C. R. Mecanique 344 (2016) 725–735

Contents lists available at ScienceDirect

Comptes Rendus Mecanique

www.sciencedirect.com

A formulation for multiple loading cases in plastic topology design of continua

Une formulation pour la conception topologique des milieux plastiques sous cas de chargement multiples

Zied Kammoun [a,b,*]

[a] *Université de Tunis El Manar, École nationale d'ingénieurs de Tunis, LR11ES16, Laboratoire de matériaux, optimisation et énergie pour la durabilité, B.P. 37, 1002 Tunis-Belvédère, Tunisia*

[b] *Université de Carthage, Institut supérieur des technologies de l'environnement de l'urbanisme et du bâtiment, 2, rue de l'Artisanat-Charguia 2, 2035 Tunis, Tunisia*

A R T I C L E   I N F O

Article history:
Received 13 May 2016
Accepted after revision 9 August 2016
Available online 29 August 2016

Keywords:
Plastic
Topology
Optimization
LADM
Limit analysis
Loading cases

Mots-clés :
Plastique
Topologie
Optimisation
Analyse limite
Cas de chargement

A B S T R A C T

In the real life, most industrial structures are subject to multiple load cases. The present paper proposes a topology optimization formulation for multiple loading cases. It is based on the recently developed Direct Method of Limit Analysis for plastic topology Design (LADM). In this formulation, a single mathematical problem is considered to optimize structures under multiple loading cases; each case acts independently at a different time. For the continuous design problem, as in LADM, a unique iteration is considered. For the discrete, i.e. black and white, topology optimization problem, the same approach used in LADM is conserved with the use of a sequence of conic programming problems of the same form as the continuous design problem. The proposed method is illustrated with continuous and discrete example design problems. Examples with multiple loading cases confirm the conservation of the LADM features.

R É S U M É

Dans la vie réelle, la majorité des structures industrielles sont soumises à des cas de charges multiples. Le présent article propose une formulation pour l'optimisation de la topologie des structures soumises à plusieurs cas de chargement. Il est basé sur une technique récente développée en utilisant une méthode directe d'analyse limite pour la conception topologique des structures plastiques (LADM). Dans cette formulation, un seul problème mathématique est généré pour optimiser les structures soumises à des cas de chargements multiples, chaque cas agissant indépendamment à différents moments. Pour le problème continu, comme dans la LADM, une seule itération est nécessaire. Pour le problème discret, l'approche utilisée dans la méthode LADM est conservée, avec l'utilisation d'une séquence de problèmes de programmation de coniques de même forme que le problème de conception continue. La méthode proposée est illustrée par des problèmes

* Correspondence to: Université de Carthage, Institut supérieur des technologies de l'environnement de l'urbanisme et du bâtiment, 2, rue de l'Artisanat-Charguia 2, 2035 Tunis, Tunisia.

E-mail address: kammounzied@yahoo.fr.

http://dx.doi.org/10.1016/j.crme.2016.08.002

continus et discrets. Les exemples de topologie avec plusieurs cas de chargement montrent la conservation des caractéristiques de la méthode LADM.

1. Introduction

Today, topology optimization techniques are highly demanded and used in the industry [1], and powerful dedicated software have emerged [2]. However, most of the work on the topology optimization of continuum structures was traditionally treated with elastic structures [2–6].

If the Solid Isotropic Microstructure with Penalization for intermediate densities (SIMP) [7] method is the most popular among the numerical FE-based topology optimization methods applied in industrial softwares [2], several other methods have been developed. Among the latter, stress-based topology optimization [8,9] represents a great interest for real applications.

According to Kiyono et al. [10], the topology optimization problem for stress design is still an open problem due to a number of difficulties.

One of these difficulties is the nonzero stresses in "void" regions; many solutions have been proposed to prevent this issue, such as the e-relaxation method, the qp-relaxation method, and the relaxed stress indicator method. Another difficulty is related to the high computational cost that reduces the efficiency of the optimization solver, and a third difficulty is related to the high nonlinear behavior of this problem, requesting an efficient and accurate optimizer. Another problematic issue is the presence of some intermediate material which requests a post-processing step, after which the stress value may increase.

In many industrial applications, the structure is loaded by Multiple Loading Cases (MLCs). Each of these loading cases acts independently at a different time. Frequently, these loading cases are completely independent of each other. And each case, when considered separately, can induce a totally different topology.

In the recent years, a great deal of research is carried out to extend topology optimization for elastic materials to the structures subject to multiple load cases (as in [11]). A review of those works is given in [12]. We can cite, for example, the work of Diaz and Bendse [13], who have extended the homogenization algorithm to MLCs. The majority of the algorithms for multiple loading cases can be solved by considering a weighted function on each load case as an objective function. Xie and Steven [14] extend the Evolutionary Structural Optimization to the MLC by considering a step-by-step optimization process. They took into account two different criteria for optimization: the extreme stress criterion and the weighted average criterion. Different results may be obtained by using one or the other.

On the other hand, as their maturity has been reached, an extension of the elastic approaches of topology optimization to elastoplastic analyses is expected. However, the high computational demands prevent this extension.

On the contrary, when only the information about the limit stress field is of interest, direct methods of limit analysis present an adequate alternative for plastic collapse analysis. Indeed, lower computational efforts are required to determine limit states in terms of stress field solutions.

Direct Method of Limit Analysis for plastic topology Design (LADM) is proposed and formulated in [15] and [16] for the minimum weight of continuum structures subject to a specified admissible loading.

The continuous topology design presented in [16] associates the optimization problem with direct limit analysis and treats them in a single mathematical programming problem. This allows the same order of magnitude of computational demand as a single static limit analysis problem. The microscopic approach [16] is used to formulate the design problem, and continuous material densities represent the design variables. The generated mathematical problem is a conic programming problem that presents interesting convergence properties, and prevents the difficulties that are commonly encountered in nonconvex elastic and elastoplastic topology design problems. Therefore, big problems can be treated in a relatively acceptable CPU time. Another consequence of convexity is the uniqueness of the optimum value and the fact that any optimum solution obtained is a global one. Moreover, by combining variable material densities with direct limit analysis, no numerical difficulties arise when densities vanish. So, we do not need to impose a finite lower bound on the density, as in continuum elastic design, to avoid singularities.

The presented work in [17] extends this method to the discrete, or black and white, topology design problem by applying a penalization scheme to the continuous one. The generated problem becomes non-convex, so it is treated by solving a sequence of continuous design problems. For the illustrative considered examples (taken from the literature), a comparison of the results of this method with those obtained by the elastic design methods shows (for the examples treated until now) a good agreement among the generated topologies, although there are large differences in the material behavior and the type of analysis. Moreover, it is shown that no checkerboard patterns appear with the direct methods of limit analysis for plastic topology design.

Throughout this work, based on [16] and [17], we will exploit the features of the LADM method to write a formulation that can give optimum topologies for a domain subject to multiple loading cases.

The outstanding feature of the formulation, largely similar to that of LADM problem, is that it considers a unique mathematical formulation to treat all loading cases. After a brief presentation of the LADM, the new formulation is proposed. The procedure is discussed and tested on the long cantilever beam problem.

2. The static method of limit analysis

Historically, the work of Lysmer in 1970 [18] was at the origin of the numerical static method of limit analysis (LA). This author proposed a formulation with the final problem coming under linear optimization. Later, Pastor and Turgeman ([19] and [20]) proposed an improvement of the numerical treatment of the static method. Since then, and for more than forty years, many research have participated in the development of the numerical static and kinematic approaches of limit analysis [21–28]...

The aim of the static Limit Analysis (LA) method is to determine the stress field at the limit state. Assume a mechanical domain Ω, composed of rigid, perfectly plastic materials, as defined in [29,30]; a stress field σ is said to be statically admissible (SA) if the field fulfills the equilibrium equations, and if stress boundary conditions and stress vector continuity are satisfied.

This stress field is said plastically admissible if the plasticity criterion of the material $f(\sigma)$ satisfies $f(\sigma) \leqslant 0$. A stress field σ is said to be "admissible" if it is both statically and plastically admissible.

A loading system $Q = Q(\sigma) \in \mathbb{R}^n$ (with n components of Q, called loading parameters) is said to be admissible when it is in equilibrium with a statically admissible stress field σ.

The limit analysis problem relative to the i^{th} loading parameter can be written as an optimization problem for an admissible stress field σ:

$$\begin{aligned} &Q_{\text{lim}} = (Q_1, \dots, \lambda_0 Q_i, \dots, Q_n) \\ &\qquad \lambda_0 = \max\{\lambda\ ,\ Q(\sigma) = (Q_1, \dots, \lambda Q_i, \dots, Q_n)\} \end{aligned} \tag{1}$$

By solving the optimization problem, a solution to the limit analysis problem is found, and the resulting loading $Q(\sigma)$ is a limit loading of the mechanical domain with respect to the loading component Q_i.

The numerical plane strain formulation of the static limit analysis problem is described in [20]. The domain Ω is discretized into triangular finite elements characterized by a linear stress field in x and y, where (x, y) represent the global reference frame. Across inter element interfaces, the stress field can be discontinuous (with respect to the stress vector continuity of the SA condition). The Tresca criterion, used in this work, can be written as:

$$f(\sigma) = \sqrt{(\sigma_x - \sigma_y)^2 + (2\tau_{xy})^2} - 2s \leqslant 0 \tag{2}$$

or equivalently as:

$$S(\sigma) = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \leqslant s \tag{3}$$

where s denotes the shear strength, or cohesion, of the material.

Introducing a change of variables such that the stress vector σ is defined by the components $\frac{(\sigma_x+\sigma_y)}{2}$, $\frac{(\sigma_x-\sigma_y)}{2}$ and τ_{xy} denoted as σ_+, σ_- and τ, respectively, the plasticity criterion can be written directly in the conic form $s \geqslant \sqrt{\sigma_-^2 + \tau^2}$. The numerical optimization problem, expressing the static limit analysis problem, can thus be written as a conic programming problem in the form:

$$\begin{aligned} &\lambda_0 = \max \lambda \\ &Q(\sigma) = (Q_1, \dots, \lambda Q_i, \dots, Q_n) \\ &S(\sigma) \leq s, \\ &\sigma\ SA \end{aligned} \tag{4}$$

and can, therefore, be solved using the conic programming code MOSEK [31] as in [28,32,33].

3. Formulation of the topology optimization problem

A domain Ω_0 of unit thickness, made out of a Tresca material having a shear strength $\bar{s}$ and a density $\bar{\rho} = 1$, subjected to a loading system $Q = (Q_1, \dots, Q_i, \dots, Q_n)$, is considered.

The goal of topology optimization is to find the structural configuration included in the domain Ω_0 achieving a minimum amount of material while maintaining a statically and plastically admissible stress field associated with the specified loading Q.

If continuous optimization consists in determining the densities in each element with acceptance of an intermediate value between the maximum of density and the void, the discrete optimization consists in determining whether each element in the designed domain should be solid or void, and whether the density of the material should eventually converge to one of the limiting values of density (0 and 1) by penalizing intermediate values.

3.1. The continuous design problem formulation

As explained in [16] and [17], in the continuous design formulation, the "amount" of material represented by the density ρ of a fictitious material is allowed to continuously span in the interval [0, 1].

The shear strength s is chosen as a function of density for the intermediate material:

$$s(\rho) = \rho \bar{s} \tag{5}$$

The condition that allows (with the plasticity criterion) the stress field to bring the stress tensor to zero when the density vanishes is expressed as follows:

$$-K\rho \le \sigma_+ \le K\rho \tag{6}$$

where K is a constant sufficiently large that the constraint tends to be activated near zero density only.

Premultiplying the objective by the real shear strength $\bar{s}$, redefining the constant K as $K\bar{s}$ and substituting s for $\rho\bar{s}$, the optimum design problem can be written in the form (Eqs. (7)), where the shear strengths replace the densities with design variables:

$$\begin{aligned}
&\min \int_{\Omega_0} s \, \mathrm{d}\Omega \\
&\text{s.t.} \quad Q(\sigma) = (Q_1, \dots, Q_i, \dots, Q_n), \\
&\qquad S(\sigma) \le s, \\
&\qquad \sigma \ \ SA \\
&\qquad 0 \le s \le \bar{s} \\
&\qquad -Ks \le \sigma_+ \le Ks
\end{aligned} \tag{7}$$

3.2. The discrete problem formulation

The technique used to drive the densities to the limits 0 and 1 is the penalization of intermediate densities employed in [17], where the penalization is performed by replacing the linear cost term ρ by the power law $\rho^{1/2}$. This leads to the following problem:

$$\begin{aligned}
&\min \int_{\Omega_0} \rho^{1/2} \, \mathrm{d}\Omega \\
&\text{s.t.} \quad Q(\sigma) = (Q_1, \dots, Q_i, \dots, Q_n), \\
&\qquad S(\sigma) \le s, \\
&\qquad \sigma \ SA \\
&\qquad 0 \le s \le \bar{s} \\
&\qquad -Ks \le \sigma_+ \le Ks
\end{aligned} \tag{8}$$

This optimization problem is not a convex one and can not be solved directly using $MOSEK$. To solve it, an iterative method is considered. It consists in solving a sequence of conic problems where the objective is a linear approximation of the power-law function. In each cycle, the problem to be solved is formed by the same constraints as in problem (8), and the cost function can be written as $\int_{\Omega_0} c_{\rho^*} \rho \, \mathrm{d}\Omega$, where $c_{\rho^*} = (\rho^*)^{(\frac{1}{2}-1)}$ and ρ^* denotes the solution of the preceding cycle. The resulting problem differs from the problem (7) only in the cost coefficients, and is solved using the code $MOSEK$.

4. Formulation for multiple loading cases

The basic idea of this approach is to write all generated mathematical optimization problems of all loading cases in a single mathematical problem that can be directly solved. Thus, the resulting topology will be valid for each of the loading cases.

Since the considered mechanical domain Ω is unique, the final density distribution must be the same in all loading cases. This is obtained by imposing the same density for each element in all loading cases (adding the equation $\rho = \rho_j$ for all loading cases j). Taking into account Eq. (5), we can also impose the same shear strength (as in Eq. (9)). As result, the objective of the optimization problem, i.e. the weight of the domain, has the same value for the different loading cases. And the continuous optimum design problem may be written as:

$$\begin{aligned} \min \int_{\Omega_0} & s \,\mathrm{d}\Omega \\ \text{s.t.} \quad & \forall j \in [1, \dots, n_c] \\ & Q(\sigma) = (Q_{1j}, \dots, Q_{ij}, \dots, Q_{nj}), \\ & S(\sigma_j) \le s, \\ & \sigma_j \; SA \\ & 0 \le s_j \le \bar{s} \\ & -K s_j \le \sigma_{j+} \le K s_j \\ & s_j = s \end{aligned} \tag{9}$$

where n_c is the number of loading cases.

The optimum design problem can be simplified in the following alternative form:

$$\begin{aligned} \min \int_{\Omega_0} & s \,\mathrm{d}\Omega \\ \text{s.t.} \quad & \forall j \in [1, \dots, n_c] \\ & Q(\sigma) = (Q_{1j}, \dots, Q_{ij}, \dots, Q_{nj}), \\ & S(\sigma_j) \le s, \\ & \sigma_j \; SA \\ & 0 \le s \le \bar{s} \\ & -K s \le \sigma_{j+} \le K s \end{aligned} \tag{10}$$

Since the generated mathematical problem retains the convex character, the continuous optimal solution obtained in multiple loading cases is a global one.

Considering similar changes for the continuous design formulation, the final discrete optimization problem can be written as follows:

$$\begin{aligned} \min \int_{\Omega_0} & \rho^{1/2} \,\mathrm{d}\Omega \\ \text{s.t.} \quad & \forall j \in [1, \dots, n_c] \\ & Q(\sigma) = (Q_{1j}, \dots, Q_{ij}, \dots, Q_{nj}), \\ & S(\sigma_j) \le s, \\ & \sigma_j \; SA \\ & 0 \le s \le \bar{s} \\ & -K s \le \sigma_{j+} \le K s \end{aligned} \tag{11}$$

5. Numerical examples

In this section, the proposed method for plastic topology optimization with multiple loading cases is tested through two examples of topology design problems based on the long cantilever beam. These tests are performed essentially to show the difference in the generated topology between the cases of one load, the maximum value of the density in each element of the mesh of these cases, and the multiple loading cases.

The finite-element meshes are all uniform. The rectangular design domain is divided into $n_x \times n_y$ elementary rectangles. Each one is divided into four triangular elements separated by the two diagonals. Before proceeding with the design, the limit load for the full density domain is determined since it represents an upper bound on the specified limit loads, for which the topology optimization problem is feasible. That is why the applied load is not larger than the limit one corresponding to the fully dense domain. All problems are solved with version 6 of $MOSEK$ code using an Intel Core i5 processor (2.4 GHz). The continuous design problem presents the initial step for all the algorithms presented next.

As reported in [16], the K factor must be chosen in the range between 3 and 40 and is chosen equal to 10 for all the simulations.

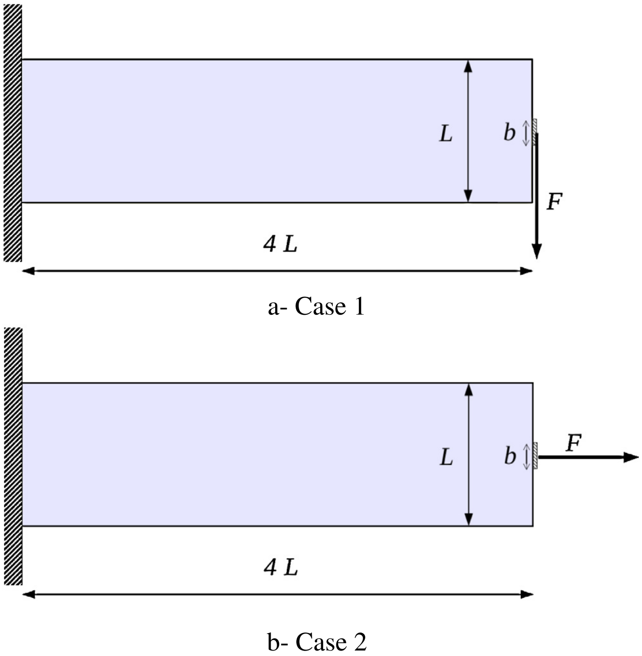

Fig. 1. Definition of loading cases 1 and 2.

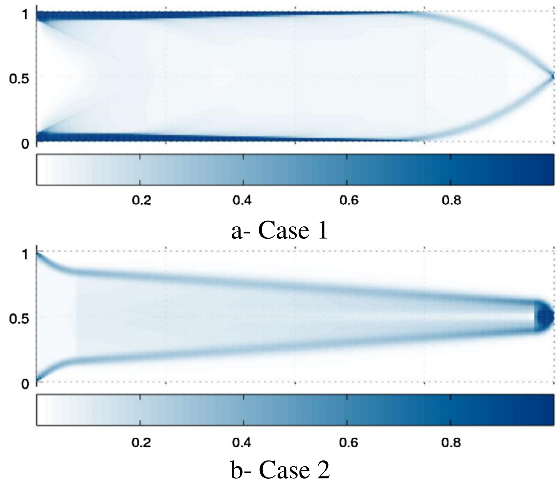

Fig. 2. Continuous topology of loading cases 1 and 2.

5.1. Two loading cases

In this section, the long cantilever beam problem (Fig. 1) with tangent centered load is treated for the first loading case. For the second loading case, the problem is defined in the same figure and the load is a centered normal one. The geometrical dimensions of the problem are $L = 1$ m and $b = 0.05$ m. For a shear strength $\bar{s} = 1$ kPa and using a mesh with $160 \times 80 \times 4 = 51,200$ elements, the limit analysis carried out for the full density domain yields a limit load $\bar{F} = 0.050$ kN in 39 s of CPU time for the first loading case and $\bar{F} = 0.248$ kN in 46 s of CPU time for the second case.

The applied load is chosen equal to 0.04 kN for case 1 and equal to 0.2 kN for the second case. Using a $160 \times 80 \times 4 = 51,200$ elements mesh, the topology of the initial step (continuous design) is represented in Fig. 2. After 30 iterations, the resulting design is shown in Fig. 3; the obtained optimum weight, cost and CPU of the continuous and discrete problem for each case are reported in Tables 1 and 2. As noted in [17], the converged solution in the discrete topology is not perfectly a 0–1 design.

One of the possible topologies that can be used for both loading cases is obtained by considering the maximum value of the density in each element of the mesh. This topology can be used as a reference (and labeled R-Topology) to compare our new technique. The new technique is used to simulate the optimal topology (called C-topology) that can be used with both

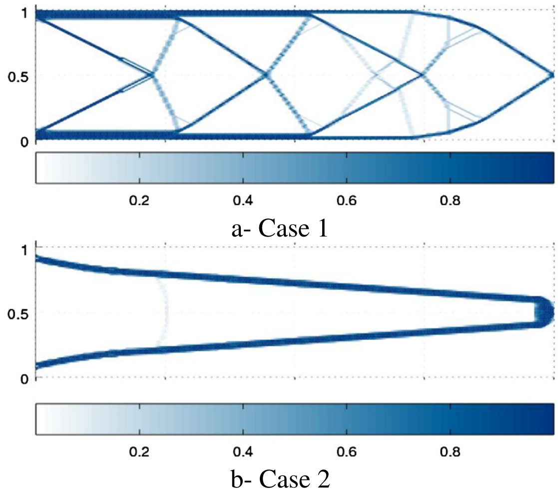

Fig. 3. Discrete topology of loading cases 1 and 2.

Table 1
Results for the continuous topology for two loading cases.

	Optimal weight	Accuracy	CPU (s)
Case 1	0.4626	$1.8{\cdot}10^{-08}$	88
Case 2	0.4055	$2.0{\cdot}10^{-08}$	82
Cases 1 and 2 (C-Topology)	0.5319	$2.8{\cdot}10^{-08}$	716

Table 2
Results after 30 iterations for two loading cases.

	Optimal cost	Optimal weight	Accuracy	CPU (s)
Case 1	0.5791	0.4919	$6.7{\cdot}10^{-07}$	112
Case 2	0.4933	0.4195	$4.7{\cdot}10^{-09}$	103
Cases 1 and 2 (C-Topology)	1.2848	0,5573	$7.7{\cdot}10^{-08}$	546

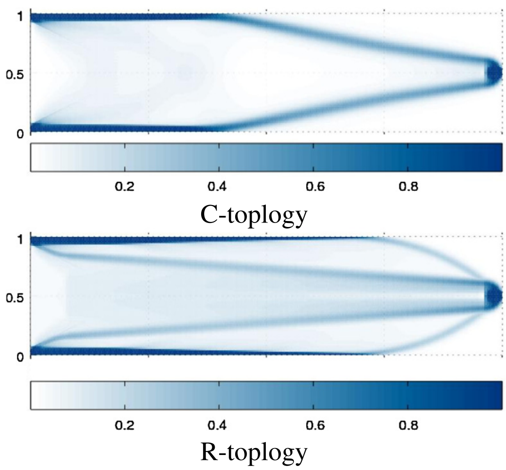

Fig. 4. Continuous topology of the two cases loading problem.

the first and the second loading cases; the optimal cost and weight are reported in Table 1 for continuous topology and in Table 2 for the discrete topology.

The weight of the R-topology is 0.7316 for the continuous topology and 0.8770 for the discrete one. This represents more than 50% of extra weight relative to the C-topology for discrete topology. The R-topology and C-Topology are represented in Figs. 4 and 5. In both cases, discrete and continuous, we can clearly see that our method was able to generate a new topology, different from that of each loading case, with an optimal weight lower than that of R-topology.

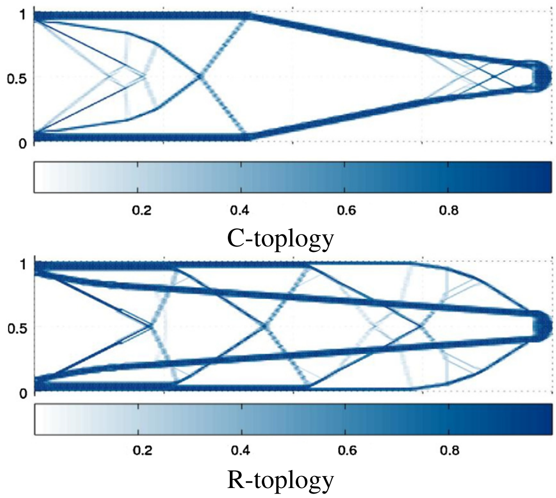

Fig. 5. Discrete topology of the two cases loading problem.

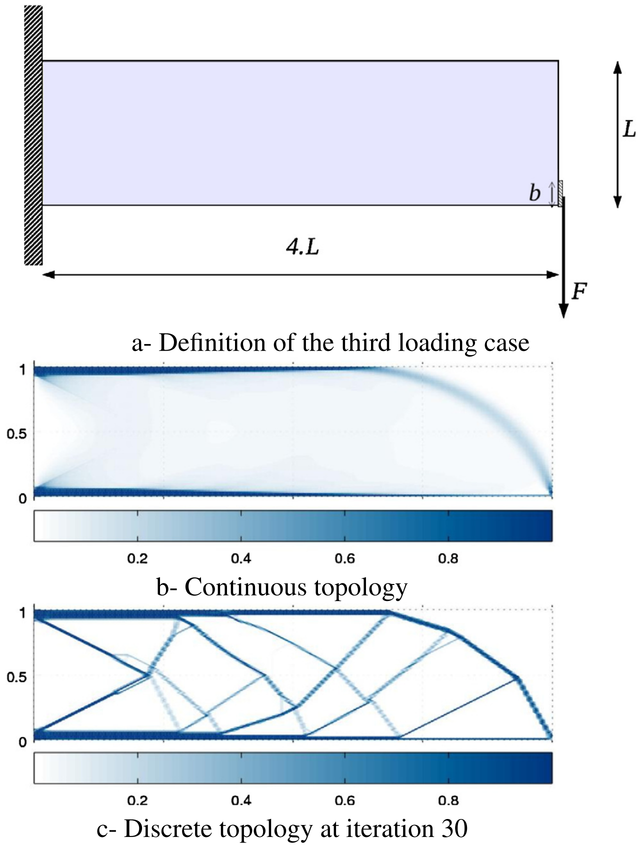

Fig. 6. Third loading case.

5.2. Three loading cases

In this section, a third loading case is considered. It is defined in Fig. 6 with a vertical load applied to the free end of the bottom of the beam, for the same shear strength and the same mesh as cases 1 and 2. The limit analysis for this third case is conducted for the full density in the domain. A limit load $\bar{F} = 0.0485$ kN is obtained in 30 s of CPU time.

Table 3
Results for case 3, and for the problem with three cases after 30 iterations.

	Optimal cost	Optimal weight	Accuracy	CPU (s)
Case 3 continuous	0.4676	0.4676	$3.9 \cdot 10^{-09}$	66
Case 3 discrete	0.5899	0.4983	$1.3 \cdot 10^{-06}$	81
3 Cases continuous (C-Topology)	1.6192	0,5397	$7.7 \cdot 10^{-08}$	2692
3 Cases discrete (C-Topology)	1.9398	0,5674	$1.3 \cdot 10^{-07}$	3327

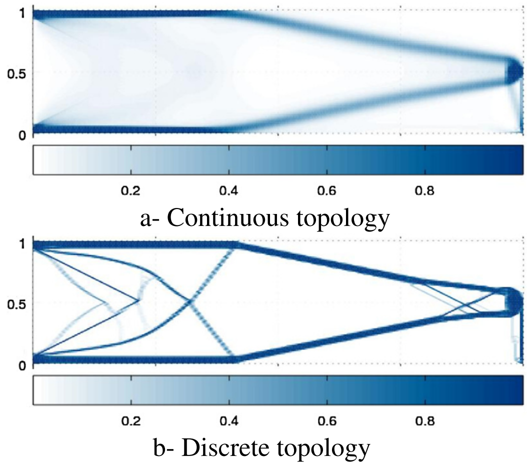

Fig. 7. Topology of the three loading cases problem.

The applied load is chosen equal to 0.04 kN for case 3, using the same 51,200-element mesh. Table 3 presents the obtained results for this case. The obtained topology for the continuous and the discrete design is represented in Fig. 6. The latter presents a completely different topology than those of cases 1 and 2.

The consideration of three loading cases at the same time, through our algorithm, enables a single topology (Fig. 7), which will support the load in all three cases. The obtained weight and cost are illustrated in Table 3. The weight of the R-topology is 0.76732 for the continuous topology and 1.0241 for the discrete one. This represents more than 80% of extra weight relative to the C-topology for discrete topology.

As expected, the CPU time of one iteration does not vary linearly with the problem's size. In fact, the CPU times of the three loading cases are equal to 5 times the CPU times of the two loading cases considered at the same time, and it is 25 times that of a single loading case. Meanwhile, the variable number grows linearly with the number of loading cases: it shifts from 665,600 variables for the single loading case to the double for the two loading cases and the triple for the three loading cases.

In return, we obtained a new topology, with a slightly higher weight than that of each individual case, which can be used in each one of the considered loading cases.

Fig. 8 shows the convergence of the total weight of the discrete solution along the iterative process. The initial iteration, or iteration 0, coincides with the solution of the continuous design. It is clear that the solution converges quickly for all the problems shown in this paper. Moreover, the convergence speed is almost the same for the individual problems and the problems dealing with multiple loading cases. It is finally noted that the number of iterations simulated in the iterative process is much higher than the required number of iterations for convergence.

6. Conclusion

A topology optimization algorithm based on limit analysis topology optimization has been developed to determine the optimal topology of a domain subjected to multiple loading cases. With this topology design, which integrates the optimization problem with direct limit analysis, we were able to consider multiple loading cases in one mathematical programming problem. Thus, a single iteration allows obtaining the continuous topology structure subject to several loading cases.

An iterative procedure identical to that conventionally used in topology optimization by limit analysis was adapted to obtain the discrete topology. The formulation is developed in plane strain using Tresca materials and is illustrated by the example of a cantilever beam subjected to two- and three loading cases.

The topologies obtained for two- and three loading cases are different from those obtained for each single case. The optimum weight of the obtained topology with multiple loading cases is slightly higher than that of the isolated case. However, it is always less than the case where the maximum of densities at each point is considered.

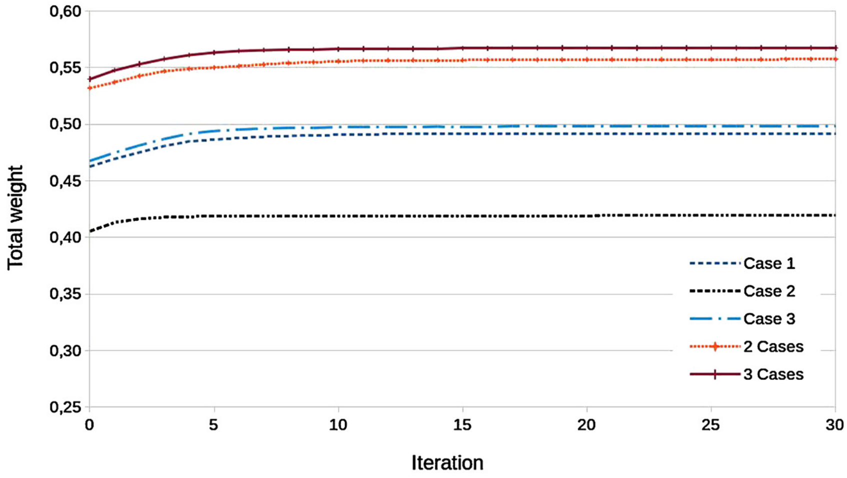

Fig. 8. Iteration history of weight.

The approach developed for multiple loading cases retains the main characteristics of the LADM, as the nonexistence of the checkerboard problem. We also note that the preservation of convexity for the continuous problem ensures the globality of the optimal solution. On the other hand, for discrete problems, we observe that a small number of iterations is necessary to converge, and that the isolated cases and multiple loading cases are within the same range of convergence rate.

References

[1] F. Duddeck, New approaches for shape and topology optimization for crashworthiness, in: NAFEMS World Congress, École centrale de Lille, France, May 2011.
[2] G.I.N. Rozvany, A critical review of established methods of structural topology optimization, Struct. Multidiscip. Optim. 37 (2009) 217–237.
[3] M.P. Bendsøe, O. Sigmund, Material interpolation schemes in topology optimization, Arch. Appl. Mech. 69 (9–10) (1999) 635–654.
[4] H.A. Eschenauer, N. Olhoff, Topology optimization of continuum structures: a review, Appl. Mech. Rev. 54 (4) (2001) 331–390.
[5] G. Allaire, Conception optimale de structures, Math. Appl., vol. 58, Springer, Berlin, 2007.
[6] P.A. Browne, Topology optimization of linear elastic structures, PhD thesis, Department of Mathematical Sciences, University of Bath, UK, May 2013.
[7] M.P. Bendsøe, Optimal shape design as a material distribution problem, Struct. Optim. 1 (4) (1989) 193–202.
[8] R.J. Yang, C.J. Chen, Stress-based topology optimization, Struct. Optim. 12 (2–3) (1996) 98–105.
[9] P. Duysinx, M.P. Bendsøe, Topology optimization of continuum structures with local stress constraints, Int. J. Numer. Methods Eng. 43 (8) (1998) 1453–1478.
[10] C.Y. Kiyono, S.L. Vatanabe, E.C.N. Silva, J.N. Reddy, A new multi-p-norm formulation approach for stress-based topology optimization design, Compos. Struct. (2016), http://dx.doi.org/10.1016/j.compstruct.2016.05.058, in press.
[11] D. Gao, D. Wang, G. Wang, L. Hao, Topology optimization of conditioner suspension for mower conditioner considering multiple loads, Math. Comput. Model. 58 (3) (2013) 489–496.
[12] W. Li, Q. Li, G.P. Steven, Y.M. Xie, An evolutionary shape optimization for elastic contact problems subject to multiple load cases, Comput. Methods Appl. Mech. Eng. 194 (30) (2005) 3394–3415.
[13] A.R. Diaz, M.P. Bendsøe, Shape optimization of structures for multiple loading conditions using a homogenization method, Struct. Optim. 4 (1) (1992) 17–22.
[14] Y.M. Xie, G.P. Steven, Optimal design of multiple load case structures using an evolutionary procedure, Eng. Comput. 11 (4) (1994) 295–302.
[15] Z. Kammoun, H. Smaoui, A direct method formulation for topology plastic design of continua, in: 4th International Workshop on Direct Methods – DM2013, University Mediterranea of Reggio Calabria, Italy, 1–2 October 2013.
[16] Z. Kammoun, H. Smaoui, A direct approach for continuous topology optimization subject to plastically admissible loading, 342 (9) (2014) 520–531.
[17] Z. Kammoun, H. Smaoui, A direct method formulation for topology plastic design of continua, in: Direct Methods for Limit and Shakedown Analysis of Structures, Springer, 2015, pp. 47–63.
[18] J. Lysmer, Limit analysis of plane problems in soil mechanics, J. Soil Mech. Found. Div. 96 (1970) 1311–1334.
[19] J. Pastor, S. Turgeman, Mise en oeuvre numérique des méthodes de l'analyse limite pour les matériaux de Von Mises et de Coulomb standards en déformation plane, Mech. Res. Commun. 3 (1976) 469–474.
[20] J. Pastor, Analyse limite: détermination numérique de solutions statiques complètes. Application au talus vertical, J. Méc. Appl. 2 (1978) 167–196.
[21] R. Negre, J. Pastor, A. Bottero, S. Turgemen, Finite element method and limit analysis theory for soil mechanics problems, Comput. Methods Appl. Mech. Eng. 22 (1980) 131–149.
[22] J. Pastor, E. Loute, T.H. Thai, On the efficiency of the limit analysis methods via the new techniques of optimization, in: P. Mesta (Ed.), Vth Eur. Conf. Num. Methods in Geotechnical Engineering, Presses des Ponts et Chaussées, Paris, 2002.
[23] J. Pastor, E. Loute, T.H. Thai, Interior point optimization and limit analysis: an application, Commun. Numer. Methods Eng. 19 (2003) 779–785.
[24] F. Pastor, M. Trillat, J. Pastor, E. Loute, P. Thoré, Convex optimization and stress-based lower/upper bound methods for limit analysis of porous polymer materials, in: J. Besson, D. Steglish, D. Moinereau (Eds.), 9th European Mechanics of Materials Conference, EMMC9, École des Mines de Paris – EDF, May 2006.
[25] K. Krabbenhoft, L. Damkilde, A general non-linear optimization algorithm for lower bound limit analysis, Int. J. Numer. Methods Eng. 56 (2003) 165–184.
[26] A.V. Lyamin, S.W. Sloan, Lower bound limit analysis using nonlinear programming, Int. J. Numer. Methods Eng. 55 (2002) 573–611.
[27] S.W. Sloan, A.V. Lyamin, Lower bound limit analysis using nonlinear programming, in: Eur. Congress on Comp. Meth. In App. Sci. and Eng., ECCOMAS-2000, Barcelona, 2000.

[28] Z. Kammoun, F. Pastor, H. Smaoui, J. Pastor, Large static problem in numerical limit analysis: a decomposition approach, Int. J. Numer. Anal. Methods Geomech. 34 (18) (2010) 1960–1980.
[29] J. Salençon, Théorie des charges limites: poinçonnement d'une plaque par deux poinçons symétriques en déformation plane, C. R. Acad. Sci. Paris 265 (1967) 869–872.
[30] J. Salençon, Théorie de la plasticité pour les applications à la mécanique des sols, Eyrolles, Paris, 1974.
[31] MOSEK ApS, C/O Symbion Science Park, Fruebjergvej 3, Box 16, 2100 Copenhagen ϕ, Denmark, www.mosek.com, 2002.
[32] Z. Kammoun, J. Pastor, H. Smaoui, Limit analysis of a soil reinforced by micropile group: a decomposition approach, in: Limit State of Materials and Structures, Springer, 2013, pp. 179–195.
[33] F. Pastor, Z. Kammoun, E. Loute, J. Pastor, H. Smaoui, Large problems in numerical limit analysis: a decomposition approach, in: Limit States of Materials and Structures, Springer, 2009, pp. 23–43.