C. R. Mecanique 345 (2017) 595–604

Contents lists available at ScienceDirect

Comptes Rendus Mecanique

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A century of fluid mechanics: 1870–1970 / Un siècle de mécanique des fluides : 1870–1970

The use of images in fluid mechanics

L'utilisation des images en mécanique des fluides

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A R T I C L E   I N F O

Article history:
Received 6 February 2017
Accepted 26 April 2017
Available online 14 July 2017

Presented by François Charru

Keywords:
Flow visualization
Images
Flow structures

Mots-clés :
Visualisation des écoulements
Images
Structures des écoulements

A B S T R A C T

Still images, photographs and drawings, as well as movies are widely used in fluid mechanics and this has been true since the very early developments of this discipline. The intrinsic geometrical complexity of fluid flows, in particular when they are turbulent, explains this necessity of using visual representations to gain a physical understanding of the phenomena involved. The aesthetic appeal of images in fluid mechanics research is another reason why their use is more prevalent than in other fields of the physical sciences.

R É S U M É

Les images fixes, photographies et dessins, aussi bien que les films et vidéos sont très largement utilisées en mécanique des fluides, et ce depuis les tous premiers développements de cette discipline. La complexité géométrique intrinsèque des écoulements, et tout particulièrement des écoulements turbulents, explique cette nécessité du recours à une représentation visuelle pour faire émerger une compréhension physique des phénomènes mis en jeu. Le caractère esthétique des images produites en recherche en mécanique des fluides est une autre raison pour laquelle leur utilisation est beaucoup plus importante que dans d'autres champs des sciences physiques.

1. Introduction

Fluid flows are complex objects evolving in a three-dimensional space and in time. Physicists and applied mathematicians manipulate the velocity, vorticity, and pressure fields as mathematical entities, but a complete understanding of the phenomena requires almost always the visualization of the flow through photographs, movies or drawings.

In addition to this necessary visual representation, there is an undeniable esthetic appeal in flow images. Unstable and turbulent flows have this remarkable capacity to generate spontaneously ordered but fluctuating structures that fascinate both laymen and specialists.

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http://dx.doi.org/10.1016/j.crme.2017.05.015

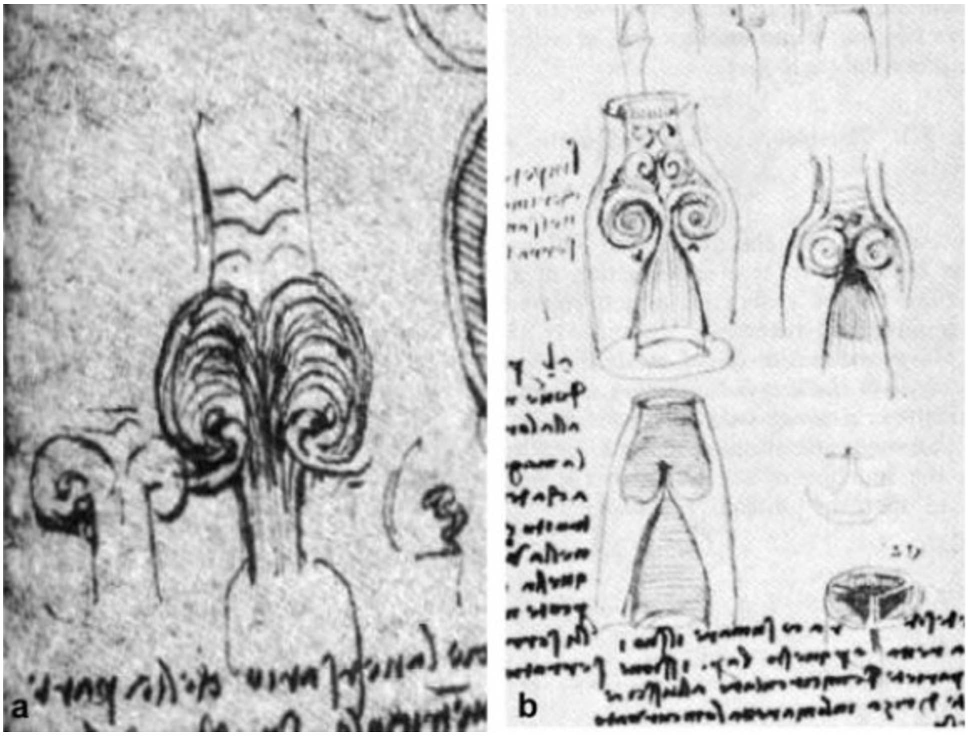

Fig. 1. Drawings by Leonardo da Vinci on the flow in a model of cardiac circulation. Images taken from [1].

Images have thus acquired a preeminent status in the work of fluid dynamicists, firstly for their own reflection and then to communicate with colleagues, students, and the general public. We describe here the multiple facets of these uses, beginning with a short summary of the developments of the techniques used to produce images. We then describe in more detail the role played by images in two major conceptual breakthroughs, namely, at the turn of the twentieth century, the separation of flows into a boundary layer and an outer flow and the discovery of permanent structures in turbulent boundary layers and shear flows in the 1960s and 1970s. Finally, we discuss the current status of images which definitely go beyond the support of a qualitative description of the flow structure.

2. From eye and pencil to ultrafast digital cameras and dpiv, the development of techniques

The need for a graphical representation of flows appears immediately when we try to analyze them. Through history, the depiction of fluid flows has gone from a simple visual observation, recorded on paper thanks to the memory and to the interpretation of the observer, to a very sophisticated quantitative analysis highly resolved in time.

The notebooks of Leonardo da Vinci give many examples of direct visual representations. For all the free surface flows that represent the major part of the drawings, Leonardo uses the deformations of the free surface and the entrainment of bubbles within the liquid to build his graphical analysis of flows. But, for “internal flows”, Leonardo develops a strategy which is still in use today: he seeds the fluid, otherwise transparent, with small particles, plant seeds, sawdust…, to reveal its motion and transcribe it to paper. Doing so, he is initiating a truly scientific analysis of flow fields. A very striking example is his study of the cardiac flow (Fig. 1). Following his anatomical observations, he designs a model of the left heart made out of glass and reveals the recirculation vortices initiated by the sudden enlargement of the aortic duct [1]. The original observations of Leonardo on this subject are very similar to modern visualizations obtained with much more sophisticated techniques. But if we consider his drawings of turbulent eddies created by jets of water falling into basins and pools, it is more difficult to separate a scientific analysis from an idealized artistic rendering.

Four centuries after Leonardo’s birth, eye and pencil are still the only means of recording the motion of fluids. A remarkable study of this kind is done by Felix Savart on liquid sheets resulting from the impact of water jets [2–4] and published in the *Annales de chimie et de physique* in 1833. Savart controls precisely the velocity of the jets and their impact parameters and a clever stroboscopic device enables him to draw very precisely the shapes of the water/air interfaces (Fig. 2).

The discovery of photography in the middle of the nineteenth century is a true revolution because it is now possible to record faithfully the motion of fluids. Photographic films on rolls of celluloid are available as soon as 1885; they are slow compared to modern ones by typically an order of magnitude, but thanks to electricity, high-speed photography became quickly a reality. A good example is given by Worthington in 1897 with a synchronizing system (Fig. 3) which is the key to record the impact of a solid sphere in a liquid and the subsequent interface deformation and generation of droplets [5].

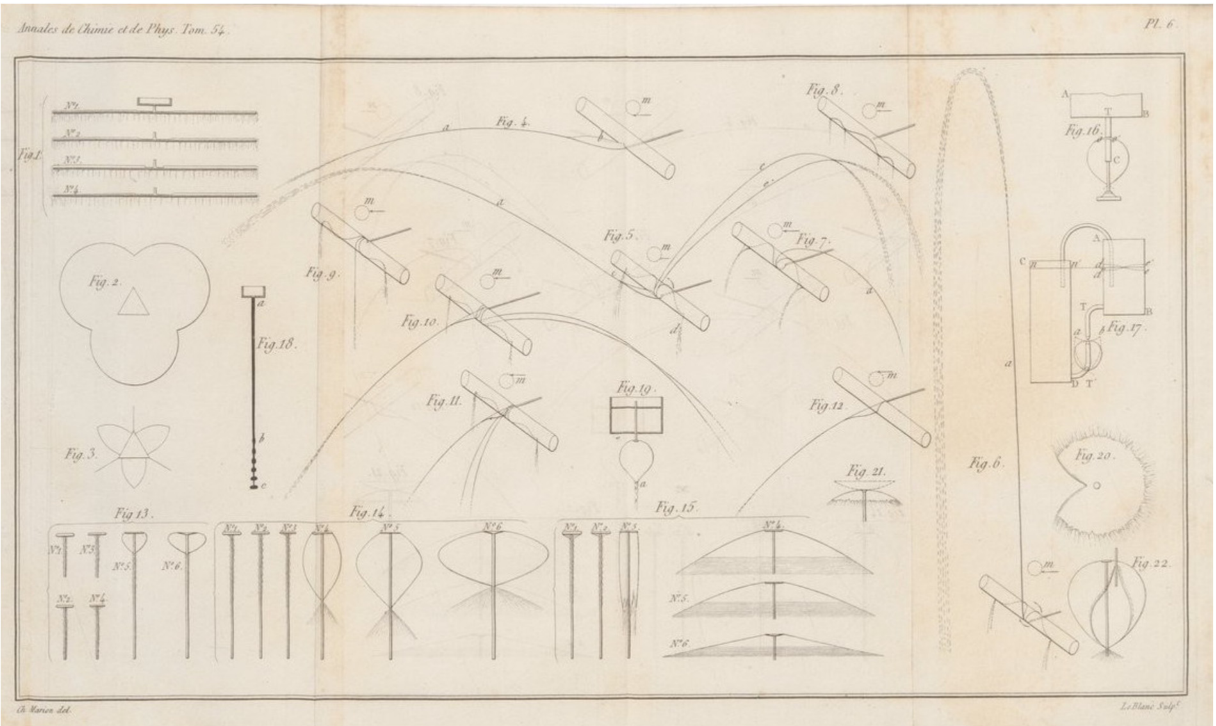

Fig. 2. Drawings by Felix Savart illustrating his study on liquid sheets. Observations of jets and sheets produced by impacts on diverse solid objects are shown on a single plate. The series of drawings on the bottom line illustrate impacts on horizontal discs and, in particular, the closed liquid bells. Image reproduced from [2].

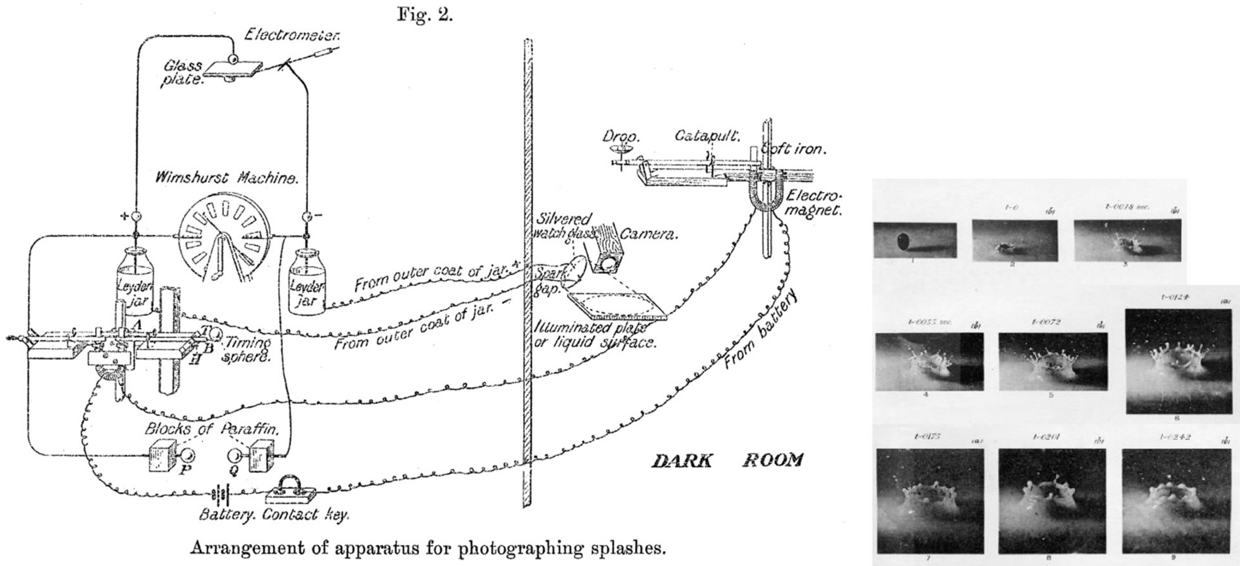

Fig. 3. Electrical synchronization device used by Worthington to record impacts of drops. On the right of the drawing we can see the camera and the electric arc illuminating the scene as well as the system emitting drops. On the left, we see the Whimshurst machine charging the condensers providing energy for the electric arc. Shown as well is the metallic sphere B that will trigger the electric arc by falling between the electrodes P and Q. By changing the height of fall of this sphere, one can change the timing of the illumination with respect to the start of the falling drop. The series of images on the left shows the splash made by a liquid drop falling onto a bath of the same liquid, at different delays after the impact. On each photograph, it is a different drop, but since the phenomenon is reproducible, the series of photographs is equivalent to a high-speed movie. Images reproduced from [5].

Recording flows on a photographic plate is a first step, but following their evolution in time is yet another crucial step thanks to chronophotography and cinema. Étienne Jules Marey is a major pioneer of these techniques. A physiologist by education, Marey develops graphical recording methods for signals like muscle motion. Similarly to Muybridge in the USA,

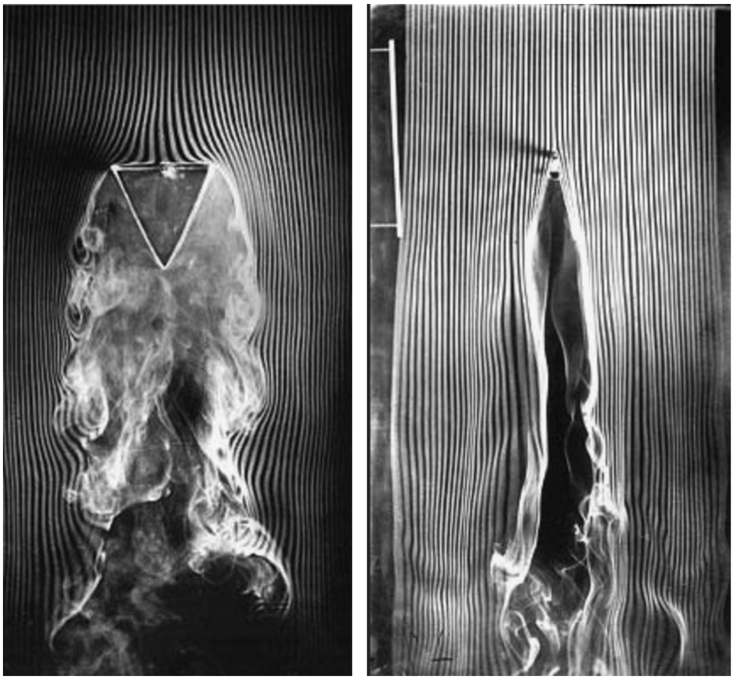

Fig. 4. Two flows visualized by Marey with the latest version of his smoke box containing 57 smoke injectors.

he is interested in the analysis of animal and human motion and, with this goal in mind, he makes great strides in the development of chronophotography and soon cinematography with the synchronization of the shutter and the motion of the film.

At the beginning of the twentieth century, Marey innovates also in fluid mechanics [6–8]. Two of the images made by Marey and his collaborators are shown in Fig. 4. They represent the flow of air around two obstacles revealed by a series of smoke lines injected upstream of the obstacle. Beyond the quality of the photographs, the concept of the smoke box is a key feature for the understanding of flow fields: smoke lines materialize, under certain conditions of steadiness, streamlines that are mathematical entities, curves everywhere tangent to the flow field. The streamlines indicate the direction of the flow, but they also reveal the changes in velocity: in regions where the streamlines are closer, the flow is more rapid, where they separate, the flow is slower. Marey went further and obtained the so-called timelines by vibrating sideways the smoke injector producing regular undulations in the smoke lines. The spacing of the undulations along a given streamline is proportional to the local velocity of the fluid, and the measurement of this spacing, thanks to a ruler in the field of view, gives the value of the velocity. Hence the photograph provides both a qualitative and a quantitative understanding of the flow.

These early developments of flow visualization are recalled in the inauguration lecture of the "Institut de mécanique des fluides de Paris" given by Henri Bénard in 1929 [9]. Bénard himself was a pioneer in the use of cinematography to analyze the development of two flow instabilities: the row of alternating vortices in the wake of a bluff body and the convection cells in a thin layer of fluid heated from below [10].

Many improvements of the visualization techniques have been done since Marey and a whole book would be needed to tell this story. However, it is worthwhile to highlight some examples, like the work of Henri Werlé at ONERA ("Office national d'études et de recherches aéronautiques"). Under his guidance, ONERA built in the 1950s several large vertical water tunnels fed by gravity. The design of these tunnels was such that the flow was virtually turbulence free despite its large scale, tens of centimeters across. This was key to the exceptional quality of the images produced by Werlé's team. These tunnels helped in particular to reveal the structure of separated flows [11] on delta wings that were designed for fighter jets and the supersonic transport plane Concorde. As advocated by Werlé, this direct visualization is a very cheap means of investigation compared to a large wind tunnel and it reveals readily the general mechanism of complex flows [12,13] (Fig. 5).

Very early on after the invention of cinematography, Lucien Bull, a collaborator of Marey, takes on the problem of recording very fast phenomena. He succeeds in 1904 in visualizing at several thousands frames/s the bursting of a soap bubble struck by a bullet. The performances of high-speed cameras will increase through the twentieth century to reach millions of frames/s, but the switch to recording on solid-sate devices instead of silver halide films is a revolution comparable to that of chronophotography [14]. High-speed cameras are now very compact and portable and their price tag makes them easily available for any fluid mechanics laboratory. Nevertheless, very high performance cameras are also built, such as the

Fig. 5. Flows visualized par H. Werlé in the vertical water tunnel of ONERA [12]. On the left, vortical structures on a delta wing evidenced by dye injected on the leading edge. On the right, turbulent wake of a sphere. A sheet of orange dye reveals the boundary layer that develops, separates from the body at the equator, and becomes unstable downstream.

Brandaris 128 [15], which was in 2003 the fastest camera recording more than 100 images at 25 Mfps. This camera using 128 synchronized CD arrays was essentially devoted to the study of the interaction of ultrasonic waves with bubbles [16].

Until roughly the last decade, one had to choose between a global analysis, essentially qualitative, of the flow by visualization and a local analysis resolved in time through velocimetry (laser Doppler or hot wire). The development of digital particle image velocimetry (DPIV) changed that completely. High-resolution digital cameras and computers performing fast Fourier transforms are now allowing the quantitative determination of whole flow fields at least in two dimensions, and in some cases in three dimensions. Among other topics, these techniques transformed the field of animal propulsion [17]. Flows generated by insects and birds wings, by fish and cetacean fins are unsteady and three-dimensional; DPIV is able to produce a complete mapping of the velocity field in these flows. From the velocity field, it is possible to compute forces generated by wings and fins by using the conservation of momentum; nevertheless, a physical understanding of the dynamics requires a reduction of thousands of data points available into a simple set of flow features. For the problem of propulsion by oscillating flexible plates, the identification of well-defined vortices is in general used to analyze the flow. In some sense, we have to go back to eye and pencil to build a simplified representation of the flow to be able to relate physical parameters such as the aspect ratio of the fins, the frequency of oscillation, and the propulsion efficiency.

3. From Hele-Shaw to Prandtl and Ahlborn, the concept of boundary layer

The beginning of the twentieth century will see the resolution of d'Alembert's paradox with the development of the boundary layer theory by Prandtl [18]. But before Prandtl's achievements, there are clearly several observations announcing the division of the flow into two regions, the boundary layer attached to a body or separated and the outer flow. One of these observations is described by Hele-Shaw in several papers in terms that are quite different from the modern description.

Henry Selby Hele-Shaw [19] is known to most fluid dynamicists for the device that he invented at the very end of the nineteenth century and which is now called a "Hele-Shaw cell". The device is made of two parallel glass plates separated by a thin gap in which a liquid flows. The thickness of the gap is so small that the Reynolds number associated with the flow is much smaller than 1 and the dynamics of the flow is dominated by viscous effects. Paradoxically, the streamlines in such a flow are identical to the streamlines calculated for a perfect fluid without viscosity, the velocity averaged over the thickness of the cell deriving from a potential that obeys Laplace's equation. Hele-Shaw used his device to solve analogically magnetic problems [20], but he developed it initially for teaching purposes: he wanted to project the streamlines on a large screen while he was lecturing the engineering students at the University College in Liverpool. In the initial experiments, the flow was visualized by air bubbles introduced upstream in the water flow and the gap was rather thick, so that the Reynolds number was actually much larger than 1 and the flow was dominated by inertial effects. In these conditions, the wake of a bluff body such as a cylinder is unstable and clearly separated from the outer flow [21] (Fig. 6).

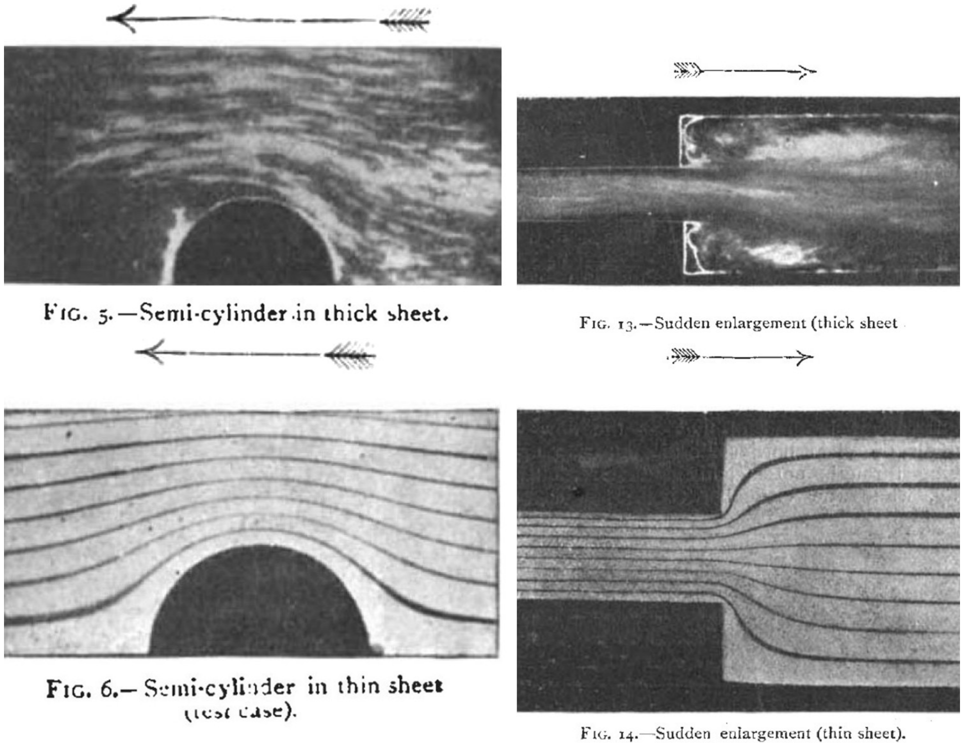

Fig. 6. Images of flow around a cylinder and in sudden expansion taken by Hele-Shaw in his visualization device, from [21].

Hele-Shaw presented some of his images at the International Congress of Naval Architects and Marine Engineers at the Imperial Institute in July 1897 [22] and said in particular: "At the time, the idea was merely to show the effect of streamlines and vortex motion upon ship-shape and other forms and in pipes, but a point which was not understood at first has been specially enlarged upon, as it appears to have an important bearing on the nature of surface resistance."

Hele-Shaw paid a special attention to the layers of liquid flowing next to the solid body introduced in the cell; he noticed that the air bubbles revealed a very regular (laminar) motion of the liquid, contrasting the sinuous motion in the wake: "The results of all these experiments has led the author to the conclusion that the clear border line represents a condition of parallel flow of layers of water past the skin of the obstacle, or the sides of a pipe, in which a state of shearing exists, while outside this, in the darker portion, the water is in a state of sinuous motion, which corresponds to the state of the higher velocity of water."

So, Hele-Shaw clearly interpreted his experiments with the notion that the fluid flow should be divided into two regions, one in which the flow is laminar and the other where the flow is unstable. He reported additional experiments at the meeting of the Liverpool Engineering Society the following year [23] and identified the laminar layer as the one where viscosity controls dynamics: "The difference of the two appearances undoubtedly represented on the one hand the condition of water in which the mere viscosity came into play, resisting the shear stress of the layers in passing over each other, and on the other hand to the breaking up of the water into eddying motion when the resistance was much greater."

However, Hele-Shaw did not go beyond this observation to propose a theoretical framework to account for the boundary layer and to provide an estimate of the drag force. Similarly, at the beginning of the twentieth century, Heinrich Ahlborn in Germany performed very remarkable visualization experiments that he presented also at meetings of the German Society of Naval Architects. Ahlborn's experiments have been analyzed in detail by Eckert [18] and also by Hinterwaldner [24]. Despite the quality of his observations, which had a better resolution than those of Prandtl, Ahlborn was not able to provide a consistent theory for the boundary layer. He even misinterpreted some of his pictures, representing the boundary layer as a series of discrete vortices (see fig. 4 and 5 in [24]).

4. Structures in turbulent flows

The intricate and fluctuating structure of turbulent flows, obvious to all observers, poses a formidable challenge to describe and predict analytically. Non-linear effects lead to the production of a hierarchy of vortical structures spanning a wide range of lengthscales. The early theories of turbulence used statistical approaches to describe the dynamics of the flow, but this approach proved to be ineffective in identifying the mechanisms of momentum transfer in boundary layers, and it also did not take into account the existence of coherent structures in the flow (see [25] for an historical overview of

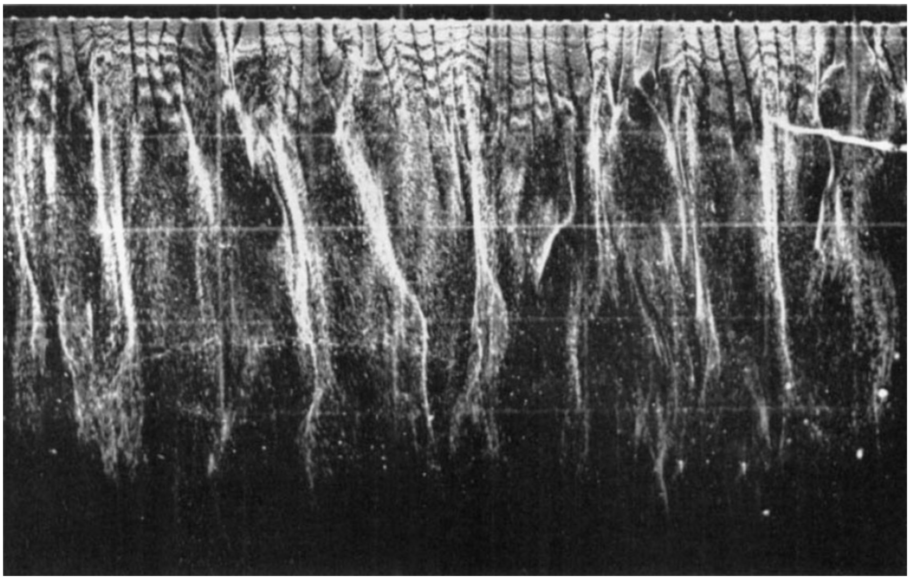

Fig. 7. Turbulent boundary layer visualized by hydrogen bubbles produced by electrolysis in water. The flow is from top to bottom and the camera looks down onto the wall. A thin Pt wire (at the top of the image) is stretched across the flow at a small distance from the bottom wall so that it lies in the laminar sublayer. Some parts of the wire are protected by a varnish to prevent the formation of bubbles and the current in the wire is pulsed so that square patches of bubbles are injected into the flow. The clear streaks in the direction of flow reveal the existence of streamwise vortices. Image reproduced from Kline et al. [26].

the concepts in turbulence). In both situations, flow visualizations proved to be decisive in advancing the understanding of turbulent flows. There were numerous papers related to this structural analysis, but I am citing essentially two of them that appear to me as representative of theses advances.

4.1. Turbulent boundary layers

At the beginning of the 1960s, there were already numerous data showing that the production of turbulent fluctuations in a boundary layer took place very close to that wall. However, there was no evidence at all for the type of flow structures that were responsible for this production. By the end of the decade, this phenomenon was clearly understood, in particular due to effort of a team in the mechanical engineering department at Stanford University. The experiments relied on sophisticated visualization techniques to analyze the near-wall region of the boundary layer. Kline et al. at Stanford [26,27] refined the technique of hydrogen bubbles generation to show that structures elongated in the direction of flow were essentially responsible for the exchange of momentum between the low-speed regions very near the wall to the outer regions. In the abstract of their paper they claim: “Extensive visual and quantitative studies of turbulent boundary layers are described. Visual studies reveal the presence of surprisingly well-organized spatially and temporally dependent motions within the so-called laminar sublayer.” They put the emphasis on the regular spacing of the streamwise streaks that they observe and which was indeed not anticipated.

In the paper, they perform a quantitative analysis of the correlation of the spanwise location of the streaks and they remark: “The peak of the ensemble average spectrum is considerably less pronounced than in the instantaneous spectra. Time-averaged correlations obtained by hot-wire measurements would appear more like the ensemble average, and hence the streakiness would not be revealed as clearly as by the (instantaneous) visual studies.” This shows that their visual studies were indeed more effective in revealing a spatial organization in the flow than correlations between velocity measurements made at different locations (Fig. 7).

4.2. Coherent structures in shear layers

One of the key advances in the 1970s is the discovery of organized structures in turbulent flows, structures that persist even in the presence of strong fluctuations. In particular, in a seminal paper [28], Brown and Roshko at Caltech reveal through their visualization technique that large eddies formed in a shear layer persist up to a very large Reynolds number. “Some of the first results obtained from this facility were instantaneous shadowgraphs of the flow which we, at first, found astonishing. [...] The coherent structure visible in the shadowgraphs was for us a most unexpected finding [...] Although the idea of a large structure in turbulent shear flow is not new, we had not expected to find it so well ‘organized’ and more or less two-dimensional.” (Fig. 8) The picture taken by Brown and Roshko is striking, but it shows only a two-dimensional view of the flow field and ignores the streamwise vortices that develop on top of the two-dimensional structure and play eventually a crucial role in the mixing of momentum between the upper and lower parts of the flow [29].

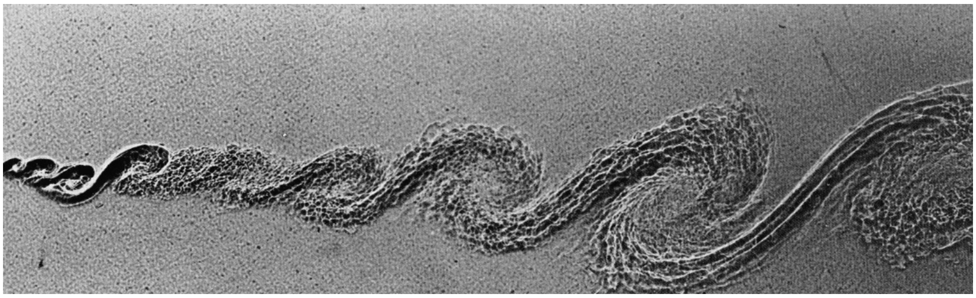

Fig. 8. Two parallel gaseous flows, one of nitrogen, the other of helium, moving at different speeds create a shear layer exhibiting large vortical structures decorated by much smaller eddies. This visualization taken from Brown & Roshko [28] is done with the shadowgraph technique, which uses the refraction of light rays caused by the difference in optical indices of helium and nitrogen. This kind of picture was instrumental in the emergence of the concept of coherent structures in turbulent flows.

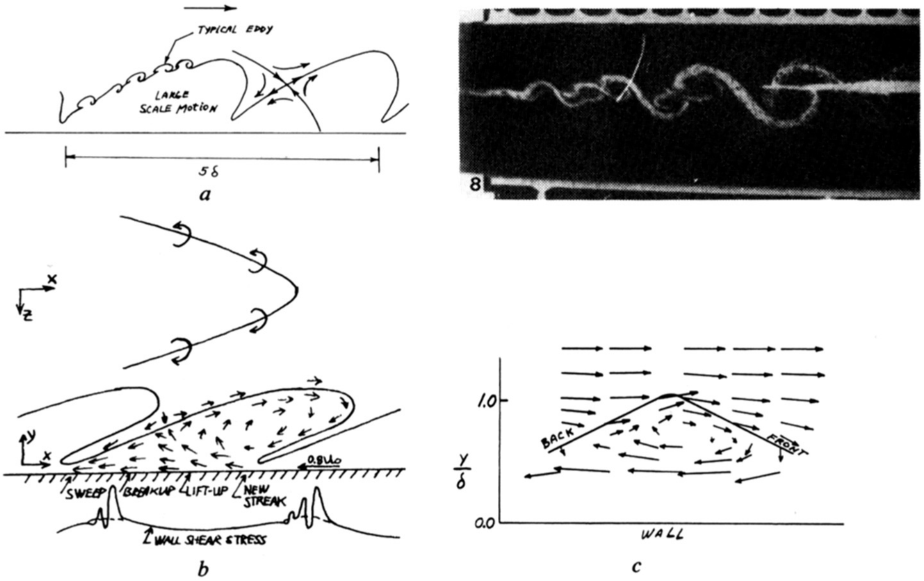

Fig. 9. Three conceptual pictures (a, b, c) of the outer structure of a turbulent boundary layer. The image on the upper right is a snapshot in an animation of a shear layer. Image reproduced from Cantwell [30].

This importance of flow visualization for the physical understanding of turbulent flows is put forward in a review article by Cantwell in 1981 [30]: “An extremely important element in current experimental research is a renewed emphasis on flow visualization and a widespread awareness that flow visualization can play a very broad role in improving our physical understanding of complicated turbulent phenomena.” This article is illustrated by many photographs, including an animation of shear layers similar to the one shown in Fig. 8. But there are also several drawings such as the one in Fig. 9, which shows a conceptual picture of the turbulent boundary layer.

Starting from visualizations and other physical measurements, the goal is to produce a conceptual description of the phenomena involved, reduced to their essential features. Fig. 9 displays three different idealized representations of a turbulent boundary layer. These three sketches coming from different authors try to represent the global structure of ejection events, bringing low momentum fluid from the wall towards the core of the flow. The spatial features of turbulent flows are so complex that one needs to reduce them to a minimal number of elements that are sufficient to interpret the physical results. Otherwise, it is impossible to really understand the mechanisms at play.

In the days of Kline and colleagues, this visual reduction was done essentially by looking at the pictures and movies. Now, as I mentioned earlier, DPIV methods provide a quantitative description of whole flow fields, and numerical algorithms can be used to identify particular structures such as vortex tubes, providing objective criteria to decompose the flow into different zones.

5. Galleries of fluid motion: science, art or advertisement?

In 1982, Milton Van Dyke, professor at Stanford University, inspired by the book *Atlas de phénomènes optiques* [31] published *An Album of Fluid Motion* [32], in which he collected hundreds of photographs organized in 11 topics of fluid

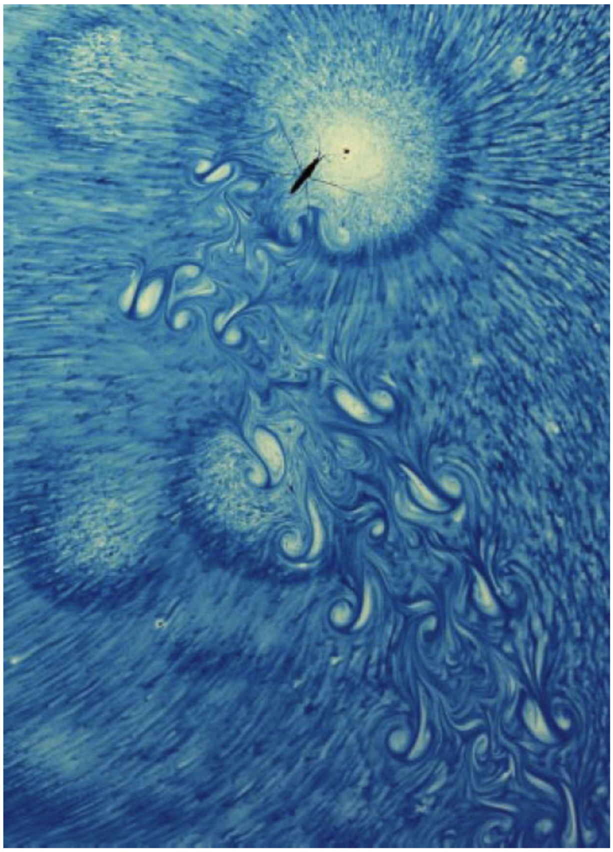

Fig. 10. Water strider by David Hu, Brian Chan, and John Bush. Colored particles sprinkled on the water surface reveal vortices generated by the motion of the insect's legs. The strider who is attracted by light moves towards the clear zone on the top of the image. The analysis of the vortices helps to distinguish different modes of propulsion. In particular for young, small sized insects, the locomotion mode was not understood. Image reproduced from Hu, Chan & Bush [33].

mechanics. The book was essentially designed for students as a supplement to textbooks, and Van Dyke took the task of publishing it himself, with a paperback version that was easily affordable. This idea of collecting and displaying flow images (which was already present in Benard's paper of 1929[1]) was extended at the 1985 meeting of the Division of Fluid Dynamics of the American Physical Society by creating a competition of the best photographs at this annual meeting, The Gallery of Fluid Motion. The current website[2] states that "The Gallery of Fluid Motion is intended to be a visual record of the aesthetic and science of contemporary fluid mechanics, to be shared both with fellow researchers and the general public. Each year, the submissions are judged for their combination of striking visual qualities and scientific interest...". The artwork of the winners is published in a specialized scientific journal, then *Physics of Fluids*, now *Physical Review Fluids*.

As said before, the images (and videos as well in the recent years) are judged essentially on their aesthetic nature, but also on the underlying technical and scientific advances. The Gallery is not just about producing pictures while revisiting known phenomena, but also about innovating on various perspectives. A very good example of an award-winning work is the photograph made in John Bush's group at MIT (Fig. 10). In order to analyze the propulsion of water striders (*Gerridae*), Hu and coworkers [33] sprinkled the surface of water with particles of a blue dye to reveal the vortices shed by the insect's legs. Was it a coincidence or was it deliberate, but it turned out that the blue and yellow striations in the image are reminiscent of Van Gogh's starry nights. The results of the work were considered significantly new to be published in Nature, but in addition to be a Gallery of Fluid Motion winner, this photograph was chosen to be displayed on the cover of the magazine. The nice colors on the image are not necessary to understand the physical phenomena involved, a black and white print would be informative enough on the dynamics of the vortices accounting for the propulsive force. Clearly, this picture has a complex status: it is not strictly speaking a science picture, since the quantitative information is somewhat pushed in the background by its visual appeal. It can be considered as an artistic object, although this definition is left to anyone's personal judgment regarding what is art and what is not art. And certainly it serves as an efficient means to

[1] "Aussi je conçus le dessein louable de faire un grand album de photographies, chronophotographies, films de cinéma pour illustrer tous ces travaux théoriques..."

[2] https://gfm.aps.org/.

advertise (in the positive meaning of the word) the related scientific results. One should note that photographs and drawings are used more and more frequently in the table of contents of scientific publications to give to the reader a very quick hint of the topic of an article.

Yet another evidence of the aesthetic character of flow images is provided by Max Ernst, who found a direct inspiration for some of his paintings in photographs published in the journal *La Nature* [34]. In particular, he used one photograph taken by Hele-Shaw and reproduced in a paper written by Lucien Bull [35] and another by Marey [36] appearing in the same issue of *La Nature*. The two pictures are merged in a composition named *Blind swimmer (effect of a touch)* painted in 1934. Max Ernst did not alter the shape of the streamlines, except that the image from Marey is used upside-down. There are several tentative explanations for the hidden meaning of the *Blind swimmer*, but at least we can claim that figures of flow appeal to us beyond their clinical description of velocity fields.

6. Conclusion

We are fortunate enough to work in a scientific field in which fundamental understanding often progresses through the production of images. The evolution of recording and processing techniques from early drawings to high-speed digital imaging has led to an enormous increase in the information contained in flow images and movies. However, the understanding of flow phenomena still requires a reduction of this information, guided by physical insight, into generic features. It turns out that most of these flow images appeal to our aesthetic senses regardless of our technical knowledge of the physical phenomena involved. This conjunction explains the significant role of images in fluid mechanics, which I think, is unparalleled in other physical sciences.

Acknowledgements

This work was initiated by a workshop organized by Étienne Guyon and Anouk Barberousse, and supported by “La fondation des Treilles” and benefitted from fruitful discussions with Eduardo Wesfreid.

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