C. R. Mecanique 344 (2016) 776–783

Contents lists available at ScienceDirect

Comptes Rendus Mecanique

www.sciencedirect.com

A simple turbulent two-fluid model

Jean-Marc Hérard [a,b,*], Hippolyte Lochon [c,d]

[a] EDF Lab Chatou, 6 quai Watier, 78400 Chatou, France
[b] I2M, UMR CNRS 7373, Technopôle Château-Gombert, 39, rue Joliot Curie, 13453 Marseille, France
[c] EDF Lab Saclay, 7 boulevard Gaspard Monge, 91120 Palaiseau, France
[d] IMSIA, UMR EDF/CNRS/CEA/ENSTA 9219, Université Paris-Saclay, 828, boulevard des Maréchaux, 91762 Palaiseau, France

ARTICLE INFO

Article history:
Received 13 July 2016
Accepted 18 October 2016
Available online 4 November 2016

Keywords:
Two-fluid model
Turbulence
Entropy inequality
Shocks

ABSTRACT

We present in this paper a simple turbulent two-phase flow model using the two-fluid approach. The model, which relies on the classical ensemble averaging, allows the computation of unsteady flows including shock waves, rarefaction waves, and contact discontinuities. It requires the definition of adequate source terms and interfacial quantities. The hyperbolic turbulent two-fluid model is such that unique jump conditions hold within each field. Closure laws for the interfacial velocity and the interfacial pressure comply with a physically relevant entropy inequality. Moreover, source terms that account for mass, momentum and energy interfacial transfer are in agreement with the entropy inequality. Particular attention is also given to the jump conditions when assuming a perfect gas equation of state within each phase; this enables us to recover expected bounds on the mean density through shock waves.

1. Introduction

We propose herein a turbulent two-fluid model for the prediction of two-phase flows. Actually, though we use the classical ensemble averaging [1,2], we follow here the approach used in a recent series of papers. Our main goal is to derive a two-fluid two-phase flow model that accounts for Reynolds stress tensors in a very simple way, so that the fundamental properties invoked in [3] are preserved. This means that the following specifications are enforced:

- (i) the model should be such that one could retrieve the standard Baer–Nunziato model in the laminar case;
- (ii) an entropy inequality should hold for smooth solutions, and meanwhile it should provide some guidelines for closure laws associated with interfacial mass, momentum and energy transfer;
- (iii) unique jump conditions should be valid so that meaningful and unique shock waves might be predicted.

These requirements are mandatory if we intend to predict relevant shock solutions in two-phase flows, such as those that arise in vapour explosions or other similar situations, while using the standard verification and validation process (see [4] for instance). A class of two-fluid models that are capable to predict unsteady situations has emerged from the recent literature, either for gas-particle flows (see [5–9] for instance), or for water-vapour flows (see [10–12] among others). These

* Corresponding author.
E-mail address: jean-marc.herard@edf.fr (J.-M. Hérard).

http://dx.doi.org/10.1016/j.crme.2016.10.010

models essentially differ from one another through the definition of the interface pressure–velocity couple. We will thus focus here on the approach suggested in [3], that gave some new enlightenment on the admissible closure laws that enable us to comply with both items (ii) and (iii) recalled above.

As we will see, the constraint (ii) will in turn provide some closure laws for the interfacial pressure (see equation (7)), whereas (iii) will enable us to define relevant closure laws for the interface velocity (see equation (6)).

Moreover, in order to account for turbulent effects in a rather simple way, we will rely on the single-phase proposal introduced in [13], which inherits from the earlier work described in [14–17]. As we will see, this will minimise the number of unknowns, and inlet/outlet boundary conditions, and meanwhile will allow complying with the above-mentioned requirements (i, ii, iii). We also emphasise that the present work should not be confused with the one of [18], which was mainly inspired by [19] and [3]. Actually, though it agrees with the former items (i) and (ii), the latter turbulent model [18] is not suitable for shock solutions. Indeed, jump conditions are not defined in a unique way, due to the occurrence of non-conservative products that are active in genuinely non-linear fields; as a result, different mesh-converged solutions issuing from distinct schemes may emerge in practical computations, as it is now quite well-known (see [15,20]).

The paper is organised as follows. We first provide the governing set of equations including source terms accounting for all possible transfers between phases. The choice of relevant interfacial velocity and pressure is discussed. Then we focus on the key property of the model, which is the entropy inequality. It is compared with the laminar case and we underline how turbulent energy affects the different relaxation terms. A third section discusses the main properties of the convective system: hyperbolicity, structure of fields and jump conditions. For simplicity, we restrict ourselves to the Baer–Nunziato closure in this section, but few remarks are given about other possible closures. Particular attention is given to the jump conditions when assuming a perfect gas equation of state within each phase. In that case, we show that density ratios are bounded through shock waves and that they are in agreement with the laminar frame. Moreover, turbulent energy ratios are bounded as well, unlike pressure ratios. The last section is devoted to a few remarks on the Riemann problem.

Though the derivation of the model is quite different, we will also address in section 3 the differences and similarities with the models proposed in [21].

2. Governing equations

The two-phase flow model has been obtained by a statistical averaging of the single-phase Navier–Stokes equations. One additional topological equation on the statistical void fraction is also needed (see [2]). For the sake of simplicity, we do not detail this averaging procedure here, but we underline the fact that the tensor involving turbulent effects is modelled following the approach of [13] for the single-phase Reynolds stress tensor. Thus the governing set of equations takes the form (when neglecting viscous terms):

$$\partial_t W + \partial_x F(W) + C(W)\partial_x W = S(W) \tag{1}$$

with W, $F(W)$, $S(W)$ in $\mathbb{R}^7$, and $C(W)$ in $\mathbb{R}^{7\times 7}$. The state variable W is:

$$W = \begin{pmatrix} \alpha_1 \\ \alpha_1\rho_1 \\ \alpha_2\rho_2 \\ \alpha_1\rho_1u_1 \\ \alpha_2\rho_2u_2 \\ \alpha_1E_1 \\ \alpha_1E_2 \end{pmatrix} \tag{2}$$

where α_k, ρ_k, u_k, p_k and E_k are respectively the statistical void fraction, the mean density, the mean velocity, the mean pressure and the mean total energy of phase k, $k = 1, 2$. The statistical void factions are such that:

$$\alpha_2 = 1 - \alpha_1$$

and the mean total energy E_k is given by:

$$E_k = \frac{1}{2}\rho_k u_k^2 + \rho_k\varepsilon_k + K_k$$

where $\varepsilon_k = \varepsilon_k(\rho_k, p_k)$ is the mean internal energy of phase k and K_k is the turbulent kinetic energy:

$$K_k = K_{k,0}\rho_k^{5/3} \tag{3}$$

(with $K_{k,0} > 0$). It is important to underline the fact that the mean internal energy only depends on the mean density and the mean pressure, which is a crude assumption on statistical thermodynamics. However, one can easily prove that it is verified in the case of simple Equation Of State (EOS) such as perfect gas or stiffened gas (see [22]). We define the set of admissible states Ω by:

$$\Omega = \left\{ W \in \mathbb{R}^7 \; ; \; \alpha_1 \in]0,1[\, , \rho_k > 0, \varepsilon_k > 0 \right\} \tag{4}$$

We also introduce the celerity of density waves c_k in the pure phase k and its temperature T_k:

$$\rho_k c_k^2 = \left(\partial_{p_k}\varepsilon_k\right)^{-1}\left(\frac{p_k}{\rho_k} - \rho_k\left(\partial_{\rho_k}\varepsilon_k\right)\right)$$
$$\frac{1}{T_k} = \left(\partial_{p_k}\varepsilon_k\right)^{-1}\left(\partial_{p_k}s_k\right)$$

where $s_k = s_k(\rho_k, p_k)$ is the specific entropy complying with the constraint:

$$c_k^2\left(\partial_{p_k}s_k\right) + \left(\partial_{\rho_k}s_k\right) = 0$$

The convective part of the system is defined by:

$$F(W) = \begin{pmatrix} 0 \\ \alpha_1\rho_1 u_1 \\ \alpha_2\rho_2 u_2 \\ \alpha_1(\rho_1 u_1^2 + p_1 + \frac{2}{3}K_1) \\ \alpha_2(\rho_2 u_2^2 + p_2 + \frac{2}{3}K_2) \\ \alpha_1 u_1(E_1 + p_1 + \frac{2}{3}K_1) \\ \alpha_2 u_2(E_2 + p_2 + \frac{2}{3}K_2) \end{pmatrix}, \quad C(W)\partial_x W = \begin{pmatrix} u_I\partial_x\alpha_1 \\ 0 \\ 0 \\ -p_I\partial_x\alpha_1 \\ -p_I\partial_x\alpha_2 \\ -u_I p_I\partial_x\alpha_1 \\ -u_I p_I\partial_x\alpha_2 \end{pmatrix} \tag{5}$$

where we use the following closure laws on interfacial velocity u_I and pressure p_I:

$$u_I = a u_1 + (1-a)\,u_2\,, \qquad a \in \left\{0, \frac{m_1}{m_1+m_2}, 1\right\} \tag{6}$$

$$p_I = b\left(p_1 + \frac{2}{3}K_1\right) + (1-b)\left(p_2 + \frac{2}{3}K_2\right), \qquad b = \frac{\frac{1-a}{T_1}}{\frac{1-a}{T_1} + \frac{a}{T_2}} \tag{7}$$

In the case when $a \in \{0, 1\}$, it corresponds to the so-called Baer–Nunziato closure [5] in the laminar case. The third choice $a = \frac{m_1}{m_1+m_2}$ has been proposed in [3,12] in the laminar case too. Those three possibilities for (u_I, p_I) have been motivated by two requirements: the enforcement of a relevant entropy inequality that will be discussed later on, and the structure of the field associated with $\lambda = u_I$, which is assumed to be linearly degenerate. Other closure laws could be found [23,8,10,24], but they will not be considered here.

The source part of the system reads:

$$S(W) = \begin{pmatrix} \Phi \\ \Gamma \\ -\Gamma \\ D + \mathcal{U}\Gamma \\ -D - \mathcal{U}\Gamma \\ Q + \mathcal{U}D + \mathcal{H}\Gamma - p_I\Phi \\ -Q - \mathcal{U}D - \mathcal{H}\Gamma + p_I\Phi \end{pmatrix} \tag{8}$$

where $\mathcal{U} = \dfrac{u_1 + u_2}{2}$, $\mathcal{H} = \dfrac{u_1 u_2}{2}$ and the relaxation terms are given by:

$$\begin{aligned} \Phi &= \alpha_1\alpha_2\delta_p\left(\left(p_1 + \tfrac{2}{3}K_1\right) - \left(p_2 + \tfrac{2}{3}K_2\right)\right) \\ \Gamma &= m_1 m_2\delta_\mu\left(\tfrac{\mu_2}{T_2} - \tfrac{\mu_1}{T_1}\right) \\ D &= m_1 m_2\delta_u\,(u_2 - u_1) \\ Q &= m_1 m_2\delta_T\,(T_2 - T_1) \end{aligned} \tag{9}$$

with:

$$\mu_k = \varepsilon_k + \frac{p_k}{\rho_k} - T_k s_k + \frac{5K_k}{3\rho_k}$$

which corresponds to the Gibbs free enthalpy in the laminar case when $K_k = 0$. The scalar functions δ_φ are not detailed here, but are assumed to be positive. For practical purposes, the pressure relaxation time scale involved in δ_p is provided by the closure law detailed in [25]; besides, other relaxation time scales $\delta_\mu, \delta_u, \delta_T$, embedded in mass, momentum and energy interfacial transfer terms, are taken from the classical two-fluid literature. Hence, we emphasise the fact that all possible transfers between phases (mass, momentum and energy) are accounted for in this model.

3. Entropy inequality

A key property of the governing set of equations (1) is the following.

Proposition 1. *Define the entropy–entropy flux pair* (η, f_η)*:*

$$\eta = -\alpha_1\rho_1 s_1 - \alpha_2\rho_2 s_2 \,, \qquad f_\eta = -\alpha_1\rho_1 s_1 u_1 - \alpha_2\rho_2 s_2 u_2$$

Then the following inequality holds for smooth solutions to (1):

$$\partial_t \eta + \partial_x f_\eta \leq 0$$

Proof. Classical manipulations of the system give us the evolution law of the specific entropy for smooth solutions:

$$\partial_t(\alpha_k\rho_k s_k) + \partial_x(\alpha_k\rho_k s_k u_k) + \frac{1}{T_k}\left(p_I - \left(p_k + \frac{2}{3}K_k\right)\right)(u_k - u_I)\,\partial_x\alpha_k$$
$$= \frac{(-1)^{k+1}}{T_k}\left\{Q + (\mathcal{U} - u_k)\,D - \mu_k\Gamma + \left(\left(p_k + \frac{2}{3}K_k\right) - p_I\right)\Phi\right\}$$

Then we use the closure laws on interfacial velocity and pressure (6)–(7) to get the evolution law of the global entropy:

$$\begin{aligned}\partial_t\eta + \partial_x f_\eta = &\left(\frac{1}{T_2} - \frac{1}{T_1}\right)Q \\ &+ \left(\frac{1}{2T_1} + \frac{1}{2T_2}\right)(u_1 - u_2)\,D \\ &+ \left(\frac{\mu_1}{T_1} - \frac{\mu_2}{T_2}\right)\Gamma \\ &+ \left(\frac{1-b}{T_1} + \frac{b}{T_2}\right)\left\{\left(p_2 + \frac{2}{3}K_2\right) - \left(p_1 + \frac{2}{3}K_1\right)\right\}\Phi\end{aligned}$$

The conclusion is now obvious using closure laws (9). □

This entropy inequality is really close to the one in the laminar case; therefore, it leads to similar source terms. Turbulent energies are involved in pressure and free enthalpy relaxation terms, but one recovers the laminar source terms when turbulent kinetic energies vanish. Temperature and velocity relaxation terms remain exactly the same as in the laminar case.

From now on, we will restrict to the case:

$$(u_I, p_I) = \left(u_1, p_2 + \frac{2}{3}K_2\right)$$

in the sequel.

Remark 1.

- The model discussed afterwards is in that case quite similar to the one detailed in [21], pages 298–299, equations (18a)–(18l), when neglecting the "turbulent entropy" dissipation $\overset{o}{\chi}_k$ (equations (18k)–(18l)), and thus retaining the obvious solution: $\chi_k(x, y, z, t) = (\chi_k)^0$, which is actually the straightforward counterpart of equation (3) in the current paper. Note that the contribution $\overset{o}{\chi}_k$ in [21], page 309, vanishes when the asymptotic pressure–velocity mechanical equilibrium is reached. We also emphasise that the problem arising in the definition of jump relations through shock waves, which is due to the occurrence of non-conservative first-order contributions in the whole system, is not addressed in [21]; this will be part of the discussion in the next section (see Proposition 4).
- On the other hand, the present model should not be confused with the Discrete Equation Method (DEM), since closure laws involved in the latter for the interface velocity and the interface pressure are indeed totally different (see [21], equations (33), page 308, to be compared with equations (6), (7) in the current paper).

4. Main properties of the convective system

We provide below some of the main properties of the convective system of equations. Propositions 2 and 4 arise as expected.

Proposition 2. *The homogeneous convective subset:*

$$\partial_t W + \partial_x F(W) + C(W)\partial_x W = 0 \tag{10}$$

is hyperbolic. It admits seven real eigenvalues:

$$\begin{cases} \lambda_{1,2} = u_1 \\ \lambda_3 = u_1 - \tilde{c}_1 \\ \lambda_4 = u_1 + \tilde{c}_1 \\ \lambda_5 = u_2 \\ \lambda_6 = u_2 - \tilde{c}_2 \\ \lambda_7 = u_2 + \tilde{c}_2 \end{cases} \quad \textit{with} \quad \tilde{c}_k^2 = c_k^2 + \frac{10\,K_k}{9\,\rho_k} \tag{11}$$

and associated vectors span the whole space $\mathbb{R}^7$, unless $|u_2 - u_1|/\tilde{c}_2 = 1$. Fields associated with eigenvalues $\lambda_{1,2}$ and λ_5 are linearly degenerate (LD). Other fields are genuinely non-linear (GNL).

We notice that the turbulent kinetic energy modifies the celerity of the GNL waves, which is straightforward when focusing on single-phase turbulent models (see [14] for instance). Thus it will also affect the resonant waves of the system.

Proposition 3 *(Riemann invariants). The five Riemann invariants of the 1–2 LD field associated with the void fraction coupling wave are the following:*

$$\begin{aligned} I^1_{1,2}(W) &= u_1 \\ I^2_{1,2}(W) &= s_2 \\ I^3_{1,2}(W) &= \alpha_2\rho_2\,(u_2 - u_1) \\ I^4_{1,2}(W) &= \alpha_1\left(p_1 + \tfrac{2}{3}K_1\right) + \alpha_2\left(p_2 + \tfrac{2}{3}K_2\right) + \alpha_2\rho_2\,(u_2 - u_1)^2 \\ I^5_{1,2}(W) &= \varepsilon_2 + \tfrac{p_2}{\rho_2} + \tfrac{5}{3}\tfrac{K_2}{\rho_2} + \tfrac{1}{2}\,(u_2 - u_1)^2 \end{aligned} \tag{12}$$

The Riemann invariants associated with the other waves read:

$$\begin{aligned} I_3(W) &= \left(\alpha_1, \rho_2, u_2, \left(p_2 + \tfrac{2}{3}K_2\right), s_1, u_1 + \phi_1\right)^T \\ I_4(W) &= \left(\alpha_1, \rho_2, u_2, \left(p_2 + \tfrac{2}{3}K_2\right), s_1, u_1 - \phi_1\right)^T \\ I_5(W) &= \left(\alpha_1, \rho_1, u_1, \left(p_1 + \tfrac{2}{3}K_1\right), \left(p_2 + \tfrac{2}{3}K_2\right), u_2\right)^T \\ I_6(W) &= \left(\alpha_1, \rho_1, u_1, \left(p_1 + \tfrac{2}{3}K_1\right), s_2, u_2 + \phi_2\right)^T \\ I_7(W) &= \left(\alpha_1, \rho_1, u_1, \left(p_1 + \tfrac{2}{3}K_1\right), s_2, u_2 - \phi_2\right)^T \end{aligned} \tag{13}$$

where $\phi_k = \displaystyle\int_{p_0}^{p_k} \frac{\tilde{c}_k}{\rho_k c_k^2}(p_k, s_k)\,\mathrm{d}p_k$.

Once again, we retrieve the classical results [12] when the turbulent kinetic energies are not accounted for (thus setting $K_k = 0$).

Proposition 4 *(Jump conditions). Within each isolated field, unique jump conditions hold. We denote $[f] = f_r - f_l$ the jump between the (l)eft and (r)ight states on each side of a discontinuity travelling at speed σ. Turning to the genuinely non-linear fields, jump conditions may be written:*

$$\begin{cases} [\alpha_k] = 0 \\ -\sigma\,[\rho_k] + [\rho_k u_k] = 0 \\ -\sigma\,[\rho_k u_k] + \left[\rho_k u_k^2 + p_k + \dfrac{2}{3}K_k\right] = 0 \\ -\sigma\,[E_k] + \left[u_k\left(E_k + p_k + \dfrac{2}{3}K_k\right)\right] = 0 \end{cases} \tag{14}$$

Those jump conditions may be easily rewritten as follows:

$$\begin{cases} [\alpha_k] = 0 \\ \sigma = [\rho_k u_k] / [\rho_k] \\ (\rho_k)_{\mathrm{R}} (\rho_k)_L [u_k]^2 = \left[p_k + \frac{2}{3} K_k \right] [\rho_k] \\ 2 \left[\varepsilon_k + \frac{K_k}{\rho_k} \right] + \left\{ \left(p_k + \frac{2}{3} K_k \right)_r + \left(p_k + \frac{2}{3} K_k \right)_l \right\} \left[\frac{1}{\rho_k} \right] = 0 \end{cases} \tag{15}$$

More over, looking at the case of a perfect gas EOS, we have the following.

Proposition 5 *(Jump conditions for perfect gas EOS). We assume that phase k complies with the perfect gas EOS: $p_k = (\gamma_k - 1)\rho_k \varepsilon_k$, $\gamma_k > 1$. We also assume that the left state is admissible $W_l \in \Omega$ according to (4) and that no vacuum occurs in the solution. Then the jump conditions (14) ensure that the right state is admissible: $W_r \in \Omega$. It also provides bounds for the density ratio whereas the pressure ratio has no bound:*

$$(\beta_k)^{-1} < \frac{(\rho_k)_r}{(\rho_k)_l} < \beta_k \tag{16}$$

with $\beta_k = \frac{\gamma_k + 1}{\gamma_k - 1}$.

Proof. We note $z_k = (\rho_k)_r / (\rho_k)_l$ the density ratio and $\pi_k = (p_k)_r / (p_k)_l$ the pressure ratio. The third equation of (15) gives the following relation between states through genuinely non-linear fields associated with λ_3 or λ_6:

$$\pi_k (\beta_k - z_k) + 1 - \beta_k z_k - g_k(z_k) = 0$$

where $g_k(z) = \frac{2}{3} \frac{(K_k)_l}{(p_k)_l} \left(z^{8/3} - 4 z^{5/3} + 4z - 1 \right)$. Moreover, $z_k > 1$ and it implies $g_k(z_k) > 0$. Then we get the pressure ratio:

$$\pi_k = \frac{\beta_k z_k - 1 + g_k(z_k)}{\beta_k - z_k}$$

and the conclusion is thus straightforward. A similar proof holds through fields associated with λ_4 or λ_7. □

We first note that we recover the bounds arising in the pure laminar case (see for instance [26]). This was expected since the instantaneous inequality:

$$\beta^{-1} \phi_{\mathrm{L}} < \phi_{\mathrm{R}} < \beta \phi_{\mathrm{L}} \tag{17}$$

(where β is constant with respect to the statistics) obviously provides the following relation between the *mean* quantities:

$$\beta^{-1} < \phi_{\mathrm{L}} > \ \leq \ < \phi_{\mathrm{R}} > \ \leq \ \beta < \phi_{\mathrm{L}} > \tag{18}$$

It is important to notice that, unlike in [14,18], we do not require any approximate jump condition. Eventually, we can easily check that the ratio of left and right turbulent kinetic energies is also bounded.

We recall that all the properties described here suppose that the Baer–Nunziato closure holds $(u_I, p_I) = \left(u_1, p_2 + \frac{2}{3} K_2 \right)$. In the case when $a = \frac{m_1}{m_1 + m_2}$, Proposition 2 is slightly different, since there is another distinct linearly degenerate field associated with $\lambda = u_I$, where $(u_I - u_1)(u_I - u_2) \neq 0$. Therefore, Proposition 3 is also different since other Riemann invariants arise. This implies a more intricate coupling wave associated with the eigenvalue $\lambda = u_I$, as it already happens in the laminar case ($K_k = 0$).

5. Remarks

We now make a few remarks on the one-dimensional Riemann problem:

$$\begin{cases} \partial_t W + \partial_x F(W) + C(W) \partial_x W = 0 \\ W(t = 0, x) = \begin{cases} W_{\mathrm{L}}, & \text{if } x < 0 \\ W_{\mathrm{R}}, & \text{if } x > 0 \end{cases} \end{cases} \tag{19}$$

where W_{L} and W_{R} are admissible states: $W_{\mathrm{L}}, W_{\mathrm{R}} \in \Omega$.

First of all, we remind that one cannot provide the solution to the general Riemann problem in the laminar case, even when a perfect gas EOS is assumed within each phase (see [12]). Nonetheless, a simple result may be given, which is the straightforward counterpart of what happens in the laminar case.

Remark 2. We assume that $(\alpha_1)_{\mathrm{L}} = (\alpha_1)_{\mathrm{R}}$ and that a perfect gas EOS holds within each phase: $p_k = (\gamma_k - 1)\rho_k \varepsilon_k$, $\gamma_k > 1$. The Riemann problem (19) has a unique entropy consistent solution involving constant states separated by shocks, rarefaction waves and contact discontinuities if and only if:

$$(u_k)_\mathrm{R} - (u_k)_\mathrm{L} \;<\; \int_0^{(p_k)_\mathrm{L}} \frac{\tilde{c}_k}{\rho_k c_k^2}(p_k, s_k)\,\mathrm{d}p_k \;+\; \int_0^{(p_k)_\mathrm{R}} \frac{\tilde{c}_k}{\rho_k c_k^2}(p_k, s_k)\,\mathrm{d}p_k \tag{20}$$

for $k = 1, 2$.

Proof. Since $(\alpha_1)_\mathrm{L} = (\alpha_1)_\mathrm{R}$, phases evolve independently. Therefore, we can use the property from [13] on the monophasic Riemann problem. □

In order to consider real void fraction coupling waves, we have to use the connection through the 1,2-wave given by the Riemann invariants $I_{1,2}$ in Property 3. This leads to the following remark.

Remark 3. We now suppose that $(\alpha_1)_\mathrm{L} \neq (\alpha_1)_\mathrm{R}$. Assume that a perfect gas EOS holds within each phase and that $W(x,t)$ is the solution to the Riemann problem (19). Then the connection of two constant states separated by a simple wave (either a shock, a rarefaction wave or a contact discontinuity) guarantees that all states of $W(x,t)$ are admissible: $W(x,t) \in \Omega,\ \forall\, x, t$.

Our last remark addresses the problem of defining $K_{k,0}$. At the initial time, we have:

$$K_{k,0} = \frac{K_k}{\rho_k^{5/3}}(x, t=0)$$

Thus, a natural choice of $K_{k,0}$ using the initial data is the mean value:

$$K_{k,0} = \frac{1}{|V|}\int_V \left(K_k \rho_k^{-5/3}\right)(x, t=0)\,\mathrm{d}x$$

The turbulent two-phase flow model introduced in this paper is in agreement with requirements (i, ii, iii), and thus allows the computation of shock solutions in turbulent two-phase flows. The extension of the current results to the framework of multiphase flows seems feasible, considering [27,28], but this point has not been investigated yet.

Acknowledgements

The authors would like to thank the reviewer for his useful comments. The second author has benefited from financial support of EDF/ANRT through a CIFRE contract number 2013/0459.

References

[1] M. Ishii, Thermo-Fluid Dynamic Theory of Two-Phase Flows, Collection de la direction des études et recherches d'Électricité de France, 1975.
[2] D.A. Drew, S.L. Passman, Theory of Multicomponent Fluids, Springer-Verlag, 1999.
[3] F. Coquel, T. Gallouët, J.-M. Hérard, N. Seguin, Closure laws for a two-fluid two-pressure model, C. R. Acad. Sci. Paris, Ser. I 334 (10) (2002) 927–932.
[4] R. Berry, L. Zou, H. Zhao, D. Andrs, J. Peterson, H. Zhang, R. Martineau, Relap-7: Demonstrating Seven-Equation, Two-Phase Flow Simulation in a Single-Pipe, Two-Phase Reactor Core and Steam Separator/Dryer, Technical Report INL/EXT-13-28750, Idaho National Laboratory (INL), Idaho Falls, ID, USA, 2013.
[5] M. Baer, J. Nunziato, A two-phase mixture theory for the deflagration-to-detonation transition (DDT) in reactive granular materials, Int. J. Multiph. Flow 12 (6) (1986) 861–889.
[6] J.B. Bdzil, R. Menikoff, S.F. Son, A.K. Kapila, D.S. Stewart, Two-phase modeling of deflagration-to-detonation transition in granular materials: a critical examination of modeling issues, Phys. Fluids 11 (2) (1999) 378–402.
[7] A.K. Kapila, S.F. Son, J.B. Bdzil, R. Menikoff, D.S. Stewart, Two-phase modeling of DDT: structure of the velocity-relaxation zone, Phys. Fluids 9 (12) (1997) 3885–3897.
[8] J. Glimm, D. Saltz, D.H. Sharp, Renormalization group solution of two-phase flow equations for Rayleigh–Taylor mixing, Phys. Lett. A 222 (3) (1996) 171–176.
[9] H. Jin, J. Glimm, D.H. Sharp, Compressible two-pressure two-phase flow models, Phys. Lett. A 353 (6) (2006) 469–474.
[10] R. Saurel, R. Abgrall, A multiphase Godunov method for compressible multifluid and multiphase flows, J. Comput. Phys. 150 (2) (1999) 425–467.
[11] S. Gavrilyuk, R. Saurel, Mathematical and numerical modeling of two-phase compressible flows with micro-inertia, J. Comput. Phys. 175 (1) (2002) 326–360.
[12] T. Gallouët, J.-M. Hérard, N. Seguin, Numerical modeling of two-phase flows using the two-fluid two-pressure approach, Math. Models Methods Appl. Sci. 14 (5) (2004) 663–700.
[13] J.-M. Hérard, Problème de Riemann pour un modèle simple de turbulence monophasique compressible, Internal EDF report CR-I81-2014-006, 2014 (in French).
[14] A. Forestier, J.-M. Hérard, X. Louis, Solveur de type Godunov pour simuler les écoulements turbulents compressibles, C. R. Acad. Sci. Paris, Ser. I 324 (8) (1997) 919–926.
[15] C. Berthon, Contribution à l'analyse numérique des équations de Navier–Stokes compressibles à deux entropies spécifiques. Applications à la turbulence compressible, PhD thesis, Université Pierre-et-Marie-Curie, Paris-6, 1999.
[16] B. Audebert, Contribution à l'analyse des modèles aux tensions de Reynolds pour l'interaction choc turbulence, PhD thesis, Université Pierre-et-Marie-Curie, Paris-6, 2006.
[17] S. Gavrilyuk, H. Gouin, Geometric evolution of the Reynolds stress tensor, Int. J. Eng. Sci. 59 (2012) 65–73.

[18] J.-M. Hérard, Numerical modelling of turbulent two phase flows using the two fluid approach, in: 16th AIAA Computational Fluid Dynamics Conference, American Institute of Aeronautics and Astronautics, 2003, http://www.aiaa.org/.

[19] P. Spalart, S. Allmaras, A one-equation turbulence model for aerodynamic flows, in: 30th Aerospace Sciences Meeting and Exhibit, Reno, NV, USA, 6–9 January 1992, American Institute of Aeronautics and Astronautics, 1992.

[20] V. Guillemaud, Modélisation et simulation numérique des écoulements diphasiques par une approche bifluide à deux pressions, PhD thesis, Université Aix–Marseille-1, 2007.

[21] R. Saurel, S. Gavrilyuk, F. Renaud, A multiphase model with internal degrees of freedom: application to shock–bubble interaction, J. Fluid Mech. 495 (2003) 283–321.

[22] J.-M. Hérard, Y. Liu, Une approche bifluide statistique de modelisation des écoulements diphasiques à phases compressibles, Internal EDF report H-I81-2013-01162-FR, 2013 (in French).

[23] V.H. Ransom, D.L. Hicks, Hyperbolic two-pressure models for two-phase flow, J. Comput. Phys. 53 (1) (1984) 124–151.

[24] R.A. Berry, R. Saurel, O. LeMetayer, The discrete equation method (DEM) for fully compressible, two-phase flows in ducts of spatially varying cross-section, Nucl. Eng. Des. 240 (11) (2010) 3797–3818.

[25] S. Gavrilyuk, The Structure of Pressure Relaxation Terms: the One-Velocity Case, EDF report H-I83-2014-00276-EN, January 2014, France.

[26] J. Smoller, Shock Waves and Reaction Diffusion Equations, Springer-Verlag, 1983.

[27] S. Muller, M. Hantke, P. Richter, Closure conditions for non-equilibrium multi-component models, Contin. Mech. Thermodyn. 28 (4) (2016) 1157–1189.

[28] J.-M. Hérard, A class of compressible multiphase flow models, C. R. Acad. Sci. Paris, Ser. I 354 (9) (2016) 954–959.