C. R. Mecanique 345 (2017) 620–626

Contents lists available at ScienceDirect

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A century of fluid mechanics: 1870–1970 / Un siècle de mécanique des fluides : 1870–1970

Turbulence from 1870 to 1920: The birth of a noun and of a concept

La turbulence de 1870 à 1920 : la naissance d'un nom et d'un concept

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ARTICLE INFO

Article history:
Received 15 November 2016
Accepted 21 March 2017
Available online 29 June 2017

Keywords:
Fluid mechanics
Turbulence
History of fluid mechanics

Mots-clés :
Mécanique des fluides
Turbulence
Histoire de la mécanique des fluides

ABSTRACT

We consider here the works of French, British, and German researchers in fluid mechanics from 1870 to the beginning of the twentieth century. Our aim is to understand how the term "turbulence" introduced by William Thomson in 1887, which was not used by the main researchers of the time, including Joseph Boussinesq, Osborne Reynolds, Lord Rayleigh, Horace Lamb in the first editions of his book, became classical in the 1920s. We trace the first introductions of the terms "turbulence", "turbulent flow" in the works of relatively unknown researchers between 1889 and 1903, until it reaches the vocabulary of mainstream researchers in fluid mechanics and physics. Our result is that the shift was in 1906–1908, when the term was used in the 1906 edition of the book of Horace Lamb, and in Lanchester's book, followed by a series of papers of German researchers before the First World War.

The use of the word "turbulence", a word used for a long time for crowds or for children, in a scientific context, corresponds to the introduction of a new concept, a new understanding of a scientific phenomenon clearly identified as being different from laminar motion. The study of the use of this term is also the study of the diffusion of a new concept among researchers of the time.

RÉSUMÉ

Nous considérons les travaux de chercheurs français, britanniques, allemands en mécanique des fluides, de 1870 au début du vingtième siècle. Notre objectif est de comprendre comment le terme « turbulence » introduit par William Thomson en 1887, qui n'a pas été utilisé par les principaux chercheurs de l'époque, incluant Jospeh Boussinesq, Osborne Reynods, lord Rayleigh, Horace Lamb dans les premières éditions de son livre, est devenu classique dans les années 1920. Nous recherchons les premières occurrences des termes « turbulence » et « écoulement turbulent » dans les travaux de chercheurs mineurs du domaine entre 1889 et 1903, avant qu'ils n'apparaissent dans le vocabulaire des spécialistes du domaine en mécanique des fluides et en physique. Notre conclusions est que la transition se situe dans les années 1906–1908, lorsque le terme a été utilisé dans l'édition de 1906 du livre de Lamb, et dans celui de Lanchester, suivis par une série d'articles par des chercheurs allemands avant la Première Guerre mondiale.

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http://dx.doi.org/10.1016/j.crme.2017.06.003

L'utilisation du mot « turbulence », un mot employé depuis longtemps pour des foules ou des enfants dans un contexte scientifique, correspond à l'introduction d'un nouveau concept, à une nouvelle compréhension d'un phénomène scientifique, clairement différencié des écoulements laminaires. L'étude de l'utilisation du terme est aussi l'étude de la diffusion d'un nouveau concept parmi les chercheurs de l'époque.

1. Introduction

The field of turbulence is an important subpart of fluid mechanics and of condensed matter physics; a Web of Science search reveals that more than 46,000 papers were published between 1995 and 2016 with "turbulence" or "turbulent" in the title, with 2884 for the year 2015 alone. Turbulence is found for high Reynolds number flows, and is characterized by erratic motion, intermittency, 3-D vortices and filaments, irreversibility, chaos, unpredictability [1–5]. Turbulence is ubiquitous in nature and is found, e.g., in the ocean [6], the atmosphere [7], geosciences [8], and also in climate models [9].

In the field of engineering, many models are used, based on the averaging of the Navier–Stokes equations, since the seminal work of Osborne Reynolds [10] at the end of the nineteenth century. Since these equations are non-linear, averaging introduces small-scale fluxes that need to be related to averaged quantities through a so-called closure. The classical closure is the one at the basis of eddy-viscosity models, and is due to Joseph Boussinesq [11]. The fact that the Boussinesq's eddy-viscosity closure was introduced in 1872 (the 1877 paper is a written report of a presentation done in front of the French Academy of Science in 1872), 25 years before the introduction of Reynolds averages, has been discussed in a paper published in this journal [12], in a thematic issue devoted to Joseph Boussinesq [13].

In the same paper, we were also surprised to see that neither Boussinesq nor Reynolds have used the word "turbulence" in their publications. In his publications in fluid mechanics, Boussinesq used "tumultuous movements", "eddy agitations", "liquid eddy theory" (in French, "mouvements tumultueux", "agitation tourbillonnaire", "théorie des tourbillons liquides") [14,11,15]. Osborne Reynolds also used many different words: "sinuous paths", "sinuous motion", "irregular eddies", "sinuous or relative disturbance" [16,10].

This shows that these two important researchers of the end of the nineteenth century did not use the word *turbulence*, while it is extremely classical nowadays and used in a very large number of academic works. Between these two periods, there is a transition to be studied. In the present paper, we are interested in the diffusion of the term "turbulence" in the scientific literature: its first introduction apparently in 1887 by William Thomson (later Lord Kelvin), its adoption and generalization in the early twentieth century. It is seen as the birth of a new concept, since its adoption as a noun is also the recognition of a new subfield of fluid mechanics.

2. From 1887 to 1903: introduction of the term in fluid mechanics by William Thomson, but no by mainstream followers

2.1. Early times, regular flows versus nonlinear or sinuous motion

As recalled by Olivier Darrigol [17], as early as 1822 Navier made some distinction between "linear" and "nonlinear" flows, and a decade later Saint-Venant made distinctions between "regular" and "tumultuous" flows. There was later much interest in the transition between the two regimes, a search of a mechanism explaining this transition, and of a criterion to characterize the "sinuous" or "irregular" or "unsteady" regime. Such characterization was made possible with the introduction by Osborne Reynolds of what was later called the "Reynolds number" in 1883 [16]. In 1884, shortly after his long experimental paper describing the transition to turbulence in a channel flow, Osborne Reynolds delivered a speech on "the two manners of motion of water" at an evening meeting of the Royal Institution of Great Britain, published in 1887 [18]. In this speech, he proposed an interesting analogy between hydrodynamics and the movement of troops. A small army has easily order and discipline in the movements of its troops; a large army has greater chance of disorder. For him, "direct or steady motion" is similar to a disciplined army, and "sinuous or unsteady motion" is like a troop whose motion is a "scramble". With such analogy between fluids and human behavior, Reynolds was close to using the word "turbulence".

2.2. Introduction of the term "turbulence" by William Thomson in 1887

The word "turbulence" had been used for a long time, either in the French (turbulence), English (turbulence) or German (Turbulenz) languages, to describe agitation and disorder. Coming from the Latin *turbulentia*, meaning perturbation, trouble, it had been used since several hundred years for animals, crowds, or children. The adjective "turbulent" came from the Latin *turbulentus* and was used for agitated, troubled, noisy phenomena. Hence the troops used as analogy by Reynolds could have been characterized as turbulent when they are scrambled. However, Reynolds did not use the term.

The adjective "turbulent", used as an image for an agitated natural phenomenon, was used in several scientific papers, for waves [19], or rivers [20]. The word "turbulence" was used as an image for a molten lava flow of the Stromboli by Charles

[272]

XXXIV. *Stability of Motion (continued from the May, June, and August Numbers).—Broad River flowing down an Inclined Plane Bed.* By Sir WILLIAM THOMSON, *F.R.S.**

41. CONSIDER now the second of the two cases referred to in § 27—that is to say, the case of water on an inclined plane bottom, under a fixed parallel plane cover (ice, for example), both planes infinite in all directions and gravity everywhere uniform. We shall include, as a sub-case, the icy cover moving with the water in contact with it, which is particularly interesting, because, as it annuls tangential force at the upper surface, it is, for the steady motion, the same case as that of a broad open river flowing uniformly over a perfectly smooth inclined plane bed. It is not the same, except when the motion is steadily laminar, the difference being that the surface is kept rigorously plane, but not free from tangential force, by a rigid cover, while the open surface is kept almost but not quite rigorously plane by gravity, and rigorously free from tangential force. But, provided the bottom is smooth, the smallness of the dimples and little round hollows which we see on the surface, produced by turbulence (when the motion is turbulent), seems to prove that the motion must be very nearly the same as it would be if the upper surface were kept rigorously plane, and free from tangential force

Fig. 1. A scanned copy of a part of the first page of the first paper of 1887 by William Thomson [23], where the word turbulence (underlined here), as a noun, is first introduced.

Sainte-Claire Deville in 1856 [21]. However, the term was used in these works as an image, an equivalent of "agitated". The noun "the turbulence" for agitated fluid was the next step. The first introduction in the scientific literature in fluid mechanics is apparently due to William Thomson, later Lord Kelvin, in two papers published in 1887.

Reynolds explained in his 1884 speech: "I am speaking of the two manners of motion of water [...]; it is my object to make clear tonight [...] that all the various phenomena of moving water may be divided into two broadly distinct classes." He has proposed some criteria to characterize the two classes, and very soon after, Lord Kelvin introduced a new noun. The first Thomson 1887 paper is, to our knowledge, usually not cited since it is absent, for unknown reasons, in the *Mathematical and Physical papers of Lord Kelvin*, a collection of his published works, which have been published after his death in 1907, and which are useful (but incomplete and sometimes not precise enough) sources for Kelvin's publications [22]. In the first 1887 paper [23], William Thomson writes about water in an inclined plane bottom: "But, provided the bottom is smooth, the smallness of the dimples and little round hollows which we see on the surface, produced by turbulence (when the motion is turbulent), seems to prove that the motion" (see Fig. 1). This shows that an adjective is needed, to explain the word turbulence in this context: when the motion is turbulent, hence it is perturbed, troubled. Later in the same paper (p. 277), he writes: "In the case of no gravity ($g \sin I = 0$), and the viscous fluid kept in "shearing" or "laminar" motion by relative motion of the two parallel planes, there is, when viscosity is annulled, no disturbing instability in the steady uniform shearing motion, with its uniform molecular rotation throughout, which viscosity would produce; and therefore our reason for suspecting any limitation of the excursions within which there is instability, and for expecting possible permanence of any kind of turbulent or tumultuous motion between two perfectly smooth planes (or between two polished planes with any practical velocities) does not exist in this case." This shows that here the author is proposing to separate the two states of the flow, on the one hand shearing or laminar flows (the term *laminar* is given with quotation marks here but it was already used earlier by his brother James Thomson [24]), and on the other hand, turbulent or tumultuous flows. This is the first time such nouns, laminar and "turbulent" flows, are so clearly used to separate the two states. These two terms are now classical in the scientific literature.

In the next issue of the same journal, Thomson published another paper [25], which is often cited as the first introduction of the word "turbulence" [17,26] since is using "turbulently" in its title, but was published several months after the first one. The first sentence of this paper is the following: "In endeavouring to investigate turbulent motion of water between two fixed planes". In the full paper, the words "turbulent motion" are used 11 times. However, the use of this adjective is here not explained, since it was first used in the previous paper. This paper was presented by William Thomson at the fifty-seventh meeting of the British Association for the Advancement of Science in Manchester in August–September 1887, and was published in the Report of this meeting in 1888 [27]. This presentation in front of many colleagues may be the beginning of the diffusion of the term in Great Britain.

William Thomson investigated many different topics, and fluid mechanics was not his main field of studies. He did not use the word turbulence in his later publications. However, it can be found in his published correspondence with George Gabriel Stokes [28], on two instances. William Thomson and George Gabriel Stokes were close friends for many decades, and they have exchanged hundreds of letters, many of those are published in two volumes. In a letter from Kelvin to Stokes, dated 27 December 1898, we read: "If now the whole fluid suddenly becomes inviscid and the globe be kept moving uniformly, the rotationally moving fluid will be washed off from it, and left moving turbulently in the wake, and mixing up irrotationally moving fluid among it." Here Thomson does not explain the word as if it is well known. It is not used in next letters of Stokes, and only two year later, we read in a letter from Stokes to Kelvin, dated 19 and 20 December 1900: "Reynolds pointed out that the dimensional relations which contain the conditions of dynamical as well as geometrical similarity are applicable to the setting in of turbulance [sic], and to the mean effects when the motion is eddying." This seems to be the only instance of the use of the word "turbulence" by G. G. Stokes, with a mistake in the spelling showing that he was not used of using it. The original letter is not reproduced, but the editor of the volume seems very precise and in many instances spelling mistakes are reproduced without correction.

3. Success of the diffusion of the new term and concept in the early twentieth century

In those years, researchers mainly published their works in their own languages and their own countries. The diffusion of ideas and results could be slow, due to the distance and also to the language barrier. Hence, in the following we review our search in different languages, since most of the research in this field, at that time was done in English, German, and French. The methodology is a search on Web of Science, on Google Books, on Gallica.fr (the website displaying the digitized archives of the French National Library) and among different historical sources, including the sources provided in several recent historical books on fluid mechanics [17,26,29].

3.1. In the English-language literature: a slow diffusion and two important publications in 1906 and 1907

After the introduction of the term *turbulence* by William Thomson in 1887, and his presentation in Manchester in front of the British scientific community, the term began to diffuse. There are two papers of George Francis FitzGerald (1851–1901), published in 1889, using the term in their title [30,31]. Some years later, the President of the Mathematical and Physical Science section of the British Society for the Advancement of Science said in his opening speech (September 12, 1895): "Lors Kelvin at the Manchester meeting (...) deduced that the velocity of propagation was $\sqrt{2}/3$ times the velocity of the mean square of the turbulent motion" [32]. Another researcher, Montaigu Browne, used the term in the field of marine geosciences: "... to denote currents of considerable turbulence such as now obtain in seas or estuaries of no great depth" [33]. Henry Selby Hele-Shaw (1854–1941), famous for his experiment visualizing streamlines in shallow flows between two plates, mainly used the term "sinuous motion" in his publications, but in one instance in 1899 he writes "The water in the thick sheet was moving with sinuous or turbulent motion" [34]. We notice also two other papers by FitzGerald in 1899 and 1900 [35,36]. Three other papers on applied topics, flows in porous media, gas flows, or marine flows may also be reported, published from 1900 to 1904 [37–39], using the term turbulence or turbulent flow, turbulent motion, without having fluid mechanics as main field of research. This shows that the term was progressively adopted in the secondary literature, on applied fields, even if prominent fluid mechanics researchers did not use the term. But it was not enough, apparently, to provide a real diffusion of the new terms in the field of fluid mechanics.

Indeed, it seems that it was not used by the main researchers in fluid mechanics at the time: Osborne Reynolds never adopted the term in his works published after 1887, George Gabriel Stokes used it only in a private letter to Lord Kelvin, it cannot be found in the published works of Lord Rayleigh (1842–1919) on the transition to turbulence in convective systems. Apparently, the first apparition of the term in the main publications was due to Horace Lamb and to Frederick Lanchester in 1906 and 1907.

The famous book by Horace Lamb (1849–1934) on *Hydrodynamics* was published in six different editions, in 1879, 1895, 1906, 1916, 1924 and 1932. The two first editions of 1879 and 1895 do not use the term turbulence. Later editions, beginning with the one of 1906 [40], contain several sections on turbulent motion, coefficients of turbulence, turbulence in the atmosphere. This book was a reference already at that time, and is still now sold by its publisher. Lanchester's book of 1907 on aerodynamics had much influence in the field of applied fluid mechanics and engineering [41]. It contains a whole section devoted to turbulence.

After the slow diffusion of the term "turbulence" or "turbulent motion" during 20 years after 1887, in the British literature and among researchers whose concerns were not primarily fluid mechanics, the adoption of the terms in these two books marked the clear adoption of the new concept in the field of fluid mechanics, in English language literature.

3.2. In the French literature

As already indicated, this word was not adopted by Boussinesq in his works in the period after 1887 (see the title of Boussinesq, 1897: Theory of tumultuous and whirling liquid flows [15]). Henri Bénard (1874–1939) apparently used the word turbulence only in his works published in the 1930s.

The word *turbulent*, in italics, was used by Raoul Pictet in 1892 to describe a chemical reaction [42]. The word turbulence appears apparently for the first time in the French fluid mechanics literature in the book of Auguste Boulanger (1866–1923) published in 1909 [43]. This book is devoted to a general explanation of the publications of Boussinesq. A chapter is called "Des fluides à l'état turbulent" (fluids in the state of turbulence) and the first use of the word *turbulence* is in italics, showing that the word was not supposed to be commonly used in French works. Henri Villat (1879–1972) published several papers in fluid mechanics from 1910 to 1918, but they contain terms such as "mouvements discontinu" (discontinuous movement), with no mention of turbulence. The first use of the term "turbulence" in a paper in fluid mechanics in the French literature may be due to Auguste Lafay (1866–1944), in two papers published in 1911 [44,45]. In the first paper, the word turbulence is written in italics in its first instance, indicating again the use of a word not well established in the French scientific literature. In the second paper, this is no more the case. These three publications are the only ones we could find in the French journals or books, before World War I.

3.3. In the German-language literature

Concerning sources in German, there is an article published in 1904 [46], devoted to a discussion around the concept of eddy viscosity developed by Boussinesq, and quite uniformly in the paper using the word "Turbulenz". It is possible that the word "Turbulenz" occurred there as a consequence of the natural German translation of "mouvement tumultueux" into "turbulente Bewegung". There are then several papers after 1907–1910 and before the First World War, some of them by well-known researchers such as Arnold Sommerfeld, Ludwig Hopf, Fritz Noether [47–55]. Some of these authors also defended a doctoral dissertation on turbulence, for instance Ludwig Hopf in 1909, a thesis entitled "Hydrodynamische Untersuchungen: Turbulenz bei einem Flusse", or Ruckes in 1908, Lechner in 1913. This collection of papers, by many different authors, shows that turbulence was already identified in Germany as a field of study in itself, in the years 1908–1914.

As in French and English, in German the word "Turbulenz" or the adjective "turbulent" can be used for a crowd, a person, an animal, but the first use in the fluid mechanics literature seems to be the 1904 paper.

3.4. After the First World War

During World War I, few papers were published in the field, the most notable ones being two papers by Geoffrey Ingram Taylor (1886–1975) [56,57]. After the war, many papers were published in English, French and German, using the terms turbulence and turbulent flow, see, e.g., in the papers in German by Ludwig Hopf, Fritz Noether, Werner Heisenberg [58–60], and in the papers in French by Marcel Brillouin, Charles Camichel, and collaborators [61–64]. Even Ludwig Prandtl and G. I. Taylor, whose first works in the 1910s did not use the word turbulence, adopted it after 1917 for G. I. Taylor and 1921 for Ludwig Prandtl [56,57,65,66]. The word turbulence as a concept and a new field was now quasi universally accepted.

4. Conclusion

The adjective turbulent has been used in different scientific contexts linked with fluid mechanics in several papers in the nineteenth century; for the noun "turbulence", it seems to have been introduced for the first time in the context of fluid mechanics in 1887 by William Thomson. The introduction of a specific noun for this subpart of fluid mechanics was made possible by the clear criterion published by Osborne Reynolds in 1883. After Reynolds and after William Thomson (Lord Kelvin), there was a clear separation between laminar flow and turbulent flows. However the diffusion of the new noun was not immediate. We reviewed here the diffusion of this term in the scientific literature, in English, in French and in German, from the beginning of the 1880s to the beginning of the 1920s. The adoption of this term was a way to recognize the specificity of high-Reynolds-number flows, and the need for a different characterization. The adoption of a given vocabulary was here a way to consider the diffusion of a new concept among researchers of the time.

We found the following items.

- Some glorious older-generation researchers never adopted the new term: Reynolds, Boussinesq, Rayleigh. Reynolds, and Boussinesq used many words such as sinuous motion, irregular motion, eddy agitation, sinuous path... The multiplicity of terms showed indeed the need for a noun to unify all these denominations.
- After William Thomson presented his results in 1887 in front of the participants of the British Association for the Advancement of Science, and during a few years, many papers, not directly in fluid mechanics, used terms such as "turbulence" or "turbulent flow". But prominent researchers in fluid mechanics have not employed it until 1906.
- The main shift seems to be the adoption of the term in two books published in 1906 and 1907: the third edition of the book of Horace Lamb, and the book of Lanchester.
- In Germany, after a paper in 1904, the shift was also 1908–1909 with many papers published before World War I, and several doctoral dissertations directly devoted to the study of turbulence.
- In France, a book is found in 1909 and two papers in 1911. The clear adoption of the terms was after World War I.
- After World War I, with the studies of G. I. Taylor, L. Prandtl, H. Heisenberg and of the German school, the very general adoption of the term *turbulence* marked the birth of a new field of fluid mechanics.

Acknowledgements

The author thanks the organizers of the "Century of Fluid Mechanics" meeting held in Toulouse (France) for this extremely interesting meeting mixing fluid mechanics and historical research, and two reviewers for very useful suggestions and references. Sandra Schmidt is thanked for her help with the German language.

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