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Effect of the defect initial shape on the fatigue lifetime of a continuous casting machine roll

Oleh P. Yasniy [a,*], Yuri Lapusta [b]

[a] Department of Mathematical Methods in Engineering, Ternopil Ivan Pul'uj National Technical University, Ruska 56, Ternopil, 46001, Ukraine
[b] French Institute of Advanced Mechanics, Institut Pascal/IFMA/CNRS/UBP/Clermont Université, Campus de Clermont-Ferrand, BP 265, 63175 Aubière cedex, France

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ABSTRACT

The article deals with the influence of the defect initial shape on the residual lifetime of a continuous casting machine roll made of 25Cr1MoV steel. Based on this approach, previously proposed by some authors, the growth of the surface fatigue crack was modeled in a roll under loading and temperature conditions that are close to operational ones, taking into account the statistical distribution of the C parameter of Paris' equation. Dependencies of the continuous casting machines roll fatigue lifetime on the initial defect shape and critical defect sizes are obtained.

1. Introduction

Lifetime prediction of structures subjected to fatigue and thermal fatigue, corrosion, and other factors is of great importance. These structures include elements of thermal and nuclear power plants [1–4], pipelines [5], rolls of continuous casting machines [6–8], railway axles [9], etc. Note that multiple cracking is a typical mechanism for all these structures and that a variety of approaches was proposed for the study of multiple cracks and defects in structural elements and related problems [10–15]. The multiple cracking is often observed in the rolls of continuous casting machines and equipment for hot rolling that undergo cyclic thermo-mechanical loading. This loading leads to the emergence and growth of multiple surface cracks [6,16,17] which can affect significantly the structure's residual lifetime.

This article studies the influence of the initial shape of such crack on the structure residual lifetime considering the example of a continuous casting machine roll (CCMR). The roll's lifetime is generally determined by surface crack growth under thermal fatigue up to a critical size. Generally, it depends on the properties of the material, on the applied cyclic mechanical stresses and temperature, on the rolling speed, and on other factors. Note that the residual lifetime of structural elements with surface defects can also significantly depend on the initial defect shape [18–21]. The effect of temperature, frequency and loading waveform on the fatigue crack growth rate of CCMR material was studied in the work [22]. The fatigue crack growth rate in 15Cr13Mo steel almost does not depend on temperature (20 °C and 600 °C). The increase of frequency loading from 0.01 to 0.1 Hz augments the fatigue crack growth rate for $\Delta K < 24$ MPa $\sqrt{\mathrm{m}}$ and reduces by more than two times for $\Delta K > 28$ MPa $\sqrt{\mathrm{m}}$ [22].

* Corresponding author. Tel.: +380 352526613; fax: +380 352254983.
E-mail address: oleh.yasniy@gmail.com (O.P. Yasniy).

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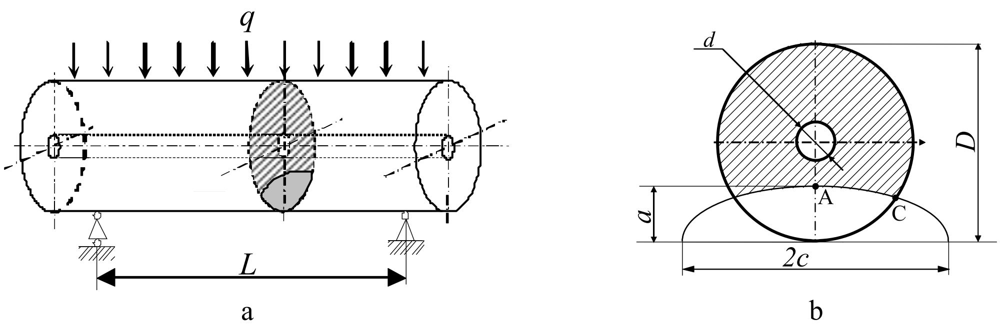

Fig. 1. a) Scheme of the roll and its loading; b) cross-section with a semi-elliptical crack.

The change in the shape of the surface fatigue cracks in cylindrical specimens of railway axles made of 30NiCrMoV12 steel induced by rotating bending has been studied in [18]. It was found that when the relative depth of the surface crack $\alpha = a/D$ is divided by 10 (from 0.025 to 0.25), the shape a/c of the propagating semi-elliptical crack decreases from 0.9 to 0.6, where a is the crack depth, D is the specimen diameter, c is the half crack length. The surface cracks in the railway axles are mostly semi-elliptical [18,21].

In the work [20], it was shown that regardless of the loading scheme (reverse or rotating bending) and initial crack shape (semi-elliptical or semi-circular) for a relative crack depth of more than 0.16, the crack shape can be described by a dependency on the relative depth.

The topology of surface cracks in entirely forged CCMR made of 25Cr1MoV steel, dismounted from service after 4500 melts, was studied in [23]. The cracks in the plane perpendicular to the roll axis are by from 1.5 to 1.8 times deeper than those in the axial one, and have crack shape 2.5 for crack depths up to 6 mm and crack shape 4 for cracks deeper than 6 mm.

Cracking of rolls and other parts of the rolling equipment under thermal fatigue was modeled, for example, in [24,25]. However, the deterministic approaches do not take into account the scatter of mechanical properties. Using them, one cannot assess the probability of reaching the critical crack size and to get the probability distribution, which is function of the structure's residual lifetime. In contrast, statistical and probabilistic fracture mechanics methods allow us to obtain the distribution function of the residual lifetime or the critical crack size [26,27]. Such approaches to fatigue crack growth modeling in structural elements are based on the analysis of the stress state and take into account the scatter of crack growth resistance characteristic of certain conditions [28,29] described by the known distribution laws. To describe the parameters of the Paris equation, the normal and the log-normal distribution are mostly used [30].

The aim of this paper is to estimate the structural elements' residual lifetime depending on the shape of the initial defect, considering the statistical scatter of the characteristics of crack growth resistance. This analysis is carried out using the example of a CCMR. Based on this approach, previously proposed by some authors, the growth of a surface fatigue crack was modeled in a roll under loading and temperature conditions that are close to the operational ones, taking into account the statistical distribution of the parameter C in Paris' equation. Dependencies of the CCMR fatigue lifetime from the shape of the initial defect are obtained.

2. Modeling of surface fatigue crack growth and residual lifetime assessment

Consider a roll that is a thick-walled hollow cylinder with an outer diameter $D = 320$ mm and a cooling hole with diameter $d = 80$ mm (Fig. 1a). The distance between the supports is 2000 mm. The semi-elliptical fatigue crack in the central cross-section of the roll perpendicular to its axis was considered (Fig. 1b). The roll is made of 25Cr1MoV steel. To predict the residual lifetime, we have employed the approach adopted in [8]. The initial conditions were the following: initial crack depth $a_0 = 15$ mm, initial crack shape $a_0/c_0 = 1/16; 1/8; 1/4; 1/2$. To simplify the model, we assume that the temperature fluctuations during one rotation are insignificant. The temperatures in the middle roll cross-section and on the surface of the roll are equal to 375 °C and 600 °C, respectively.

The stresses in the roll are caused by the pressure of the liquid metal and the weight of the slab. The semi-elliptical fatigue crack growth was modeled under stress ratio $R = K_{min}/K_{max} = 0$, where K_{min}, K_{max} are the minimum and maximum stress intensity factors (SIFs), respectively. The stress range $\Delta\sigma = \sigma_{max} - \sigma_{min} = 257$ MPa, where σ_{min} and σ_{max} are the minimum and maximum normal stresses of the loading cycle, perpendicular to the crack plane. The stress range was determined at the surface of the roll.

The SIF at the deepest point (Fig. 1b) and at the point on the surface of the semi-elliptical crack in the hollow cylinder was calculated according to the data of Carpinteri [31]. The SIF range for mode I at points A and C was calculated by the formula

$$\Delta K_{A(C)} = \Delta\sigma\sqrt{\pi a} Y_{A(C)}$$

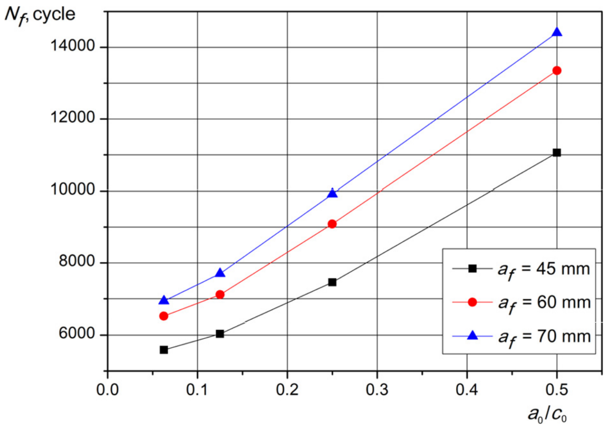

Fig. 2. Dependency of the median lifetime on the initial crack shape at 600 °C.

where $Y_{A(C)}$ is the dimensionless SIF, which is a function of the relative crack depth $\zeta = a/D$ and the crack front shape $\lambda = a/c$. The tabular representation of $Y_{A(C)}$ can be found in [31]. These data were interpolated using bicubic splines in two dimensions. The fatigue crack growth rate in 25Cr1MoV steel was determined under uniaxial loading of compact tension specimens with a thickness of 5 mm [8]. The steel mechanical properties at a temperature of 600 °C are the following: yield strength $\sigma_Y = 280$ MPa and tensile strength $\sigma_{UTS} = 380$ MPa [33]. The experimental fatigue crack growth dependencies in 25Cr1MoV steel were approximated by Paris equation [34]:

$$da/dN = C(\Delta K)^m$$

where C and m are the constants of the material.

For a temperature of 600 °C at $R = 0$ and a loading frequency $f = 0.1$ Hz with a triangular waveform, we have $C = 6.6 \cdot 10^{-9}$ (mm/cycle)/(MPa $\sqrt{\text{m}}$)^m, $m = 3.26$ [8]. The material properties obtained under these loading and temperature conditions are close to the operational ones. The maximum temperature of the roll on the surface reaches 600 °C and decreases with the distance from the surface. Though the roll is being operated in conditions of thermal mechanical loading, when the distance from the surface increases, the main role is played by the stresses caused by mechanical loading due to the pressure of the liquid metal. Therefore, the cyclic crack resistance properties obtained in isothermal conditions were used for modelling crack growth. The above-mentioned type of rolls is being operated under a rotation frequency from 0.01 Hz to 0.1 Hz. The influence of creep on the fatigue crack growth rate was not taken into account in the model, since a hold time under maximum loading of $t = 10$ s decreases the fatigue crack growth rate in the roll material at 600 °C as compared to the application of a triangular waveform [22].

The parameter C for 25Cr1MoV steel was treated as a normally distributed random variable with the following parameters: mean $\mu = 7.45 \cdot 10^{-9}$ (mm/cycle)/(MPa $\sqrt{\text{m}}$)^m, and standard deviation $s = 6.09 \cdot 10^{-10}$ (mm/cycle)/(MPa $\sqrt{\text{m}}$)^m [8] using the approach proposed in the paper [32]. The assumptions about the unknown mean and standard deviation were tested and accepted according to Anderson–Darling's test [35].

The fatigue crack growth rate of surface semi-elliptical crack in a CCMR at radial (point A) and circumferential (point C) directions was calculated from the system of Paris-type equations:

$$\begin{cases} \frac{da}{dN} = C(\Delta K_A)^m \\ \frac{dc}{dN} = C(\Delta K_C)^m \end{cases}$$

where K_A, K_C are the SIF of mode I in points A and C of the crack front, respectively.

The fatigue crack growth in CCMR with semi-elliptical initial crack at the temperature of 600 °C was modeled in two different ways. In the first one, the values of the final crack depth were preset: ($a_f = 45$, 60, and 70 mm). The crack depth $a_f = 70$ mm corresponds to the critical crack depth with $K = K_C = 78.0$ MPa $\sqrt{\text{m}}$. As a result of our simulations, lifetime distributions were obtained.

In the second modeling, the median lifetimes to reach the final crack depth were fixed ($N = 5588$, 6931, 11,060, and 14,405 cycles), and the distributions of final crack depth were constructed. In both cases, 100 simulations were carried out. In each simulation, the random values of C were chosen according to an inverse distribution function of C.

Fig. 2 shows the dependencies of the median lifetime N_f on the initial crack shape for different values of a_f. It was found out that with the increase in the crack shape a_0/c_0 of the initial semi-elliptical crack with the depth $a_0 = 15$ mm, the median CCMR fatigue lifetime augments. For example, by increasing the crack shape a_0/c_0 from 0.0625 to 0.5, the lifetime increases in about 2 times.

Fig. 3 shows the results of probabilistic fatigue crack growth modeling. These are the distributions of fatigue lifetime (the number of loading cycles) needed for the crack to reach the depths $a_f = 45$ mm and 70 mm for the initial shape of the defect.

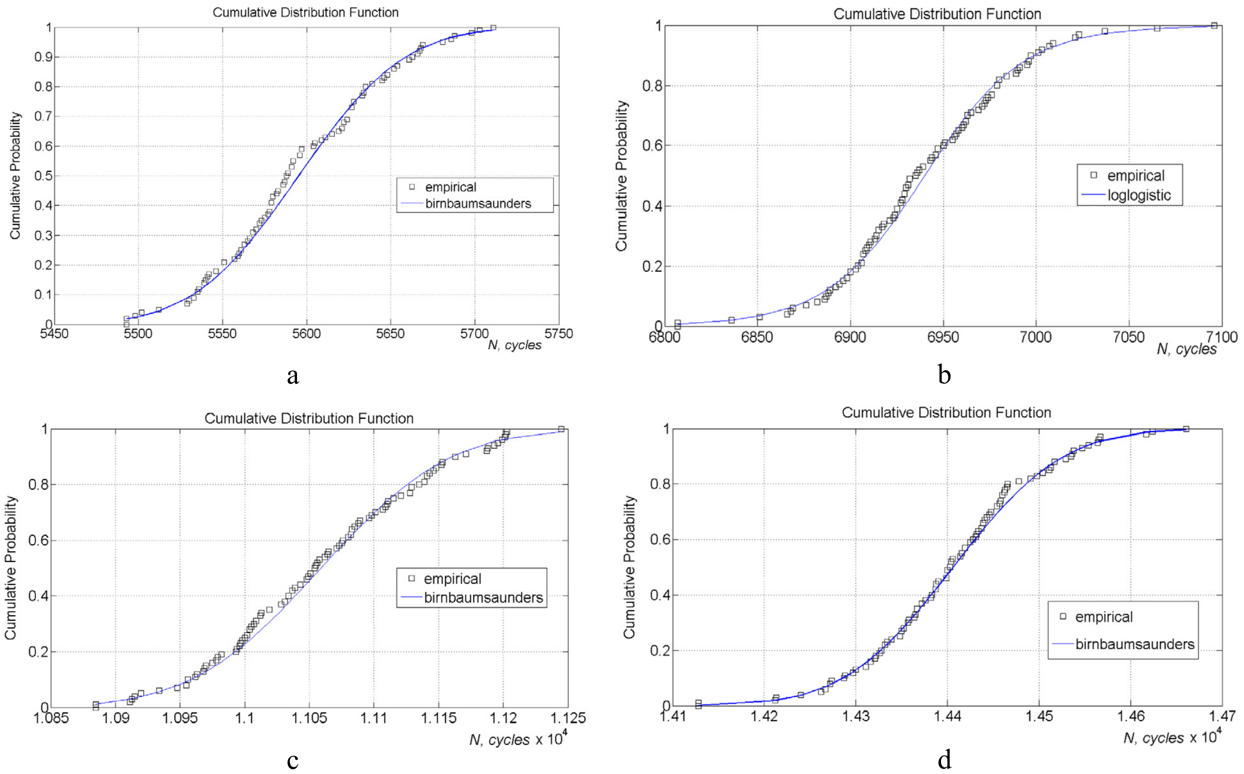

Fig. 3. The distributions of CCMR lifetime for fixed initial crack depth $a_0 = 15$ mm and crack shape $a_0/c_0 = 1/16$ (a, b); 1/2 (c, d) and $a_f = 45$ mm (a, c); 70 mm (b, d).

Unlike in the study [8], where the non-linear least-square method was used to determine the unknown parameters of the distributions, another approach was employed in present paper to obtain the most preferred model among the set of available ones. Namely, the distributions of fatigue lifetime were chosen by means of the Akaike information criterion (AIC) [36]:

$$AIC = 2k - 2\ln(L)$$

where L is the maximized value of the likelihood function for the model, k is the number of estimated parameters in the model. Given a set of available models for the data, the preferred model is the one with the minimum value of AIC.

The empirical data are the result of the calculation of the crack growth with a probabilistic distribution of C and for a fixed initial geometry. In each simulation, the parameter C was chosen according to its inverse distribution function and was constant during this simulation.

In three cases (Fig. 3a, c, d), the most preferred model among a set of 12 models was Birnbaum–Saunders' distribution [37], which has the following cumulative distribution function:

$$F(x;\alpha;\beta) = \Phi\left(\frac{1}{\alpha}\left[\left(\frac{x}{\beta}\right)^{0.5} - \left(\frac{\beta}{x}\right)^{0.5}\right]\right)$$

Here α is the shape parameter, β is the scale parameter, $\Phi(x)$ is the standard normal distribution.

In the case when $a_0/c_0 = 1/16$ and $a_f = 75$ mm, the most preferred model was log-logistic. Its probability density function is:

$$f(x;c;d) = \frac{1}{d}\frac{1}{x}\frac{e^z}{(1+e^z)^2}$$

where $z = \frac{\ln(x)-c}{d}$, c is the log mean parameter, d is the log scale parameter.

The types and parameters of the determined distributions and values of AIC are presented in Table 1.

Fig. 4 presents the probability density functions of crack depth after $N_f = 5588$, 6931, 11,060 and 14,405 cycles for the defect's initial shape. In all simulation cases, the distributions of crack depth are consistent with the Anderson–Darling test for normal, log-normal, and Weibull distributions.

Fig. 5a shows the surface cracks fronts during crack propagation. The left crack has initial shape $a_0/c_0 = 1/2$, and the right one has a shape of 1/16. Fig. 5b presents the dependencies of semi-elliptical crack shape a/c upon a/t, where $t = (D - d)/2$ is the roll's wall thickness. The calculations were performed for $a_0/c_0 = 1/16$; 1/2.

Table 1
The distributions of lifetime and their parameters.

a_0/c_0	a_f	Distribution	Parameters	AIC
1/16	45	Birnbaum–Saunders	$\alpha = 0.0088$, $\beta = 5595.3$	1067.9
1/16	70	log-logistic	$c = 8.8451$, $d = 0.0038$	1059.1
1/2	45	Birnbaum–Saunders	$\alpha = 0.0071$, $\beta = 11059$	1161.7
1/2	70	Birnbaum–Saunders	$\alpha = 0.0065$, $\beta = 14406$	1197.1

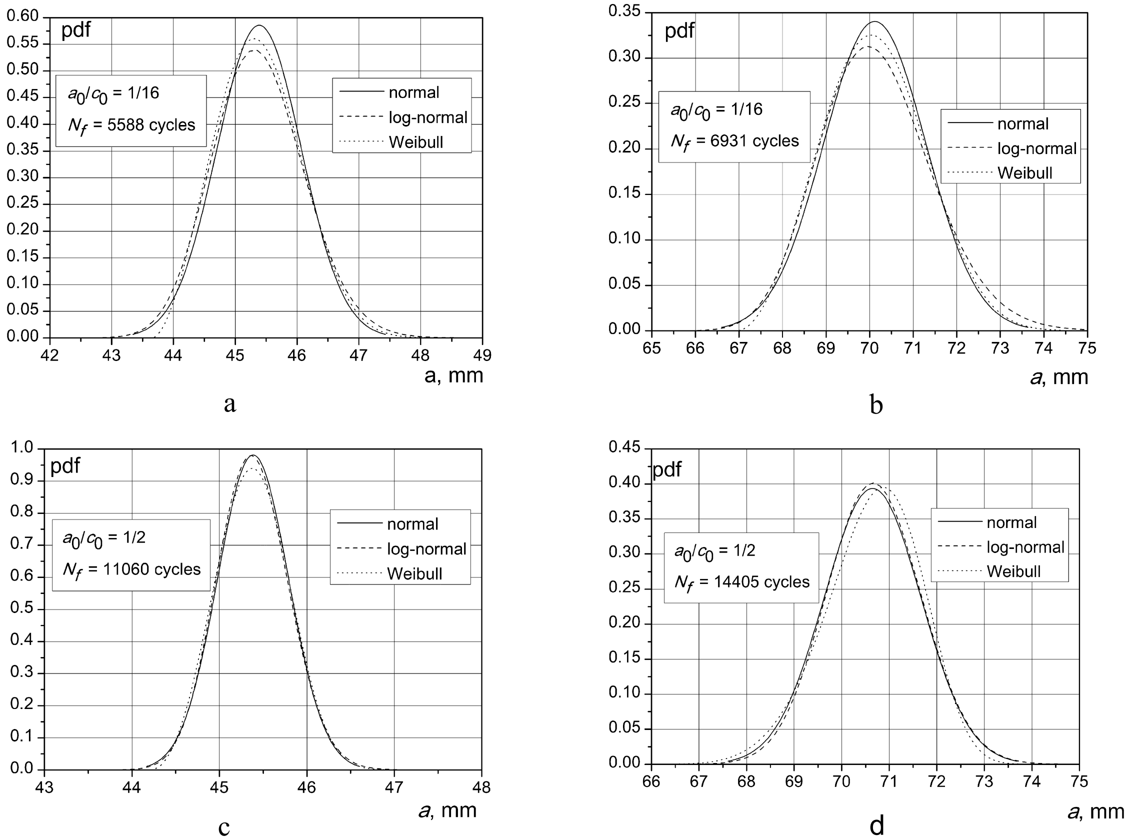

Fig. 4. The probability density functions of CCMR crack depth for initial crack shape $a_0/c_0 = 1/16$ (a, b); 1/2 (c, d) and $N_f = 5588$ cycles (a); 6931 cycles (b); 11 060 cycles (c); 14 405 cycles (d).

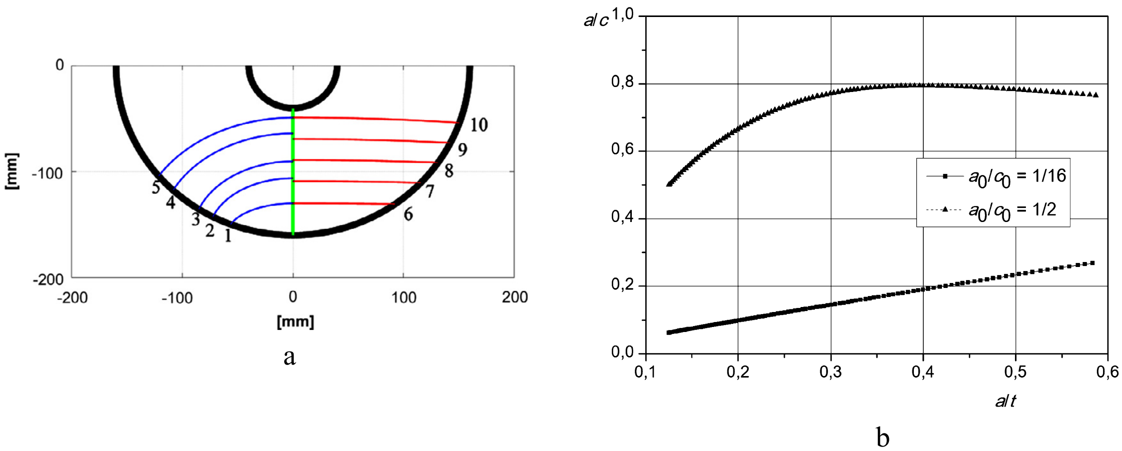

Fig. 5. a) Crack front geometry as a function of its depth ($a_0 = 15$ mm). On the left-hand side, $a_0/c_0 = 1/2$; on the right-hand side, $a_0/c_0 = 1/16$, $1N = 0$ (1), 5917 (2), 8583 (3), 9426 (4), 12 638 (5), 0 (6), 3104 (7), 4615 (8), 5598 (9), 6273 (10); b) dependencies of semi-elliptical crack shape a/c on a/t for $a_0/c_0 = 1/16$; 1/2.

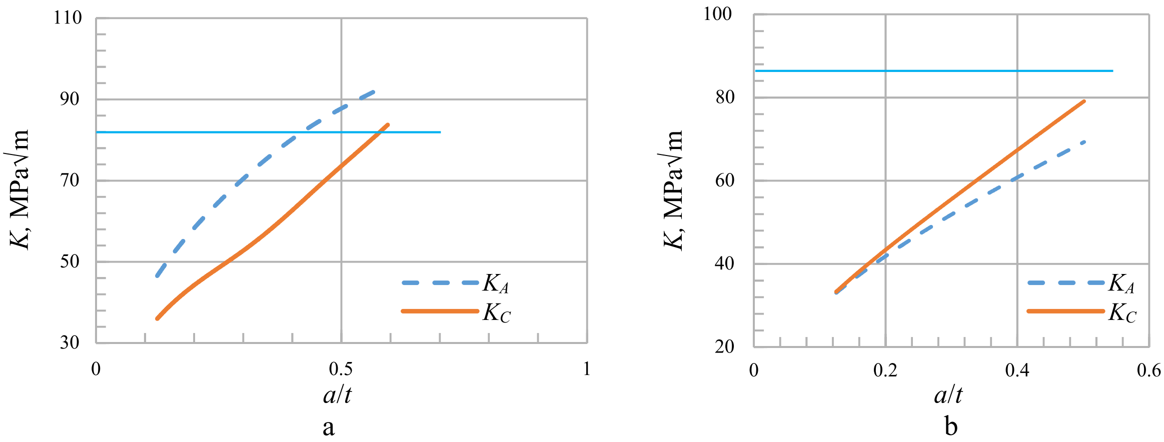

Fig. 6. The evolution of the SIF with the growth of the crack a) $a_0/c_0 = 1/16$; b) $a_0/c_0 = 1/2$.

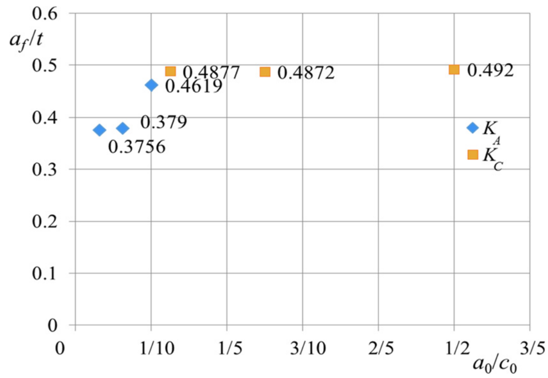

Fig. 7. The relative final crack depth a_f/t for the preset a_0/c_0.

It can be seen from Fig. 5 that with increasing the number of loading cycles, the crack shape a/c augments for $a_0/c_0 = 1/16$. When $a_0/c_0 = 1/2$, the crack shape a/c first increases up to approximately 0.8, and decreases after that.

In the studied range of a/t (0.125–0.58), surface cracks with significantly different initial shape retained this distinction with the increase of crack depth.

Fig. 6 presents the evolution of the SIF with the growth of the crack. As it can be seen from Fig. 6a, when $a_0/c_0 = 1/16$, K_A is bigger than the respective K_C. This means that the fracture of the roll occurs because SIF at the deepest point reaches the critical value K_c. On the contrary, when $a_0/c_0 = 1/2$, K_C is greater, than K_A. Therefore, the modelled roll fractures because SIF on the point on the surface takes the critical level K_c.

The fatigue crack growth was modeled for the initial crack depth $a_0 = 15$ mm and the shapes $a_0/c_0 = 1/32$; $1/16$; $1/10$; $1/8$; $1/4$; $1/2$. Fig. 7 presents the dependency of relative final crack depth a_f/t on the initial crack shape a_0/c_0. This dependency can be divided into two parts regarding the point of the crack front, where the critical SIF is reached. When $a_0/c_0 \leq 1/10$, the SIF K_A at the deepest point A becomes equal to the critical SIF value K_c earlier than the SIF K_C at point C. In this case, failure starts from the deepest point of the crack front. Conversely, when $a_0/c_0 \geq 1/8$, the limit state is reached earlier at the point C of the surface.

The obtained results are very important for the further detailed assessment of the limit state of the structural elements that follow a temperature gradient through their wall thickness. In this case, the fracture toughness of the material will change along the front of crack, which should be taken into account.

3. Conclusion

In this work, the influence of the initial surface crack shape on the lifetime of a continuous casting machine roll submitted to fatigue was studied. Based on this approach, previously proposed by some authors, the growth of a surface fatigue crack was modeled in a roll under loading and temperature conditions that were close to the operational ones, taking into account the statistical distribution of the parameter C of Paris' equation. Dependencies of the lifetime of a continuous casting machine roll submitted to fatigue upon the initial defect shape at the temperature of 600 °C were obtained. It was seen that with increasing the initial crack shape from 1/16 to 1/2, the lifetime augments in about 2 times. Therefore, these results evidence that the defect shape that can be found using nondestructive control during in-service inspection operations must be taken into account. Cumulative distribution functions of the lifetime of a continuous casting machine roll submitted to fatigue in the assumption of a normal evolution law of the parameter of the Paris equation C for 25Cr1MoV steel using different initial crack shapes and final crack depths were constructed. The probability density functions for the final crack depth in a continuous casting machine roll for different initial shapes and median lifetimes were built. The evolution of the SIF with crack growth of was built, as well as the dependency of relative final crack depth on the initial crack shape.

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