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Simulation of the shear strength for unsaturated soils

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ABSTRACT

The simulation of the hydro-mechanical behavior of unsaturated soils is becoming a subject of major importance in soil mechanics. However, unlike the laws governing the behavior of saturated soils, those used to describe the behavior of unsaturated soils still lack of simplicity for common engineering practice. This is why it is important to reconcile saturated and unsaturated soil mechanics and establish a unified theory. In this paper, we use the same strength equation of saturated soils in unsaturated materials and verify that a single failure surface is obtained for any value of suction in wetting and drying paths.

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1. Introduction

Shear strength is one of the most important engineering properties of soils. This is because most civil engineering works require the knowledge of the soil resistance to design safe structures.

The shear strength of soils can be measured in equipments that apply three-dimensional pressures (directions 1, 2 and 3). In this case, the triaxial cell or the true triaxial apparatus can be used. When different pressures are applied in two or three directions, they cause shear stresses that are supported by the internal structure of soils. When these acting stresses exceed the strength of the material, it is said that the soil fails. In the case of saturated materials, this failure state is identified through the final deviator stress ($q_f = \sigma_1 - \sigma_3$) and the final mean effective stress ($p' = (\sigma'_1 + 2\sigma'_3)/3$) where the effective stress (σ') is defined as the difference between the total stress and the pore pressure. If we plot these values (p'_f, q_f) under different confining stresses for a saturated soil, they exhibit a single failure line with a slope called *M*. In this sense, the failure deviator stress is equal to Mp'_f and represents the maximum internal resistance in a failure plane when the soil sample is subjected to external loads:

$$q_{\rm f} = M p_{\rm f}'$$

(1)

In contrast, the shear strength in unsaturated soils can be predicted using two different approaches. The first one is based on the independent stress state variables and considers that the strength due to mechanical loading is independent of the strength due to suction [1]. The variables used to establish the governing equations in this approach are the mean net stress $p_{\text{net}} = (\sigma_1 + 2\sigma_3)/3$, the deviator stress $q = \sigma_1 - \sigma_3$ and suction s. Matric suction $s = u_a - u_w$ represents the pressure difference of the fluids in the soil pores: air (u_a) and water (u_w). Suction is included as a third variable in order to take







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into account correctly the effect of moisture on the strength of the material. This approach considers the stress state as a function of these three variables $f = f(p_{net}, q, s)$.

The second approach is similar to the saturated case, where the strength of the material is linked to the shear stress and the mean effective stress. Models written in terms of effective stress couple the mechanical and hydraulic behavior of the material in this single variable. For the case of unsaturated soils, the most popular effective stress relationship is Bishop's effective stress equation, which in tensorial form is written as:

$$\sigma_{ij}' = \sigma_{ij}^{\mathrm{r}} - u_a \delta_{ij} + \chi s \delta_{ij} \tag{2}$$

where σ_{ij}^t represents the total stress applied at the boundaries of the soil, δ_{ij} is the Kronecker delta and χ is Bishop's parameter, closely related to the degree of saturation of the soil [2]. More recent studies relate this parameter to saturated, dry, and unsaturated fractions of soil [3].

When relationship (2) is multiplied by the Kronecker delta, the mean effective stress is expressed in terms of the mean net stress (p_{net}) and suction:

$$p' = p_{\text{net}} + \chi s \tag{3}$$

Several researchers have made attempts to establish analytical equations to obtain parameter χ . The most simplistic of them assumes that it is equal to the degree of saturation of the material ($\chi = S_r$).

This paper focuses on the prediction of the shear strength of unsaturated soils using the effective stress concept as it shows some advantages when compared to the independent stress state variables approach. Among other advantages, effective stress models require less constitutive parameters to be formulated. Furthermore, in order to calibrate these parameters, more simplistic and economical laboratory tests are required [4]. In this paper, we only consider the effect of matric suction and neglect those produced by other types of suction.

A model to predict the strength of unsaturated soils is proposed herein. First, an analytical equation to determine Bishop's parameter based on the analysis of the equilibrium of the soil phases (solid, liquid, gas) is presented. Then it is demonstrated that it can be applied to any type of soil. Later, a probabilistic porous model is developed to generate the parameters required to obtain the effective stresses in unsaturated materials. Additionally, proper details regarding the calculation of the variables that intervene in the effective stress equation proposed herein are provided. Finally, some numerical and experimental results comparisons are presented.

2. Bishop's parameter determination

Fig. 1(a) outlines a porous structure composed of solids and pores. These pores can be of two different types: sites and bonds. The sites contain the largest part of the voids and can be subdivided in macropores and mesopores. The bonds or throats are the elements that interconnect the sites.

Consider that a soil is initially subjected to a high suction and all pores are dry. If suction is reduced by steps, at certain point, water intrudes into the system. Initially, only the smaller bonds located at the boundaries of the sample saturate. With further reductions in suction, other larger elements start to saturate. Fig. 1(b) shows the distribution of water in the system at a certain stage of the wetting process. It can be observed that the system is now composed of pores that may be either dry or saturated (both sites and bonds). Additionally, it can be observed that solids can be completely surrounded by either saturated or dry pores, or even they can be surrounded by a combination of them. For example, solid 6 in Fig. 1(b) is completely surrounded by dry pores and in that sense it is called a dry solid. Instead, solids 1 and 3 are completely surrounded by a combination of saturated and dry pores, then they are called unsaturated solids. Notice that the definitions of the different fractions involve both solids and pores and not the pores alone. The total volume of solids added by the volume of their surrounding pores forms the volume of the fraction, be it saturated, dry, or unsaturated.

It will be seen later that the value of Bishop's parameter as proposed herein depends on the quantification of the different fractions. Because the saturated and the dry fractions do not share common elements, these two fractions are the first ones to be quantified. Then, the unsaturated fraction is quantified from the difference with the total volume of bonds, sites and solids. In this form we ensure that neither element is counted twice.

Let us define the saturated fraction f^s as the ratio between the saturated volume (saturated solids and its surrounding pores) and the total volume of the sample V; the saturated (f^u) and the dry fraction (f^d) are defined in the same way: the unsaturated volume divided by the total volume and, the dry volume divided by the total volume, respectively.

$$f^{s} = (V_{SOL}^{s} + V_{S}^{s} + V_{B}^{s})/V$$

$$f^{d} = (V_{SOL}^{d} + V_{S}^{d} + V_{B}^{d})/V$$
(4a)
(4b)

$$f^{\rm u} = \left(V_{\rm SOL}^{\rm u} + V_{\rm S}^{\rm u} + V_{\rm B}^{\rm u}\right)/V \tag{4c}$$

In this way, the condition $f^{s} + f^{d} + f^{u} = 1$ must be satisfied at any value of the degree of saturation. Variables in Eq. (4) show subscripts that refer to either solids (SOL), sites (S), or bonds (B). On the other hand, superscripts indicate whether



Fig. 1. Sketch of the porous structure of a soil in (a) dry and (b) wetting process conditions.

the referred volume V belong to a saturated (s), dry (d), or unsaturated (u) fraction. Notice that, for an initially saturated soil subjected to a drying process, f^u and f^d should be initially cero ($f^s = 1$, $f^d = 0$, $f^u = 0$). Then when suction increases, some pores will dry, consequently, the unsaturated and, eventually, the dry fraction will increase their value. Finally, at the end of the drying process, condition $f^s = 0$, $f^d = 1$, $f^u = 0$ should be accomplished.

The volume of pores surrounding the unsaturated solids is called the unsaturated volume of voids V_v^u (no matter they are saturated or dry); moreover, this unsaturated volume of pores is formed by the unsaturated volume of sites and bonds, in the form $V_v^u = V_S^u + V_B^u$. Notice that, according to Fig. 1, the unsaturated voids volume V_v^u is the sum of all dry and saturated pores surrounding the unsaturated solids.

Because the unsaturated fraction shows both saturated and dry pores, a degree of saturation for this fraction can be defined. Let us define the degree of saturation of the unsaturated fraction S_r^u as the volume of water inside the pores surrounding the unsaturated solids divided by the total volume of those pores ($S_u^r = V_w^u/V_v^u$). From the analysis of the equilibrium between the different phases of a loaded soil sample, an equation to quantify the

From the analysis of the equilibrium between the different phases of a loaded soil sample, an equation to quantify the effective stress in unsaturated materials is obtained [5]:

$$\sigma' = \sigma_{\text{net}} + \left(f^{\text{s}} + f^{\text{u}}S^{\text{u}}_{\text{r}}\right)s \tag{5}$$

Notice that Eq. (5) can be arranged in the form of Bishop's stress with $\chi = f^s + f^u S_r^u$. The physical meaning of parameter χ can be found when it is expressed under the form $\chi = f^s S_r^s + f^u S_r^u + f^d S_r^d$, where S_r^s and S_r^d represent the degree of saturation of the saturated and the dry fractions, respectively. Therefore, parameter χ represents the weighted degree of saturation of the sample, as all three fractions are multiplied by their corresponding degree of saturation.

Parameters f^s , f^u and S^u_r can only be evaluated when the distribution of water in the soil pores is known. This distribution of water can be approximately determined through a solid-porous model able to simulate the hydraulic behavior of soils during wetting-drying cycles. Such a model requires the pore size distribution and the grain size distribution of the soil as data.

The pore size distribution can be obtained from mercury intrusion porosimetry or scanning electron microscopy tests. In the first method, the volume of mercury intruding the pores of the soil is quantified. The size of cavities being filled is obtained from Laplace's law: $R_c = 2T_s/p$, where T_s is the interfacial surface tension between air and mercury, p is the mercury pressure [6], and R_c is the critical radius, representing the maximum size of pores being filled with mercury. Because, in general, pores show very complex geometries, R_c represents in fact an equivalent size of the real pore.

Both mercury intrusion and scanning electron microscopy show similar results to those presented in Fig. 2 (black squares) in the axis of cavity size versus relative volume. In this figure, the relative volume is the volume of pores of certain size divided by the total volume of pores. It can be observed that this distribution approximates a logarithmic normal distribution, hence it can be modeled with the aid of just two statistical parameters: the standard deviation (σ) and mean value (μ). In the case of doubled structured soils, a double logarithmic normal distribution can be used.

The solid–porous model can be built in a regular network where nodes represent the sites and the connectors represent the bonds. As stated before, sites are subdivided in macropores and mesopores. The main difference between these two elements is that macropores are responsible for most of the volumetric deformation of soils when subjected to loading or suction increase while mesopores maintain their size [7].

The porous model can be built using a bidimensional or tridimensional network. In the first case, sites are represented by circles and bonds by rectangles. For the tridimensional model sites are represented by spheres and bonds by cylinders. Because the network is built with elements of different sizes, some constrains must be established in order to ensure that the model is physically possible. These constraints are summarized in the following construction principle: when two bonds with radius r_{b1} and r_{b2} are concurrent (they meet at 90°) to a site with radius r_{st} , then their size must satisfy the condition $r_{st} \ge \sqrt{r_{b1}^2 + r_{b2}^2}$. This principle avoids the overlapping between concurrent bonds and also guaranties that all sites are larger than their concurrent bonds.



Fig. 2. Experimental (black squares) and theoretical (solid line) relative volume distribution.

Some computational codes to build virtual porous networks have been developed. These networks usually focus on the simulation of water distribution. In general, these models require a minimum size for the network to guarantee the convergence of results. This may be a problem for the case of soils, because their pores sizes involve several orders of magnitude (from hundreds to thousands of micrometers); therefore, not all pore sizes can be adequately represented on small grids. This is why computational models require large networks and large memory resources (above those of a common PC) to simulate properly drying–wetting processes, and thus they become impractical for the case of soils.

Percolation models represent an alternative to computational models [3]. Basically, these models simulate the distribution of water in the porous structure by analyzing the probability of a pore of certain size to saturate or dry during a wetting or drying process, respectively. The equations of the model are derived from the analysis of the basic units for sites and bonds. A basic unit is represented by the minimum number of elements required to determine if a site or a bond saturates or dries at certain suction during a wetting or drying process, respectively. Then, the equations of these two basic units are simultaneously solved and the probabilistic equation for the wetting or drying process of soils can be established. If the pore-size distribution of the material is known, then it is possible to determine the volume of water intruding or withdrawing the pores of soil at certain suction and, finally, the soil–water retention curves can be obtained. The mathematical expressions to obtain these curves can be easily programmed and the results are obtained in seconds.

It is assumed that the basic units repeat indefinitely in the system and therefore the mass of soil is treated as homogeneous. This means that boundary conditions in the porous–solid model are not taken into account. In any case, the influence of the boundary conditions is negligible in the case of real soils [3]. For example, if it is assumed that at the borders of the network, only solids and bonds can be found, then the basic unit for bonds differs from the one considered, as, in such a case, bonds are connected with a single site. In a cubic network made of n sites per side, the proportion of bonds at the border with respect to the total is approximately 3/n, which represents a very small proportion if it is acknowledged that nis of the order of several thousands to several billions per gram of material, depending on the type of soil.

In conclusion, computational models for soils require large networks and sophisticated software that need several hours to produce results. In contrast, probabilistic models are simple and produce immediate results. In the following section, the equations for a probabilistic solid–porous model are derived.

3. Probabilistic model

Equations to model the water distribution need to be established for both the wetting and the drying conditions. In order to establish these equations, the distribution of the relative volume of pores is required. During a wetting process, the smallest pores (commonly bonds) are the first to saturate, while the largest are the last (Fig. 3). The maximum size of pores that can be intruded by water is given by the Laplace equation, which establishes a relationship between suction and the maximum size of pores (*R*) able to saturate, in the form $R \leq 2T_s/s$. In this case, the contact angle between water and soil is considered to be zero, while adsorption phenomena are neglected.

Fig. 3 shows the distributions of the relative volume versus pore radius for sites (V_{RS}) and bonds (V_{RB}). The area below each one of these curves is unity meaning that they represent in fact the distribution of probabilities for sites and bonds. The relative volume at the critical radius R_c represent the addition of all relative volumes smaller than or equal to the critical radius and is given by the equations $S(R_c) = \int_0^{R_c} V_{\text{RS}} dR$ and $B(R_c) = \int_0^{R_c} V_{\text{RB}} dR$, for sites and bonds, respectively. The critical radius R_c represents the scanning limit given by the Laplace equation. In the same way, it is possible to define the relative volume distribution for solids in the form $Sol(R) = \int_0^R V_{\text{SOL}} dR$.

In addition, connectivity C is defined as the number of bonds concurrent to a site. A regular bidimensional network has a connectivity of 4, while a regular tridimensional network has a connectivity of 6.

Consider the basic unit for sites in the bidimensional network depicted in Fig. 4(a). It consists of a central site with four concurrent bonds connected with the same number of external sites. Consider now that the basic unit is initially dried and subjected to a wetting process. By inspection, we can determine that this site may saturate when the following two



Fig. 3. Wetting scanning to simulate S_r increments.



Fig. 4. Basic unit for (a) sites and (b) bonds.

conditions are met: (a) its radius is smaller or equal to the critical radius so water can intrude it and (b) at least one bond connected with this site is already saturated and connected with the bulk of water.

The first condition can be written as S(R) (see Fig. 3) and the second one as $[1 - (1 - L_{BI}(R_c))^c]$ where $L_{BI}(R_c)$ is the probability for a bond to be intruded by water. So, if $L_{SI}(R_c)$ represents the probability for a site to be saturated during a wetting process, its value will be given by the product of the two aforementioned conditions because they must occur simultaneously:

$$L_{\rm SI}(R_{\rm c}) = S(R_{\rm c}) \left[1 - \left(1 - L_{\rm BI}(R_{\rm c}) \right)^{\rm c} \right]$$
(6)

In a similar manner, Fig. 4(b) depicts the basic unit for bonds. It consists of a central bond linked to two sites, each of them connected with C - 1 additional bonds. With Fig. 4(b) in mind, it can be established that a bond must satisfy the following two conditions to saturate: (a) its radius is smaller or equal to the critical radius, and (b) at least one of its two sites is already saturated. The first condition can be written as $[1 - (1 - L_{SI}(R_c))^2]$. Then, if $L_{BI}(R_c)$ represents the probability for a bond to saturate during a wetting process, its value will be given by the product of the two aforementioned conditions as they must occur simultaneously:

$$L_{\rm BI}(R_{\rm c}) = B(R_{\rm c}) \left[1 - \left(1 - L_{\rm SI}(R_{\rm c}) \right)^2 \right]$$
(7)

Eq. (6) can be rewritten in the form $L_{SI}(R_c) = S(R_c)F_{SI}(R_c)$, where $F_{SI}(R_c)$ is a non-constant factor defining the fraction of saturated sites during a wetting process. Thus, $L_{SI}(R_c)$ represents the distribution of probabilities of saturated sites when the critical radius reaches value R_c (see Fig. 3). These considerations are helpful to establish the following equation to compute the total volume of saturated sites during a wetting process: $V_{SI}^s = F_{SI}(R_c)(\int_0^{R_c} V_{RS} dR)V_S$, where V_S represents the total volume of sites of the soil sample. In the same way, Eq. (7) can be rewritten as $L_{BI}(R_c) = B(R_c)F_{BI}(R_c)$, where $F_{BI}(R_c)$ represents a non-constant factor defining the fraction of saturated bonds during a wetting process. Threefore, $L_{BI}(R_c)$ is the distribution of the probabilities of saturated bonds when the critical radius reaches the value R_c . In the same way as the of sites, the total volume of saturated bonds during a wetting process is obtained through the relationship $V_{BI}^s = F_{BI}(R_c)(\int_0^{R_c} V_{RB} dR)V_B$, where V_B represents the total volume of bonds of the soil sample. These equations can be solved for any value of succion and thus, the total volume of water intruding the sites during

These equations can be solved for any value of suction and thus, the total volume of water intruding the sites during a wetting process can be obtained. This procedure can also be applied to a drying process (Fig. 5) and therefore, the



Fig. 5. Drying process.

conditions to determine the probabilities for a site $L_{SD}(R_c)$ or a bond $L_{BD}(R_c)$ to be saturated during a drying process can be established:

$$L_{\rm SD}(R_{\rm c}) = S(R_{\rm c}) + \left[1 - S(R_{\rm c})\right] \left[L_{\rm BD}(R_{\rm c})\right]^{\rm c}_{2}$$
(8a)

$$L_{\rm BD}(R_{\rm c}) = B(R_{\rm c}) + \left[1 - B(R_{\rm c})\right] \left[L_{\rm SD}(R_{\rm c})\right]^2 \tag{8b}$$

Eq. (8a) can be rewritten in the form $L_{\text{SD}}(R_c) = S(R_c) + [1 - S(R_c)]F_{\text{SD}}(R_c)$, where $F_{\text{SD}}(R_c)$ denotes the fraction of saturated sites with radius smaller than critical during drying. In the same way, (8b) can be rewritten in the form $L_{\text{BD}}(R_c) = B(R_c) + [1 - B(R_c)]F_{\text{BD}}(R_c)$, where $F_{\text{BD}}(R_c)$ represents a non-constant factor defining the fraction of saturated bonds with radius larger than critical.

Parameters $L_{SD}(R_c)$ and $L_{BD}(R_c)$ in Eqs. (8a) and (8b) can be obtained by solving these equations simultaneously.

Once the total volume $(V_S^s + V_B^s)$ of saturated sites and bonds is obtained for wetting and drying, the retention curves can be plotted. Also, the theoretical relative volume distributions for both elements can be obtained. Nevertheless, at this stage, the model has no information regarding the saturated, dry, or unsaturated fractions of the soil. These parameters are obtained in the next section.

It can be stated [8,9] that pore-size distribution can be derived from the capillary pressure. This implies that retention curves and pore-size distribution maintain a one-to-one relationship. Hysteresis appears in the retention curves because the sites control the wetting process as they are larger than bonds and require lower suction values to saturate. In contrast, the drying process is controlled by bonds because they are smaller than the sites and require larger suction values to dry. This is why different degrees of saturation are reached in wetting and drying for the same value of suction in a soil showing a single PSD.

3.1. Saturated, dry and unsaturated fractions

Using soil's grain size distribution, it is possible to define the parameters μ and σ of the solid fraction. In this case, it is also possible to use double- or triple-logarithmic normal distributions for samples made of mixtures of different types of soil.

Consider now that the basic unit for solids is the one represented by the shaded elements shown in Fig. 6. It is formed by a solid, four surrounding (BU) sites, eight external (E) sites and 16 farther external sites (FE), with their respective bonds. Notice that Fig. 6 shows only one FE bond and one FE site in dotted lines. Assume that initially all pores are dry and that the soil undergoes a wetting process. It can be established that all surrounding sites (BU) saturate when the following two conditions are fulfilled: (a) all bonds and sites surrounding a solid are smaller than the critical radius and (b) at least one external bond (E) has saturated already and is connected with the bulk of water. Since 2(C - 2) and 4(C - 3) represent the number of sites and bonds related to the solid in Fig. 6, respectively, the first condition may be expressed as $S(R_c)^{2(C-2)}B(R_c)^{4(C-3)}$. Then, if $L_{BI_1}(R_c)$ represents the probability for an external bond (E) to saturate, the second condition may be written as $\{1 - [1 - L_{BI_1}(R_c)]^{C(C-2)}\}$. In this case, C(C - 2) represents the number of E bonds.

Now, if $L_{PI}(R_c)$ represents the probability that all (BU) sites of the basic unit depicted in Fig. 6 are saturated, then:

$$L_{\rm PI}(R_{\rm c}) = S(R_{\rm c})^{2(C-2)} B(R_{\rm C})^{4(C-3)} \left\{ 1 - \left[1 - L_{\rm BI_1}(R_{\rm c}) \right]^{C(C-2)} \right\}$$
(9)

Eq. (9) may be rewritten in the form $L_{\text{PI}}(R_c) = F_{\text{SI}}^{\tilde{S}}(R_c)S(R_c)$, where $F_{\text{SI}}^{\tilde{S}}(R_c) = S(R_c)^{2(C-5)}B(R_c)^{4(C-3)}\{1 - [1 - L_{\text{BI}_1}(R_c)]^{C(C-2)}\}$ is a factor analogous to the factors $F_{\text{SI}}(R_c)$ and $F_{\text{BI}}(R_c)$ previously defined. This factor can be used to obtain the volume of saturated solids up to the critical radius R_c .

Parameter $L_{Bl_1}(R_c)$ may be obtained in the following form: combining Eqs. (6) and (7) results in:

$$L_{\rm BI}(R_{\rm c}) = B(R_{\rm c}) \left[1 - \left(1 - S(R_{\rm c}) \left[1 - \left(1 - L_{\rm BI}(R_{\rm c}) \right)^{C-1} \right] \right)^2 \right]$$
(10)

Notice that exponent *C* in Eq. (7) has been replaced in the above equation by C - 1. This is because the bond under consideration is connected with two sites each one with only C - 1 additional bonds able to saturate (see Fig. 4).

Now, it can be shown that the number of FE bonds connected with the E sites in Fig. 6 is $C(C-2)^2 + 2(6-C)$. Furthermore, the number of E sites in this same unit is C(C-2), thus, if $L_{BI_1}(R_c)$ represents the probability for a single site E to be saturated, exponent C-1 in Eq. (10) will be replaced by $\frac{C(C-2)^2+2(6-C)}{C(C-2)}$, resulting in:



Fig. 6. Basic unit for solids.

$$L_{\text{BI}_{1}}(R_{\text{c}}) = B(R_{\text{c}})S(R_{\text{c}}) \Big[1 - \left(1 - L_{\text{BI}_{1}}(R_{\text{c}})\right)^{\frac{C(C-2)^{2} + 2(6-C)}{C(C-2)}} \Big]$$
(11)

Notice that exponent 2 in Eq. (10) has been replaced with 1 in the above equation, because all external bonds are connected with a single E site. Eq. (11) can be substituted in Eq. (9) to obtain $L_{PI}(R_c)$ representing the probability of solids to be saturated during a wetting process.

Furthermore, as 2(C - 2) represents the number of sites surrounding a solid (Fig. 6), then, the probability $G_{PI}(R_c)$ for a solid to remain dry during a wetting process can be obtained with the relationship:

$$G_{\rm PI}(R_{\rm c}) = \left(1 - L_{\rm SI_1}(R_{\rm c})\right)^{2(C-2)}$$
(12)

The procedure to derive $L_{SI_1}(R_c)$ is similar to that used to obtain $L_{BI_1}(R_c)$ and it starts by combining Eqs. (6) and (7). In this case, connectivity *C* is substituted by *C*/2 because it represents the number of *E* bonds connected with a single *S* site (see Fig. 6):

$$L_{\rm SI_1}(R_{\rm c}) = S(R_{\rm c}) \left\{ 1 - \left[1 - B(R_{\rm c}) L_{\rm SI_1}(R_{\rm c}) \right]^{C/2} \right\}$$
(13)

The procedure to obtain the saturated, the dry, and the unsaturated fractions during a drying process is similar to that employed for a wetting process. For example, consider a soil undergoing a drying process. If $L_{PD}(R_c)$ represents the probability that all sites surrounding a solid remain saturated, it can be calculated with the following relationship:

$$L_{\rm PD}(R_{\rm c}) = L_{\rm SD_1}(R_{\rm c})^{2(C-2)}$$
(14)

with $L_{SD_1}(R_c)$ given by:

$$L_{\rm SD_1}(R_{\rm c}) = S(R_{\rm c}) + \left[1 - S(R_{\rm c})\right] \left\{ B(R_{\rm c}) + \left[1 - B(R_{\rm c})\right] L_{\rm SD_1}(R_{\rm c}) \right\}^{C/2}$$
(15)

Parameter $L_{SD_1}(R_c)$ represents the probability for a surrounding site (BU) to remain saturated.

In order to compute the total volume of dry sites and bonds surrounding a solid during a drying process, it is necessary to establish its corresponding probability distribution $G_{PD}(R_c)$ as follows. A solid belongs to the dry fraction when it complies with the following two conditions: all its surrounding sites and bonds must be able to dry $([1 - S(R_c)]^{2(C-2)}[1 - B(R_c)]^{4(C-3)})$ and, at least one external bond should already be dry and connected with the bulk of gas $[1 - L_{BD_1}(R_c)^{C(C-2)}]$. These two conditions are multiplied as they must occur simultaneously, therefore:

$$G_{\rm PD}(R_{\rm c}) = \left[1 - S(R_{\rm c})\right]^{2(C-2)} \left[1 - B(R_{\rm c})\right]^{4(C-3)} \left[1 - L_{\rm BD_1}(R_{\rm c})^{C(C-2)}\right]$$
(16)

where $L_{BD_1}(R_c)$ represents the probability for a surrounding bond to remain saturated during a drying process. Eq. (16) can be rearranged to obtain $G_{PD}(R_c) = F_{SD}^d(R_c)[1 - S(R_c)]$, where $F_{SD}^d(R_c)$ represents the fraction of those dry sites surrounding dry solids during a drying process; consequently, their total volume can be computed as $V_{SD}^d(R_c) = V_S F_{SD}^d(R_c) (\int_{R_c}^{R_{max}} S(R_c) dR) = V_S F_{SD}^d(R_c)[1 - S(R_c)]$. This very same Eq. (16) can be rearranged to obtain the volume of dry bonds surrounding a solid during a drying process $V_{BD}^d(R_c) = V_B F_{BD}^d(R_c) (\int_{R_c}^{R_{max}} B(R_c) dR)$ where parameter $F_{BD}^d(R_c)$ can be obtained by isolating factor $[1 - B(R_c)]$ from Eq. (16). Notice that parameter $G_{PD}(R_c)$ can be applied to sites or bonds indistinctly because the probability to have one dry solid is the same as to have four dry sites connected through four dry bonds (in the bidimensional case).



Fig. 7. Barcelona clay grain size distribution.

The value of $L_{BD_1}(R_c)$ can be computed by combining Eqs. (8a) and (8b), thus obtaining $L_{BD}(R_c) = B(R_c) + [1 - B(R_c)][S(R_c) + [1 - S(R_c)][L_{BD}(RC)]^C]^2$. Here, exponent *C* must transform into $\frac{C(C-2)^2 + 2(6-C)}{C(C-2)}$ which represents the number of FE bonds connected with a single external site, while exponent 2 is changed into 1 because it represents the number of external bonds connected with a single external site. Additionally, the factor $[1 - B(R_c)]S(R_c)$ is removed because this product implies that some sites are connected with larger bonds, condition forbidden by the constructive principle. These considerations lead us to finally obtain:

$$L_{\text{BD}_{1}}(R_{\text{c}}) = B(R_{\text{c}}) + \left[1 - B(R_{\text{c}})\right] \left\{ \left[1 - S(R_{\text{c}})\right] L_{\text{BD}_{1}}(R_{\text{c}})^{\frac{C(C-2)^{2} + 2(6-C)}{C(C-2)}} \right\}$$
(17)

Moreover, the degree of saturation of the unsaturated fraction for any suction value can be calculated in the form $S_r^u = V_w^u/V_v^u$, where V_w^u represents the volume of water of the unsaturated fraction, which is obtained from relationship $V_w^u = V_w - V_w^s$, where V_w is the total volume of water of the sample and V_w^s is the volume of water of the saturated fraction. The volume of voids of the unsaturated fraction (V_v^u) can be obtained from the following equation: $V_v^u = V_v - V_v^s - V_v^d$, where V_v , V_v^s and V_v^d represent the total volume of voids, the volume of voids of the saturated fraction and that of the dry fraction, respectively. Notice that in this form, the volume of any site or bond is assigned to a single fraction.

Once f^s , f^u and S^u_r are obtained, the mean effective stress can be calculated using Eq. (3) and Eq. (1) can be used to compute the slope of the failure envelope M.

4. Numerical and experimental comparisons

4.1. Experimental data of a Barcelona clay

To evaluate the capability of the proposed model to simulate the strength of unsaturated soils, experimental data concerning the Barcelona clay reported by Buenfil [10] have been used. These data come from a series of suction controlled triaxial tests on remolded soil samples showing sand, silt, and clay contents of 40, 42, and 18%, respectively. The aim of the whole experimental study was to observe the hydro-mechanical behavior of samples subjected to different suctions. Basic properties of tested samples are detailed in Table 1. According to Barrera [11], the clay fraction consists primarily of illite minerals. Table 1 contains some mass-volume properties as well as the USCS classification for the Barcelona clay.

The grain-size distribution, obtained from sieve analysis complemented with the hydrometer technique, is shown in Fig. 7. This figure also shows the best fitting of the numerical curve using proper μ and σ parameters. The best fit was accomplished by considering that the soil shows a bimodal distribution where $V_{SOL} = V_{SOL_1} + V_{SOL_2}$ (see Fig. 8). The parameters used to represent these distributions were: $\mu_{SOL_1} = 0.7$, $\sigma_{SOL_1} = 4.2$; $\mu_{SOL_2} = 0.0005$, $\sigma_{SOL_2} = 6.6$.

The procedure to prepare compacted soil specimens of Barcelona clay for the triaxial compression tests was as follows. The soil was air dried under laboratory conditions and then carefully mixed with water to reach a water content of 12%. The mixture was sealed in airtight bags for at least 24 hours to allow homogenization. Static compaction was performed using a compression speed of 0.2 mm/min in moulds of 3.8 cm diameter and 7.6 cm height. Each sample was packed in three layers under a static pressure of 0.27 MPa to achieve the gravimetric and volumetric characteristics shown in Table 1. Suction was measured in compacted soil samples using a high-range tensiometer which produced an average value of 270 kPa. However, some specimens reduced their volume when they were subjected to smaller suction (e.g. samples shrunk when suction was brought to 200 kPa). These experimental observations among others suggest that the initial suction for Barcelona clay ranged between 100 and 200 kPa. Buenfil [10] suggests that initial suction was close to 100 kPa.



Fig. 8. Particle size distribution for Barcelona clay.

Table 2 Confining stresses σ_3 for different specimens (kPa).





Fig. 9. Theoretical (T) and experimental (E) soil-water retention curves.

Once samples were set up in the triaxial test system, a suction equalization stage was applied through the axis translation technique. Water pressure u_w varied while air pressure u_a was maintained constant until the desired suction value was reached. Mean net and deviator stresses of 23 and 10 kPa, respectively, were applied in order to guarantee continuous contact between triaxial apparatus and soil specimen. The equalization stage was assumed to be completed when water content as well as volume deformations were stabilized (or when the change in water content was less than 0.04% per day).

During the equalization stage, volumetric collapse was observed in those specimens reaching the saturated condition (s = 0). In the case of these samples, the volumetric collapse reached an average value of 4.9%. Sample IWD also showed collapse during saturation although its equalization stage was somewhat different. Initially this sample was brought to a suction of 200 kPa. This generated a small volumetric reduction in the sample. Then, the suction was reduced to 0 kPa (i.e. saturated conditions). At this last stage, a volumetric collapse of 5.6% was reported, mainly occurring at the range between 10 to 0 kPa of suction. Finally, the sample was subjected to a suction increment of 100 kPa which led the soil to shrink an additional 12.8%. This volumetric deformation would generate a radical change in the structure of the sample, producing a substantial change in parameters μ and σ of the pore-size distribution and on the values of parameter χ . Additionally, according to the critical state theory, the shear strength of soils depends on the mean net stress p_{net} and the preconsolidation stress which evolves with the plastic volumetric strains [12]. With this in mind, it can be pointed out that specimen IWD suffered larger plastic volumetric strains during the equalization stage which is why the strength of this sample cannot be directly compared with the other specimens.

At the end of the equalization stage, the isotropic net stress p_{net} was increased in drained conditions (constant suction), while the volumetric deformation was recorded until the isotropic net stress reached the value indicated in Table 2 for each test. Finally, the deviator stresses q_f was increased while the confining stresses σ_3 and suction remained constant.

The soil-water retention curves of the Barcelona clay are depicted in Fig. 9, including the wetting (circles) and the drying (squares) branch. These data were obtained from suction controlled odometric tests using samples 5 cm in diameter and 10 cm in height subjected to a constant vertical net stress of 20 kPa. These samples were obtained using the same compaction procedure as for the other samples. At the end of compaction, these samples had the characteristics indicated in Table 1 and exhibited a suction of around 100 kPa. From this value, suction was reduced by steps to 10 kPa using the axis translation technique. Then, additional suction reductions up to almost complete saturation were imposed to the sample using the negative water column technique [13]. Finally, wetting paths were conducted from values of suction near 0 kPa to 8 kPa using the negative water column technique.

Fig. 9 also shows the fitting of the numerical soil-water retention curves obtained from the probabilistic solid-porous model. Buenfil [10] pointed out that there is strong evidence that the initial part of the wetting path (identified with



Fig. 10. Theoretical (line) and experimental (circles) pore-size distributions for the Barcelona clay.

| Та | bl | e | 3 |
|----|----|---|---|
| | | | |

| Parameters e | employed to | reproduce | the grain and | pore-size | distribution | of the | Barcelona | clav. |
|--------------|-------------|-----------|---------------|-----------|--------------|--------|-----------|-------|
| | | | | P | | | | , |

| | Standard deviation σ | Mean value of the distribution μ |
|------------|-----------------------------|--------------------------------------|
| Solids 1 | 4.2 | 0.7 |
| Solids 2 | 6.6 | 0.0005 |
| Macropores | 3.5 | 1.5 |
| Mesopores | 3.8 | 0.005 |
| Bonds 1 | 3.5 | 2.0 |
| Bonds 2 | 9.0 | 0.005 |

rhombus in Fig. 9) is in fact a wetting scanning curve (see, e.g. [14]), since the slope of the curve is rather small at this suction range (100 to 10 kPa) compared with the slope at the range from 10 to 1 kPa.

In addition, Buenfil [10] carried out mercury intrusion porosimetry tests (MIP) on compacted samples of the Barcelona clay. These results are shown by circles in Fig. 10. This figure also shows the numerical pore-size distribution obtained for the best fit of the soil-water retention curves shown in Fig. 9. As in the case of the grain size distribution, the fitting of the soil-water retention curves was achieved under the consideration that the porosimetry of the Barcelona clay shows a bimodal distribution, as in fact it was experimentally observed according to Fig. 10. Other researchers [15,16] also report bimodal pore-size distributions for compacted kaolin.

In Fig. 10 it can be observed that the sizes of sites corresponding to the peaks of the numerical and experimental distributions coincide. However, relative volumes for mesopores and macropores are larger and smaller than the experimental values, respectively. These discrepancies can be attributed to the diverse structure of samples used for MIP test and retention curves. While the soil sample used to obtain the retention curves was subjected to the equalization stage, the sample used to obtain the pore-size distribution was not. It is known that volumetric deformation is mainly generated by the shrinkage of macropores [7]. In such a case, the pore size distribution shows a reduction in the relative volume of macropores while that of mesopores seems to increase. Because the experimental retention curves were used to obtain the numerical pore-size distribution of the material, this would explain the differences between these curves.

Furthermore, notice that theoretical bonds volume (bonds 1 and bonds 2) can be neglected from the total volume. More detailed descriptions concerning the entities used to model unsaturated soil structures can be found elsewhere [17,18].

The parameters used to reproduce the fits in Figs. 7 and 9 are included in Table 3.

At this stage of the process, it is now possible to calculate parameters f^s , f^u and S^u_r , and, therefore, it is possible to apply Eq. (5) in order to obtain the effective stresses for any degree of saturation. These parameters are plotted on Fig. 11 as a function of the degree of saturation S_r .

The values of parameters f^s , f^u and S^u_r shown in Fig. 11 were used to calculate Bishop's parameter ($\chi = f^s + f^u S^u_r$) included in Eq. (5).

The results of the shear triaxial tests are presented in Fig. 13 in the axes of net stress against deviator stress. Air and water pressures in the triaxial chamber were controlled to maintain a constant suction of 0, 0.4, 10, 100 and 600 kPa. Samples subjected to the same suction approximately align with a slope similar to that observed for saturated samples.

When these results are plotted on the effective stress axis instead of the net stress axis, Fig. 14 is obtained. For this plot, the data presented in Figs. 12, 13 and Eqs. (1) and (5) have been used.

Fig. 14 shows that all failure points (p_f , q_f) align, independently of whether the soil is in saturated or unsaturated conditions. With this in mind and recalling that Buenfil [10] reported that the initial suction of compacted samples was between 100 and 200 kPa, it can be pointed out that the specimen subjected to 600 kPa in suction followed a drying path, while those subjected to suctions of 100, 10, 0.4 and 0 kPa followed a wetting path prior to failure. These results indicate that the effective stress concept can be applied to the strength of soils under any suction and drying–wetting path.

A slight scattering can be observed in the experimental results presented in Fig. 14, in particular the failure point of test IWD. This scattering may be explained by the fact that samples subjected to failure experience important volumetric deformation. This reduction in volume makes both branches of the soil–water retention curves displace to larger suctions, modifying the values of parameter χ , which in turn affects the value of the effective stress. In that sense, volumetric strains



Fig. 11. (a) Saturated fraction, (b) unsaturated fraction and (c) degree of saturation of the unsaturated fraction, as a function of the Barcelona clay saturation degree.



Fig. 13. Barcelona clay shear strength.

(kPa)

should be taken into account in the porous model to correctly determine the effective stresses. However, at its present stage, the porous model does not take into account volumetric strains, and therefore the simulation presented herein can still be improved.

Notice from Table 4 that all samples suffered negligible volumetric strains at the end of the equalization stage, except for saturated specimens SI1 and SI2, which experienced volumetric collapse of about 5% during this stage. Likewise, sample IWD collapsed during saturation, but, unlike SI1 and SI2, it was taken through new progressive suction increments during equalization (s = 0 to s = 100 kPa), which led to additional volumetric strains. This second equalization stage resulted in a total volumetric strain of 18.38% (5.6% and 12.7% due to volumetric collapse and drying process respectively). Furthermore, in contrast with the other unsaturated specimens, IWD underwent a testing program (i.e. isotropic and triaxial compressions) under considerably larger volumetric strains compared with other unsaturated specimens (11, 12, 13, A1) (i.e. 20%



Fig. 14. Barcelona clay shear strength simulation through effective stresses.

Table 4Total volumetric strains (%).

| Specimen | After equalization | After applied isotropic mean net stress | After applied deviator stress |
|----------|--------------------|---|-------------------------------|
| I1 | 0.87 | 10.10 | 18.67 |
| | (unsaturated) | | |
| 12 | -0.27 | 5.90 | 14.60 |
| | (unsaturated) | | |
| 13 | -0.33 | 10.69 | 20.01 |
| | (unsaturated) | | |
| A1 | 0.00 | 8.06 | 21.49 |
| | (unsaturated) | | |
| IWD | 18.38 | 20.90 | 25.77 |
| | (unsaturated) | | |
| SI1 | -0.55 | 16.71 | 21.71 |
| | (saturated) | | |
| SI2 | 4.90 | 19.12 | 24.12 |
| | (saturated) | | |
| SI3 | 4.92 | No information | No information |
| | (saturated) | | |
| SM1 | 1.64 | No information | No information |
| | (saturated) | | |



Fig. 15. Experimental and numerical fitting for (a) the soil-water retention curves and (b) the grain-size distribution from residual undisturbed gneiss.

against 10% average). These considerations, among others described previously, may explain why IWD shifts away from the failure line in Fig. 14.

4.2. Strength of a residual gneiss

In this section, some experimental results for a residual gneiss [19] are presented and compared with the numerical simulations obtained from the porous–solid model. Undisturbed soil samples were obtained with initial water contents between 31 and 32%. Clay, silt and sand fractions were of the order of 46%, 9% and 45%, respectively. Retention curves were obtained using two different techniques: filter paper and axis translation techniques (see Fig. 15(a)). The experimental grain size distribution along with the numerical fitting are presented in Fig. 15(b). The parameters used to fit this figure are presented in Table 5.

Notice that experimental retention curves (Fig. 15) show that the drying process ended at a degree of saturation of 44.6% and then the samples were subjected to wetting up to a suction of 0.012 MPa.

The parameters used to reproduce the soil–water retention curves are shown in Table 5. Their corresponding relative volume distributions are depicted in Fig. 16 along with the experimental pore-size distribution of the material.

| | Mean value of the distribution | Standard deviation | |
|------------|--------------------------------|--------------------|--|
| | μ | σ | |
| Solids 1 | 6 | 3.5 | |
| Solids 2 | 0.0001 | 6 | |
| Macropores | 10 | 2.5 | |
| Mesopores | 0.0005 | 3 | |
| Bonds 1 | 8 | 4 | |
| Bonds 2 | 0.00001 | 13 | |

Table 5Parameters employed to reproduce the grain and pore-size distribution of residual gneiss.



Fig. 16. Theoretical (line) and experimental (circles) pore-size distributions for residual gneiss.



Fig. 17. Experimental soil-water retention curves from residual undisturbed gneiss.

The fitting of the experimental soil-water retention curves using the porous-solid model is shown in Fig. 17. Notice the typical shape of retention curves for a bimodal pore size distribution: first, an abrupt desaturation is underwent by the soil when suction ranges between 0.001 and 0.01 MPa. Then, further increase in suction only reduces slightly the degree of saturation (S_r). Finally, when suction becomes larger than 5 MPa, abrupt changes in the degree of saturation occur once more. This behavior is explained by the relative volume distribution shown in Fig. 16: the first step in the drying retention curve is produced by the drainage of macropores and the second by the drainage of mesopores.

Using these results, it is now possible to compute parameters f^s , f^u and S^r_n , and apply Eqs. (1) and (5) to obtain the effective stress for any degree of saturation the material. These parameters are plotted in Fig. 18 as a function of the saturation degree S_r .

Notice the influence of the shape of the retention curves (Fig. 17) on the parameters shown in Figs. 18 and 19. An abrupt change in the drying curves for the three parameters is observed as the degree of saturation reduces from 100 to 67%. For the wetting curves, a slight change in these curves can be seen when the degree of saturation becomes larger than 40%.

Along with the experimental results for suction-controlled triaxial tests for samples subjected to different confining stresses, the numerical simulations are presented in Fig. 20. The undisturbed samples of residual gneiss were initially subjected to an equalization stage and then their suction was increased to 100 or 300 kPa. Two to six hours were necessary to stabilize air and water pressures, then, the samples were loaded to the initial net mean stresses indicated in Table 6.

Then the samples were subjected to an increase in the deviator stresses in drained conditions up to failure. These results are plotted in Fig. 20 along with the results of samples tested in saturated conditions [20].

Once parameter χ has been defined, it is possible to obtain the shear strength envelope in the effective stress plane, as represented in Fig. 21. Again, failure stress states align with those obtained in saturated conditions, just as effective stress concept establishes.

Notice that samples subjected to 100 kPa shift away from general alignment. It is important to point out that even though critical state was intended to be achieved [21], this condition was not reached on some samples as volumetric strains were still occurring at the end of these tests as it can observed in Fig. 22.

At its present stage, the porous-solid model presented herein does not consider the collapse of macropores during loading or suction increase. This behavior can be included into the porous-solid model and is the subject of future work.



Fig. 18. (a) Saturated fraction, (b) unsaturated fraction and (c) degree of saturation of the unsaturated fraction, as a function of the degree of saturation.



Fig. 19. Parameter χ for wetting and drying paths.

Table 6Initial net mean stresses for suction controlled tri-axial compression tests.

| Suction | Initial net mean stress | |
|---------|--------------------------|--|
| (kPa) | (kPa) | |
| 100 | 25, 50, 100, 200 and 400 | |
| 300 | 25, 50, 100, 200 and 500 | |



Fig. 20. Residual gneiss shear strength.



Fig. 21. Residual gneiss shear strength simulation through effective stresses.



Fig. 22. Volumetric and axial strains of samples subjected to s = 100 kPa.

5. Conclusions

A probabilistic solid-porous model able to reproduce both branches of the soil-water retention curve is presented. The model can be generated from the grain and the pore-size distributions of the material. When the pore-size distribution is missing, it can be generated by fitting both branches of the numerical soil-water retention curve with the experimental ones. Comparisons between experimental and numerical results for the two soils show that the predicted pore-size distributions using the probabilistic solid-porous model are accurate.

A single failure line is obtained for samples tested at different suctions and following wetting or drying paths when plotted in the effective stress-versus-shear stress plane.

An improvement that needs to be included in the probabilistic porous–solid model is the effect of the volumetric deformation on the soil–water retention curves, which in turn affect the value of parameter χ .

References

- D. Sheng, Constitutive modelling of unsaturated soils: Discussion of fundamental principles, in: E.E. Alonso, A. Gens (Eds.), Unsaturated Soils, CRC Press, Boca Raton, FL, 2011, pp. 91–122.
- [2] N. Lu, J.W. Got, D.T. Wu, A closed-form equation for effective stress in unsaturated soil, Water Resour. Res. (2010) 1–14.
- [3] E. Rojas, J. Horta, T. López-Lara, J.B. Hernández, A probabilistic solid-porous model to determine the shear strength of unsaturated soils, Probab. Eng. Mech. 26 (2011) 481-491.
- [4] A. Uchaipichat, A Hydro-mechanical model for unsaturated soils, Int. J. Eng. Appl. Sci. 6 (2010) 207–211.
- [5] E. Rojas, Equivalent stress equation for unsaturated soils. I: Equivalent stress, Int. J. Geomech. 8 (2008) 285-290.
- [6] ASTM D4404, Standard Test Method for Determination of Pore Volume, Pore Volume Distribution of Soil, Rock by Mercury Intrusion Porosimetry, ASTM International, 2010.
- [7] P.H. Simms, E.K. Yanful, Measurement and estimation of pore shrinkage and pore distribution in a clayey till during soil-water characteristic curve tests, Can. Geotech. J. 38 (2001) 741–754.
- [8] F.A.L. Dullien, Porous Media: Fluid Transport and Pore Structure, Academic Press, San Diego, CA, 1992.
- [9] W.B. Haines, Studies in the physical properties of soils: IV. A further contribution to the theory of capillary phenomena in soil, J. Agric. Sci. 17 (1927) 264–290.
- [10] C. Buenfil, Caracterización experimental del comportamiento hidromecánico de una arcilla compactada, PhD thesis, Universidad Politécnica de Cataluña, 2007 (in Spanish).
- [11] M. Barrera, Estudio experimental del comportamiento hidro-mecánico de suelos colapsables, PhD thesis, Universidad Politécnica de Cataluña, 2002 (in Spanish).
- [12] E.J. Murray, V. Sivakumar, Unsaturated Soils A Fundamental Interpretation of Soil Behaviour, Wiley-Blackwell, Chichester, UK, 2010.
- [13] E.E. Alonso, E. Romero, Collapse behavior of sand, in: UNSAT-ASIA, 2003, pp. 325-334.

- [14] N. Lu, W.J. Likos, Unsaturated Soil Mechanics, John Wiley & Sons, New York, 2004.
- [15] R. Thom, R. Sivakumar, V. Sivakumar, E.J. Murray, P. Mackinnon, Pore size distribution of unsaturated compacted kaolin: the initial states and final states following saturation, Geotechnique 57 (2007) 469–474.
- [16] V. Sivakumar, R. Sivakumar, E.J. Murray, P. MacKinnon, J.L. Boyd, Mechanical behaviour of unsaturated kaolin with isotropic and anisotropic stress history. Part 1: Wetting and compression behaviour, Geotechnique 60 (2010) 581–594.
- [17] E. Rojas, A. Zepeda, M.L. Pérez-Rea, J. Leal, G. Gallegos, A four elements porous model to estimate the strength of unsaturated soils, Geotech. Geol. Eng. 29 (2009) 193-202.
- [18] J. Horta, E. Rojas, M.L. Pérez-Rea, T. López, J.B. Zaragoza, A random solid-porous model to simulate the retention curves of soils, Int. J. Numer. Anal. Methods Geomech. 37 (2013) 932–944.
- [19] M.M. Futai, M.S.S. Almeida, An experimental investigation of the mechanical behaviour of an unsaturated gneiss residual soil, Geotechnique 55 (2005) 201–213.
- [20] M.M. Futai, M.S.S. Almeida, W.A. Lacerda, Yield, strength and critical state conditions of a tropical saturated soil, J. Geotech. Geoenviron. Engng. 130 (2004) 1169–1179.
- [21] K.H. Roscoe, A.N. Schofield, C.P. Wroth, On the yielding of soils, Geotechnique 8 (1958) 22-53.