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Topological approach to solve frame structures using topological collections and transformations

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ABSTRACT

The present work tackled the modeling of frame structures using a topological approach based on the concepts of topological collections and transformations. The topological collections are used to specify the interconnection law between the frame structures and the transformations that are used to describe their behavior. As a language allowing the application of this approach, we applied the MGS (Modeling of General System) language. To validate this approach, we studied the case of two- and three-dimensional frame structures. Then, the results obtained using the MGS language are presented and compared to those obtained by the structural calculation software by the finite-element method RDM6. For both studied cases, we find that the results obtained by MGS language based on the notions of topological collections and transformations and those obtained by the RDM6 software based on the finite element method are very close, which validates our approach. Using this topological approach, any structure can be characterized by local relations between its elements, thus making it possible to dissociate its topology and its physics. Indeed, in our topological approach, we separately define the topology of the studied frame structure and the local behavior law as well as the equilibrium equations of its various components. Therefore, this topological approach might be generalized to model complex systems which can be considered as a set of local elements linked by a neighborhood relationship.

1. Introduction

This work is a part of a general research in which we tried to adapt a topological approach to separate the topology and the behavior of the studied system in order to have a generic local model allowing the optimization of the system behavior according to the whole system. In particular, we are interested in the topological modeling of beam structures. In fact, structural analysis using a topological approach has been the research topic of several work studies. Lind [1] showed how to present trusses by graph and then analyzed them, Kron [2] applied topology and graph theory to the analysis of elastic networks (structures), Shai [3–5] developed a Combinatorial Representation (CR) based on graph and matroid theory

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and applied them to different engineering fields, in particular to the analysis of trusses, Plateaux [6,7] applied a topological graph to model multi-physics systems and in particular truss structure . . .

In this paper we are interested in applying a topological approach for the modeling of frame structures. This topological approach is based on topological collections and transformations and it remains valid, even in the case of bar structures (only axial loads) [8] or beam structures (only shear and bending loads). As a language allowing the application of this approach, we applied the MGS (Modeling of General Systems) modeling language. MGS (http://mgs.spatial-computing.org/) embedded the idea of topological collections and their transformations into the framework of a simple dynamically typed functional language [9,10], and it is devoted to the simulation of biological processes [11,12].

The originality of this work is the use of topological collections in order to present the topological structure (interconnection law) of the studied system and its transformations in order to describe its behavior. Thanks to the topological approach, any structure can be characterized by local relations between its elements, thus making it possible to dissociate its topology and its physics. Indeed, using this approach, we do not need to search the global stiffness matrix of the studied structure. In fact, we define the local behavior law and the equilibrium equations of a beam element across transformation independently of the topological structure, which is declared through the topological collections based on the neighborhood relationship between the different elements of a beam structure. Therefore, we study a structure from a local point view and not from a global point of view.

The remaining of this paper is organized as follows. In Section 2, we present a state of the art on modeling using the topological approach (graph theory), and then we introduce our approach based on the concepts of topological collections and transformations. In Section 3, we present the theoretical background of frame structures as well as the modeling steps using topological collections and transformations. In Section 4, we studied two cases of a 2D and a 3D frame structures, and the results obtained by MGS language are presented and compared to those obtained by RDM6 software. Finally, we drew our conclusions in Section 5.

2. Modeling using a topological approach

In this section, we begin with a brief review of the graph theory and its various applications, in particular in the modeling of mechanical systems. Then, we present our topological approach based on the concepts of topological collections and transformations.

2.1. State of the art: graph theory

The first known application of the graph was Euler's treatment of the Bridges of Königsberg (Kaliningrad) [13]. It is significant because it established the graph theory and showed that the abstractions of reality can indeed be useful to solve problems in the real world. Since the 1930s, the graph theory has known very significant developments on the theoretical level as well as on the practical level and has become an established branch of mathematics dedicated to the study of system topologies. The Graph theory has been extremely useful in computer sciences, electrical engineering, communication networks, and transport networks (rail, road, air) as well as in a number of other applied disciplines.

The first application of topology and graph theory to the analysis of structures seems to be due to Kron [2,14], who used an analogy between electrical networks and elastic structures. Kron applied the method of piecewise solutions of large-scale systems, the so-called "diakoptics". In this method, a physical system is torn into an appropriate number of small subdivisions, each of which is analyzed and solved separately. The partial solutions are then interconnected step by step until the solution for the entire system is obtained.

Branin [15] relied on the work of Kron validated mathematically by Roth [16] on the electrical networks and machines, using the same topological structure to describe the physical parameters of multi-physics system: the generic modeling approach consists in representing the system by a topological structure (graph) with which we associate an algebraic structure. This structure connects the various topological entities (nodes, branches and meshes) of the graph. Then according to problem specification, we associate physical data with corresponding topological entities. These data are related with physical tensors.

Relying on these works, Plateaux [7] applied the topological graph, named "KBR" in honor of their creators (Kron, Branin and Roth), for the modeling of multi-physics systems and in particular for truss structures. Thus this graph allows us to get any relationship between its elements whatever the unknown parameters and specifications of the studied systems. Also, it allows the distinction between the topological structure and the associated physics of the studied system. On the other hand, it is limited to structure without moment (truss structure) and consequently cannot be applied for the modeling of a beam or a frame structure. Plateaux used MODELICA language (http://www.modelica) that allows the application of this KBR topological graph. In fact, MODELICA is an object-oriented language. It helps define models in a declarative manner and combine electrical, mechanical, hydraulic, thermodynamic, etc., within the same application model. It has also the advantage of being implemented in other types of software such as MATLAB/Simulink, Maple/MapleSim and CATIA V6... On the other hand, its topological nature is limited to 0 and 1 complexes and the access to higher dimensions can be done only via transformations. Besides, it associates topology with physics, which limits the generalization of the studied model [7].

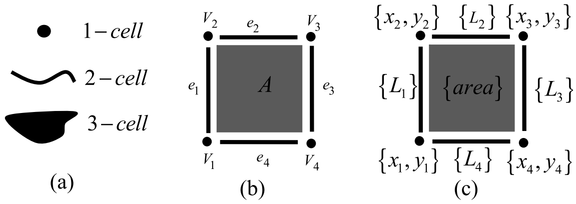

Fig. 1. Example of (a) k-cells, (b) a cellular complex, (c) a topological collection.

2.2. Our topological approach: topological collections and transformations

The topological approach suggested in this paper is based on the concepts of topological collections and transformations. The idea is then to see a mechanical system as a set of values equipped with a neighborhood relation called "topological collection". The behavior law, the equilibrium equations and the compatibility equations will be seen as functions defined by a set of rewritten rules using the topological relation, called "transformations".

In the following section, we present the concepts of topological collections and transformations.

The interested reader can referred to Spicher [9] for further information.

2.2.1. Topological collections

A topological collection is a collection in which the structure is captured by a neighborhood relationship among the data, that is to say, from providing one of the elements in the collection, we can provide all the other data which are directly related. The organization of a topological collection is founded on a cellular complex where values are associated with each cell.

A cellular complex is made up of elements of various dimensions called topological cells of dimension n or n-cells. Fig. 1(a) shows examples of k-cells, 0-cells represent vertices, 1-cells represent edges, 2-cell represent faces, etc. These basic elements are organized following the incidence relationship that relies on the notion of boundary: let c_1 and c_2 be respectively an n_1-cell and an n_2-cell with $n_1 < n_2$, c_1 is incident to c_2 if c_1 belongs to the border of c_2. More precisely, if $n_1 = n_2 - 1$, c_1 is called a face of c_2, and c_2 is a coface of c_1. Fig. 1(b) shows an example of a cellular complex. It is made up of four 0-cells (V_1, V_2, V_3, V_4), four 1-cells (e_1, e_2, e_3, e_4), and a 2-cell A. The boundary of A is formed by its incident cells $V_1, V_2, V_3, V_4, e_1, e_2, e_3$ and e_4. Especially, the four edges are the faces of A, and therefore, A is the coface of e_1, e_2, e_3 and e_4.

In conclusion, we can say that a topological collection is a cellular complex where the values are associated with each cell. An example of a topological collection is given in Fig. 1(c), in which positions are associated with the vertices, lengths with the edges and area with the face.

2.2.2. Transformations

A transformation is a function operating on the topological collections. It is defined by a set of rewritten rules of the form $m \Rightarrow e$. The left-hand part of a rule is called pattern and the right-hand part is the expression that will replace the instances of m. We note a transformation as follows $\{m_1 \Rightarrow e_1, m_2 \Rightarrow e_2, ..., m_n \Rightarrow e_n\}$.

The application of a transformation to a collection is as follows: a number of non-intersecting occurrences of the first pattern m_1 are selected and then replaced with the appropriate element calculated from e_1. When we cannot find a new instance of the first pattern, we select a number of non-intersecting occurrences of the second pattern among the elements which were not already selected, and so on. When this process is finished, we replace the selected elements by the new corresponding elements and the new collection is thus created. We distinguish:

- the functions defined by the case;
- the path transformations—they allow the update of values associated with cells;
- patches: they allow modifications of the topological structure of a collection.

3. Modeling of frame structure using topological collections and transformations

In this section, we begin with a brief review of the frame structure theory [17]. Then we present the modeling steps based on the notion of topological collections and transformations using the MGS language.

3.1. Frame structure theory

The frame structure is seen to possess the properties of both truss and beam structures. In fact they are capable of carrying both axial and transverse forces, as well as moments. Thereby, a frame structure can be found in most of our real world structural problems.

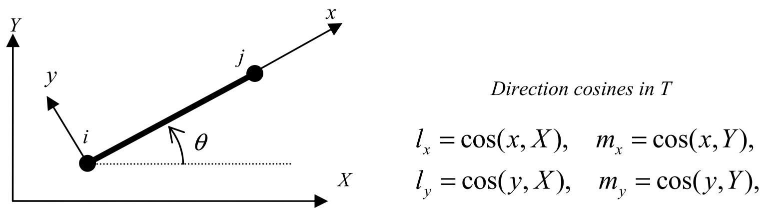

Fig. 2. Two-dimensional frame element.

For a frame structure, the force displacement relationship in local coordinates can be expressed as:

$$\{\vec{\tau}_{\mathrm{p}}\}_{\mathrm{l}} = [K]_{\mathrm{l}}\{\overrightarrow{\Delta P}\}_{\mathrm{l}} \tag{1}$$

where $\{\vec{\tau}_{\mathrm{p}}\}_{\mathrm{l}}$, $\{\overrightarrow{\Delta P}\}_{\mathrm{l}}$, $[K]_{\mathrm{l}}$ respectively represent the element nodal force vector, the element nodal displacement vector and the stiffness matrix in local coordinates.

For local–global correspondence, we use a transformation matrix $[T]$.

Therefore, the force–displacement relationship in global coordinate is given as:

$$\{\vec{\tau}_{\mathrm{p}}\}_{\mathrm{g}} = [K]_{\mathrm{g}}\{\overrightarrow{\Delta P}\}_{\mathrm{g}} \tag{2}$$

where $\{\overrightarrow{\Delta P}\}_{\mathrm{g}}$, $\{\vec{\tau}_{\mathrm{p}}\}_{\mathrm{g}}$ and $[K]_{\mathrm{g}}$ respectively represent the element nodal displacement vector, the element nodal force vector and the stiffness matrix of the bar element global coordinate system given as:

$$\{\overrightarrow{\Delta P}\}_{\mathrm{g}} = [T]^{\mathrm{t}}\{\overrightarrow{\Delta P}\}_{\mathrm{l}} \tag{3}$$

$$\{\vec{\tau}_{\mathrm{p}}\}_{\mathrm{g}} = [T]^{\mathrm{t}}\{\vec{\tau}_{\mathrm{p}}\}_{\mathrm{l}} \tag{4}$$

$$[K]_{\mathrm{g}} = [T]^{\mathrm{t}}[K]_{\mathrm{l}}[T] \tag{5}$$

3.1.1. Frame element reformulation

Two-dimensional frame element reformulation Let us consider the planar frame element shown in Fig. 2.

In a local coordinate system, the displacement vector, the force vector, and the stiffness matrix are given as:

$$\{\overrightarrow{\Delta P}\}_{\mathrm{l}} = \{u_1 \quad v_1 \quad \theta_1 \quad u_2 \quad v_2 \quad \theta_2\}^{\mathrm{T}} \tag{6}$$

$$\{\vec{\tau}_{\mathrm{p}}\}_{\mathrm{l}} = \{Fx_1 \quad Fy_1 \quad M_1 \quad Fx_2 \quad Fy_2 \quad M_2\}^{\mathrm{T}} \tag{7}$$

$$[K]_{\mathrm{l}} = \begin{bmatrix} \frac{AE}{L} & 0 & 0 & -\frac{AE}{L} & 0 & 0 \\ & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ & & \frac{4EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ & & & \frac{AE}{L} & 0 & 0 \\ & sy. & & & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ & & & & & \frac{4EI}{L} \end{bmatrix} \tag{8}$$

The transformation matrix is given as:

$$[T] = \begin{bmatrix} [T_2] & [0] \\ [0] & [T_2] \end{bmatrix}; \qquad [0] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}; \qquad [T_2] = \begin{bmatrix} l_x & m_x & 0 \\ l_y & m_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{9}$$

In global coordinate system, the displacement vector and the force vector are given as:

$$\{\overrightarrow{\Delta P}\}_{\mathrm{g}} = \{U_1 \quad V_1 \quad \Theta_1 \quad U_2 \quad V_2 \quad \Theta_2\}^{\mathrm{T}} \tag{10}$$

$$\{\vec{\tau}_{\mathrm{p}}\}_{\mathrm{g}} = \{Fx_1 \quad Fy_1 \quad M_1 \quad Fx_2 \quad Fy_2 \quad M_2\}^{\mathrm{T}} \tag{11}$$

Three-dimensional frame element reformulation Considering the space frame element shown in Fig. 3.

In local coordinate system, the displacement vector, the force vector and the stiffness matrix for a three-dimensional frame element are given as:

$$\{\overrightarrow{\Delta P}\}_{\mathrm{l}} = \{u_1 \quad v_1 \quad w_1 \quad \theta_{x1} \quad \theta_{y1} \quad \theta_{z1} \quad u_2 \quad v_2 \quad w_2 \quad \theta_{x2} \quad \theta_{y2} \quad \theta_{z2}\}^{\mathrm{T}} \tag{12}$$

$$\{\vec{\tau}_{\mathrm{p}}\}_{\mathrm{l}} = \{Fx_1 \quad Fy_1 \quad Fz_1 \quad M_{x1} \quad M_{y1} \quad M_{z1} \quad Fx_2 \quad Fy_2 \quad Fz_2 \quad M_{x2} \quad M_{y2} \quad M_{z2}\}^{\mathrm{T}} \tag{13}$$

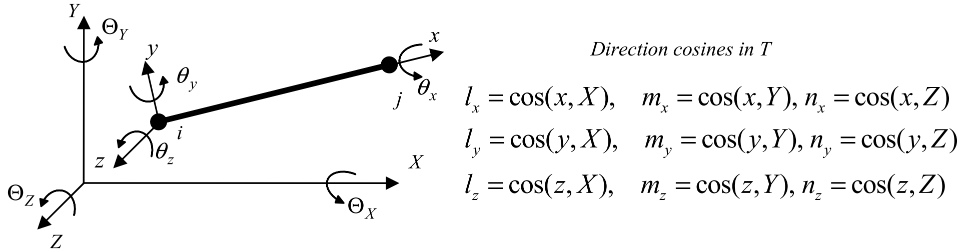

Fig. 3. Three-dimensional frame element.

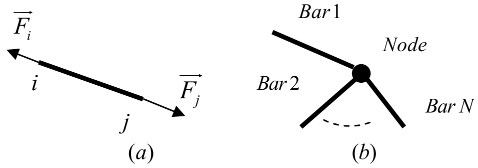

Fig. 4. Equilibrium of a frame structure: (a) equilibrium of a bar, (b) equilibrium of a node.

$$[K]_\mathrm{l}=\begin{bmatrix}
\frac{EA}{L} & 0 & 0 & 0 & 0 & 0 & \frac{-AE}{L} & 0 & 0 & 0 & 0 & 0\\
 & \frac{12EI_z}{L^3} & 0 & 0 & 0 & \frac{6EI_z}{L^2} & 0 & \frac{-12EI_z}{L^3} & 0 & 0 & 0 & \frac{6EI_z}{2}\\
 & & \frac{12EI_y}{L^3} & 0 & \frac{-6EI_y}{L^2} & 0 & 0 & 0 & \frac{-12EI_y}{L^3} & 0 & \frac{-6EI_y}{L^2} & 0\\
 & & & \frac{GJ}{L} & 0 & 0 & 0 & 0 & 0 & \frac{-GJ}{L} & 0 & 0\\
 & & & & \frac{4EI_y}{L} & 0 & 0 & 0 & \frac{6EI_y}{L^2} & 0 & \frac{2EI_y}{L} & 0\\
 & & & & & \frac{4EI_z}{L} & 0 & \frac{-6EI_z}{L^2} & 0 & 0 & 0 & \frac{2EI_z}{L}\\
 & & & & & & \frac{EA}{L} & 0 & 0 & 0 & 0 & 0\\
 & & & & & & & \frac{12EI_z}{L^3} & 0 & 0 & 0 & \frac{-6EI_z}{L^2}\\
 & & sy. & & & & & & \frac{12EI_y}{L^3} & 0 & \frac{6EI_y}{L^2} & 0\\
 & & & & & & & & & \frac{GJ}{L} & 0 & 0\\
 & & & & & & & & & & \frac{4EI_y}{L} & 0\\
 & & & & & & & & & & & \frac{4EI_z}{L}
\end{bmatrix} \tag{14}$$

The transformation matrix is given as:

$$[T]=\begin{bmatrix}[T_3] & [0] & [0] & [0]\\ [0] & [T_3] & [0] & [0]\\ [0] & [0] & [T_3] & [0]\\ [0] & [0] & [0] & [T_3]\end{bmatrix};\qquad [0]=\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix};\qquad [T_3]=\begin{bmatrix}l_x & m_x & n_x\\ l_y & m_y & n_y\\ l_z & m_z & n_z\end{bmatrix} \tag{15}$$

In global coordinate system, the displacement vector and the force vector are given as:

$$\{\overrightarrow{\Delta P}\}_\mathrm{g}=\{U_1\quad V_1\quad W_1\quad \Theta_{X1}\quad \Theta_{Y1}\quad \Theta_{Z1}\quad U_2\quad V_2\quad W_2\quad \Theta_{X2}\quad \Theta_{Y2}\quad \Theta_{Z2}\}^\mathrm{T} \tag{16}$$

$$\{\vec{\tau}_\mathrm{p}\}_\mathrm{g}=\{F_{X1}\quad F_{Y1}\quad F_{Z1}\quad M_{X1}\quad M_{Y1}\quad M_{Z1}\quad F_{X2}\quad F_{Y2}\quad F_{Z2}\quad M_{X2}\quad M_{Y2}\quad M_{Z2}\}^\mathrm{T} \tag{17}$$

3.1.2. Equilibrium of frame structure

To study the equilibrium of a frame structure, we study the equilibrium of each one of its bars and nodes (Fig. 4).

$$\textit{Equilibrium of a bar:}\quad \begin{cases}\vec{F_i}+\vec{F_j}=\vec{0}\\ \vec{M_i}+\vec{M_j}+\vec{F_j}\wedge\vec{L_{ij}}=\vec{0}\end{cases} \tag{18}$$

$$\textit{Equilibrium of a node:}\quad \begin{cases}\displaystyle\sum_{j=1}^{N}\vec{F_j}=\vec{0}\\ \displaystyle\sum_{j=1}^{N}\vec{M_j}=\vec{0}\end{cases} \tag{19}$$

where N is the number of concurrent bars to the node.

3.2. Modeling steps based on the notion of topological collections and transformations

In this section we present the different modeling steps by applying the topological collections and their transformations. This approach consists in representing the studied system by a cellular complex with which we associate the variables of interest. Then, we specify the local behavior law and equilibrium equations of the different components of the studied structure via the transformations and, finally, the generation of the system of equations is done by sweeping all the cells representing the system.

Step 1. *Definition of the k-cells representing the frame structure*

We use only 0-cells and 1-cells: 0-cells represent the nodes noted V_i $(i = 1...N_\mathrm{n})$; 1-cells represent the bars noted e_i $(i = 1...N_\mathrm{b})$, the forces noted $e_{\mathrm{f}i}$ $(i = 1...N_\mathrm{force})$ and the frames noted $e_{\mathrm{b}i}$ $(i = 1...N_\mathrm{frame})$ with N_n, N_b, N_force and N_frame represent respectively the number of nodes, of bars, of forces, and of frames.

The 1-cells representing the bars differ from those representing the frames and the forces in that they are bounded by two 0-cells.

Step 2. *Definition of the functions of the association of the physical parameters*

We define functions that associate the physical parameters (displacement, rigidity, length ...) with the corresponding *k*-cells representing the frame structure.

Step 3. *Association of the physical parameters with the k-cells*

After defining these functions, we associate the physical parameters with the corresponding *k*-cells.

Step 4. *Definition of the function of the creation of the system equations*

We generate the system equations of the frame structure which is written in MODELICA format. We distinguish two functions:

- the function of creation of the variables of the frame structure,
- the function of creation of the equations by sweeping all the cells representing the frame structure.

We have two cases:

- if the cell is of dimension 1, it can represent a force or a frame, in this case we are limited to Eq. (2). If not, that it is to say, it represents a bar, we add the force deformation relation (Eq. (1)), but we take only the superior part of the stiffness matrix (Eq. (8) for a 2D frame structure and Eq. (14) for a 3D frame structure);
- if the cell is of dimension 0, it represents a node; we then sum the forces induced for each incidental arc with this node (Eqs. (3) and (4)).

Step 5. *Resolution*

As indicated previously, the equations are written in format MODELICA, and we use DYMOLA as a solver [18].

These steps are the same in the various cases of frame structure. The only difference is at the level of the definition of the cells representing the system (nodes and arcs) and the parameters which are associated with it because these two steps vary according to the geometry and the data of the frame structure. Therefore, the declaration of the local law behavior as of the equilibrium equations of the principal components of a frame structure through the transformations is independent of the topological structure declared through the topological collections, and it is invariable in the various studied cases (Fig. 5).

4. Case studies

In this section, we consider two cases: the first presents an example of 2D frame structure which is a portal frame and the second presents an example of 3D frame structure which is a two-bar frame structure. In order to validate our topological approach, we compare the results obtained by the MGS language and the results obtained by structural calculation software RDM6.

For the first case, we presented the classical approach in order to show the advantages of our approach (local model, no assembling).

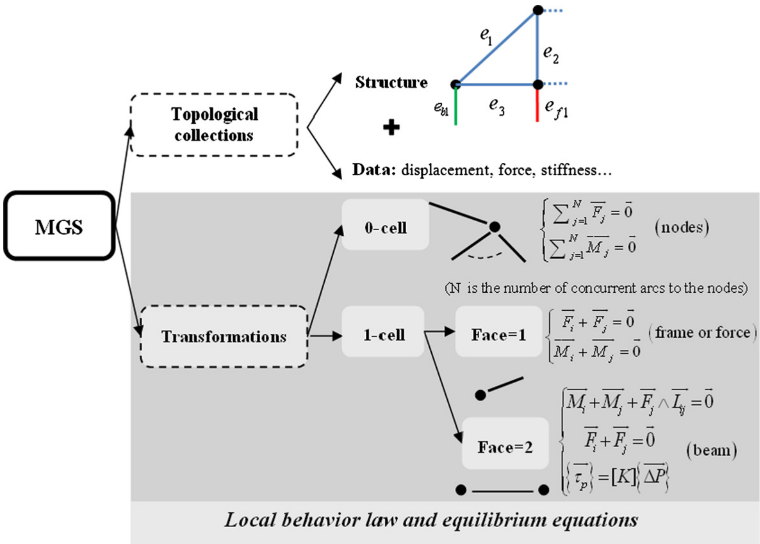

Fig. 5. (Color online.) Application of topological collections and their transformations to the modeling of frame structures.

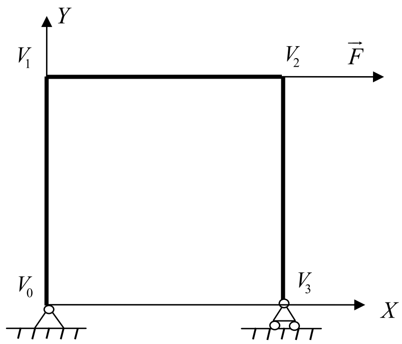

Fig. 6. Portal frame.

Table 1
The coordinates of the nodes.

	X	Y
V_0	0	0
V_1	0	4
V_2	4	4
V_3	4	0

4.1. Planar frame structure: portal frame

We consider the portal frame shown in Fig. 6 comprised of three bars and linked to the outside by a simple support in V_3 and an articulation in V_0.

For the sake of simplicity, the three bars have the same cross-section area A and even Young modulus. Young's modulus: $E = 200$ GPa, cross sectional area: $A = 0.01\ \text{m}^2$, inertia: $I = 8.33 \times 10^{-6}\ \text{m}^4$.

The portal frame is subjected to an external force $\vec{F}(N) = [2000; 0]$ on V_2.

We associate a reference $(O, \vec{X}, \vec{Y}, \vec{Z})$ with point V_0 in the initial position. The coordinates of the nodes are given in Table 1.

4.1.1. Classical approach using the displacement method

The bar's parameters are given in Table 2 ($L = 4$).

The total number of degree of freedom of the structure is 12 (four nodes at each node three degree of freedom).

$$\{\overrightarrow{\Delta P}\}_{\text{g(portique)}} = \{U_0 \quad V_0 \quad \Theta_0 \quad U_1 \quad V_1 \quad \Theta_1 \quad U_2 \quad V_2 \quad \Theta_2 \quad U_3 \quad V_3 \quad \Theta_3\}^{\text{T}}$$
$$\{\overrightarrow{\Delta \tau}\}_{\text{g(portique)}} = \{Fx_0 \quad Fy_0 \quad M_0 \quad Fx_1 \quad Fy_1 \quad M_1 \quad Fx_2 \quad Fy_2 \quad M_2 \quad Fx_3 \quad Fy_3 \quad M_3\}^{\text{T}}$$

Table 2
Bar properties.

	$L_i(m)$	l_x	m_x	l_y	m_y
Bar 1	L	0	1	−1	0
Bar 2	L	1	0	0	1
Bar 3	L	0	−1	1	0

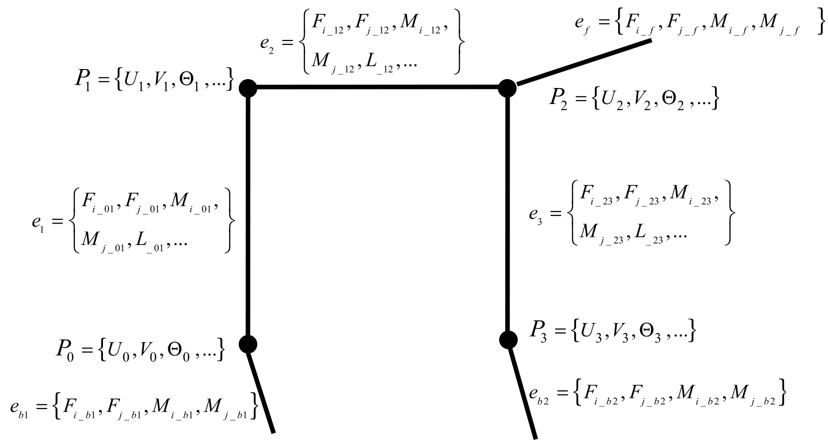

Fig. 7. Topological representation of the portal frame.

After calculating the global stiffness matrix of each beam $[K]_{g(\text{bar1})}$, $[K]_{g(\text{bar2})}$ and $[K]_{g(\text{bar3})}$ (Eq. (8)), they are assembled. The stiffness matrix in the global coordinate system of the portal is then written as follows:

$$[K]_{g(\text{portique})} = \begin{bmatrix} k_2 & 0 & -k_4 & -k_2 & 0 & -k_4 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & k_1 & 0 & 0 & -k_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -k_4 & 0 & k_3 & k_4 & 0 & \frac{k_3}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ -k_2 & 0 & k_4 & k_2+k_1 & 0+0 & k_4+0 & -k_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -k_1 & 0 & 0+0 & k_1+k_2 & 0+k_4 & 0 & -k_2 & k_4 & 0 & 0 & 0 \\ -k_4 & 0 & \frac{k_3}{2} & k_4+0 & 0+k_4 & k_3+k_3 & 0 & -k_4 & \frac{k_3}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & -k_1 & 0 & 0 & k_1+k_2 & 0+0 & 0+k_4 & -k_2 & 0 & k_4 \\ 0 & 0 & 0 & 0 & -k_2 & -k_4 & 0+0 & k_2+k_1 & -k_4+0 & 0 & -k_1 & 0 \\ 0 & 0 & 0 & 0 & k_4 & \frac{k_3}{2} & 0+k_4 & -k_4+0 & k_3+k_3 & -k_4 & 0 & \frac{k_3}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & -k_2 & 0 & -k_4 & k_2 & 0 & -k_4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -k_1 & 0 & 0 & k_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & k_4 & 0 & \frac{k_3}{2} & -k_4 & 0 & k_3 \end{bmatrix}$$

where $k_1 = \frac{EA}{L}$; $k_2 = \frac{12EI}{L^3}$; $k_3 = \frac{4EI}{L}$; $k_4 = \frac{6EI}{L^2}$.

The selected items in the global stiffness matrix present the assembling terms.

Using the force–displacement relationship, we calculate the displacement of each node. Then, we calculate the support reaction and force in each bar.

4.1.2. Topological approach using topological collections and transformations

As indicated in Section 3, we need only to define cells representing the structure as well as the variables with which they are associated (Fig. 7).

The equations obtained by the MGS language are generated by sweeping all cells representing the portal frame. For example:

– for node V_2: the equilibrium equations are given by Eq. (20),

$$\begin{cases} \overrightarrow{F_{i\_23}} + \overrightarrow{F_{j\_12}} + \overrightarrow{F_{i\_f}} = \vec{0} \\ M_{i\_23} + M_{j\_12} + M_{i\_f} = 0 \end{cases} \tag{20}$$

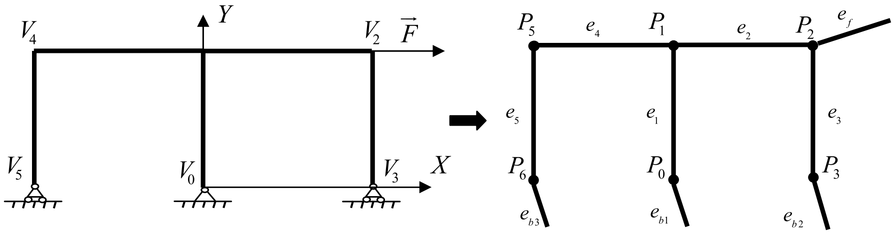

Fig. 8. The new studied beam structure and its topological representation.

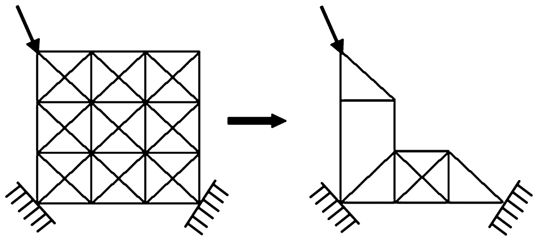

Fig. 9. Example of topological optimization of a beam structure.

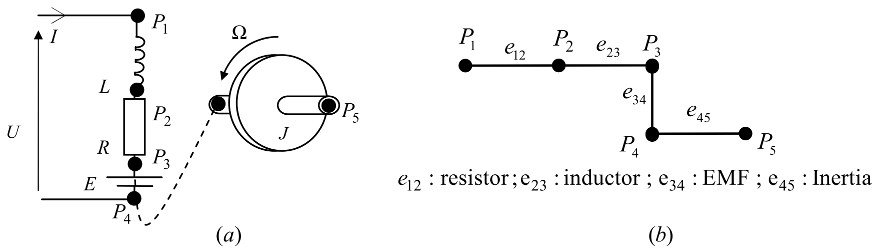

Fig. 10. (a) Equivalent diagram of the DC motor. (b) The topological representation of the DC motor.

– for beam 3: the local equilibrium equations and the local behavior law are respectively given by Eqs. (21) and (22),

$$\begin{cases} \overrightarrow{F_{i\_23}} + \overrightarrow{F_{j\_23}} = \vec{0} \\ M_{i\_23} + M_{j\_23} + \overrightarrow{F_{j\_23}} \wedge \overrightarrow{L\_23} = 0 \end{cases} \tag{21}$$

$$\begin{Bmatrix} \overrightarrow{F_{i\_23}} \\ M_{i\_23} \end{Bmatrix} = \begin{bmatrix} k_2 & 0 & k_4 & -k_2 & 0 & k_4 \\ 0 & k_1 & 0 & 0 & -k_1 & 0 \\ k_4 & 0 & k_3 & -k_4 & 0 & \frac{k_3}{2} \end{bmatrix} \begin{Bmatrix} U_2 \\ V_2 \\ \Theta_2 \\ U_3 \\ V_3 \\ \Theta_3 \end{Bmatrix} \tag{22}$$

We can clearly see that we consider the portal frame as local elements and that there are no assembling terms. The assembly is done indirectly through the equilibrium equations deduced from the neighborhood relations. For example, if we add two beams to the studied portal frame (Fig. 8), Eq. (22) remains the same and we only need to change the topological structure by the declaration of the new cells related to the addition of these two beams, whereas using the stiffness method we should modify the global stiffness matrix of the structure.

Also, to reduce the number of beams of any structure, we need only to modify its topological structure declared through the topological collections. Then, the advantage of this topological approach compared with the stiffness method is that we declare the local behavior law of the beams (physics) independently of their number and their connection (topology), which allows the optimization of the global behavior of any structure according to the local behavior law of its components. Fig. 9 shows an example of optimization problem.

Indeed, the separation of the topology (interconnection law) and the behavior law (physics) allow the simplification of the modeling of complex systems that can be described as a set of local interactions between elementary entities. For example, using topological collections and transformations, a DC motor (Fig. 10(a)) can be seen as a set of local elements (resistor, inductor, EMF and inertia). Fig. 10(b) shows the topological representation of the DC motor. Each component of the DC motor can be represented by an arc noted e_{ij} bounded by two nodes P_i and P_j.

Table 3
Nodal displacements of the portal frame.

Nodal displacement (m, rad)		
V_0	MGS	$\{0 \quad 0 \quad -1.524 \times 10^{-2}\}$
	RDM6	$\{0 \quad 0 \quad -1.524 \times 10^{-2}\}$
V_1	MGS	$\{4.8769 \times 10^{-2} \quad 3.809 \times 10^{-6} \quad -6.097 \times 10^{-3}\}$
	RDM6	$\{4.877 \times 10^{-2} \quad 3.810 \times 10^{-6} \quad -6.097 \times 10^{-3}\}$
V_2	MGS	$\{4.877 \times 10^{-2} \quad -3.809 \times 10^{-6} \quad 3.045 \times 10^{-3}\}$
	RDM6	$\{4.876 \times 10^{-2} \quad -3.810 \times 10^{-6} \quad 3.046 \times 10^{-3}\}$
V_3	MGS	$\{6.0956 \times 10^{-2} \quad 0 \quad 3.0457 \times 10^{-3}\}$
	RDM6	$\{6.096 \times 10^{-2} \quad 0 \quad 3.046 \times 10^{-3}\}$

Table 4
Support reactions of the portal frame.

Support reactions (N, N·m)		
in V_0	MGS	$\{-2000 \quad -2000 \quad 0\}$
	RDM6	$\{-2000 \quad -2000 \quad 0\}$
in V_3	MGS	$\{0 \quad 2000 \quad 0\}$
	RDM6	$\{0 \quad 2000 \quad 0\}$

Table 5
Stress resultants at the two end cross sections of the portal frame.

Stress resultants at the two end cross sections (N, N·m)			
Beam 1	MGS	0	$\{2000 \quad -2000 \quad 0\}$
		1	$\{2000 \quad -2000 \quad 8000\}$
	RDM6	0	$\{2000 \quad -2000 \quad 0\}$
		1	$\{2000 \quad -2000 \quad 8000\}$
Beam 2	MGS	1	$\{2000 \quad 2000 \quad 8000\}$
		2	$\{2000 \quad 2000 \quad 0\}$
	RDM6	1	$\{2000 \quad 2000 \quad 8000\}$
		2	$\{2000 \quad 2000 \quad 0\}$
Beam 3	MGS	2	$\{-2000 \quad 0 \quad 0\}$
		3	$\{-2000 \quad 0 \quad 0\}$
	RDM6	2	$\{-2000 \quad 0 \quad 0\}$
		3	$\{-2000 \quad 0 \quad 0\}$

The global behavior law of the DC motor is given as follows (p represent the Laplace variable):

$$\begin{Bmatrix} I_{12} \\ I_{23} \\ I_{34} \\ \theta_{34} \\ \theta_{45} \end{Bmatrix} = \begin{bmatrix} 1/R & 0 & 0 & 0 & 0 \\ 0 & p/L & 0 & 0 & 0 \\ 0 & 0 & 1/K_c & 0 & 0 \\ 0 & 0 & 0 & 1/pK_e & 0 \\ 0 & 0 & 0 & 0 & 1/p^2 J \end{bmatrix} \begin{Bmatrix} U_{12} \\ U_{23} \\ U_{34} \\ M_{34} \\ M_{45} \end{Bmatrix} \tag{23}$$

where R is the motor winding resistance, L the motor inductance, K_e the motor's back electromagnetic force constant, K_c the motor torque constant, J the rotor moment of inertia, θ_{ij} the angular displacement, U_{ij} the voltage, M_{ij} the torque and I_{ij} the current.

In Eq. (23), the global stiffness matrix does not contain any assembling terms. The assembly is done through the equilibrium equations of each component.

4.1.3. *Numerical results of the portal frame*

Tables 3, 4, 5 show, respectively, the nodal displacements $\{U \ V \ \Theta\}$, the support reactions $\{R_x \ R_y \ M_z\}$, the stress resultants at the two end cross sections $\{N \ T_y \ Mf_z\}$, with N the normal force, T_y the shear force, and Mf_z the bending moment.

We can clearly see that the results obtained by the MGS language based on topological collection and transformations are similar to those obtained using software RDM6 based on the finite-element method.

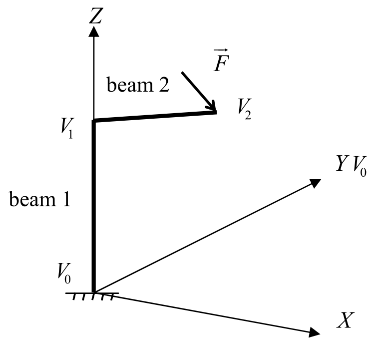

Fig. 11. The two-bar frame.

Table 6
Node coordinates of the two bars frame structure.

	X	Y	Z
V_0	0	0	0
V_1	0	0	4
V_2	2	2	4

Table 7
Nodal displacements of the two-bar frame structure.

Nodal displacement (m, rad)		
V_1	MGS	$\{9.14286 \times 10^{-3}$ 2.13333×10^{-2} -1.90476×10^{-10} -9.14286×10^{-3} 4.57143×10^{-3} $7.04541 \times 10^{-3}\}$
	RDM6	$\{9.143 \times 10^{-3}$ 2.133×10^{-2} -1.905×10^{-10} -9.143×10^{-3} 4.571×10^{-3} $7.046 \times 10^{-3}\}$
V_2	MGS	$\{-7.10227 \times 10^{-3}$ 3.75798×10^{-2} -3.17405×10^{-2} -1.07591×10^{-2} 6.18767×10^{-3} $8.66165 \times 10^{-3}\}$
	RDM6	$\{-7.103 \times 10^{-3}$ 3.758×10^{-2} -3.174×10^{-2} -1.076×10^{-2} 6.188×10^{-3} $8.662 \times 10^{-3}\}$

Table 8
Support reactions of the two-bar frame structure.

Support reactions (N, N·m)		
in V_0	MGS	{0 −1000 1000 6000 −2000 −2000}
	RDM6	{0 −1000 1000 6000 −2000 −2000}

Table 9
Stress resultants at the two end cross sections of the two-bar frame structure.

Stress resultants at the two end cross sections (N, N·m)			
Bar 1	MGS	0	{−1000 0 −1000 2000 6000 −2000}
		1	{−1000 0 −1000 2000 2000 −2000}
	RDM6	0	{−1000 0 −1000 2000 6000 −2000}
		1	{−1000 0 −1000 2000 2000 −2000}
Bar 2	MGS	1	{707.107 −1000 −707.107 0 2000 −2828.43}
		2	{707.107 −1000 −707.107 0 0 0}
	RDM6	1	{707.1 −1000 −707.1 0 2000 −2828.4}
		2	{707.1 −1000 −707.1 0 0 0}

4.2. Two bar frame structure

We consider the two bar frame structures shown on Fig. 11 embedded in V_0. The two bars have the same cross-section area A and even Young modulus. Young's modulus: $E = 200$ GPa and cross sectional area: $A = 0.01$ m^2. Poisson's ratio and the inertia of the bars are $\upsilon = 0.3$; $J = 1.40 \times 10^{-6}$ m^4; $I_y = 8.33 \times 10^{-6}$ m^4; $I_z = 8.33 \times 10^{-6}$ m^4.

This structure is subjected to an external force $\vec{F}(N) = [0\ -1000\ 1000]$ on V_2.

We associate a reference $(O, \vec{X}, \vec{Y}, \vec{Z})$ with point V_0 in the initial position. The coordinates of nodes are given in Table 6. Tables 7, 8 and 9 respectively show nodal displacements $\{U\ V\ W\ \Theta_x\ \Theta_y\ \Theta_z\}$, support reactions $\{R_x\ R_y\ R_z\ M_x\ M_y\ M_z\}$ and stress resultants at the two end cross sections, $\{N\ T_y\ T_z\ Mt\ Mf_y\ Mf_z\}$ with N the normal force, $(T_y\ T_z)$ the shear force, Mt the twisting moment, $(Mf_y\ Mf_z)$ the bending moment.

Comparison of the results obtained by the MGS language based on topological collections and transformations with those obtained by software RDM6 based on the finite-element method evidenced that they were very close.

5. Conclusion

In this work, we presented a topological approach for the modeling of a frame structure on the basis of topological collections and transformations. This approach makes it possible to dissociate the topology and the physics of the studied system to get a generic local model that allows the optimization of the global behavior. To validate this approach, we have studied two cases of planar and space frame structure and compared the results using software RDM6 based on the finite-element method. The advantage of the application of the topological collections and of their transformations for the modeling of frame structures compared to the other approach is that we declare the local behavior law of beams independently of their number and the way in which they are connected, i.e. of their topology. Indeed, a beam is considered as a local element.

This topological approach opens up very large perspectives for the modeling of complex systems because these systems can be considered as sets of local elements connected by a neighborhood relationship. Also, this approach can be used as a unification basis for the modeling of mechatronic systems, and this is particularly true when the same topological structure can support different physics. So, the case of mechatronic systems might constitute a good topic for our future articles.

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