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Ludwig Prandtl and the growth of fluid mechanics in Germany

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ABSTRACT

Ludwig Prandtl (1875–1953) has been called the father of modern aerodynamics. His name is associated most famously with the boundary layer concept, but also with several other topics in 20th-century fluid mechanics, particularly turbulence (Prandtl's mixing length). Among his disciples are pioneers of modern fluid mechanics like Heinrich Blasius, Theodore von Kármán, and Walter Tollmien. Furthermore, Prandtl founded the Aerodynamische Versuchsanstalt (AVA) and the Kaiser-Wilhelm-Institut für Strömungsforschung in Göttingen, nuclei for the growth of fluid mechanics in Germany. In this article I trace this development on the basis of my recent biography of Prandtl.

1. Introduction

At the turn of the 19th century, 'fluid mechanics' in Germany (and elsewhere) was a discipline with a dual identity. Under the label 'hydrodynamics', it belonged to the realm of theoretical physics and was considered first of all as a mathematical challenge. The equations of motion, both in the form of Euler's equations for ideal fluids and the Navier–Stokes equations for viscous fluids, had been studied for decades—without arriving at solutions for most practical applications. Hydrodynamics suggested concepts such as "vortex atoms" and offered one or another argument for an electrodynamic ether, but actual flow phenomena usually eluded this sort of fluid mechanics. As the author of a textbook on hydrodynamics observed in 1900, practical applications were to such an extent beyond its reach "that technology has adopted its own procedure to deal with hydrodynamical problems, which is usually called hydraulics" [1, p. III]. Hydraulics, in contrast to hydrodynamics, was an engineering discipline in the realm of technical mechanics. It was taught at technical colleges (*technische Hochschulen*) rather than universities and disposed of its own corpus of textbooks (e.g., [2]). The renowned *Enzyklopädie der mathematischen Wissenschaften* assigned hydrodynamics and hydraulics to different authors [3,4]. From the perspective of a hydrodynamic textbook writer, hydraulics lacked "so much of a strict method, in its foundations as well as in its conclusions, that most of its results do not deserve a higher value than that of empirical formulae with a very limited range of validity" [1, p. III].

This situation changed in the early decades of the 20th century. "The solution to the dilemma, of course, lay in Ludwig Prandtl's 1904 proposal that flow around immersed bodies be approximated by a boundary zone of viscous influence and a surrounding zone of irrotational motion, and in his insistence that theory and experiment go hand in hand." Thus the renowned hydraulician Hunter Rouse pointed to the boundary layer concept as the crucial step which bridged the gap between hydraulics and hydrodynamics, and to the man who initiated this change towards modern fluid mechanics. And he added "the fact that nearly all who became the charter fluid machinicists were originally mechanical engineers" [5, p. 2].

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In this paper, I analyze this change in more details. Section 2 is dedicated to Prandtl's early career and the origins of the boundary layer concept. Section 3 sheds light on the environment in which practical concerns were addressed by academics, with particular emphasis of a seminar organized by the Göttingen mathematician Felix Klein. The next step to bring theory and practice in closer contact happened with the advent of aeronautics, first with wind tunnel investigations of airship models (section 4) then with a theory of airfoils (section 5). Prandtl's role for the growth of modern fluid mechanics was not limited to theories and experiments. By the end of the First World War, he was head of a huge aerodynamic testing facility, and a few years later he founded the Kaiser-Wilhelm-Institut für Strömungsforschung—an establishment in which fluid dynamics was pursued with a focus on basic research (section 6). But Prandtl's "basic research" in fluid mechanics was close to applications. This becomes apparent in the quest for a theory of turbulence (section 7), a specialty in which Prandtl and his school left a particular mark. Prandtl also served as a science advisor in the political arena—from the German Empire through the Weimar Republic to the "Third Reich". In World War II he served Göring's Air Ministry as chairman of a four-men board (*Forschungsführung*) charged with the direction of aeronautical research; abroad he assumed the role of a goodwill ambassador for Nazi Germany (section 8). His performance calls for a critical discussion of a scientist in the political arena. I conclude (section 9) with some remarks on Prandtl's legacy. For a more detailed account of Prandtl's life, I refer to [6].

2. The practical origins of the boundary layer concept

The idea of a boundary layer may already be found in the 19th century, when "surfaces of discontinuity" were introduced in an attempt to explain some failures of hydrodynamic theory—most clearly in discussions between George Gabriel Stokes and William Thomson (Lord Kelvin) on the stability of plane flows [7, pp. 204–206]. Prandtl's approach to the boundary layer concept, however, was motivated rather by practical concerns. He had studied mechanical engineering at the Technische Hochschule in Munich before he was employed in 1901 as an engineer in a machine factory. Charged with the inspection of an exhaust system for removing shavings and dust from workbenches, he discerned a pressure loss in the exhaust tubes as the cause of undue expenditure of power. The pressure resulted from diverging connections between tubes of different diameters where the flow separated from the wall. "The question why a flow, instead of flowing along the wall becomes detached from it, did not go out of my mind until three years later the boundary layer theory brought the solution," Prandtl recalled this episode from the year 1901 many years later [8, p. 90]. At the time, Prandtl solved the problem by a more appropriate design for joining exhaust tubes. His firm patented this design and Prandtl obtained for many years a share of the firm's profit [6, chapter 2.1].

There were other engineering problems that amounted to the same question why a flow becomes detached from a solid surface. In 1902, by now Prandtl was professor of technical mechanics at the Technische Hochschule in Hanover, steam flow was a major theme at the annual meeting of the Association of German Engineers (*Verein Deutscher Ingenieure*, VDI). Prandtl paid tribute to this subject with several papers. In a "Contribution to the Theory of Steam Flow through Nozzles',' he remarked that an unexplained behavior of steam flow could be due to flow detachment from the nozzle wall as a result of pressure rise in the flow direction. He referred to an "heretofore unpublished hydrodynamic investigation" in which he had analyzed this behavior more closely [9, p. 349].

Prandtl left some notes which shed light on this investigation [6, chapter 2.3]. They reveal a primary interest in a qualitative understanding of the gradual process of flow separation. Prandtl built a special water channel for observing the detachment of vortices from curved bodies. When he finally presented the boundary layer concept at the Third International Congress of Mathematicians in August 1904 in Heidelberg, he illustrated it with photographs from this channel [10]. For a modern reader of this landmark paper, it appears strange that there is little mathematical elaboration. Prandtl derived the boundary layer equations for the flow along a flat plate by canceling terms from the Navier–Stokes equations and presented without proof the velocity profile in the boundary layer and an approximate formula for the resistance of this flow. He left it to Heinrich Blasius, one of his first doctoral students, to elaborate in 1907 the mathematical details [11]. When Prandtl was asked later why he had kept the Heidelberg paper so short—after all this was a congress of mathematicians—Prandtl excused this with the lack of time [12, p. 11]. But, from the perspective of his practical approach, it is more likely that by 1904 he was not yet aware of the mathematical challenges lurking behind the boundary layer concept.

3. Felix Kein's seminar on hydrodynamics in 1907

In 1904, the Göttingen mathematician Felix Klein lured Prandtl away from Hanover to Göttingen University as professor of "technical physics" and director (together with Carl Runge) of a new Institute for Applied Mathematics and Mechanics. Klein's initiative was part of a long-term effort to open the door of Göttingen University to applied sciences—traditionally the domain of technical colleges [13]. In the same vein Klein organized seminars that demonstrated the uses of mathematics in other sciences. He involved Prandtl and Runge already during their first Göttingen semester in the winter 1904/1905 as co-organizers in a seminar "On selected topics of elasticity theory." In summer 1905 another seminar was held "On Electrical Technology," co-organized by Klein, Prandtl, Runge and Theodor Simon, who was appointed in 1901 as director of a new Institute for Applied Electricity. Prandtl's move from Hanover to Göttingen, therefore, did not alienate him from the engineering topics with which he had been concerned before. Prandtl's students, too, would be confronted with technological subjects, although they were otherwise confined to an academic curriculum and would not leave the university as engineers.

In the winter semester 1907–1908 Klein involved Prandtl, Runge and the geophysicist Emil Wiechert as co-organizers of another seminar—this time on hydrodynamics. “Connection of theory with practice (observation, experiment, construction),” Klein remarked about his intention in the beginning of the seminar [14]. He also referred to Prandtl’s lectures and the Institute for Applied Mechanics where those attending would find pertinent practical applications. Among the seminarists were Prandtl’s advanced students like Theodore von Kármán, who had come to Prandtl for a doctoral dissertation on buckling strength. Kármán’s seminar presentation was titled “On discontinuous potential motion.” It dealt with problems such as the drag of a plate exposed at an oblique angle to an airstream. Kármán contrasted the “Newtonian” approach based on the impact of particles with “Kirchhoff’s” concept of discontinuity surfaces. He showed that Newton’s approach was wrong, but he did not qualify the solution obtained by the concept of discontinuity surfaces. Many years later, this example would become a textbook lesson in order to show that the surface of discontinuity also was “a blind alley for drag predictions” [15, pp. 100–106].

The seminar covered a broad spectrum of problems from fluid mechanics. Georg Fuhrmann, for example, talked about “Boundary layers and vortex detachment,” referring to Prandtl’s Heidelberg presentation from 1904 and the doctoral dissertation of Blasius, which was accomplished by the same time. Blasius himself chose “Turbulent flows” as the subject of his seminar presentation. Three presentations were dedicated to ship waves, ship propellers, and ship resistance. Originally Klein had titled the entire seminar “hydrodynamics of ships” [14]. In a period when the German Empire expanded its fleet in a frantic pace in order to catch up with the British Royal Navy, the choice of this theme was certainly not accidental.

4. Airship models in the wind tunnel

Besides the naval fleet there was another technology in which the German military had a keen interest by this time: airships. In May 1906, representatives from industry, politics and the military formed a “Study Group for Motorized Airships” (*Motorluftschiff-Studiengesellschaft*) “in order to develop aeronautics as technology and industry.” A few months later, test cruises with a Zeppelin suggested future military uses of the airship if it could be “technically improved.” The other German airship of “type Parsival” was also regarded as “usable for war,” but the study group considered it “absolutely necessary that it is further developed.” Klein, with his connections to industry and politics, perceived this as another opportunity to establish application-oriented research at Göttingen University. In December 1906, he presented the study group with a proposal for an airship testing facility. It was scheduled to operate for a duration of three years, during which it would perform aerodynamic measurements on airship models in a wind tunnel. Prandtl was supposed to direct this facility besides his obligation as director of the university’s institute for applied mechanics, together with Georg Fuhrmann as assistant [6, chapter 3.2].

What Klein and Prandtl originally regarded as a three-year project, however, developed into a permanent effort. The Motorluftschiffmodell-Versuchsanstalt (MVA), installed in 1908 in a small hut at the outskirts of Göttingen, became the nucleus of aeronautic research on a much larger scale. In the First World War, it was moved to a new site which offered the opportunity for ever-growing expansion. Originally confined to measurements on airship models, it developed into a facility for a broad range of aerodynamic investigations. In 1915, the war ministry financed the construction of a new wind tunnel housed in a new building. In 1919, the facility was renamed as Aerodynamische Versuchsanstalt (AVA) [16].

From the very beginning, when the MVA aimed at the improvement of airships, Prandtl pursued this applied aerodynamic research with an interest in basic questions of fluid mechanics. He asked Fuhrmann not only to analyze various airship models in the wind tunnel in order to find a shape with minimal air resistance, but also to compare his measurements with theory. Fuhrmann resorted to Rankine’s method of sources and sinks for the construction of cigar-like shapes. As he wrote in a letter to Prandtl in September 1909, he combined sources and sinks in such a way that the resulting stream-lines enclosed bodies with equal diameter and equal volume [17]. “The shape of the models was determined by a mathematical procedure that allowed us to compute by a special approach the flow and particularly the distribution of pressure under the assumption of a frictionless fluid in order to arrive at a comparison between these hydrodynamic methods and the measurements,” Prandtl reported about Fuhrmann’s use of Rankine’s method. “It may be added that the agreement between theory and experiment is very satisfactory” [18, p. 44].

In his final treatise on this subject, Fuhrmann compared six different airship models with respect to the distribution of pressure and velocity according to Rankine’s method with measurements in the wind tunnel. Around the front part of the models there was close agreement between theory and experiment, but at the stern there were more or less severe differences dependent on the respective shape. “According to the theory of flow detachment by Prof. Prandtl this is completely understandable,” Fuhrmann explained these discrepancies, “because in the rear part, where the flow is slowed down by friction and enters a region of higher pressure, the conditions for flow detachment and vortex formation are fulfilled; these vortices correspond to what is called backwash at ships” [19, p. 105]. Thus Fuhrmann discerned two different sources of flow resistance, skin friction and form drag. In the front part of the models, where the distribution of flow velocity and pressure outside the boundary layer obeys the laws of ideal flow theory, the flow resistance is only due to skin friction exerted inside the boundary layer; in the rear part, where the flow separated from the surface, the resistance was due to vortex shedding. In other words: ideal flow theory (by means of Rankine’s method) combined with Prandtl’s boundary layer concept became instrumental to investigate the causes of drag.

Prandtl and his collaborators at the MVA soon extended the drag measurements. In 1912, they found a blatant discrepancy for the drag of spheres as measured in Gustave Eiffel's wind tunnel in Paris, and supposed at first that Eiffel had omitted a factor of 2 in the final evaluation of the data. Provoked by this claim, Eiffel performed a new test series and found that the discrepancy was not the result of an erroneous data evaluation, but of a new phenomenon that could be observed only at higher air speeds than those attained in the Göttingen wind tunnel: at a critical air speed the drag coefficient suddenly dropped to a much lower value. By inserting a nozzle into their wind tunnel Prandtl and his collaborators confirmed Eiffel's discovery—and offered an explanation which at first appeared paradoxical: turbulence diminishes the drag. When the initially laminar boundary layer around the sphere becomes turbulent beyond a critical air speed, it entrains fluid from the wake so that the boundary layer stays attached to the surface of the sphere longer than in the laminar case. Hence the onset of turbulence in the boundary layer reduces the wake behind the sphere and thus also its drag. The boundary layer concept, which had been limited so far to laminar flow, thus became extended to turbulent flow. In 1914, Prandtl demonstrated this effect by exciting the transition to turbulence in the boundary layer with a thin "trip wire" around the sphere, visualizing the reduction of the wake with smoke [6, chapter 3.7], [20].

5. The lift and drag of airfoils

Not the least because of the Göttingen results concerning airship models with minimal resistance, airships had become "usable for war." But in the First World War the aerodynamic measurements in Prandtl's wind tunnel—by modern standards a small instrument with a closed circuit—were no longer focused on airship improvements. Although the same sort of airship model testing was used in order to determine the optimal shape of bombs [6, chapter 4.2], the major part of Prandtl's wind tunnel measurements during the war were concerned with airplanes. In the First World War, the airplane became a weapon. A comparison before, during and after the war shows how much the airplane changed in the course of only few years. There was also a change in quantities. Before the war airplanes were produced in workshops of individuals or small companies, with an annual production in the hundreds. In 1918, wartime delivery of airplanes amounted to several ten thousands [21]. The German Army established a special branch for testing new airplanes in Berlin–Adlershof. They also submitted contracts to Prandtl's Göttingen facility, which had grown during the war into a big-science establishment with a huge wind tunnel. It became operational in March 1917. As in the first wind tunnel (which was modernized and kept in use in an attached building), the air flowed in a closed circuit; compared to the 30 horsepower fan of the first wind tunnel, the new wind tunnel had a ten times more powerful motor that blew the air at a maximal speed of 200 km/h through a test section of circular cross section with a diameter of 2 m. It was the "most powerful and largest facility of its kind" in the world, Prandtl noted after the war, which "we owe, of course, to the generosity of our military administration" [22, p. 87].

Prandtl's research was almost exclusively oriented towards war-related goals. When the Deutsche Mathematiker-Vereinigung asked the directors of mathematical institutes and allied disciplines about their involvement in the war effort, Prandtl answered that he and his students were "practically completely" working "for the interests of the Army (aerodynamical measurements, mainly on models of airplanes, parts of airplanes etc., calibration of instruments for measuring the air speed)." The students employed for these tasks performed "test work and extensive numerical and graphical elaboration of test results." Although most of it was routine work such as "reading the scales of balances," he revealed that there were also some challenges. "A few advanced mathematicians deal with more difficult calculations (evaluation of integrals etc. for hydrodynamic problems etc.)" [23, quoted on p. 59].

Among these "hydrodynamic problems," the flow around wings became a particular challenge. Prandtl's own theoretical efforts in this regard may be traced back to a lecture in the summer 1909 when he characterized the flow behind a wing by a vortex around the wing and a pair of vortices trailing backwards from the wing-tips [16, p. 190]. The two-dimensional flow around a wing profile had been the subject of a theory by Wilhelm Martin Kutta and Nikolai Joukowski from which the lift or certain profiles ("Joukowski profiles") could be computed by conformal mapping. The extension of this concept to wings of finite span, however, implicated enormous mathematical problems. A number of Prandtl's students and assistants (Carl Wieselsberger, Albert Betz, Ernst Pohlhausen, Max Munk) became involved in this effort. What became famous after the war as Prandtl's airfoil theory resulted from aerodynamic measurements of hundreds of wing profiles in the Göttingen wind tunnels, performed as a result of war contracts. During the war, the theory was only informally communicated among Prandtl and his students. In May 1918, an airplane manufacturer wondered why certain results were presented "without any reference to a publication." Prandtl responded: "The theory of the monoplane for which you are asking has not been published so far in print, it was presented only in lectures and seminars" [23, quoted on p. 79].

Prandtl published the airfoil theory after the war in the proceedings of the Göttingen Academy of Sciences [24,25]. By the same time the doctoral dissertations of his assistants Albert Betz [26] and Max Munk [27] revealed more details. Prandtl's "lifting line" concept, as the airfoil theory at this stage became labeled, marked the beginning for the mathematization of flight [28]. Once the scientific principles of the theory had been published, Prandtl presented a synopsis together with the wartime wind tunnel measurements in a new series which was specifically addressed to engineers [29].

Although the paradoxical use of ideal flow concepts in a theory that addressed real flow problems such as the airflow around wings provoked controversial debates [30], the theory immediately attracted the curiosity of aeronautical engineers. In 1920, the National Advisory Committee for Aeronautics (NACA) in the USA published short versions of the theory in its Technical Notes, and in 1922 a comprehensive report. Despite the boycott of German science after the war by the former Entente, NACA entertained close relations with Prandtl. Other foreign relations developed in 1921 with the Royal Aircraft

Establishment in Farnborough, and a year later with the Royal Aeronautical Society in London. Prandtl also used his contacts with a NACA-representative in Paris in an attempt to renew the friendly relationship with Eiffel, whose laboratory he had visited in 1913. But Eiffel's cool response prevented further attempts to get in contact with French scientists. In Prandtl's voluminous correspondence, there are few letters from French colleagues, in contrast to his exchange with NACA officials in Paris who remained important addressees throughout the 1920s and 1930s [31].

6. From aerodynamics to fluid mechanics

After the war, Prandtl was an authority of modern aerodynamics. The catalog of wing profiles measured in the wind tunnels of the AVA ("Göttingen profiles") became an important tool for aeronautical engineers worldwide, and Prandtl and his pupils were sought-after experts—both for the design of wind tunnels and the aerodynamic theory of lift and drag. Yet Prandtl was not happy with a reputation focused on aerodynamics only. He had planned a large institute for aerodynamics *and* hydrodynamics since 1911, after the foundation of the Kaiser-Wilhelm-Society for the Advancement of Sciences (*Kaiser-Wilhelm-Gesellschaft zur Förderung der Wissenschaften*, KWG). After the war, Prandtl hoped once more that he could re-orient his research towards more fundamental fluid mechanics. When he received in 1920 a call to his alma mater, the Technische Hochschule in Munich, he reminded the KWG of his old plans. In view of the dire economic situation in Weimar Germany, chances appeared slim, but Prandtl did not give up. Göttingen University was determined to keep him, and the KWG in Berlin promised to look for funds in order to fulfill his wishes. Prandtl used the call to Munich and his international reputation as an asset in a tug-of-war in which he negotiated with the involved administrations in Berlin, Munich, and Göttingen for several years in order to achieve his goals. Finally, a wealthy industrialist offered a considerable donation if Göttingen University presented him with an honorary doctoral degree. The university agreed "if he enables the development of an important specialty in Göttingen and to keep a highly esteemed colleague here by erecting a hydrodynamical institute." With these additional funds the KWG signaled green light. In July 1925, Prandtl became director of a new "Kaiser-Wilhelm-Institute for Fluid Dynamics Associated with the Aerodynamic Testing Establishment in Göttingen" (*Kaiser-Wilhelm-Institut für Strömungsforschung, verbunden mit der Aerodynamischen Versuchsanstalt in Göttingen*) [16, pp. 234–244].

During his negotiations about the Kaiser-Wilhelm-Institut (KWI) Prandtl had argued that he would stay in Göttingen only if he would be given there the opportunity to tackle "a new life task" ("eine neue Lebensaufgabe"). At the age of fifty, he was eager to leave a mark in fluid mechanics at large. The "hydrodynamic institute," as he called it in distinction to the associated AVA, would investigate a broad spectrum of flow phenomena. "Hydrodynamics is understood as the combination of scientific issues of fluid mechanics as a whole," Prandtl defined its scope. Aerodynamics, by distinction, was just "a branch of hydrodynamics that is concerned with the laws of motion and actions of forces in a free stream of air." Hydrodynamics, perceived in Prandtl's sense, involved "the research of certain fundamentally important phenomena like the formation of vortices, turbulence, wave motion and the like" [6, quoted on pp. 138–139].

For the experimental investigation of these flow phenomena, Prandtl furnished his new KWI with a variety of laboratory equipment, from pumps and pipes to water channels and wind tunnels for special purposes. A "rotating laboratory", for example, was used for the study of flows in rotating systems—from rotating machinery like turbines to geophysical applications; Prandtl's children occasionally used it as a carousel. Pressurized and evacuated tanks (*Windkessel*) served for experiments on cavitation and gas dynamics. Cinema technology was used and extended in order to record the formation of cavitation bubbles in slow motion with up to 6,000 images per second. Within a few years, Prandtl's "sanctum sanctorum"—as two of his disciples recalled their early career in the KWI [32]—became the source of aspiring and innovative research in fluid mechanics. This is also evident from the contributions to the four volumes on hydro- and aerodynamics in the *Handbuch der Experimentalphysik*, with more than half of all contributions authored by Prandtl and his collaborators. Prandtl introduced these volumes with a review on the principles of fluid mechanics; in addition, he contributed an article on the "construction of flawless air flows (wind tunnels)", Jacob Ackeret authored the chapter on cavitation, Adolf Busemann on gas dynamics, Walter Tollmien on boundary layers and turbulent flow—to mention just a few examples [6, pp. 199–200].

By the same time, Oskar Tietjens elaborated his notes from Prandtl's lectures into a two-volume textbook "Hydro- und Aeromechanik, nach Vorlesungen von L. Prandtl", which appeared in English translation in 1934 and became an international classic of modern fluid mechanics. Another example of Prandtl's rise as an international authority of fluid mechanics was his textbook-like contribution on "The Mechanics of Viscous Fluids" published in 1935 in volume III of William Fredrick Durand's famous series *Aerodynamic Theory*. Most admired became another textbook presentation that started in 1931 as *Abriss der Strömungslehre* ("Summary of Fluid Mechanics") and was renamed a few years later as *Ein Führer durch die Strömungslehre* ("A Guide through Fluid Mechanics"). It went through many new editions. The English translation was titled *Essentials of Fluid Dynamics*. "If I have to consult Prandtl's *Essentials* in future, it is to my well-thumbed 1952 edition that I shall return," Keith Moffatt concluded in 2005 a critical book review of a later edition of this legendary textbook that attempted to pursue Prandtl's tradition with debatable success [33, p. 379].

By the mid-1930s Prandtl's reputation as an international pioneer of modern fluid mechanics was at its climax. Geoffrey Ingram Taylor, for example, felt "very strongly," as he confided to Prandtl in a personal letter in November 1935, "that if the Nobel Prize is open to non-atomic physicists it is definitely insulting to us that our chief—and I think that in England and USA at any rate means you—should never have been rewarded in this way." Prandtl regarded fluid mechanics as belonging "not really to physics," but rather as "an independent field between mathematics and engineering sciences." Therefore he raised “no hope whatsoever” to receive the Nobel prize. “If the Swedes classify the sciences in a similar way as here,” he responded to Taylor, “then I will be taken under consideration as little as the mathematicians, and for the rest I will know how to console myself just like the mathematicians.” Actually Prandtl had been nominated for the Nobel prize already in 1928 by German colleagues, and again in 1936 by the British Nobel prize laureate William Lawrence Bragg, whose letter of nomination hints at Taylor’s initiative in the background [6, pp. 238–239].

7. Turbulence

In the 1930s Prandtl’s research was no longer focused on the lift and drag of airfoils or boundary layer theory. If there was one field among the many research topics in fluid mechanics that rose to prominence as a particular challenge—this was turbulence. Prandtl had penciled a “working program for a theory of turbulence” already in 1916, but it developed into a research program only during the 1920s and 1930s. Besides its practical aspects, turbulence was (and still is) regarded as “one of the great unsolved mysteries of science,” to quote Theodore von Kármán, Prandtl’s master pupil and, since the 1920s, rival and competitor in this field. As Kármán recalled [34, p. 135],

> my old professor and I were in a kind of world competition. The competition was gentlemanly, of course. But it was first-class rivalry nonetheless, a kind of Olympic Games, between Prandtl and me, and beyond that between Göttingen and Aachen. The “playing field” was the Congress of Applied Mechanics. Our “ball” was the search for a universal law of turbulence.

The beginning of this rivalry may be dated to 12 February 1921, when Kármán presented Prandtl with a sketch of a theory for the turbulent boundary layer. “Dear master, colleague, and former boss,” he addressed Prandtl in this five-page-long letter, in which he derived from the empirical “Blasius law” for turbulent friction in pipe flow (friction coefficient proportional to $Re^{-1/4}$, where Re is the Reynolds number) a formula for the velocity distribution in the turbulent boundary layer, $v \sim y^{1/7}$, where v is the mean flow velocity and y the distance from the wall. Kármán recalled that Prandtl had mentioned this formula earlier without derivation, and was hesitant to publish his theory without Prandtl’s consent. Prandtl was busy with other things at that time and did not object to Kármán’s wishes for a swift publication. “I will see if I can gain recognition by my own right with my different derivation,” he responded. “Ultimately, I can get over it if the precedence of publication has gone over to friendly territory” [23, quoted on p. 119].

Thus began the quest for a universal “wall law” of turbulence. The final result involved a logarithmic velocity distribution with growing distance from the wall rather than the “1/7th law”—and a universal constant which was first published by Kármán in 1930 and became named after him. Prandtl’s slightly different version appeared only in 1932. Kármán’s undisputed priority in publication, however, tends to blur the actual achievements in this competition. By the mid 1920s, Prandtl had introduced his “mixing length” approach for fully developed turbulence. Furthermore, despite the fierce rivalry between Göttingen and Aachen, there was also a close collaboration and exchange of experimental results. Kármán’s theory benefited from the mixing length approach and, in particular, from precision measurements in Göttingen (performed at Prandtl’s request by Johann Nikuradse), in which the logarithmic law first appeared in 1929 [20, pp. 54–62], [35, pp. 104–109].

From the perspective of practical applications, the “wall law” for turbulent skin friction can hardly be overrated. This is illustrated by a dispute between Prandtl and Kármán in the aftermath of a conference convened in May 1932 by the Hamburgische Schiffbau-Versuchsanstalt (Hamburg testing establishment for naval architecture). A section of this conference was dedicated to the recent theories and experiments about turbulent skin friction. Kármán could not attend (since 1930 he spent half of his time each year in Pasadena as director of the new Guggenheim Aeronautical Laboratory of the California Institute of Technology, GALCIT) and contributed only in the form of a paper which was read by another attendee. Franz Eisner, a scientist from the Preussische Versuchsanstalt für Wasserbau und Schiffbau in Berlin (Prussian testing establishment for hydraulics and naval architecture) with close connections to Prandtl’s institute, addressed the same theme from a broader perspective, and Günther Kempf from the Hamburg Schiffbau-Versuchsanstalt presented recent results about friction on smooth and rough plates. Prandtl was invited as a commentator. Two months after this conference, the Schiffbautechnische Gesellschaft published in its journal *Werft, Reederei, Hafen* a diagram about plate resistance taken from Eisner’s conference presentation. It showed experimental data in accordance with the logarithmic wall law “after Prandtl” with a reference to a forthcoming publication in the *Ergebnisse der Aerodynamischen Versuchsanstalt zu Göttingen*, and the outdated 1/7th power law with the comment “Prandtl–Kármán 1921” with a meager fit to the data at high Reynolds numbers. When Kármán saw this article, he was upset. He felt that his breakthrough to the logarithmic law from 1930 was ignored. A reader of this article would get the impression “as if I had given up to work on this problem after 1921,” he complained in a letter to Prandtl, “and that everything has been done in 1931/1932 in Göttingen.” He asked Prandtl to correct this erroneous view. “I write so frankly how I think in this matter because I know you as the role model of a just man,” he appealed to Prandtl’s fairness. But he had little sympathy for “your lieutenants who understandably do not know other gods beside you. They wish to claim everything for Göttingen.” He was not satisfied with a mere acknowledgment of his priority by reference to his pertinent academic publications from the year 1930, but wished to be rehabilitated also in Prandtl’s forthcoming volume of the *Ergebnisse der Aerodynamischen Versuchsanstalt zu Göttingen*, which he regarded as the “standard for practitioners” [6, chapter 6.8].

Another item on Prandtl's "working program" concerned the onset of turbulence. What finally became categorized under hydrodynamic stability was by 1920 considered as "The turbulence problem," to quote the title of an article in the first volume of the *Zeitschrift für Angewandte Mathematik und Mechanik (ZAMM)*. It climaxed in efforts to account for the stability of plane laminar flows by an approach conceived by the Irish mathematician William McFadden Orr and the German theoretical physicist Arnold Sommerfeld [36]. Prandtl paid tribute to these efforts for a number of years with little success, until in 1929 his doctoral student Walter Tollmien was able to derive from the Orr–Sommerfeld approach an instability diagram for a special plane flow (close to laminar boundary layer flow along a flat plate). Tollmien's solution was further studied in Prandtl's school in the 1930s by Hermann Schlichting. The limit of stability in laminar boundary layer flow became known as "Tollmien–Schlichting instability." However, it remained disputed until its experimental corroboration by Hugh Dryden and his collaborators in a wind tunnel of the National Bureau of Standards in World War II [37]. After this discovery, the onset of turbulence became subject of another rivalry—this time between Tollmien, who succeeded Prandtl in 1947, and Kármán's doctoral student Chia-Chiao Lin, whom Kármán charged with a thorough scrutiny of theories based on the linear stability approach after Orr and Sommerfeld. Tollmien complained that Lin misrepresented the historical development of this field. "In 1945 many foreigners believed that German science was dead and that one can get away with plundering in our garden," Tollmien wrote to a colleague. Kármán had come to Germany after the war with an intelligence mission of the US Army (operation LUSTY, Luftwaffe Secret Technology) and found little sympathy with his former colleagues. As Tollmien wrote in a letter to Lin: "In spite of the fact that you have done grave injustice to me in your paper I do not bear any grudge against you because I think you were ill advised." In a letter to Werner Heisenberg, who had himself contributed to linear stability theory in 1924 in his doctoral dissertation under Sommerfeld, Tollmien regarded Lin as "a promising young scientist, who was only somewhat poorly advised in 1945 to tackle a task whose difficulty he did not oversee" [38].

By the early 1930s, Prandtl's own turbulence research became focused almost exclusively on fully developed turbulence. Excited by a paper of G.I. Taylor in which Prandtl's mixing length approach—portrayed as momentum transport theory—was compared with Taylor's older vorticity transport theory [39, chapter 4.3.2], Prandtl wrote in July 1932 to Taylor: "In the last weeks I studied your old papers from 1915 and 1922 with the greatest interest. I think that if I had known these papers, I would have found the way to turbulence earlier." Prandtl and Taylor had corresponded earlier on a variety of subjects, but now their correspondence showed a growing mutual enthusiasm for a closer investigation of fully developed turbulent flow. By the end of 1932, they planned hotwire experiments for measuring turbulent velocity fluctuations. Taylor aimed at the "spectrum of turbulence" by measuring correlations between fluctuating quantities, and Prandtl was "as convinced as you that from the study of those correlations as well as between the direction and magnitude of fluctuations, for which we have prepared a hotwire setup, very important insights into turbulent flows can be gained." In 1934, Prandtl and his main collaborator in this research, Hans Reichardt, published the first results from these measurements. Within a few years, such measurements became the standard tool for obtaining the data upon which the statistical theory of turbulence was founded [20, chapter 2.7], [39, chapter 4.5].

The culmination of this development was reached in 1938. With a paper on "The spectrum of turbulence" Taylor added in this year the final touch to the "Statistical Theory of Turbulence", as he had titled a series of articles in 1935 in the *Proceedings of the Royal Society in London*. The Fifth International Congress for Applied Mechanics, held at Harvard University and the Massachusetts Institute of Technology in Cambridge, Massachusetts, from 12 to 16 September 1938, dedicated to this topic a special symposium chaired by Prandtl. "The Organizing committee is grateful to Professor Prandtl and considers his Turbulence Symposium not only the principal feature of this Congress, but perhaps the Congress activity that will materially affect the orientation of future research," the conference organizer wrote to Kármán, who had also left his mark on the statistical theory of turbulence with key contributions (e.g., the Kármán–Howarth equation, see [35, Chapter 3.3]). Prandtl distinguished in his contribution to this symposium [40] four types of fully developed turbulence: wall turbulence, free turbulence, turbulence in stratified flows and decaying isotropic turbulence as generated behind grids in a wind tunnel—but each one seemed to involve different theoretical approaches. While the mixing length concept was successful for wall turbulence, isotropic turbulence was beyond its reach. Prandtl was not satisfied with this state of affairs in turbulence research. After his return to Germany, he confided to a colleague that the symposium on turbulence has "not particularly succeeded" according to his opinion. "There have been gathered from all sides a whole lot of bricks and other raw materials, but it has not yet become apparent how the edifice should look like which can be built from it. Perhaps one has to prepare later a second symposium in which the construction plan can be clarified" [6, p. 247].

Despite his misgivings Prandtl's efforts to cope with fully developed turbulence in 1938 contained the seeds for a theory published in 1945 together with his collaborator Karl Wieghardt which was later regarded as an early example of turbulence modeling. As revealed by some unpublished notes sketched during the final months of the war, Prandtl also came close to the universal laws that are linked today with the name of Andrey Nikolaevich Kolmogorov and labeled as K41 theory [20, chapter 2.8–2.10].

8. Prandtl—a nonpolitical scientist?

At the time of the turbulence symposium, the Sudeten Crisis was in the international headlines. On 15 September 1938, Neville Chamberlain agreed with Adolf Hitler's plans to transfer the Sudetenland—since the end of World War I a part of Czechoslovakia—to the German Reich. The events surrounding this crisis and its "solution" by appeasing Hitler were also debated by the scientists in Cambridge and left some traces in their correspondence after the conference. Taylor admired Prandtl as a scientist, as he had revealed in his correspondence about the Nobel prize, but had to notice during the discussions in Cambridge that with regard to politics, Prandtl behaved like a goodwill ambassador for Nazi Germany. “I realized that you know nothing of what the criminal lunatic who rules your country has been doing,” he wrote to Prandtl on 27 September 1938, “so you will not be able to understand the hatred of Germany which has been growing for some years in every nation which has a free press” [6, p. 257].

This exchange of letters raises the question of Prandtl’s political attitude, all the more because he served as a science manager, expert and advisor for aeronautical research since the foundation of the MVA before World War I. This role brought him in close contact with politics from Imperial Germany and the Weimar Republic to the “Third Reich”. Nevertheless, Prandtl characterized himself as a nonpolitical scientist. After the defeat of Nazi Germany he titled a memorandum addressed to the British Military Government (Göttingen belonged to the British occupation zone) “Thoughts of a nonpolitical German about denazification.” Prandtl’s daughter, too, in a personal account on her father’s life, characterized him as nonpolitical. She excused Prandtl’s pro-Nazi statements during the Sudeten crisis as mere patriotic statements [41, p. 173].

Upon closer inspection, however, Prandtl’s role and actions were far from nonpolitical, whatever his own awareness may have been in this regard. The closeness of aeronautics to and dependence on ministries made it impossible to adopt the role of a national research leader as a nonpolitical scientist. Prandtl’s correspondence with the ministry for traffic in Weimar Germany and the air ministry in Nazi Germany, with Hermann Göring as minister and Erhard Milch as state secretary, belies the self-image as a nonpolitical scientist. This does not mean that Prandtl was a follower of the ruling politicians at all times. In the early years of the “Third Reich,” Prandtl even openly opposed Nazi politics. He criticized the dismissal of jewish colleagues by writing letters to influential Nazi authorities. When the Nazis invaded his own sphere he was upset: “You are also one of these brown fanatics,” he is reported to have said to an assistant [6, p. 208]. A Nazi official characterized Prandtl in 1937 as “the type of honorable, sedulous scholar of the old time, who is anxious about his integrity and reputability, whom, however, we cannot and should not spare with regard to his extraordinary valuable scientific accomplishments for the set-up of the air force” [6, p. 218]. By this time Prandtl’s appraisal of the Nazi rule was changing. “I believe that Fascism in Italy and National Socialism in Germany constitute rather good beginnings of the new form of thought and economic system,” he wrote to a former NACA-representative in May 1937. He believed that “nations which do not wish to fall to Bolshevism have to go very similar ways, the sooner the better” [6, p. 236].

There were tangible reasons for this change of attitude. In the preceding year Prandtl celebrated the inauguration of a new wind tunnel for which he had pleaded in vain during the Weimar Republic. When the Nazi established the air ministry under Göring they regarded Prandtl and his Göttingen facilities as an asset for their plans. Aeronautical research was an essential part of the preparation for war. “Our wishes were far from modest,” Prandtl said at the ground-breaking ceremony of the new wind tunnel, “but we could realize with satisfaction that for the new ministry only the best was good enough” [6, p. 223]. The ministry also fostered aeronautical research in a German Academy for Aeronautical Research (*Deutsche Akademie für Luftfahrtforschung*) and a Lilienthal Gesellschaft for Aeronautical Research (*Lilienthal Gesellschaft für Luftfahrtforschung*). Both organizations provided the framework for the self-mobilization of German scientists and engineers on a broad scale—including propaganda. The ministry assigned Prandtl a leading role in both organizations. At the annual meetings and banquets, Prandtl could be seen in the company of leading Nazi representatives like Göring, Heinrich Himmler and others. Such closeness also paid off in the form of Nazi distinctions like the Göring Medal in March 1939 and the Goethe-Medal for Art and Science awarded by Hitler to Prandtl’s 65th birthday in February 1940. In October 1939, shortly after the German invasion of Poland that launched the Second World War, Prandtl supported Nazi propaganda efforts by sending Hitler speeches to foreign colleagues [6, chapter 7.11].

In World War II, Prandtl’s involvement in Nazi politics extended beyond propaganda. In 1942 the air ministry assigned the organization and control of aeronautical research to a group of four men, the so-called “Research Leadership of the Reich’s Air Minister and Commander in Chief of the Air Force” (*Forschungsführung des Reichsminister der Luftfahrt und Oberbefehlshaber der Luftwaffe*), FoFü for short, with Prandtl as the chairman. In this capacity, Prandtl assumed far-reaching responsibilities, such as the construction of a giant wind tunnel in the Ötztal Alps and other projects that could be realized in this phase of the war only by forced labor from prisoners of war or concentration camp inmates [6, chapter 8.6]. When Göring awarded him in March 1945 with the War Merit Cross (*Ritterkreuz des Kriegsverdienstkreuzes mit Schwertern*), Prandtl hesitated to accept it because such distinctions were usually reserved for the military. But he overcame his doubts and accepted it in his capacity as head of the FoFü. “May the many beautiful results that it [the FoFü] could accomplish just recently become effective for the defense of the German fatherland and thus contribute its share to the hoped for final victory,” Prandtl thanked Göring on 7 March 1945, a few weeks before the American troops reached Göttingen [6, p. 293].

When, in summer 1945, British and American experts visited the facilities for aeronautical research in Göttingen and elsewhere, they were astonished about the extent and advanced state of research. Leslie E. Simon, director of the Ordnance Ballistic Research Laboratory of the US Army in Aberdeen, Maryland, regarded the FoFü as “the most powerful scientific organization of the world” and the AVA in Göttingen as the source of “almost all the aerodynamics research on the ballistics of projectiles.” Ben Lockspeiser from the British Ministry of Supply, which assumed the responsibility of the Göttingen research establishments, recalled in a meeting of the Royal Aeronautical Society in October 1946 that “there was quite a pilgrimage after the war from this country—from the Royal Aircraft Establishment, from the universities and from the industry by people who talked to the scientists and learned much about the work they had done.” Even Theodore von Kármán, who visited the site where he had started his own career as Prandtl’s assistant now as leader of an American intelligence mission, was surprised. “They got funds to pursue almost any scheme they wanted to follow. I did not think this was a healthy sign, that scientists should pursue their own ends in this way, but it was perhaps the least unhealthy state of affairs in an unhealthy state" [34, p. 274], [6, chapter 9.1].

9. Conclusion

After World War II, fluid mechanics was no longer a discipline of mere theoretical interest on the one side, or a bunch of practical formulae without scientific underpinning on the other—like hydrodynamics versus hydraulics half a century earlier. Fluid mechanics *à la Prandtl* could be basic science *and* applied engineering, with short ways from one to the other. The British Military Government forbade further research disregarding whether it was considered basic or applied, and committed the staff of Prandtl's former institutes to write reports about their wartime research. In October 1945, Prandtl asked G.I. Taylor to support a request to the Royal Society to which he had turned as a foreign member in the hope that basic research at the Kaiser-Wilhelm-Institute—the AVA with its war-related research was completely dismantled—would be exempted from the research prohibition. Taylor, however, did not respond.

In May 1946, the Allied Control Council specified the control of scientific research by Law No. 25 which confined the research prohibition in fluid mechanics to basic research of a military nature and to applied aerodynamics and applied hydrodynamics. Prandtl, now in his 70s, once more drafted a research program for future investigations at his institute. Frail health and uncertainties about the orientation of the institute under his successors made it difficult to plan for a new era. Nevertheless, turbulence remained in the focus of the Max Planck Institute for Fluid Mechanics (*Max-Planck-Institut für Strömungsforschung*), as the institute was renamed. Another topic concerned meteorological fluid mechanics, the last field to which Prandtl contributed with own research papers. Otherwise, he spent his final years with the update of his *Führer durch die Strömungslehre*. In 1948 it appeared in English translation (*Essentials of Fluid Dynamics*) and in 1952 in French translation (*Guide à travers la mécanique des fluides*) [6, chapters 9.6 and 9.7]. This book demonstrates like few others to what extent fluid mechanics has grown into a versatile science during Prandtl's lifetime.

Prandtl's legacy is also preserved by the Göttingen institutions which celebrate him as their founding father. "The early history of the Max Planck Institute for Dynamics and Self-Organization is closely linked to the work of the famous physicist Ludwig Prandtl," the institute pays tribute to its beginnings [42]. The name change in 2004 from Fluid Mechanics to Dynamics and Self-Organization hints at a broadening research orientation since the 1950s. Yet fluid mechanics still constitutes a considerable part of its agenda. The Göttingen institutes of the German Aerospace Center (*Deutsches Zentrum für Luft- und Raumfahrt*, DLR) that emerged from the dismantled AVA since the 1950s also trace their origins back to Prandtl. "Ludwig Prandtl is today considered one of the key founding fathers of institutionalized aerospace research," the DLR celebrated in 2007 the centenary of the foundation of the MVA, "which would later become the Aerodynamische Versuchsanstalt, AVA, [Institute for Aerodynamic testing] which Ludwig Prandtl would head from 1907 to 1937. Ludwig Prandtl held the post of Chairman of the Board of AVA from 1938 to 1945" [43].

Last, but not least, Prandtl's name provides "symbolic capital" (to speak with Pierre Bourdieu) for organizations and scholars who render outstanding services to fluid mechanics. The Deutsche Gesellschaft für Luft- und Raumfahrt (DGLR) awards since 1957 a "Ludwig-Prandtl-Ring" for excellent work in the sciences of flight [44]; together with the Gesellschaft für Angewandte Mathematik und Mechanik (GAMM), the DGLR furthermore invites each year an outstanding scholar to present a "Ludwig Prandtl Memorial Lecture" [45]. Laureates like Theodore von Kármán, Hugh Dryden, Philip Saffman or Keith Moffatt demonstrate that these awards honor merits without regard of nationality. But symbolic capital with the name of Prandtl also raises questions: Is Prandtl's political attitude in Nazi Germany irrelevant for awarding achievements in fluid mechanics? Or do these awards reflect an adherence to the myth of the nonpolitical scientist? Whatever lessons we draw from this case: Prandtl's role for the growth of fluid mechanics in Germany may not be separated from his performance in the political arena.

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