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From the water wheel to turbines and hydroelectricity. Technological evolution and revolutions

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ABSTRACT

Since its appearance in the first century BC, the water wheel has developed with increasing pre-industrial activities, and has been at the origin of the industrial revolution for metallurgy, textile mills, and paper mills. Since the nineteenth century, the water wheel has become highly efficient. The reaction turbine appeared by 1825, and continued to undergo technological development. The impulsion turbine appeared for high chutes, by 1880. Other turbines for low-head chutes were further designed. Turbine development was associated, after 1890, with the use of hydropower to generate electricity, both for industrial activities, and for the benefits of cities. A model "one city + one plant" was followed in the twentieth century by more complex and efficient schemes when electrical interconnection developed, together with pumped plants for energy storage.

1. Introduction

The appearance of water power was, for mankind, a major technological revolution. This history starts as early as classical Antiquity. Water technologies for gravity irrigation had been used in the Middle and Far East since the fourth millennium BC, and aqueducts have been developed in the countries around the Aegean Sea, prior to their use in the Roman world. A new climate favorable to water technology innovation appeared from the third century BC, during the Hellenistic period, around knowledge centers as Alexandria (Egypt) and Pergamon (in today's Turkey), with new water lifting devices as the tympanum, the saqqya, the Archimedes screw, the Ctesibios pump [1].

The vertical water wheel was born somewhere in the Middle East during the first century BC, as suggested by the earliest textual mention by Strabo [2] of a water mill in Mithridate's palace in Cabeira (north of present-day Turkey). Water wheels further developed to a large extent in the empires of Rome and of China during the first to third centuries AC. In the Roman world, they were mostly used to grid grains, and sometimes also, in the fourth century, for sawing marble. In the Chinese empire, it is known from textual sources [3] that water wheels were used for many industrial purposes (moving bellows for metallurgy, sawing ...). Roman water wheels used to be vertical wheels (the so-called Vitruvius wheel), with diameters between 1.5 and 3.5 m [4,5]. Mills with multiple water wheels existed, for instance in Barbegal, close to Arles, in the south of France [6]. In the Middle Ages, the water wheel continued to develop, in both western and eastern civilizations, together with the demographic development which lasted until the thirteenth century (and ended with the wars and great plague of

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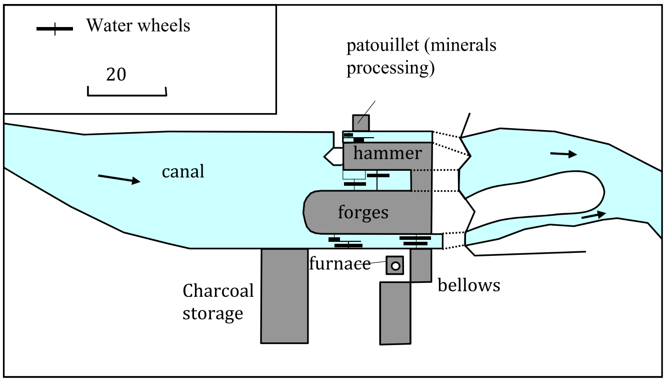

Fig. 1. The Buffon forge in 1828, installed on a canal derived from the Armançon River (France), with eight water wheels.

the fourteenth century). Today's landscape of watermills in the western countryside is mostly inherited from this expansion period.

2. The economic and industrial importance of water power, from the Middle Ages to the nineteenth century

At the end of the Middle Ages and at the eve of the industrial revolution, the water wheel was the support to many economical pre-industrial activities [7–9]: grinding grain, sawing, fulling for textile manufacturing and paper mills, treading cane for sugar processing, moving bellows and water hammers for metallurgy (forges). Water wheels were also used in the mining industry for lifting water from mine pits, fulling minerals, etc. As for Western Europe, the dominant technology was the vertical wheel north of a line between La Rochelle and Lyon and the horizontal water wheel south of this line. Mills on boats anchored on rivers or suspended from bridges, and tidal mills also developed in the same period.

Monks (especially Benedictines and Cistercians) were among the most active communities in the development of watermills, because they had the capacity to develop technological skills, together with available manpower. It is known for example than 84 waterwheels were owned by the abbey of Saint-Germain-des-Prés in France, one of the major abbeys owning lands between the Loire and the Rhine Rivers, by the ninth century [10]. In 1086, after the Norman conquest of England, the Domesday Book identified no less than 5624 water mills in this country.

During the period between the seventeenth and the nineteenth century, there was a progressive development or larger and larger industrial sites using water power as the source of energy [8,11]. Water power was at the origin of the early industrial revolution, the use of coal as a source of energy developed only as a second phase.

Forges using water powered hammers and bellows were installed on multiple canals derived from rivers, with many water wheels powering the different workshops of the chain between raw materials to terminated steel products. It is known from a survey ordered by a minister of King Louis XV that about 140 such integrated forges were in operation in France in 1772 [12]. Fig. 1 shows the example of one of those integrated forges, which was built by 1770 by Buffon, the naturalist and scientist.

The mechanical machine for cotton clothing manufacturing, the *spinning jenny* was invented in England in 1767. It was water powered after 1769, and after that time water powered mills quickly developed in England, in France and slightly later in the USA. While in England the use of coal replaced hydropower after 1830 for cotton manufactures, water power remained the main source of energy for cotton spinning mills in France and the USA. In this country, in New England and Massachusetts, very large mills were built upon canals, which were powered by a large number of water wheels, further to be replaced by hydraulic turbines. In 1823, a group of persons called the *Boston Associates* decided to concentrate upon the site of Lowell, where an existing navigation canal, called the Pawtucket canal and by-passing the falls of the Merrimack River already existed. From 1823 to 1847, ten textile mills were built on this site, as shown in Fig. 2, using 191 water wheels (11).

Many other textile mills were built in the USA. Labor was hard and dangerous for the girls who were working in these textile mills [13].

Water power was also very important for the mining industry. In Germany, a large number of dams and canals were built between the sixteenth and the eighteenth century, in order to supply power to mines in the area of the Metal and Harz mountains. In the Harz mountains, situated between Hanover and Leipzig, more than 100 earth dams were built in the above-mentioned period, with more than 400 km in total length of canals. Those technologies were further transferred to eastern and northern Europe. There were used also in the new world (for instance in the silver mines of Potosi in Bolivia). Very interesting illustrations of the use of the waterwheels in the mining industry of that time may be found in the book written in 1565 by a German scholar (Agricola), and translated into English in 1950 by former US President Hoover and his wife [14].

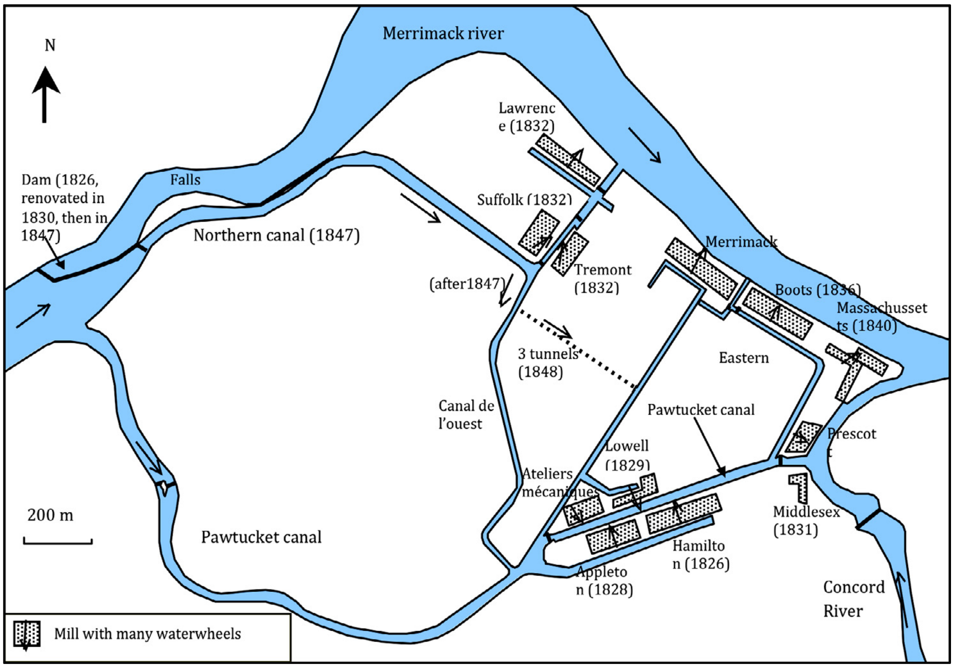

Fig. 2. The industrial site of Lowell (New England, USA), by 1850. Ten cotton mills were using in total 191 water wheels. Those wheels were transformed after 1850 into Fourneyron, Boyden or Francis turbines.

3. Improvements to the vertical water wheel: 1824–1890

The large economic importance of the water wheel in the 19th century was a strong motivation for innovation, with the objective to improve the efficiency of the traditional vertical water wheel, which was only about 30 to 40% [15]. Scientists analyzed the fluid mechanics of the water wheel, for instance Antoine Parent (1704), Bernard Forrest de Bélidor (1737), John Smeaton (1752). Using scale models experiments, Smeaton concluded that the tip velocity of the paddles should be 2/5 of the water incoming flow velocity. In France, the work was continued by Jean-Charles Borda (1767), Charles Bossut (1771), and Claude Louis Henri Marie Navier [16]. In his commented 1819 edition of *L'architecture hydraulique* by Bélidor,[2] Navier supported Borda's assessment that the paddles' velocity should be half of the flow velocity [17]. In 1824, the French "Académie des sciences" launched a prize in order to encourage innovation on water wheels. This prize was awarded in 1825 to Jean-François Poncelet, for the design of the water wheel shown on Fig. 3a. In 1855, another French engineer, Alphonse Sagebien, developed another efficient water wheel, shown in Fig. 3b, with about 85% efficiency in the optimal conditions ([11], from a publication dated 1866), and fitted to low-head chutes (less than 4 m). In Switzerland, Walter Zuppinger proposed in 1883 a new design combining the previous innovations (Fig. 3c), ensuring more than 80% efficiency over a large range of inflow conditions (15).

As a result of these innovations, the vertical water wheels, now manufactured in iron, had reached by the end of the nineteenth century a state of nearly optimal design with very good efficiency (80 to 90%) for low to moderate head chutes, and in the power range of 10 to 50 kW per unit. But the remaining weakness of the vertical water wheel was that there was too much influence of the downstream water level, and that it was impossible to use high chutes. Thus, despite of the perfection of the new water wheels, there was still room for innovation.

4. The turbines century: 1826–1926

4.1. Premises: 1744–1754

The first concept of a reaction turbine was developed separately in 1744 by a Dr Barker in England, and in 1750 by Johan Andreas Segner from Göttingen in Germany, a concept very close to the rotary sprinkler used today to water gardens. Leonhard Euler studied the Segner invention, and published in 1754 a paper called *Théorie plus complète des machines qui sont mises en mouvement par la réaction de l'eau*, with a new design developed from Segner's one, with in addition a fixed distributor above the wheel allowing to optimize the angle of the flow reaching the rotating blades of the rotor [18].

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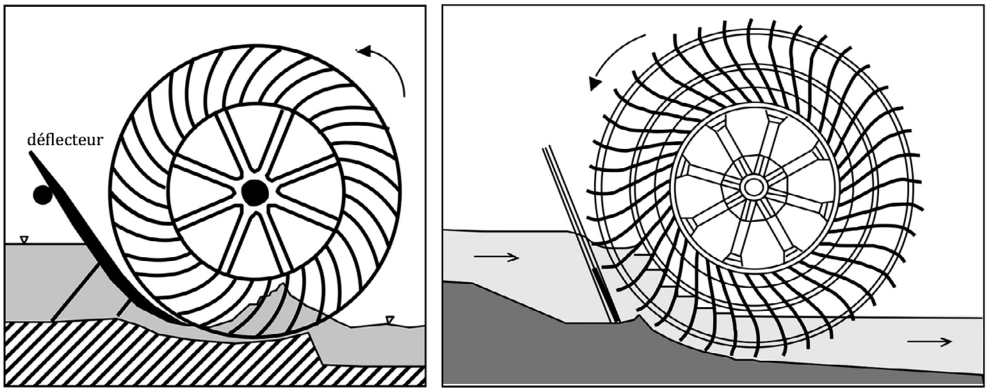
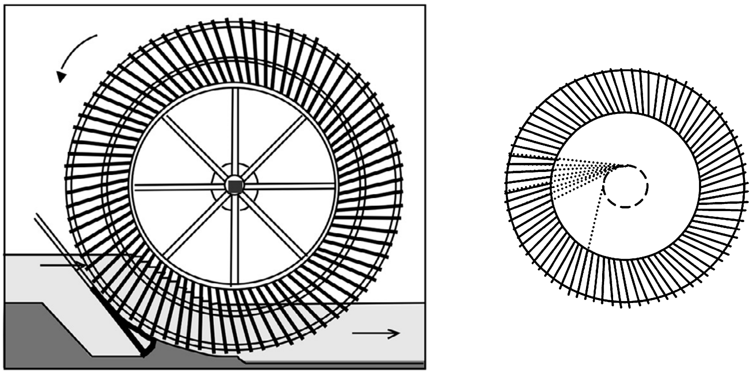
[2] See notes db, dl, dm of Navier's edition of Bélidor.



3a. Left : Poncelet (1825).

3c. Right : Zuppinger (1899)



3b. Sagebien (1858)

Fig. 3. Improvements of water wheel efficient designs in the 19th century.

4.2. Success (Burdin and Fourneyron): 1820–1840

During the years 1820–1824, Claude Burdin, Professor at the "École des mines de Saint-Étienne" was trying to design a practically oriented wheel from the principle of the Euler wheel. Recognizing the importance of the distributor proposed by Euler, he tried to integrate the distributor *inside* the wheel, allowing the resulting machine to work submerged inside water. He proposed the name *turbine* for the concept [19]. In 1826, The French "Société d'encouragement à l'industrie nationale" proposed a prize for a turbine having proved to be fully operational on a real industrial site. A young French engineer, having studied in Saint-Étienne with Burdin, called Benoit Fourneyron, made experiments with a small turbine model (4.5 kW) derived from the Burdin one, on the forge of Pont-sur-l'Ognon (Franche-Comté) where he used to work as an engineer. The major characteristic of the Fourneyron turbine is that water was flowing from the inside to the outside of the turbine (Fig. 4).

In 1832, his first industrial turbine (37 kW) was installed for the bellows of a metallurgical furnace in Dampierre (Jura). Fourneyron won the prize of the "Société d'encouragement", and from that moment an increasing number of Fourneyron turbines were installed on industrial sites. A major step was fulfilled in 1837–1838 when two industrial sites in the Black Forrest were equipped with Fourneyron turbines (45 kW each, about 80% efficiency) on high chutes (108 m and 114 m) (from Morin (20)). In 1843, there were 129 Fourneyron turbines working on European industrial sites. In 1895, three Fourneyron turbines with 3700 kW each were used for the first large hydropower plant in the USA on the Niagara Falls.

4.3. The successors of Fourneyron (1840–1847)

Following the success of the Fourneyron turbine, a number of trials of alternative design were attempted, with new patents: Fontaine-Baron (1840), Jonval (1841), Koechlin (1843). In the Fontaine-Baron's and Koechlin's design, the distributor was situated above the rotor, as it used to be with the Euler initial idea. By 1850, this process used to be applied to a number of chutes, with head between 3 to 5 m. James Thomson in England (Lord Kelvin's brother) designed another "vortex" turbine by 1850, which was applied to a number of industrial sites (19).

In 1843, the Fourneyron turbine was introduced in the USA, with some improvement patented by Uriah Boyden. This Boyden turbine was used in the Appleton mills in Lowell and to other textile mills in the USA, i.e. the Harmony mills in Cohoes, as shown in Fig. 5 [20].

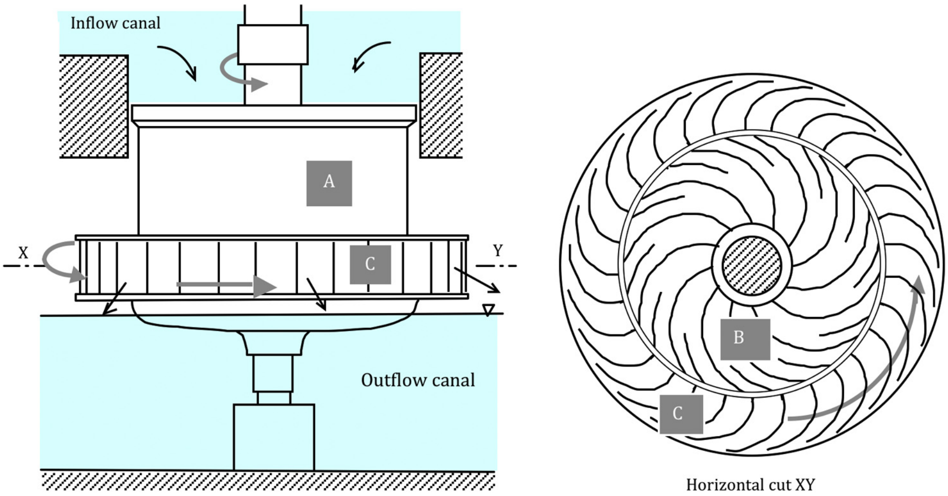


Fig. 4. The Fourneyron turbine, by 1835, the first industrial reaction turbine. In the Fourneyron turbine concept, the distributor (B) is inside the rotor (C).

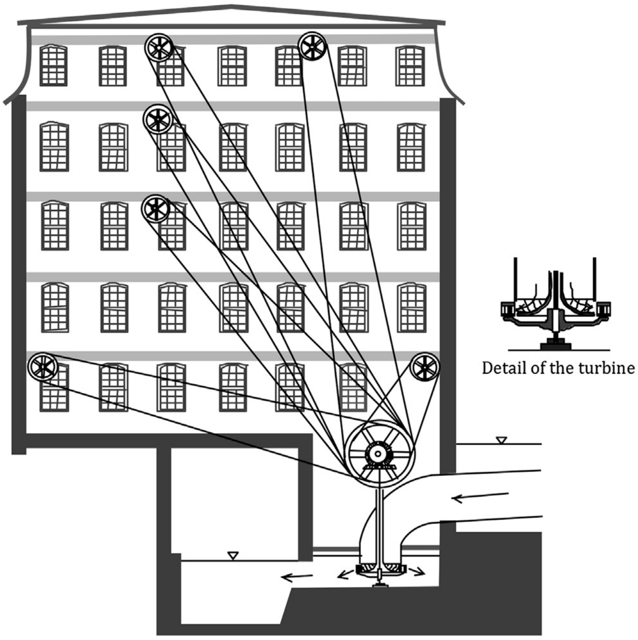


Fig. 5. The Harmony mills in Cohoes (USA), by 1871, with two Boyden turbines (600 kW each).

4.4. *The Francis turbine (1855)*

The chief-engineer in Lowell, between 1837 and 1885, was named James B. Francis, an immigrant who had arrived in the USA in 1833, at the age of 18. Francis made a large number of experiments and tests starting from the Fourneyron turbine, and trying to improve the design of the reaction turbine. He published in 1855 the result of his *Lowell Hydraulic Experiments*. His new turbine design used a principle adverse to the Fourneyron turbine's one, with the rotor inside and the distributor outside, and a flow strictly radial in the turbine (Fig. 6). Later, the concept of the Francis turbine moved in such a way that the flow would no longer be strictly radial, but progressively changing its direction downward (Fig. 7), which is still the principle of today's Francis turbines (11).

Initially designed for moderate head chutes, the application range of the Francis turbine slowly extended towards higher chutes: more than 400 m by 1950 on hydropower sites in Norway. Today, 700-MW Francis turbines are working on the largest hydropower plants such as Grand Coolee (USA), Itaipu, Three Gorges, and 1000-MW turbines are planned for a project in China.

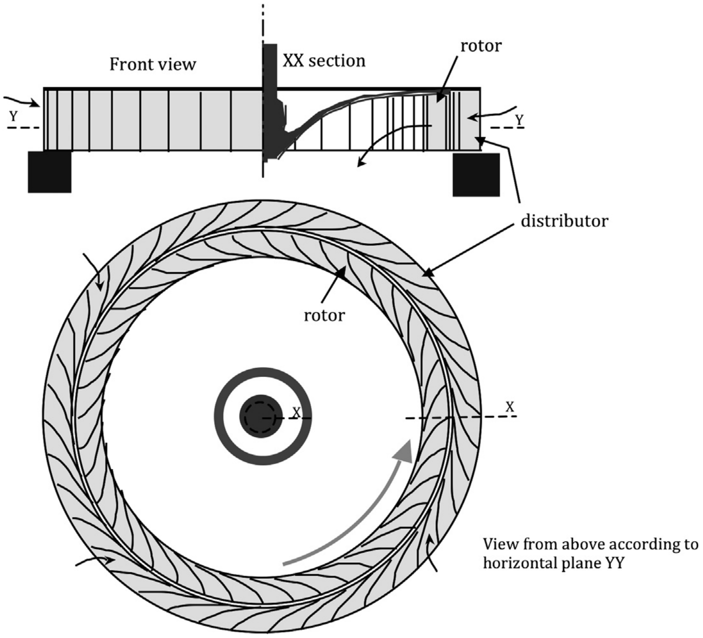


Fig. 6. The initial design of the Francis turbine (1855), with a strictly radial flow in the rotor.

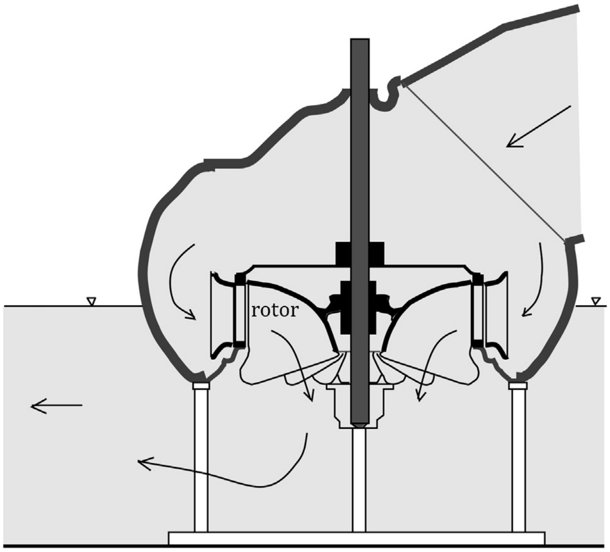


Fig. 7. Simplified view of a 180-kW Francis turbine manufactured in 1882 by the Humphrey Machine Company and installed at the Tremont & Suffolk mills in Lowell, replacing an older 120 kW Fourneyron turbine. The efficiency of this turbine was about 82%, according to measurements published by Francis in 1884.

4.5. Impulsion turbines for very high chutes: Girard, Bergès, and Pelton

By 1850–1880, the Fourneyron and Francis turbines did not yet allow one to deal with chutes higher than 120 m. There was still a demand for turbines which would allow one to use smaller mountain flows, with higher chutes, especially in the Alps. The impulsion turbine principle appeared to be the key technology for high chutes, with a water jet directly impacting the blades of the rotor, as it was the case in pre-industrial horizontal water wheels. The major problem with high chutes impulsion turbines was the rapid erosion of the wheel caused by cavitation phenomena. The technological developments were aimed at solving this problem, together with improving turbine efficiency. In France, Louis-Dominique Girard designed in 1854 an impulsion turbine with a horizontal axis and a controlled water jet impacting the rotor blades. Its efficiency was probably about 60%. This turbine became the reference turbine in the French Alps during the key period 1860–1880 of development of the paper industry in the Alps. Between 1869 and 1882, Aristide Bergès developed the exploitation of

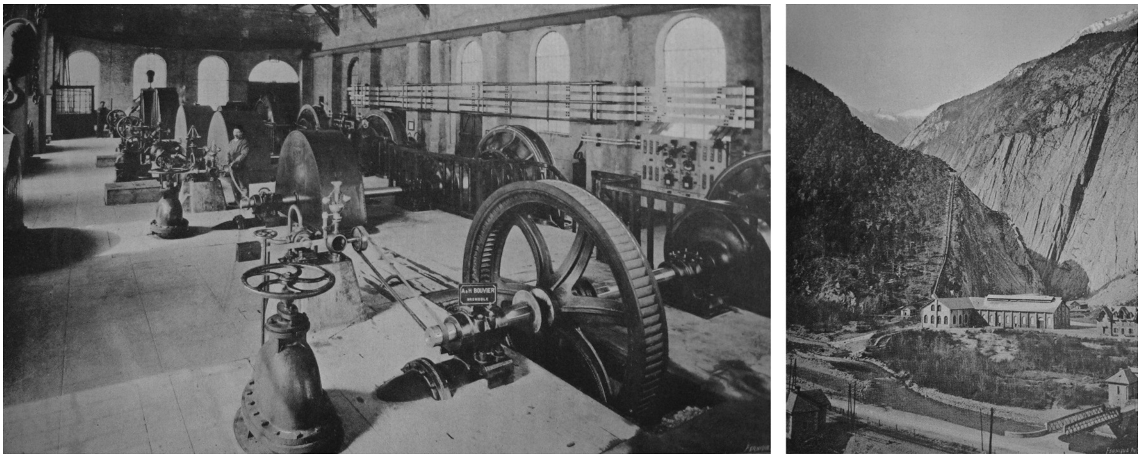


Fig. 8. The Calypso plant by 1902, in the French Alps. Photos from the 1902 "Congrès de la houille blanche" – archives of the SHF.

high chutes up to 480 m for his paper mills in Lancey, close to Grenoble, using a turbine derived from the Girard one, with still many difficulties tied to cavitation erosion [21]. Bergès sometimes disregarded the rights of other water users, and his development was subject to a number of disputes and trials.

By this time, the use of hydropower for electricity generation was beginning to develop, with the need to illuminate the cities, move tramways, and develop new electrolytic industrial processes for chemical industry (electrochemistry), and electrometallurgy (11). In 1884, a 30-kW Girard turbine was used for pubic lighting in Greenock in Scotland. The Calypso electrochemistry plant (Fig. 8) was built in 1891 in Valloirette in France, with a 135-m-high chute, probably using Girard turbines. The Chedde plant, built in 1896 used a 140-m high chute, with 12 Girard turbines (630 kW each). Still cavitation was a limit for using higher chutes.

In the USA, during the gold mines development period, water jets used to be used to move water wheels. In 1878, Lester Allan Pelton extended the idea and developed the impulsion turbine bearing his name. In 1891, Bergès successfully used Pelton turbines for his new 470-m-high chute in Lancey. Since improvements patented by William Doble in 1899, the Pelton turbine has become until nowadays the reference turbine for high chutes. In 1902, the above-mentioned Calypso plant used Pelton turbines with a new 600 m high chute.

The highest chute today is the Dixence–Bieudron chute, in Switzerland, with an 1883-m head: the Bieudron plant at the foot of the chute is equipped with three Pelton turbines (400 MW each). A break on the penstock at 1234 m altitude happened in 2000, and the site was re-opened only in 2014.

4.6. *Efficient turbines for very low-head chutes: the Kaplan and bulb turbines*

Francis turbines are not adequate for delivering large power from very low chutes, and a patent was deposited in 1912 by Victor Kaplan, from the *Wien Technische Hoschschule*. The rotor of this turbine was shaped as a helix, with a horizontal axis. The efficiency was excellent (90%), but due to the high velocity angular rotation of the turbine under high flow rate conditions, cavitation problems were encountered. The solution came in 1926 when a Swedish engineer called Olov Englesson defined a command allowing one to adjust the angle of the blades. After this date, the Kaplan turbine became the reference for low head run of the river plants, as in the Kembs plant on the Rhine, built in 1933 (Fig. 9).

An improvement of the Kaplan turbine for very low-head chutes is the so-called bulb turbine, with the alternator integrated inside the turbine, allowing almost zero flow deviation when passing through the turbine. Bulb turbines were used on the Rance tidal plant in 1960–1966 (Fig. 10), and, following the success, bulb turbines were used after 1967 for new plants on the Rhine and Rhône Rivers.

5. Turbines and the hydropower development

Electricity generation from water power began from 1880, with a 7-kW Thomson turbine on a 8.8-m-high chute, in Cragside (Northumberland). From that date, plants developed quickly, using different types of turbines. And since 1889, all hydropower projects have been devoted to electricity generation. The two major drivers for this development used to be:

- industry, with a special mention to electrochemistry and electrometallurgy, with examples of the Calypso and Chedde plants in the Alps given earlier, but also plants built on large rivers, with key examples of Rheinfelden on the Rhine river in 1899 (11.2 MW), or Shawinigan on the Saint-Maurice (Canada), developed from 1901 to 1948 (providing also electricity to Montréal);
- providing cities with electricity, for public light and tramways, as explained below.

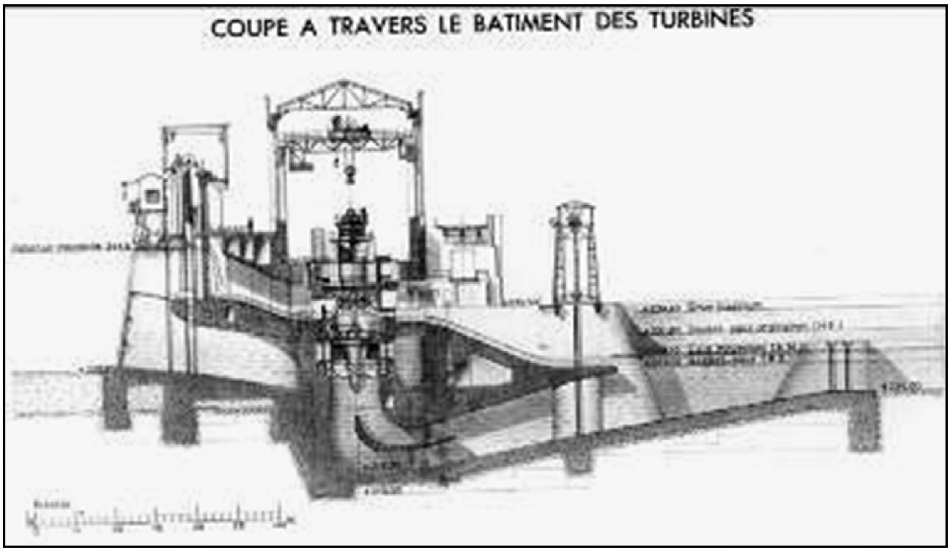


Fig. 9. View across the old Kembs plant, with sketch of one of the six Kaplan turbines, from a technical brochure dated 1932.

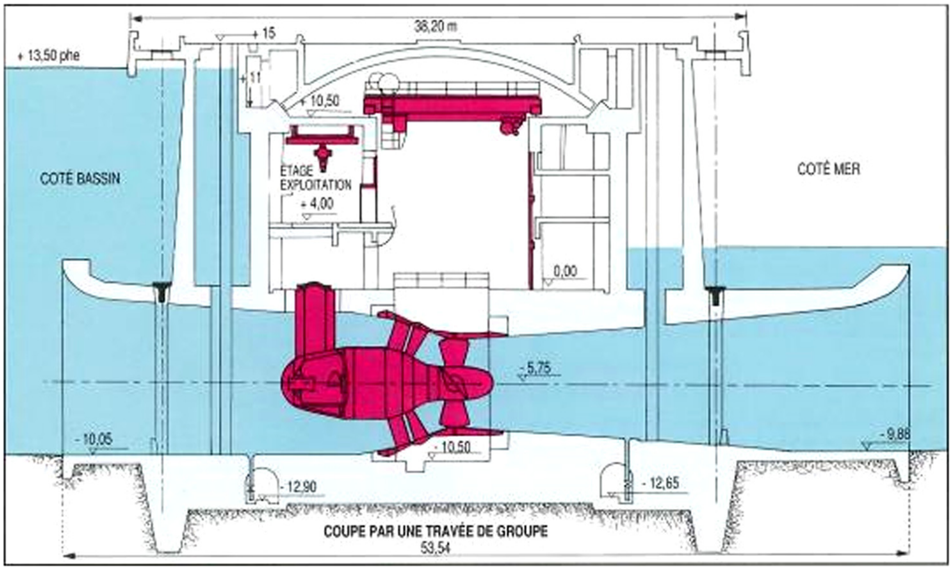


Fig. 10. Cross section of the La Rance tidal plant (1960), showing one of the 24 bulb turbines. Total peak power is 240 MW (courtesy EDF).

5.1. A hydropower plant for every city (1895–1920)

As long-distance electricity transport was not developed at that time, a number of couples city + hydropower plant appeared, for every city having access to water power resource (11). Here are some key pioneer developments according to this model.

Buffalo and Niagara falls. In 1895, the first industrial plant on the US side of the Niagara falls was installed, in order to supply the Buffalo city with electricity. The plant used the full 60-m height of the falls, with three, and later ten Fourneyron turbines installed at the base of vertical pits (Fig. 11). The total flow rate was 250 m^3/s, and water was evacuated downstream trough a 2-km-long underground canal. The plant delivered 11 MW (and later 37 MW) electrical power to the city of Buffalo. Another plant was built in 1901 inside an excavation at the very bottom of the falls, with 11 turbines delivering 46 MW; this plant was destroyed in 1956 after a spectacular explosion and casualties.

Geneva and Chèvres. In 1896, the run-of-river plant of Chèvres (Fig. 12) on the Rhône River (Fig. 10), 7 km downstream from Geneva, with 8.8-m head, began operating. It was delivering 4-MW electrical power to the city of Geneva, which was owning the plant. The power was increased in 1899 to 13 MW, with 15 Francis turbines (two turbines in the same axis).

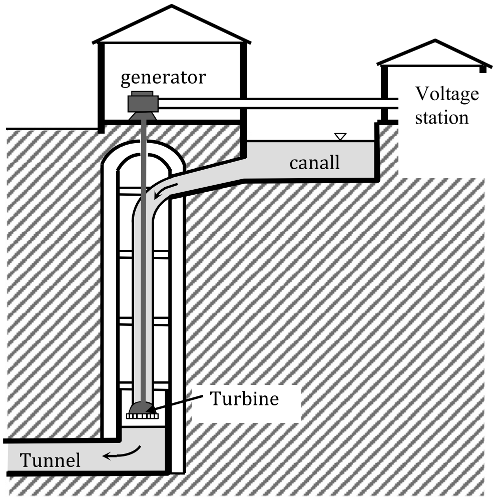


Fig. 11. The first industrial plant upstream of the Niagara falls, for Buffalo, with Fourneyron turbines at the basis of 60-m-high pits, operating since 1895.

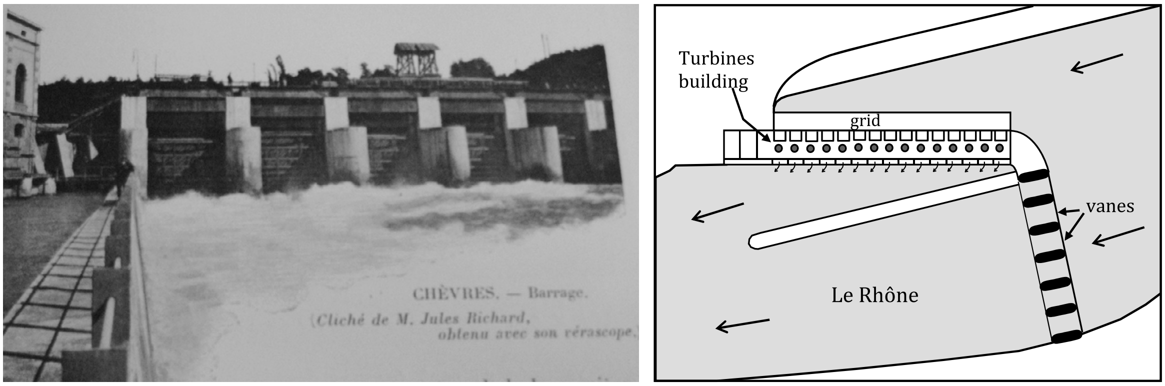


Fig. 12. The Chèvres plant commissioned in 1896 for Geneva (from the 1902 “Congrès de la houille blanche” – archives of the SHF), with the scheme of the 15 turbines.

Milan and Paderno. In 1898, the Paderno plant was commissioned in order to provide the city of Milan with electricity. The plant used a 45 m^3/s water flow derived from the Adda River, with four, and later seven Francis turbines, delivering 1.5 MW each, which was at that time the record. This plant is still existing today.

Lyon and Cusset. The Cusset plant was commissioned in 1899 by the “Société des forces motrices du Rhône”, delivering 16 MW of electrical power to Lyon. The plant was on the Jonage canal, derived from the Rhône, with a 12-m-high chute, and used 16 Francis turbines. This historical plant is still in operation today.

After 1920–1930, the development of electric interconnections allowed one to share hydropower at larger scale than the local networks associated with the above-mentioned examples [22,23]. For example, the La Truyère plant in the French Massif Central was commissioned in 1932–1934 in order to supply Paris (but also Lyon and Clermont-Ferrand) with electricity.

5.2. Towards complex water management systems

As electric power generation is to follow power consumption, the concept of water storage early began to appear.

Under the model one city–one plant described before, the “Énergie électrique du Rhin” company commissioned in 1933 the Kembs plant on a canal derived from the Rhine. The purpose was to provide the city of Mulhouse with electricity. The plant, shown in Fig. 9, was on a 15-m-high chute, and used six Kaplan turbines (4.6 MW each). In order to store energy during low-demand hours and generate additional energy at peak hours, a storage plant was installed using two lakes, the Lac Blanc and the Lac Noir, with a 130-m altitude difference between the two lakes. This storage plant was supposed to be commissioned in 1934, but an explosion of a penstock caused casualties, and the plant was finally commissioned in 1938.

The concept of pumped storage plants further developed to a large extent from that period, and is now the dominant electric energy storage technology, with two development periods:

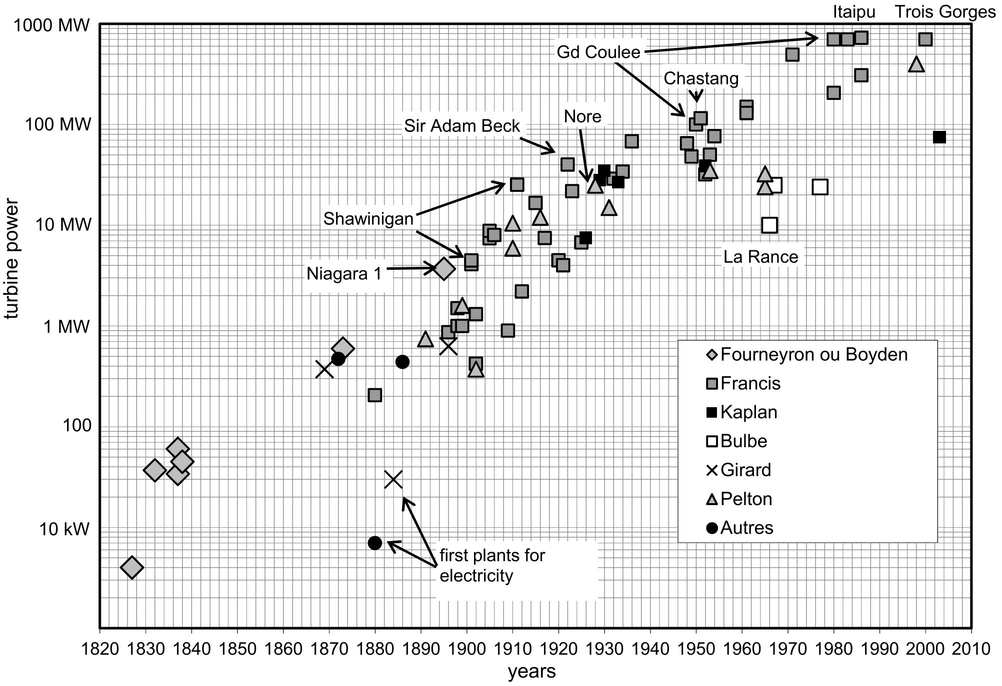


Fig. 13. The increase of turbines unit power between 1825 and 2000.

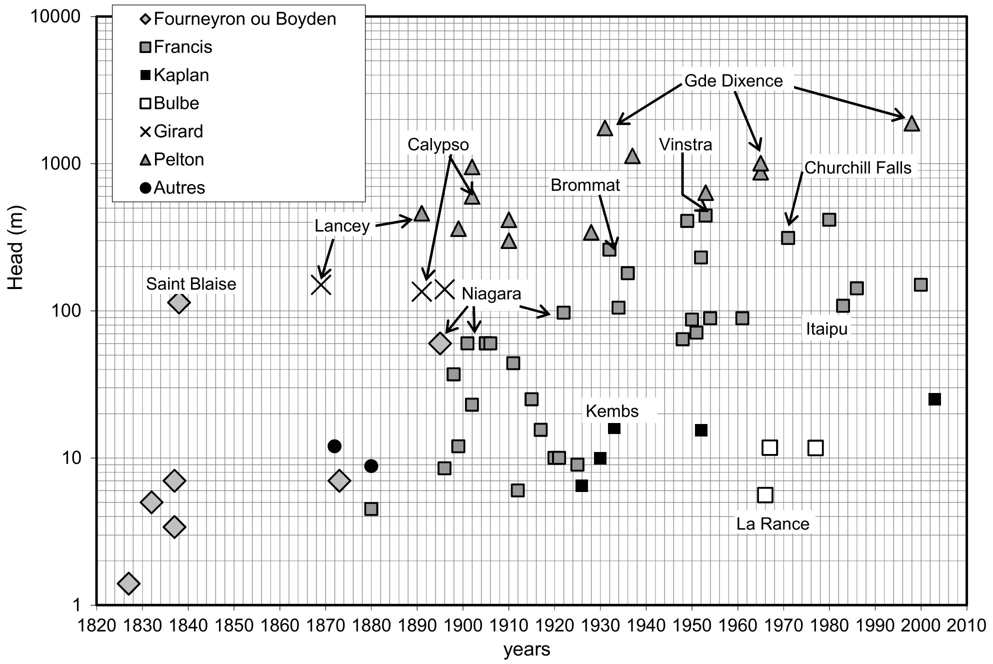


Fig. 14. The increase of turbines head from 1825 to 2000.

- 1970–1990, associated with the development of large plants,
- since 2000, associated wind the development of wind and solar energies.

6. Conclusion

The water wheel has been the companion of human activities since its early development in the first century AC. It allowed the development of pre-industrial activities in the Middle Ages, and was at the source of the industrial revolution –

coal was later used, as a substitution to hydropower in some countries (UK). In the 19th century, water wheels had reached a state of quasi-optimal design.

Turbines that were developed from 1850 onwards allowed the plants to be less dependent on downstream boundary conditions, to use high chutes, and to increase power, with a variety in the design allowing one to cover the full range between low-head chutes (a few meters high) to very high chutes (up to 1800 m), and unit power from a few kW up to 700 MW (and probably soon 1000 MW). Figs. 13 and 14 show the historical progress of a century of turbines development, in terms of chute head and unit power, showing the different turbine designs.

Contributing to the development of renewable sources of energy, hydropower is strongly developing in countries having the potential for such development: Asia (especially in China), South America. It could be developed to a considerable extent in Africa.

Hydraulic machineries today face the challenge of flexibility. The electricity markets short-term issues together with the development of versatile wind and solar energies are demanding the hydropower plants to provide more and more flexibility services. The highly unsteady regimes associated with these services are hard for the machines, and are bad for the aquatic environment.

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