caused by the development of thermal waves. In the case of the negative Soret effect, the instability of the flow with respect to such perturbations becomes possible at smaller Prandtl number values than in the case of single-component fluid. For positive Soret effect, the growing thermosolutal waves become possible for any value of the Prandtl number.

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