C. R. Mecanique 344 (2016) 375–387

Contents lists available at ScienceDirect

Comptes Rendus Mecanique

www.sciencedirect.com

Dynamic behaviour of a wind turbine gear system with uncertainties

Moez Beyaoui, Manel Tounsi, Kamel Abboudi, Nabih Feki, Lassaad Walha, Mohamed Haddar

Laboratory of Mechanics, Modelling and Manufacturing (LA2MP), National Engineering School of Sfax, Ministry of Higher Education and Research of Tunisia, BP 1173, 3038 Sfax, Tunisia

ARTICLE INFO

Article history:
Received 13 July 2015
Accepted 2 January 2016
Available online 22 March 2016

Keywords:
Gearbox system
Uncertainty
Random aerodynamic torque
Polynomial chaos

ABSTRACT

In this paper, a new methodology for taking into account uncertainties in a gearbox transmission system of a horizontal-axis wind turbine is proposed. Gearbox transmission is the major part of the wind turbine's drive train. For a more reasonable evaluation of its dynamic behaviour, the influence of the uncertain parameters should be taken into consideration. The dynamic equations are solved by using the Polynomial Chaos method combined with the ODE45 solver of Matlab. The effects of the random perturbation caused by the aerodynamic torque excitation on the dynamic response of the studied system are discussed in detail. The proposed method is an efficient probabilistic tool for uncertainty propagation. For more accuracy, the Polynomial Chaos results are compared with direct simulations.

1. Introduction

Wind energy is one of the most efficient renewable energies. Wind turbines harvest the kinetic energy of air and convert it into a usable power such as electricity power. For this, the capacity of wind turbines has increased and they have become the fastest-growing new sources of electricity generation.

Many scientific studies have investigated the dynamic behaviour of wind turbines [1–3]. In their analysis, gear power transmission was considered as the perfect system. However, the gearbox system presents constantly precocious failures [4]. Therefore, in a wind turbine, an adequate knowledge of the dynamic characteristics of gearboxes system is necessary.

In this context, several studies have been developed to study the dynamic behaviour of wind turbines. The dynamic behaviour of a two-stage gear reducer in the presence of aerodynamic excitation has been investigated by Abboudi et al. [5] and a lamped mass dynamic model with 12 DOFs has been developed. Under wind speed fluctuations and system disturbances, the dynamic behaviour and transient stability of fixed-speed wind turbines has been studied by Rahimi et al. [6]. The combined effects of gravity, input torque, bending moment and bearing clearance of planetary wind turbine gearboxes are reported by Guo et al. [7]. In 2011, Helsen et al. [8] investigated the modal behaviour of a wind turbine gearbox using flexible multi-body modelling techniques.

All previous studies have investigated the dynamic behaviour of the wind turbine considering the deterministic parameters as the system's parameters. However, the instability of rotor inflow caused by the atmosphere creates persistent

E-mail address: walhalassaad@yahoo.fr (L. Walha).

http://dx.doi.org/10.1016/j.crme.2016.01.003

Nomenclature

$k_1(t)$	Meshstiffness of the first stage	C_L	Lift coefficient
$k_2(t)$	Meshstiffness of the second stage	C_D	Drag coefficient
x_j and y_j	Translations of each block j ($i = 1$ to 3).	a	Axial induction factor
θ_{ji}	Angular displacements of the component i in block j ($i = 1$ and 2, $j = 1$ to 3).	a'	Tangential induction factor
α	Pressure angle (generally adopted equal to 20°)	V_0	Wind velocity far up stream
rb_{ji}	Base radius of the gear (m)	k_{xj}	Stiffness to bending according to the X direction (N·m)
I_{ji}	Moments of inertia of gears	k_{yj}	Stiffness to traction – compression according to Y direction (N·m)
$\delta_1(t), \delta_2(t)$	Displacements along the line of action	k_m	Average mesh stiffness (N·m)
r	Radius of the rotor (m)	$k_{\theta j}$	Torsional stiffness of the shaft (Nm/rad)
n_p	Number of blades	$Z_{(12)}$, $Z_{(21)}$ $Z_{(22)}$, $Z_{(31)}$	Number of teeth
c	Chord (m)	$\varepsilon_{\alpha 1}$, $\varepsilon_{\alpha 2}$	Contact ratio
ϕ	Inflow angle (rad)	$C_g(t)$	Electromechanic torque
D	Rotor diameter (m)	$Q_{aero}(t)$	Aerodynamic torque
ρ_{air}	Air density (Kg/m^3)		
Ω	Turbine rated speed (rad/s)		

variations of blade loads and rotor torque. Therefore, an increased penetration of wind turbine systems calls for a suitable modelling of the system parameter and incorporates the model into various uncertainty parameters. Until now, system parameters present a random parameter and suffer from a lack of accuracy focusing on the measurement of the parameters. The choice of the design parameters is very critical to optimise the performance of the system. Therefore, it becomes necessary to take into account uncertainty parameters [9,10]. In this context, advanced techniques and methods of uncertainties are developed. Monte Carlo simulation is a well-known technique in this field [11]. For reasonable accuracy, it requires a great number of samples; therefore, it is too costly. The Polynomial Chaos (PC) method is considered as the best framework in dealing with uncertainty quantification. This method is more attractive and more efficient compared to other methods such as Monte Carlo approaches [12,13].

Ghanem and Spanos [14,15] have used successfully the Polynomial Chaos (PC) method in their study of uncertainties in the structural mechanics and vibration fields. The PC method represents the random state and input parameter variables as a probability distribution in the stochastic system state governed by the differential equations of motion. Indeed, the PC method is defined as a spectral representation of the uncertainty in random space in terms of an expansion of orthogonal polynomials that are functions of the random input variables.

The computational accuracy and efficiency supplied by the Polynomial Chaos method in nonlinear problems is reported through scientific works in many fields such as in fluid dynamics [16–19], in solid mechanics [20,21], in chemical reactions [22], in terramechanics [23,24], etc. Due to the accuracy and efficiency of the Polynomial Chaos method in previous studies with different fields, this method is considered to investigate the dynamical behaviour of a wind turbine taking into account the parameters of the uncertainty system.

The originality of this study is to investigate the effects of the uncertainty input gear system parameter of a horizontal-axis wind turbine. The main objective is to capture the dynamical behaviour of a two-stage spur gearbox transmission system subjected to an uncertain input parameter. In order to calculate the dynamical response of the studied model, the PC method is used to deal with uncertainty and to discuss the capabilities of this new methodology. Monte Carlo simulations are reserved to the treatment of reference examples in order to test the validity and the properties of the Polynomial Chaos method.

2. Dynamic model

The wind turbine is composed by the rotor, the transmission power system and the generator. The transmission model implemented in this work is a two-stage spur gearbox system with twelve degrees of freedom (12 DOF) and three main blocks ($j = 1$ to 3) as shown in Fig. 1.

Each block is supported by a flexible bearing characterised by two stiffness parameters k_{xj} and k_{yj} according to the x and y directions, respectively. The connecting shafts admit a torsional stiffness parameter $k_{\theta j}$ according to the x direction. The four gears (gear 12, gear 21, gear 22 and gear 31) are considered as spur gears.

The gear system is subject to a random aerodynamic torque $Q_{aero}(t)$, which represents the effect of wind on the three-bladed rotor. The aerodynamic torque expression is given in paragraph 3. The output electromechanic torque is defined by $C_g(t)$.

In addition to the external excitation, the system is also submitted to two internal excitations, which are the periodic fluctuations of mesh stiffness $k_1(t)$ and $k_2(t)$. In fact, every contact is modelled by a variable stiffness represented by a linear spring following the line of action, whose temporal expression is described in paragraph 4.

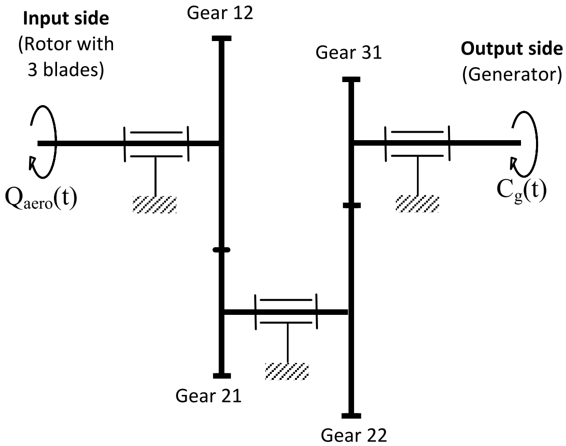


Fig. 1. Two-stage gear system in a wind turbine.

According to Lagrange's formalism, the kinematic differential equations governing the system motion are given by Equation (3):

$$\begin{cases} I_{11}\ddot{\theta}_{11} + k_{\theta 1}(\theta_{11} - \theta_{12}) = Q_{\text{aero}} \\ I_{12}\ddot{\theta}_{12} - k_{\theta 1}(\theta_{11} - \theta_{12}) + k_1(t) rb_{12}\delta_1(t) = 0 \\ I_{21}\ddot{\theta}_{21} + k_{\theta 2}(\theta_{21} - \theta_{22}) + k_1(t) rb_{21}\delta_1(t) = 0 \\ I_{22}\ddot{\theta}_{22} - k_{\theta 2}(\theta_{21} - \theta_{22}) + k_2(t) rb_{22}\delta_2(t) = 0 \\ I_{31}\ddot{\theta}_{31} + k_{\theta 3}(\theta_{31} - \theta_{32}) + k_2(t) rb_{31}\delta_2(t) = 0 \\ I_{32}\ddot{\theta}_{32} - k_{\theta 3}(\theta_{31} - \theta_{32}) = -C_r(t) \\ m_1\ddot{x}_1 + k_{x1}x_1 - k_1(t)\delta_1(t)\sin\alpha_1 = 0 \\ m_2\ddot{x}_2 + k_{x2}x_2 + k_1(t)\delta_1(t)\sin\alpha_1 + k_2(t)\delta_2(t)\sin\alpha_2 = 0 \\ m_3\ddot{x}_3 + k_{x3}x_3 - k_2(t)\delta_2(t)\sin\alpha_2 = 0 \\ m_1\ddot{y}_1 + k_{y1}y_1 + k_1(t)\delta_1(t)\cos\alpha_1 = 0 \\ m_2\ddot{y}_2 + k_{y2}y_2 - k_1(t)\delta_1(t)\cos\alpha_1 - k_2(t)\delta_2(t)\cos\alpha_2 = 0 \\ m_3\ddot{y}_3 + k_{y3}y_3 + k_2(t)\delta_2(t)\cos\alpha_2 = 0 \end{cases} \tag{1}$$

x_j and y_j are the translations of each block j ($i = 1$ to 3). θ_{ji} are the angular displacements of the component i in block j ($i = 1$ and 2, $j = 1$ to 3). α is the pressure angle, it is generally adopted equal to 20°. The base radius of the gear is rb_{ji} and the moments of inertia of gears are I_{ji}.

The displacements $\delta_1(t)$ and $\delta_2(t)$ along the line of action are expressed by:

$$\delta_1(t) = (x_1 - x_2)\cdot\sin(\alpha_1) + (y_1 - y_2)\cdot\cos(\alpha_1) + rb_{12}\theta_{12} + rb_{21}\theta_{21} \tag{2}$$

$$\delta_2(t) = (x_2 - x_3)\cdot\sin(\alpha_2) + (-y_2 + y_3)\cdot\cos(\alpha_2) + rb_{22}\theta_{22} + rb_{31}\theta_{31} \tag{3}$$

3. Aerodynamic modelling

The developed method in the aerodynamic part is the blade element theory (BEM) [25]. It has been introduced by Glauert in 1930 [2]; it takes into account the rotation of the air flow in order to calculate the aerodynamic loads and to investigate the evaluation of the performance of the wind turbine. This theory is often used in the fields of the wind industry and it is the most frequently used by science and industry [26–29].

Blade Element Theory enables us to design the rotor blade by fluid dynamics, to choose the geometric parameters of the turbine and to evaluate the forces acting on the blades. By using this theory, the torque applied and the turbine performance can be modeled by uncertain parameters.

Blade element theory consists in dividing up the blade into many elements. For each element, characterized by a radius r, a thickness dr and a section $dA = 2\pi r dr$, we calculate the flow and the aerodynamic elementary forces generated by this flow by applying momentum and angular momentum conservation equations.

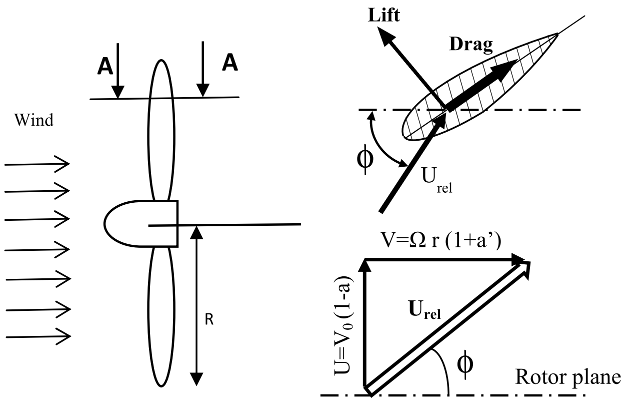


Fig. 2. Components of the relative wind velocity into the blade section.

Then a summation of these actions is applied along the blade. The aerodynamic force (torque) acting on the blades is calculated to evaluate the performance of the rotor.

Blade element theory relies on some key assumptions:

- there are no aerodynamic interactions between the different blade elements,
- the forces on the blade elements are solely determined by the lift and drag coefficients,
- static pressure is considered equal to the atmospheric pressure downstream of the rotor.

By applying the BEM theory to the fluid dynamic wind turbine design, it is possible to evaluate the torque $\mathrm{d}Q_{\mathrm{aero}}$ [30] for each blade element as given in equation (4):

$$\mathrm{d}Q_{\mathrm{aero}} = \frac{\rho_{\mathrm{air}}}{2} n_{\mathrm{p}} c U_{\mathrm{rel}}^{2} (C_{\mathrm{L}} \sin\phi - C_{\mathrm{D}} \cos\phi) r\, \mathrm{d}r \tag{4}$$

In this work, only the relative velocity of wind U_{rel} is considered, which is the result of an absolute velocity V and a training velocity U.

Axial and tangential induction coefficients (respectively a and a') are introduced. These coefficients significantly affect the real value of the velocities. Fig. 2 shows the components of the relative wind velocity (U_{rel}) on a section of the blade. ϕ is the inflow angle, r is the radius, V_0 is the wind velocity far up stream and Ω is the angular velocity of the rotor.

The wind, flowing around the blade, creates the resultant aerodynamic force. The last force (Fig. 2) could be split into two components called Lift and Drag. The lift force acts on the blade in a direction perpendicular to the relative wind. The drag force is the resistance that opposes the motion of the airfoil through the air. It acts on the blade in a direction parallel to that of the relative wind. Lift and drag forces acting on the blade element are written respectively as follows [30]:

$$C_{\mathrm{L}} = 2C_{\mathrm{L\,max}} \sin\phi \cdot \cos\phi \tag{5}$$

$$C_{\mathrm{D}} = C_{\mathrm{D\,max}} \sin^2\phi \tag{6}$$

where $C_{\mathrm{L\,max}}$ and $C_{\mathrm{D\,max}}$ are constants determined from the graphs presented in [31].

The aerodynamic torque expression for each blade element can be written in this form, taking into account the modelling of the wind velocity and lift and drag forces.

$$\mathrm{d}Q_{\mathrm{aero}} = \frac{\rho}{2} n_{\mathrm{p}} \frac{V_0(1-a)}{\sin\phi} \frac{\Omega r(1+a')}{\cos\phi} \big((2C_{\mathrm{L\,max}} \sin\phi\cos\phi)\sin\phi - \big(C_{\mathrm{D\,max}} \sin^2\phi\big)\cos\phi\big) c r \mathrm{d}r \tag{7}$$

In this paper, we suppose that the inflow angle ϕ is the uncertain parameter. We are interested in studying the effects of this uncertainty on the dynamic behaviour of the gear transmission system.

4. Mesh stiffness modelling

In this work, the gear mesh stiffness function $k_i(t)$ is modelled by a square wave form (Fig. 3). The periodic square wave form is the most representative of the real phenomenon of gearing systems according to Walha et al. [32] and Jairo et al. [33].

Two main components establish the stiffness of the mesh. The first one is noted $K_{\mathrm{m},i}$. It is constant over time and represents the mean value of the stiffness. To this component is added a second variable component, whose extreme values are written as follows [5]:

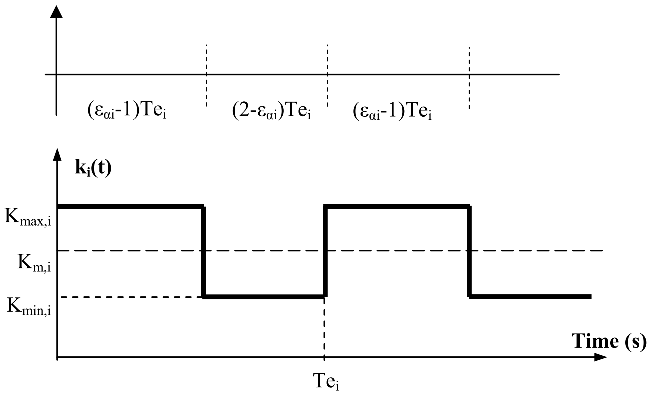


Fig. 3. Gear meshing modelling.

Table 1
Gear meshing values.

	Number of teeth	Meshing values (N/m)
First gear meshing	72/18	$k_{m,1} = 2 \cdot 10^8$ $k_{min,1} = 1.3926 \cdot 10^8$ $k_{max,1} = 2.3321 \cdot 10^8$
Second gear meshing	54/18	$k_{m,2} = 2 \cdot 10^8$ $k_{min,2} = 1.3966 \cdot 10^8$ $k_{max,2} = 2.3143 \cdot 10^8$

$$k_{\min,i} = k_{\mathrm{m},i}\left(1 - \frac{1}{2\varepsilon_{\alpha i}}\right) \tag{8}$$

$$k_{\max,i} = k_{\mathrm{m},i}\left(1 + \frac{2-\varepsilon_{\alpha i}}{2\varepsilon_{\alpha i}(\varepsilon_{\alpha i}-1)}\right) \tag{9}$$

ε_α represent the contact ratio.

The variation of the mesh stiffness over time is explained by the number of pairs of teeth in contact at a given time t.

In this work, we present the gear meshing values in Table 1.

5. Polynomial chaos method

Different techniques exist to model the propagation of uncertainty. These methods are generally classified into three categories: the simulation methods, the perturbation methods, and the spectral methods.

Monte Carlo simulations are considered as reference methods for calculations on systems with uncertain parameters. Its major disadvantage is the very large quantity of calculations, which complicates the use of these methods.

The perturbation method is based on a Taylor series development of the response around its mean. The main problem of this method comes from the conditions ensuring the convergence of these series. The variables must have low dispersion. This method presents a difficulty during the calculations of the dynamic responses.

For this, the method adopted to take into account uncertainty in this work is the Polynomial Chaos method. The fundamental idea of the Polynomial Chaos method, coined by Norbert Wiener in 1938, is to establish a separation between the stochastic components of a random function and its deterministic components.

The random process of interest is approximated by sums of orthogonal Polynomial Chaos of random independent variables. In this context, any uncertain parameter can be viewed as a second-order random process. So, a second-order random variable u can be expanded in terms of orthogonal Polynomial Chaos as [34]:

$$u = \sum_{i=0}^{\infty} \bar{u}_i \psi_i(\xi) \tag{10}$$

where ξ is a vector of standard normal random variables, $\bar{u}_i$ are the stochastic modes of the random variables u and ψ_i are the multivariate orthogonal polynomials, such as Hermite, Legendre, etc. The choice of the polynomial family depends on the density distribution of the uncertain input parameter. Indeed, optimal correspondences between the families of probability laws and the families of orthogonal polynomials have been established.

The orthogonal polynomials satisfy the orthogonally relation [34]:

$$\langle \psi_{\mathrm{n}}, \psi_{\mathrm{m}} \rangle = \int_{-1}^{1} \psi_{\mathrm{n}}(\xi)\psi_{\mathrm{m}}(\xi) W(\xi)\,\mathrm{d}\xi \tag{11}$$

where $\langle\cdot\rangle$ means the internal product operator and $W(\xi)$ is the probability density function (PDF) of the random variables that make up the vector ξ.

In practice, the generalized Polynomial Chaos expansion is truncated to a finite number of terms. The truncation of the infinite series is necessary to keep the problem computationally feasible. In this work, we will truncate the series in such a way that all expansion polynomials up to a certain maximum degree, denoted by p, are included. The number of terms $(P+1)$ in the expansion now follows from this maximum degree p and the dimensionality n of the random vector according to:

$$P = \frac{(p+n)!}{p!n!} \tag{12}$$

By using the method of Polynomial Chaos, three fundamental steps are essential to study the propagation of uncertainties in stochastic models namely:

- choosing the appropriate polynomial basis to the problem studied,
- fixing the order p of Polynomial Chaos,
- calculate the coefficients of the expansion of the Polynomial Chaos (the stochastic modes).

The last step consists in characterizing the solution $u(x,t,\xi)$ of a differential equation whose general form is as follows:

$$D\big[u(x,t,\xi)\big] = f\big[u(x,t,\xi), x, \xi\big] \tag{13}$$

where

- D is a differential operator,
- f is a given function,
- $u(x,t,\xi)$ is the solution depending on the space (x), the time (t) and the uncertainty (ξ).

The solution of the differential equation is expressed as a series expansion of orthogonal polynomials.

$$u(x,t,\xi) = \sum_{i=0}^{P} \bar{u}_i(x,t)\psi_i(\xi) \tag{14}$$

The intrusive approach used in this study is presented through the following steps:

a. Substitute the expression (14) of the solution of the problem in the differential equation (13)

$$D\left[\sum_{i=0}^{P} \bar{u}_i(x,t)\psi_i(\xi)\right] = f\left[\sum_{i=0}^{P} \bar{u}_i(x,t)\psi_i(\xi), x, \xi\right] \tag{15}$$

b. Apply Galerkin's projection. The two members of equation (15) will be multiplied by the polynomials of the chosen base. Then the mean statistical will be applied.

$$\left\langle D\left[\sum_{i=0}^{P} \bar{u}_i(x,t)\psi_i(\xi)\right], \psi_L(\xi)\right\rangle = \left\langle f\left[\sum_{i=0}^{P} \bar{u}_i(x,t)\psi_i(\xi), x, \xi\right], \psi_L(\xi)\right\rangle; \quad L = 0, \ldots, P \tag{16}$$

Due to the orthogonality property, a deterministic system that defines the dynamic of the stochastic modes $\bar{u}_L(x,t)$ is obtained:

$$\dot{\bar{u}}_L(x,t) = \left(\frac{1}{\langle \psi_L(\xi), \psi_L(\xi)\rangle}\right)\left\langle f\left[\sum_{i=0}^{P} \bar{u}_i(x,t)\psi_i(\xi), x, \xi\right], \psi_L(\xi)\right\rangle; \quad L = 0, \ldots, P \tag{17}$$

c. Use an appropriate algorithm to determine the vector of the stochastic modes $\bar{u}_L(x,t)$.

In this study, the input torque (equation (7)) of the gearbox transmission system is considered with uncertainty according to the inflow angle.

According to the literature, both uniform and normal (Gaussian) law distributions are treated to describe uncertainty in both lift coefficient and natural frequencies for the blade, which are considered as random parameters in the model developed by Pourazarm et al. [35]. Desai et al. [36] studied the aeroelastic instability system using the Polynomial Chaos method to describe random parameters normally distributed. Duck Kwon [37] demonstrate that the variation of normalised wind velocity at wind tunnel may be described by a normal distribution. Liu et al. [38] studied the performance evaluation of an horizontal-axis wind turbine with consideration of the angle of attack and the wind speed as uncertain parameters in a Gaussian distribution.

The steps presented previously will be applied to study the propagation of this uncertainty. The decomposing of the random in the polynomial basis allows generating a differential equation system:

$$\begin{cases} \ddot{\bar{\theta}}_{11,l} = -\dfrac{k_{\theta 1}}{\mathrm{I_m}}(\bar{\theta}_{11,l} - \bar{\theta}_{12,l}) + \dfrac{\rho n_{\mathrm{p}} c}{2\mathrm{I_m}\langle \psi_l^2(\xi)\rangle} \cdot V_0(1-a)(1+a') \cdot \\ \qquad (2C_{\mathrm{L\,max}} - C_{\mathrm{D\,max}}) \cdot \dfrac{R^3 - r_{\mathrm{moy}}^3}{3} \cdot \displaystyle\sum_{j=0}^{p} \dot{\bar{\theta}}_{11,j} \langle \sin\phi(\xi)\ \psi_j(\xi)\ \psi_l(\xi)\rangle \\ \ddot{\bar{\theta}}_{12,l} = \dfrac{k_{\theta 1}}{I_1}(\bar{\theta}_{11,l} - \bar{\theta}_{12,l}) - \dfrac{rb_1}{I_1} \cdot k_1(t) \cdot \delta_{1,l} \\ \ddot{\bar{\theta}}_{21,l} = \dfrac{k_{\theta 2}}{I_2}(-\bar{\theta}_{21,l} + \bar{\theta}_{22,l}) - \dfrac{rb_2}{I_2} \cdot k_1(t) \cdot \delta_{1,l} \\ \ddot{\bar{\theta}}_{22,l} = \dfrac{k_{\theta 2}}{I_3}(\bar{\theta}_{21,l} - \bar{\theta}_{22,l}) + \dfrac{rb_3}{I_3} \cdot k_2(t) \cdot \delta_{2,l} \\ \ddot{\bar{\theta}}_{31,l} = \dfrac{k_{\theta 3}}{I_4}(-\bar{\theta}_{31,l} + \bar{\theta}_{32,l}) + \dfrac{rb_4}{I_4} \cdot k_2(t) \cdot \delta_{2,l} \\ \ddot{\bar{\theta}}_{32,l} = \dfrac{k_{\theta 3}}{I_r}(\bar{\theta}_{31,l} - \bar{\theta}_{32,l}) - C_g(t) \\ \ddot{\bar{x}}_1 = -\dfrac{k_{x1}}{M_1}\bar{x}_{1,l} + \dfrac{\sin\alpha_1}{M_1} \cdot k_1(t) \cdot \delta_{1,l} \\ \ddot{\bar{x}}_2 = -\dfrac{k_{x2}}{M_2}\bar{x}_{2,l} - \dfrac{\sin\alpha_1}{M_2} \cdot k_1(t) \cdot \delta_{1,l} - k_2(t)\dfrac{\sin\alpha_2}{M_2} \cdot \delta_{2,l} \\ \ddot{\bar{x}}_3 = -\dfrac{k_{x3}}{M_3}\bar{x}_{3,l} + k_2(t)\dfrac{\sin\alpha_2}{M_3} \cdot \delta_{2,l} \\ \ddot{\bar{y}}_1 = -\dfrac{k_{y1}}{M_1}\bar{y}_{1,l} - \dfrac{\cos\alpha_1}{M_1} \cdot k_1(t) \cdot \delta_{1,l} \\ \ddot{\bar{y}}_2 = -\dfrac{k_{y2}}{M_2}\bar{y}_{2,l} + \dfrac{\cos\alpha_1}{M_2} \cdot k_1(t) \cdot \delta_{1,l} + \dfrac{\cos\alpha_2}{M_2} \cdot k_2(t) \cdot \delta_{2,l} \\ \ddot{\bar{y}}_3 = -\dfrac{k_{y3}}{M_3}\bar{y}_{3,l} - \dfrac{\cos\alpha_2}{M_3} \cdot k_2(t) \cdot \delta_{2,l} \end{cases} \tag{18}$$

6. Numerical results

In this section, the dynamic behaviour of a two-stage gearbox system is investigated using the Polynomial Chaos (PC) method. The PC results are compared with those obtained with the Monte Carlo (MC) method (100,000 simulations). The PC results are calculated using the ODE45 solver of software MATLAB.

The dimensional parameters of the two-stage gearbox system studied are summarised in Table 2 [5].

The fluctuation of the aerodynamic torque is given in Fig. 4, with consideration of the inflow angle of the blade as an uncertain input parameter.

Both uniform and normal probability distributions are treated to describe the random parameter. The objective is to determine the most appropriate distribution of probability for this application. Indeed, to our knowledge, there is not a study that confirms the validity of a clearly defined probability distribution for our case study. Our judgement will be based on the results of the Monte Carlo method taken as a reference.

6.1. Uniform distribution of the uncertainty

In this section, a uniform probability law is supposed to describe the dispersion of the inflow angle ϕ:

$$\phi(\xi) = \frac{l_1 + l_2}{2} + \frac{l_2 - l_1}{2}\xi \tag{19}$$

The inflow angle varies in the interval $[l_1\ l_2] = [0°\ 30°]$ and ξ is distributed uniformly within the orthogonally interval $[-1\ 1]$.

Table 2
System parameters.

Description	Symbol	Value
Air density (kg·m^{-3})	ρ_{air}	1.225
Turbine rated speed (rad·s^{-1})	Ω	13
Rotor diameter m	D	12
Stiffness to bending (N·m^{-1})	k_{xj}	$7\cdot10^8$
Stiffness to traction–compression (N·m^{-1})	k_{yj}	$6\cdot10^8$
Average mesh stiffness (N·m^{-1})	k_m	$2\cdot10^8$
Torsional stiffness of the shaft (N·m^{-1}·rad^{-1})	$k_{\theta j}$	$5\cdot10^6$
Number of teeth	$Z(12)$	72
	$Z(21)$	18
	$Z(22)$	54
	$Z(31)$	18
Pressure angle	α	20
Contact ratio	$\varepsilon_{\alpha 1}$	1.67
	$\varepsilon_{\alpha 2}$	1.64

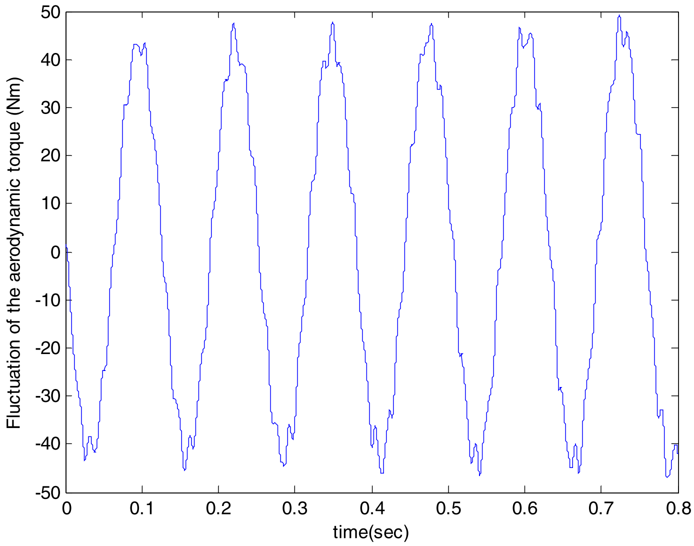


Fig. 4. Fluctuation of the aerodynamic torque.

According to the state of the art, the Legendre polynomials are the best suited to deal with uniform uncertainties. The Legendre polynomials are calculated using the recurrence relation as follows:

$$\begin{cases} (n+1)L_{n+1}(x) = (2n+1)xL_n(x) - nL_{n-1}(x) \\ L_0(x) = 1,\ L_1(x) = x \end{cases} \tag{20}$$

The validity of the method and the influence of the order p of the Polynomial Chaos on the response will be studied. To achieve this objective, the relative displacements of the three bearings of the system in the two directions x and y will be determined by varying the value of p. A correct analysis requires a comparison of the results with those obtained by the Monte Carlo method (100,000 simulations).

Figs. 5–7 show a remarkable difference between the curves obtained by the PC method and those obtained by Monte Carlo simulations for low values of the order p. This seems obvious. Indeed, in this case, there are not enough terms of chaos to correctly represent the random response of the system.

By increasing the value of p, the curves are similar to reference ones (Monte Carlo). For $p = 5$, the error is almost null, the curves are superposed almost perfectly with those of Monte Carlo. With this value of the order of Polynomial Chaos, the dynamic response of the various bearings is correctly modelled, taking into account the uncertainty of the inflow angle ϕ.

The same conclusion is also confirmed in the case of the angular displacements (Fig. 8).

6.2. Normal distribution of the uncertainty

In this part, a normal (Gaussian) probability law is supposed to describe the dispersion of the inflow angle ϕ

$$\phi = \phi_0 + \sigma_\phi \xi \tag{21}$$

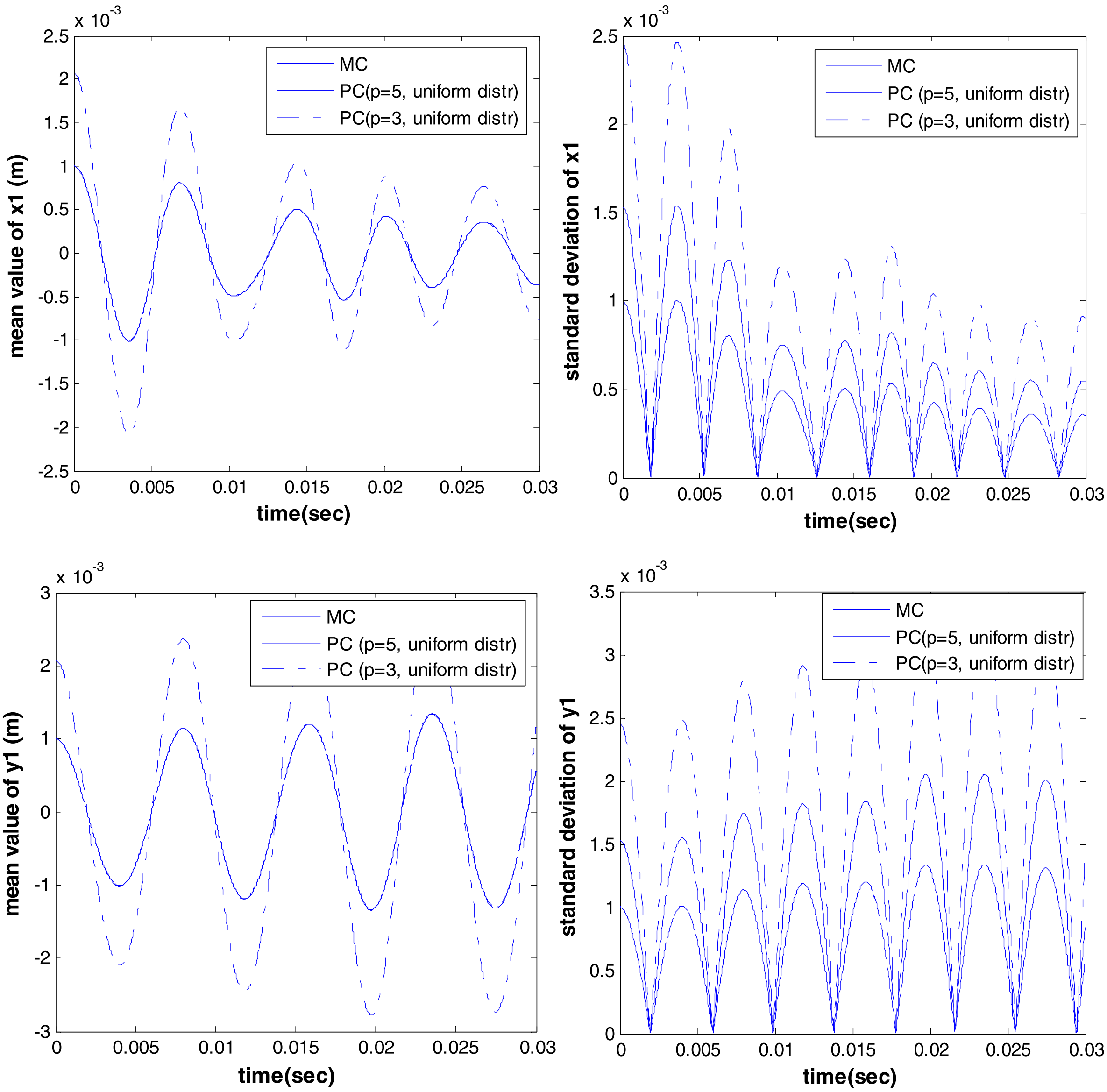


Fig. 5. Mean value and standard deviation of the linear displacements of the first bearing.

where ξ is a zero mean value Gaussian random variable, ϕ_0 is the mean value of the inflow angle and σ_ϕ is the standard deviation of this parameter.

In this case, the Hermite polynomials are the best suited to deal with normal uncertainties. The Hermite polynomials are calculated using the recurrence relation as follows:

$$\begin{cases} H_0(x) = 1 \\ H_n(x) = xH_{n-1}(x) - \dfrac{\mathrm{d}H_{n-1}(x)}{\mathrm{d}x} \end{cases} \tag{22}$$

The order of the Polynomial Chaos being fixed ($p = 5$: the optimal value allowing one to have a sufficient number of terms to correctly represent the solution). In this section, the sensibility of the solution to the standard deviation of the normal distribution of the uncertain parameter will be studied.

To achieve this objective, the mean value and the standard deviation of the linear displacement of the first bearing in the x direction will be determined by varying the value of the standard deviation $\sigma_\phi = 2\%$ and $\sigma_\phi = 5\%$. The mean value being fixed $\phi_0 = 15°$, the results are compared with Monte Carlo simulation (100,000 simulations).

As shown in Fig. 9, the mean value and the standard deviation of the displacement $x_1(t)$ have a good accuracy compared to Monte Carlo simulations for the value of the standard deviation ($\sigma_\phi = 2\%$) by increasing the value of the standard

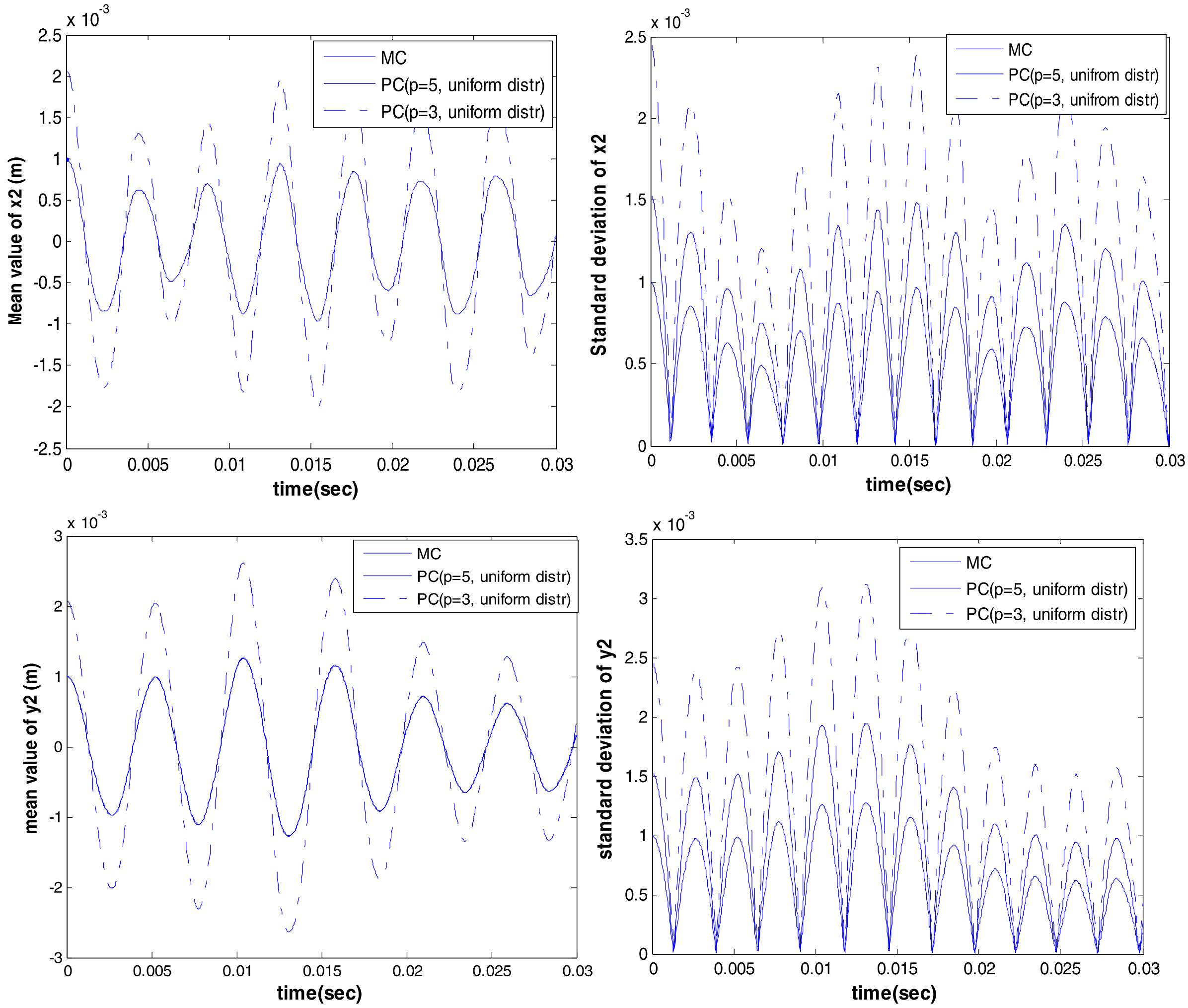


Fig. 6. Mean value and standard deviation of the linear displacements of the second bearing.

deviation ($\sigma_\phi = 2\%$), the solutions obtained oscillate in a more remarkable way around the solutions obtained by the Monte Carlo method. Consequently, the error increases when the standard deviation increases.

It should be noted that the same conclusion is also confirmed in the case of the other linear and angular displacements corresponding to different bearings of the system.

6.3. Optimal probability distribution

The efficient modelling of the propagation of uncertainty in the model requires the choice of the optimal probability distribution. Indeed generally, if the mean and the standard deviation of the uncertain parameter are known and its variation range is unbounded, a normal distribution must be chosen. In the case where the variation of the uncertain parameter is bounded, the uniform law will be more effective.

In this work, the problem has been studied by using both types of distribution. For each case, a hypothesis is proposed, according to which the parameters of the distribution will be defined. By taking the example of the linear displacement of the first and third bearings in the direction x, the order of the Polynomial Chaos being fixed ($p = 5$), Fig. 10 shows that the results obtained with normal distribution are much closer to those obtained by Monte Carlo simulations. The normal distribution is thus more effective for the modelling of the propagation of the uncertainty related to the inflow angle ϕ through the studied model.

7. Conclusion

In order to ensure the robustness of the dynamic response of a gear transmission in a wind turbine, it is very important to take into account uncertainties in the phase of design modelling. In the present work, we have performed the Polynomial

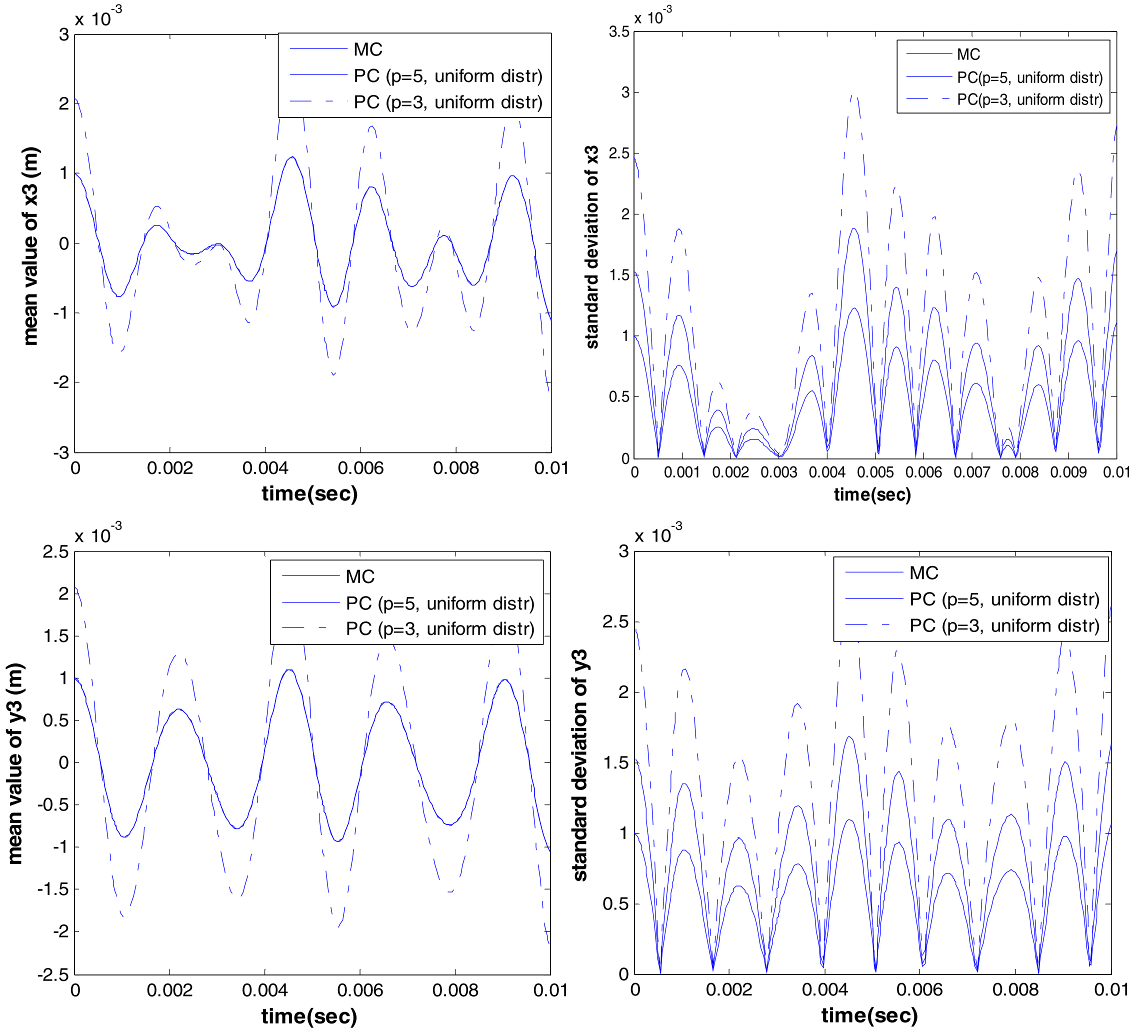


Fig. 7. Mean value and standard deviation of the linear displacements of the third bearing.

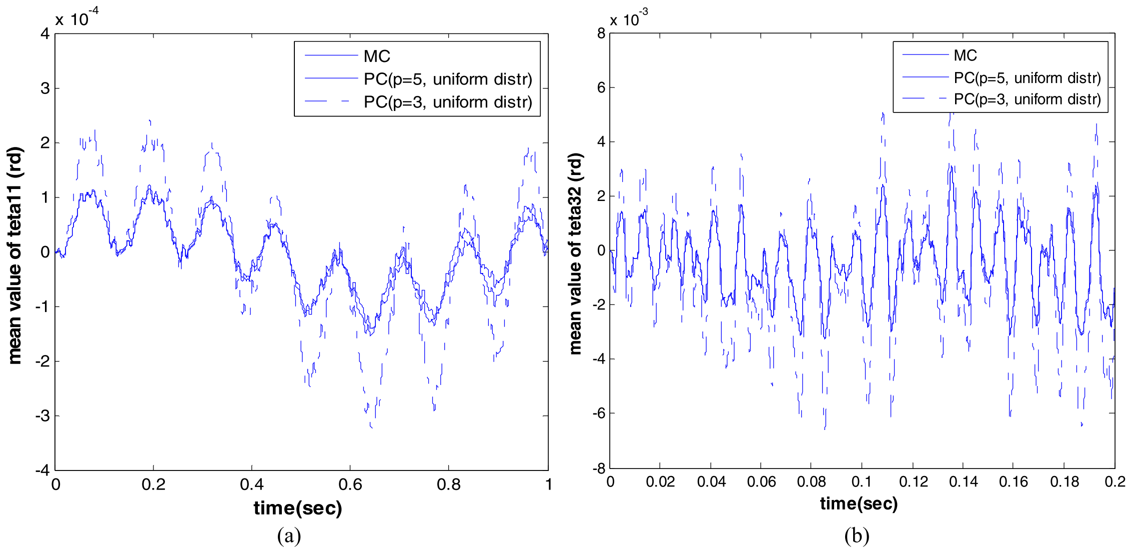


Fig. 8. Mean value of the angular displacement: (a) θ_{11}, (b) θ_{32}.

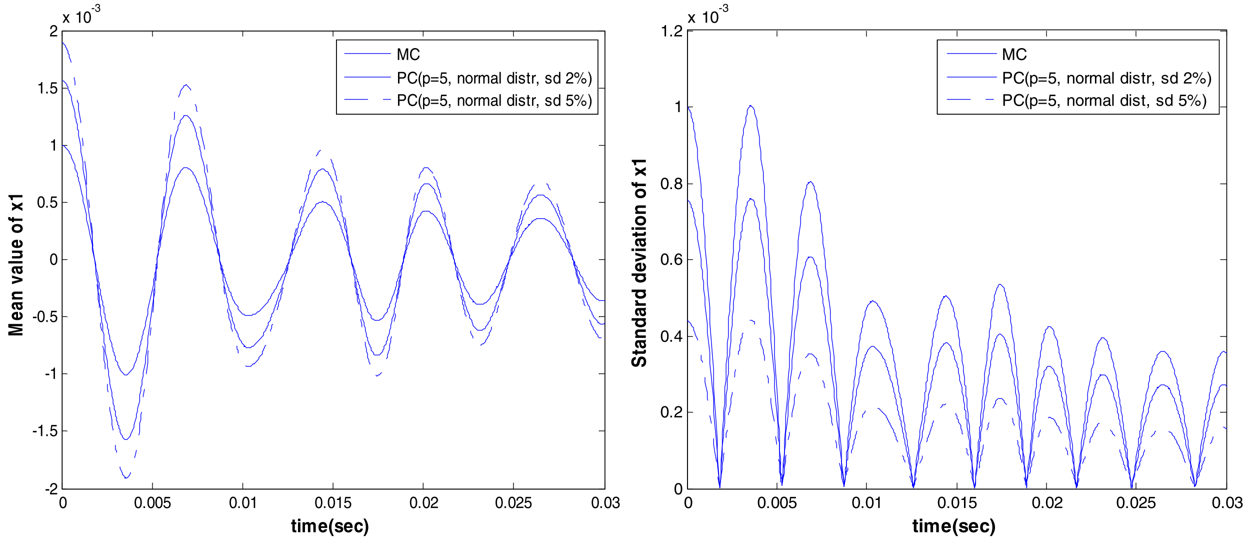
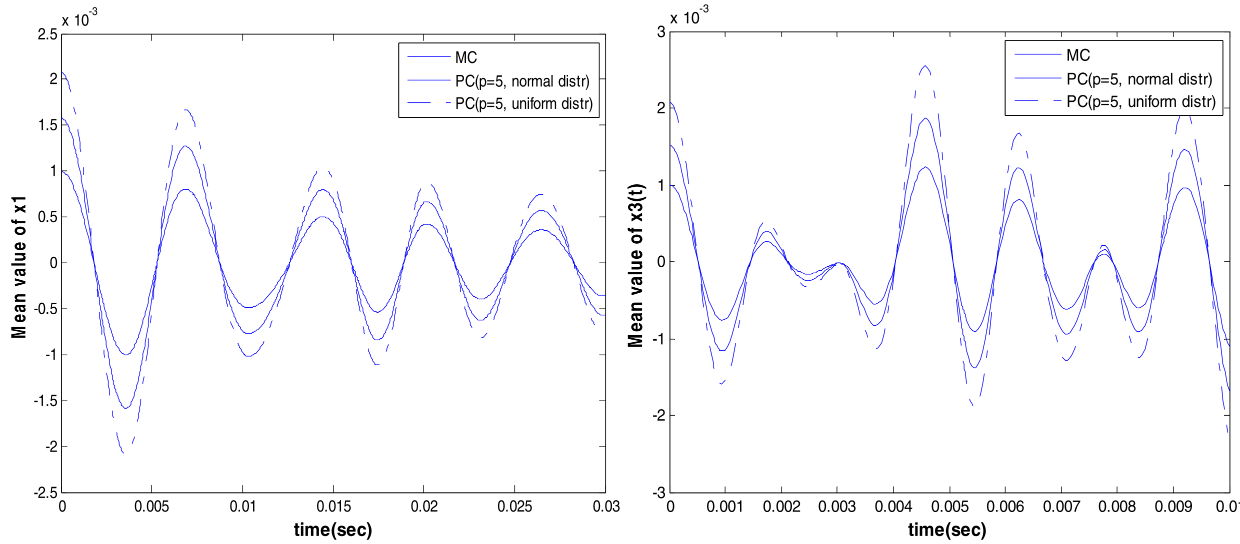


Fig. 9. Mean value and standard deviation of the linear displacements of the first bearing.



(a) (b)

Fig. 10. Mean value of the linear displacements (a) of the first and (b) of the third bearings.

Chaos method to study the dynamic behaviour of the gearbox transmission system of a horizontal-axis wind turbine with 12 degrees of freedom. Dynamic responses of the studied system describing a two-stage spur gear model with an uncertain input parameter are presented. The Polynomial Chaos (PC) method has been developed to deal with uncertainty according to the inflow angle of the blade when the wind attacks the blade. Three main results were deduced from this work. By referring to Monte Carlo simulations, the effectiveness of the polynomial chaos method for a modelling of the dynamic behaviour in the presence of uncertainty has been demonstrated. The results of this modelling strongly depend on the order of the Polynomial Chaos. An increase in this order is accompanied by a better projection of the solution. Finally, it has been found that a normal distribution is more appropriate for modelling the propagation of the uncertainty of the inflow angle ϕ through the model.

Acknowledgements

The present research work has been supported by the Laboratory of Mechanics, Modelling and Manufacturing (LA2MP), the National Engineering School of Sfax and by the Ministry of Higher Education and Research of Tunisia. The authors gratefully acknowledge the support of these institutions.

References

[1] A.G. González Rodríguez, A. González Rodríguez, M. Burgos Payán, Estimating wind turbines mechanical constants, in: Proc. International Conference on Renewable Energies and Power Quality, ICREPQ'07, 28–30 March, Sevilla, Spain, 2007.

[2] M. Gabriele, A.D. Hansen, T. Hartkopf, Variable speed wind turbines – modeling, control and impact on power systems, in: Proc. European Wind Energy Conference and Exhibition, Brussels, 2008, pp. 100–104.

[3] F. AbdalRassul Abbas, M. Abdulla Abdulsada, Simulation of wind-turbine speed control by MATLAB, Int. J. Comput. Electr. Eng. 2 (5) (2010) 915–917.

[4] Z. Hameed, Y.S. Hong, Y.M. Cho, S.H. Ahn, C.K. Song, Condition monitoring and fault detection of wind turbines and related algorithms: a review, Renew. Sustain. Energy Rev. 13 (1) (2009) 1–39.

[5] K. Abboudi, L. Walha, Y. Driss, M. Maatar, T. Fakhfakh, M. Haddar, Dynamic behavior of a two-stage gear train used in a fixed-speed wind turbine, Mech. Mach. Theory 46 (2011) 1888–1900.

[6] M. Rahimi, M. Parniani, Dynamic behavior and transient stability analysis of fixed speed wind turbines, Renew. Energy 34 (12) (2009) 2613–2624.

[7] Y. Guo, J. Keller, W. LaCava, Combined effects of gravity, bending moment, bearing clearance, and input torque on wind turbine planetary gear load sharing, in: Proc. American Gear Manufacturers Association Fall Technical Meeting, Dearborn, MI, USA, 2012.

[8] J. Helsen, F. Vanhollebeke, B. Marrant, D. Vandepitte, W. Desmet, Multibody modelling of varying complexity for modal behaviour analysis of wind turbine gearboxes, Renew. Energy 36 (11) (2011) 3098–3113.

[9] W. Sha, Z. Jingshan, H. Qinkai, C. Fulei, Dynamic response analysis on torsional vibrations of wind turbine geared transmission system with uncertainty, Renew. Energy 78 (2015) 60–67.

[10] W. Jingyue, W. Haotian, G. Lixin, Analysis of effect of random perturbation on dynamic response of gear transmission system, Chaos Solitons Fractals 68 (2015) 78–88.

[11] G.S. Fishman, Monte Carlo, Concepts, Algorithms and Applications, first edition, Springer Verlag, 1996.

[12] R.Y. Rubinstein, Simulation and the Monte Carlo Method, John Wiley & Sons Inc., New York, 1981.

[13] M.H. Kalos, P.A. Whitlock, Monte Carlo Methods, Basics, vol. 1, Wiley-Interscience, New York, 1986.

[14] R.G. Ghanem, P.D. Spanos, A stochastic Galerkin expansion for nonlinear random vibration analysis, Probab. Eng. Mech. 8 (3) (1993) 255–264.

[15] R.G. Ghanem, P.D. Spanos, Polynomial chaos in stochastic finite element, J. Appl. Mech. 57 (1990) 197–202.

[16] P. Pettersson, G. Iaccarino, J. Nordstrom, Numerical analysis of the Burgers' equation in the presence of uncertainty, J. Comput. Phys. 228 (2009) 8394–8412.

[17] T. Chantrasmi, P. Constantine, N. Etemadiz, G. Iaccarino, Q. Wang, Uncertainty quantification in simple linear and non-linear problems, Center for Turbulence Research, Annual Research Briefs, 2006.

[18] T. Chantrasmi, A. Doostan, G. Iaccarino, Padé–Legendre, approximants for uncertainty analysis with discontinuous response surfaces, J. Comput. Phys. 228 (2009) 7159–7180.

[19] H.N. Najm, Uncertainty quantification and Polynomial Chaos techniques in computational fluid dynamics, Annu. Rev. Fluid Mech. 41 (2009) 35.

[20] M.S. Eldred, Recent advances in non-intrusive Polynomial Chaos and stochastic collocation methods for uncertainty analysis and design, in: Proceedings of the 11th AIAA Non-deterministic Approaches Conference, Palm Springs, CA, USA, 2009, No. AIAA-2009-2274.

[21] D. Ghosh, C. Farhat, Strain and stress computations in stochastic finite element methods, Int. J. Numer. Methods Eng. 74 (8) (2008) 1219–1239.

[22] H.N. Najm, B.J. Debusschere, Y.M. Marzouk, S. Widmer, O.P. Le Maître, Uncertainty quantification in chemical systems, Int. J. Numer. Methods Eng. 80 (2009) 789–814.

[23] C. Sandu, A. Sandu, L. Li, Stochastic modeling of terrain profiles and soil parameters, SAE 2005 Trans. J. Commer. Veh. 114 (2) (2006) 211–220.

[24] A. Sandu, C. Sandu, M. Ahmadian, Modeling multibody dynamic systems with uncertainties. Part I: theoretical and computational aspects, Multibody Syst. Dyn. 15 (4) (2006) 369–391.

[25] R. Gasch, J. Twele, Wind Power Plants. Fundamentals, Design, Construction and Operation, James & James Science Publishers Ltd, 2002.

[26] L. Buhl Jr., A new empirical relationship between thrust coefficient and induction factor for the turbulent windmill state, Technical report, NREL/TP-500-36834, August 2005.

[27] G.P. Corten, Flow separation on wind turbine blades, Ph.D. thesis, Utrecht University, Utrecht, The Netherlands, 2001.

[28] P.J. Moriarty, A.C. Hansen, AeroDyn theory manual, Technical report, NREL/TP-500-36881, January 2005.

[29] D.A. Sphera (Ed.), Wind Turbine Technology: Fundamental Concepts of Wind Turbine Engineering, ASME, 1998.

[30] S. Rajakumar, D. Ravindran, Iterative approach for optimising coefficient of power, coefficient of lift and drag of wind turbine rotor, Renew. Energy 38 (2012) 83–93.

[31] R. Lanzafame, M. Messina, Fluid dynamics wind turbine design: critical analysis, optimization and application of BEM theory, Renew. Energy 32 (14) (2007) 2291–2305.

[32] L. Walha, T. Fakhfakh, M. Haddar, Nonlinear dynamics of a two-stage gear system with mesh stiffness fluctuation, bearing flexibility and backlash, Mech. Mach. Theory 44 (2009) 1058–1069.

[33] R.B. Jairo Alberto, L.L. Juan Fernando, Q.R. Héctor Fabio, Design, modeling and dynamic simulation of three double stage gearboxes with different module, mesh stiffness fluctuation and different level tooth breakage, Rev. Fac. Ing. Univ. Antioquia 74 (2015) 117–131.

[34] L. Nechak, S. Berger, E. Aubry, A Polynomial Chaos approach to the robust analysis of the dynamic behavior of friction systems, Eur. J. Mech. A, Solids 30 (4) (2011) 594–607.

[35] P. Pourazarm, L. Caracoglia, M. Lackner, Y. Sadeghi, Stochastic analysis of flow-induced dynamic instabilities of wind turbine blades, J. Wind Eng. Ind. Aerodyn. 137 (2015) 37–45.

[36] A. Desail, J.A.S. Witteveen, S. Sarkar, Uncertainty quantification of a nonlinear aeroelastic system using Polynomial Chaos Expansion with constant phase interpolation, J. Vib. Acoust. 135 (2013) 1–13.

[37] S. Duck Kwon, Uncertainty of bridge flutter velocity measured at wind tunnel tests, in: Proc. Fifth International Symposium on Computational Wind Engineering, CWE2010, Chapel Hill, NC, USA, 2010.

[38] Z.Y. Liu, X.D. Wang, S. Kang, Stochastic performance evaluation of horizontal axis wind turbine blades using non-deterministic CFD simulations, Energy 73 (2014) 126–136.