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Micromechanics of granular materials – A tribute to Ching S. Chang

Particle size and boundary geometry effects on the bulk friction coefficient of sheared granular materials in the inertial regime

Andrés D. Orlando [a,*,1], Hayley H. Shen [b,2]

[a] Jenike & Johanson, Inc., 400 Business Park Drive, Tyngsboro, MA 01879, USA
[b] Department of Civil and Environmental Engineering, Clarkson University, 8 Clarkson Ave., P.O. Box 5710, Potsdam, NY 13699-5710, USA

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ABSTRACT

Glass beads of varying diameters ($d = 2, 3, 4$, and 5 mm) are used to measure the ratio of shear-to-normal stress, or bulk friction coefficient, generated inside an annular shear cell at high shearing rates. The effects of the particle size, the solids concentration, and the shear rate are explored. It is found that (1) for a given particle size, the magnitude of the bulk friction coefficient decreases with increasing solids concentration, (2) for a given solids concentration, the bulk friction coefficient decreases with increasing particle size, and (3) the bulk friction coefficient is independent of the shear rate except for cases with low solids concentration, where it decreases with increasing shear rate. The boundary geometry is found to affect bulk friction only for dilute (low solids concentration) flows involving small particles.

1. Introduction

In geotechnical engineering, the shear strength of a soil is measured using the angle of internal friction. In the absence of cohesion, the tangent of this angle, also known as the bulk friction coefficient of the soil, is equal to the ratio of shear-to-normal stress at the plane of failure. This is usually the failure criterion used in engineering design. A similar idea was adopted by Bagnold [1] in his work in sediment transport. In such flows the major momentum transfer is through particle collisions, referred to as the "inertial" regime. Bagnold assumed that a dynamic yield criterion is satisfied at the boundary between movable and immovable grains. He based this on measurements of shear and normal stresses generated by shearing a neutrally buoyant dispersion of spheres between two concentric cylinders [2]. In his experiments the bulk friction coefficient varied from 0.32 to 0.75 depending upon the flow conditions and characteristics of the granular material. This yield criterion for granular fluid flows was later validated by Hanes and Inman [3,4] using an annular shear cell similar to the one employed in the current study. During annular shear cell experiments using glass beads of 1.1 and 1.85 mm diameter they noted that in many cases the flow was divided into two distinct regions: (1) a shearing (upper) region and (2) a non-shearing (lower) region. Their data showed a nearly constant (with respect to shear rate) bulk friction coefficient at the boundary between a rapidly flowing granular layer and a stationary but movable granular bed. Their data shows that

* Corresponding author.
E-mail addresses: aorlando@jenike.com (A.D. Orlando), hhshen@clarkson.edu (H.H. Shen).
[1] Project Engineer.
[2] Professor.

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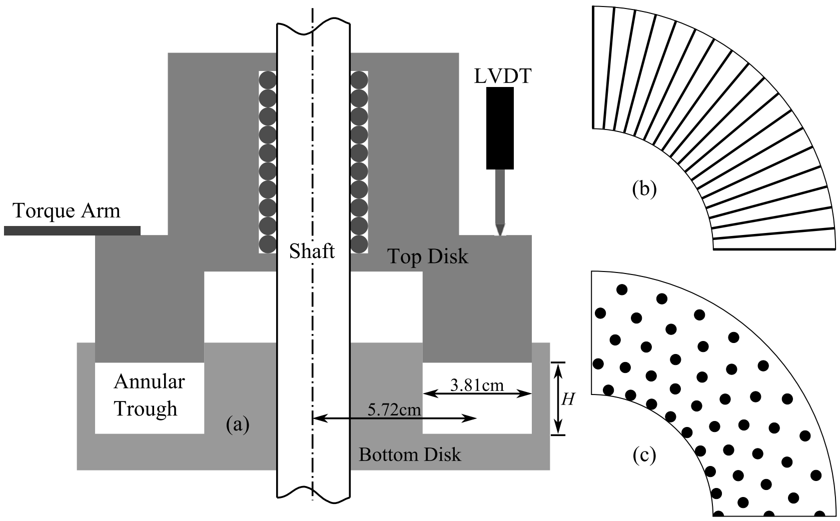

Fig. 1. Schematic of (a) the annular shear cell used in the experiments, (b) boundary type 1, and (c) boundary type 2.

the bulk friction coefficient is nearly independent of particle diameter, for that narrow range in size. More recently Orlando and Shen [5] showed that the particle size and boundary geometry have a marked effect on the shear stress measured on the upper boundary of an annular shear cell at high shearing rates. However, the effects of particle size and boundary geometry on the bulk friction coefficient of a granular material were not explored. In the current work their data is analyzed in terms of the bulk friction coefficient and the effects that parameters such as particle size, solids concentration, shear rate, and boundary geometry have on it.

2. Experiments and results

Orlando and Shen [5] describe annular shear cell experiments used to study the effect of particle size and boundary geometry on the measured shear stress at the upper boundary. The upper and lower boundaries were roughened by machining either 72 equally distributed half-cylinders (boundary type 1) of 1.59 mm diameter along the radial direction or hemispheres (boundary type 2) of 3.18 mm diameter in a polar hexagonal close packed configuration. The materials used in their work were glass beads of nominal diameter $d = 2, 3, 4$, and 5 mm with a density of 2.6 g/cm^3. In the current study their data is presented in terms of the bulk friction coefficient measured at the upper boundary. A schematic of the shear cell used in the experiments and the boundary geometries are shown in Fig. 1. The annular trough was machined out of aluminum into the bottom circular disk and was driven by a 3/4 hp DC servo-controlled motor. The top disk was mounted on the shaft using bearings to allow smooth vertical displacement and rotation, and alongside a set of counterweights was used to adjust the normal load on the granular material (0–4.5 kPa). The torque developed by the flow was measured using a full bridge thin beam load cell attached to the torque arm, and the displacement of the top disk was measured using a Linear Variable Differential Transformer (LVDT). Data from the LVDT and the load cell were recorded simultaneously. The angular velocity of the bottom disk was measured using a tachometer.

The experiments were run by depositing a known mass of glass beads in the annular trough. The top disk was allowed to rest fully on the granular bed, and the initial height of the bed was measured using Vernier calipers. After the normal load was adjusted to a desired value by using counterweights, the motor was started and the speed of the annular trough increased until the desired height H in Fig. 1, and thereby the desired average solids concentration ν was reached. After readings of the torque and rotational speed were taken, the normal load was increased. The rotational speed was also increased until the same height H was obtained, and new readings of the torque and rotational speed were taken for the current normal load. This was done until enough data points were collected. The shear and normal stresses generated by the flow at the upper boundary and the apparent shear rate were calculated as in Savage and Sayed [6]:

$$\tau = \frac{3}{2}\frac{T}{\pi(R_o^3 - R_i^3)} \tag{1}$$

$$\sigma = \frac{N}{\pi(R_o^2 - R_i^2)} \tag{2}$$

$$\gamma = \frac{\omega(R_o + R_i)}{2H} \tag{3}$$

where τ, σ and γ are respectively the shear and normal stress at the upper boundary and the apparent shear rate. T is the measured torque, N is the applied normal load, ω is the angular velocity of the annular trough, and R_o and R_i are the

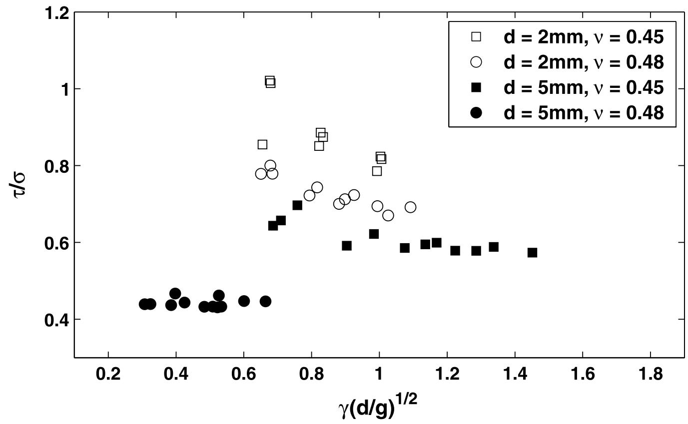

Fig. 2. Results of bulk friction coefficient vs. non-dimensional shear rate for $d = 2$ and 5 mm, different solids concentration $\nu = 0.45$ and $\nu = 0.48$, and boundary type 1.

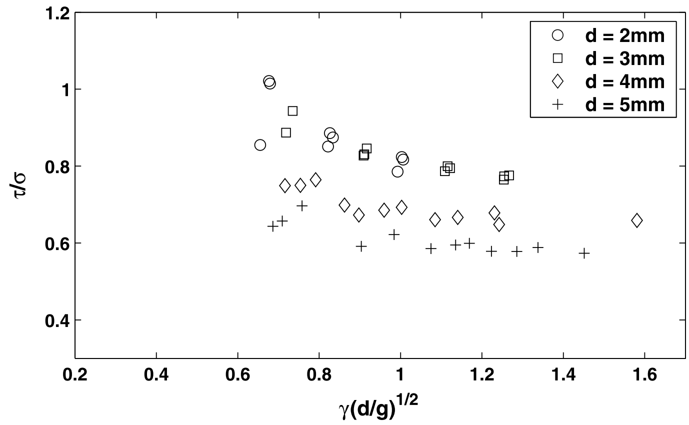

Fig. 3. Results of bulk friction coefficient vs. non-dimensional shear rate for $d = 2, 3, 4$, and 5 mm, solids concentration $\nu = 0.45$, and boundary type 1.

outer and inner radii of the annular trough. The solids concentration is determined as the ratio of the total particle volume in the trough to the total cell volume [5].

In the following figures results of three different experiments per solids concentration/particle diameter/boundary type combination are included. The results of the bulk friction coefficient are presented as a function of the non-dimensional shear rate $\gamma(d/g)^{1/2}$, which is the standard parameter for rapid granular flows. Fig. 2 shows results for experiments using glass beads of $d = 2$ and 5 mm and boundary type 1. It can be seen that the sensitivity of bulk friction coefficient relative to shear rate varies with particle size. Bulk friction of larger particles is insensitive to the shear rate, but not so for smaller particles. Due to equipment limitations, experiments with high solids concentration and large particles could only be run at relatively low shearing rates. Also noteworthy is that the bulk friction coefficient decreases with increasing solids concentration. A similar trend was observed by Savage and Sayed [6] using a similar annular shear cell and glass beads of $d = 1.8$ mm. They explained this phenomenon by noting that during a continuous, fully developed granular shear flow (in the dense limit) the particles tend to line up into distinct layers, thus facilitating the shearing motion within the flow. Since the current apparatus was opaque, we could not directly verify this flow kinematics. Numerical simulations conducted have confirmed this idea [7]. Experiments using boundary types 2 yielded similar trends.

Figs. 3 and 4 show results of the bulk friction coefficient for solids concentrations $\nu = 0.45$ and 0.48 respectively, particle diameters $d = 2, 3, 4$, and 5 mm and boundary type 1. It can be seen that the particle diameter plays a role in the overall behavior of the flow, with smaller particles having a higher bulk friction coefficient than larger ones. This phenomenon is very clear in the denser flows of Fig. 4, where the bulk friction coefficient for particles with $d = 4$ and 5 mm is almost 50% less than for particles with $d = 2$ and 3 mm. The same layering argument used by Savage and Sayed [6] may explain the dependence of the bulk friction coefficient on the solids concentration. Since all tests were run using the same total mass (400 g) of glass beads, larger particles resulted in fewer particles than tests run with smaller ones. With fewer particles, the layering phenomenon mentioned earlier would be reached more easily, thereby reducing the shearing resistance of the flow. Comparing Fig. 3 to Fig. 4, the particle size effect is more pronounced for higher solids concentration. At $\nu = 0.48$ there appears to be a sudden transition between particles of $d = 2$, 3 mm, and $d = 4$, 5 mm.

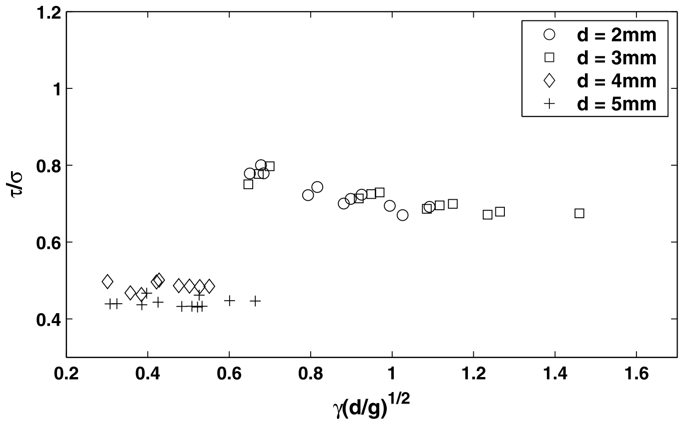

Fig. 4. Results of bulk friction coefficient vs. non-dimensional shear rate for $d = 2, 3, 4$, and 5 mm, solids concentration $\nu = 0.48$, and boundary type 1.

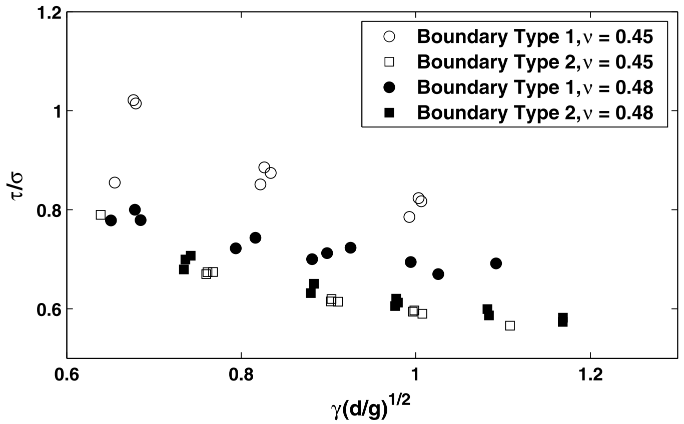

Fig. 5. Results of bulk friction coefficient vs. non-dimensional shear rate for $d = 2$ mm, solids concentration $\nu = 0.45$ and $\nu = 0.48$ for boundary types 1 and 2.

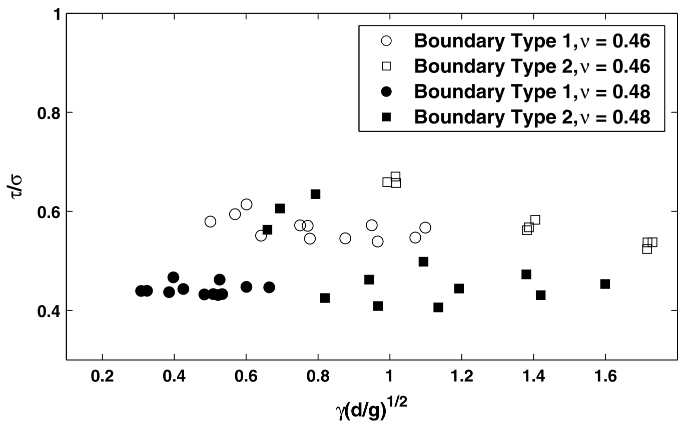

Fig. 6. Results of bulk friction coefficient vs. non-dimensional shear rate for $d = 5$ mm, solids concentration $\nu = 0.46$ and $\nu = 0.48$ for boundary types 1 and 2.

The effect of the boundary geometry on the measured bulk friction coefficient is shown in Figs. 5 and 6 for particle diameters $d = 2$ and 5 mm, respectively. Note that due to equipment limitations, solids concentrations of 0.45 were not attainable with 5-mm particles, instead a solids concentration of 0.46 is used for the comparison in Fig. 6. The bulk friction coefficient is dependent on the boundary geometry, especially for the smaller particles (Fig. 5). The same layering argument used earlier explains this behavior [5]. Since the bulk friction coefficient is lower with boundary type 2, one would be tempted to suggest that this type of boundary facilitates the layering phenomenon.

3. Discussion and conclusions

Results of annular shear cells experiments are presented. The bulk friction coefficient is found to decrease with increasing shear rate, particle size, and solids concentration. This phenomenon is present for all boundary types tested. It is found that the bulk friction coefficient is highly dependent on the particle diameter, particularly for flows in the dense limit. The results can vary by up to 50% depending on the particle size and flow conditions. It is hypothesized that a heightened ability of larger particles to form distinct layers during a continuous, fully developed granular shear flow may contribute to the reduction of shearing resistance. The results are also dependent on the solids concentration, with the bulk friction coefficient decreasing with increasing solids concentration regardless of the boundary geometry. The shear rate seems to have a negligible effect in most cases. Only in the more dilute flows with smaller particles a clear dependence on the shear rate is observed, where the bulk friction coefficient decreases with increasing shear rate. A plausible explanation for this behavior is that at low shearing rates, the normal stress (or confining pressure) is relatively low. This results in the two regions of flow previously described by Hanes and Inman [3] where a layer of particles is sheared above a non-shearing layer. With a shallower shearing region, less energy is lost due to friction between adjacent layers and therefore more of the kinetic energy supplied from the rotating annular trough is delivered to the upper boundary, thereby increasing the rate of exchange of momentum between particles and boundary and consequently the shear stress. This and the layering hypotheses mentioned earlier have been confirmed using discrete element simulations of the annular shear cell experiments [7]. The effect of the boundary geometry on bulk friction seems to be noticeable only in the more dilute flows and flows involving small particles.

Through a systematic laboratory study of different sizes of particles and two boundary types we found that the bulk friction coefficient is inter-related to the relative size of the particle to the apparatus, and the property of the boundary. There appear to be two regimes. In one regime the bulk friction coefficient is independent of the shear rate, while in the other it decreases with increasing shear rate. The transition from one regime to the other is over a slight change of the solid concentration. This phenomenon suggests that shear flow of granular fluid materials at a dense phase is unlikely to remain steady. Furthermore, due to the inter-relationship between the particles and the boundary, the bulk friction coefficient needs to be considered as a combined parameter between the materials and the measuring apparatus.

From this study we also observe the strong effect of the relative size of the particles and the apparatus. The effect of particle size on the bulk behavior of granular materials is of particular importance to researchers using discrete element simulations. Many times in an attempt to drive the simulation time down, the particles are "coarsened" by making the simulation particles larger than their physical counterparts. By doing this, the bulk behavior of the flow is being inadvertently changed. When using simulation results in the design of bulk solids handling equipment, the effect of coarsening particles should be taken into consideration. In addition, our results show that boundary type may also contribute to the coarsening effect on bulk friction coefficient.

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