CrossMark

A century of fluid mechanics: 1870–1970 / Un siècle de mécanique des fluides : 1870–1970

Intermittency as a transition to turbulence in pipes: A long tradition from Reynolds to the 21st century

Les intermittencies comme transition vers la turbulence dans des tuyaux : Une longue tradition, de Reynolds au XXIe siècle

Christophe Letellier

Normandie Université, CORIA, avenue de l'Université, 76800 Saint-Étienne-du-Rouvray, France

A B S T R A C T

Intermittencies are commonly observed in fluid mechanics, and particularly, in pipe flows. Initially observed by Reynolds (1883), it took one century for reaching a rather full understanding of this phenomenon whose irregular dynamics (apparently stochastic) puzzled hydrodynamicists for decades. In this brief (non-exhaustive) review, mostly focused on the experimental characterization of this transition between laminar and turbulent regimes, we present some key contributions for evidencing the two concomittant and antagonist processes that are involved in this complex transition and were suggested by Reynolds. It is also shown that a clear explicative model was provided, based on the nonlinear dynamical systems theory, the experimental observations in fluid mechanics only providing an applied example, due to its obvious generic nature.

© 2017 Académie des sciences. Published by Elsevier Masson SAS. This is an open access article under the CC BY-NC-ND license (http://creativecommons.org/licenses/by-nc-nd/4.0/).

R É S U M É

Les intermittencies sont communément observées en mécanique des fluides et, plus particulièrement, dans les écoulements dans des conduites cylindriques. Initialement obervées par Reynolds en 1883, il a fallu un siècle pour parvenir à une compréhension plutôt complète de ce phénomène dont la dynamique irrégulière (apparemment stochastique) déconcerta les hydrodynamiciens durant plusieurs décades. Par cette brève revue (non exhaustive), essentiellement focalisée sur la caractérisation expérimentale de cette transition entre régimes laminaire et turbulent, nous présentons quelques contributions clés ayant conduit à mettre en évidence les deux processus concomittants et antagonistes impliqués et qui avaient déjà été suggérés par Reynolds. Il est également montré qu'un modèle explicatif clair fut proposé, sur la base de la théorie des systèmes dynamiques non linéaires, les observations expérimentales en mécanique des fluides ayant servi uniquement d'exemple, et ce en raison de son caractère générique évident.

© 2017 Académie des sciences. Published by Elsevier Masson SAS. This is an open access article under the CC BY-NC-ND license (http://creativecommons.org/licenses/by-nc-nd/4.0/).

1. Introduction

In the development of science, it often happens that a concept that has a long tradition in one scientific community is rediscovered by another one, being considered as fully new in its early phase. It also happens that a relevant result in a field has at the end a stronger impact in a different field. Such a situation arose with *Les méthodes nouvelles de la mécanique céleste* written by Henri Poincaré [1], which resulted from a technical problem encountered in celestial mechanics for more than one century (the small divisors problem, see [2] for details) and which lead to the birth of the nonlinear dynamical systems theory. Today, astronomers, as Michelle Chapront-Touzé in her lunar theory [3], refer to Charles Delaunay [4,5], George William Hill [6] or Ernst Brown [7], and are thankful to Kepler, but not to Poincaré. Jacques Laskar read Laplace and Le Verrier but do not too much Poincaré's contribution. For instance, in his researches on the stability of the solar system [8], Laskar refers a lot to Laplace [9–11] and Le Verrier [12] and only quotes Poincaré [1] for the lack of integrability of the three body problem and the existence of "chaotic" solutions. Contrary to this, Poincaré's book was the starting point for "dynamicists" or "chaoticians" as George Birkhoff [13], Vladimir Arnold [14], or Edward Lorenz [15]. Based on a technical problem in celestial mechanics, Poincaré's contribution was relevant for another field, "chaos theory".

In this paper, I will show that the transition to turbulence *via* intermittency follows more or less this example. First, there is a long tradition in investigating the route to turbulence, and intermittencies were already observed in the early experiments made by Reynolds. Roughly one may say that a century was needed for clarifying this transition (see, for instance, Barkley's review [16]). Second, in the 1980s, another approach to this problem came from numerical experiments and provided simple and generic models to describe such transition. If it contributed to popularized the idea of intermittency as a route to turbulence in fluid mechanics, this theoretical explanation had a much more significant impact in the nonlinear dynamical systems theory and opened our minds to the possibility to have intermittent behaviors in well-conducted experiments, that is, without any external perturbations.

2. The historical approach

2.1. Early experimental evidences

Henry Darcy (1803–1858), preoccupied by the engineering of water supply, investigated in details the law providing the head loss h per unit of length in a tube of diameter d. Darcy started from a general law for the head loss h per unit of length made of the first two power of the mean velocity V, that is, from

$$\frac{dh}{4} = AV + BV^2 \tag{1}$$

He clearly understood that the second term, BV^2, was sufficient to explain the flow with large velocities (thus corresponding to the turbulent regime in our present terminology). This "turbulent" component was for him the most important since directly related to the roughness of pipes. Darcy underconsidered the cases where the term AV was negligible, that is, when the velocity was small (less than 0.10 m s^{-1}) or when the roughness of pipes was important as mostly encountered in a working system for water supply, pipes are "very quickly covered by slits, tubers or limescale" [17, p. 91]. Consequently, Darcy mostly focused his attention on the turbulent regime, arguing that "the experiments for which the slope or the velocity was small are in general the less accurate ones and, contrary to this, those for which slope and velocity are large provide more reliable results." Nevertheless, Darcy remarked that there is a threshold velocity beyond which "removing the first term (AV) does not affect the value of the flow" [17, p. 120]. He even added that "the first term was therefore relevant only for velocities ($V < 0.10 \text{ m s}^{-1}$) and for pipes with sufficiently smooth walls" [17, p. 120]: in this case, the second term can be removed. Darcy mentioned that for intermediate velocities, the friction of water against walls becomes proportional to a binomial made of the first and the second power of the velocity" [17, p. 121], thus suggesting that there are two concomitant underlying phenomena. He was convinced that beyond a velocity equal to 0.10 m s^{-1}, "a new phenomena occurs" [17, p. 214]. When he kept the two terms, Darcy recommended to use

$$\begin{cases} A = 0.000\,031\,655 + \dfrac{0.000\,000\,007\,5112}{D} \\[2mm] B = 0.000\,442\,939 + \dfrac{0.000\,012\,402}{D} \end{cases} \tag{2}$$

In 1854, Gotthilf Hagen (1797–1884) investigated the influence of temperature on water flow in pipes [18]. In order to do this, Hagen used different pipes and varied the temperature of water: in doing so, he was in fact varying very slowly the properties of the water flow. He thus remarked that, for certain conditions (pressure and diameter of the pipe, for instance), the water flow, when plotted *versus* the temperature, was presenting a maximum followed by a minimum (Fig. 1). Hagen observed the evolution of the water flow by looking at the jet at the end of the tube. He remarked that between the maximum and the minimum velocity ($17 < T < 30\,°C$) in the case of Fig. 1, the jet was jerking. Initially, he believed that such a feature was due to a badly conducted experiment, but finally convinced himself that this phenomenon was in fact "normal". He also understood that this was a transition from one regime to another one. Since he developed a theory
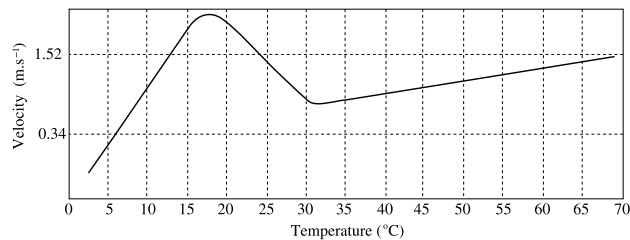
Fig. 1. Water velocity in a pipe ($d = 0.28$ cm, $l = 47.2$ cm) with a load of 11.08 water inch (2934.7 Pa when the old French inch is taken equal to 27 mm) when the temperature is varied. The values for the velocity where assessed with the help of Poiseuille's law. Redrawn from Hagen, 1854.
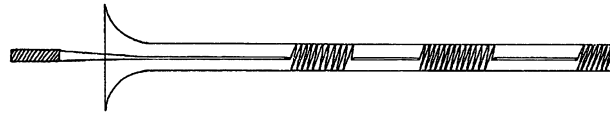
Fig. 2. "The flashes would often commence successively at one point in the pipe. The appearance when the flashes succeeded each other rapidly [is here] shown." From Reynolds, Plate 72, Fig. 16, 1883.

based on the hypothesis that the velocity was proportional to the distance from the wall, he was not able to provide a full agreement between his theoretical rules and observational data.

Osborne Reynolds (1842–1912) is mostly known for having evidenced that there are two very different types of flow, namely the laminar and turbulent flows [19, p. 935]. He understood that

the general character of the motion of fluids in contact with solid surfaces depends on the relation between a physical constant of the fluid and the product of the linear dimensions of the space occupied by the fluid and the velocity.

In order to distinguish laminar from turbulent flows, he thus derived a dimensionless number, the "Reynolds number" as named by Arnold Sommerfeld [20]. It is more rarely known that Reynolds already described "*the intermittent character of the disturbance [...] giving the appearance of flashes*" (Fig. 2). For Reynolds, the concept of the critical Reynolds number was clear, in spite of the experimental difficulties encountered for managing the initial conditions of the flow [19, p. 955]:

The only idea that I had formed before commencing the experiments was that at some critical velocity the motion must become unstable, so that any disturbance from perfectly steady motion would result in eddies.

[...] I had not been able to form any idea as to any particular form of disturbance being necessary. But experience having shown the impossibility of obtaining absolutely steady motion, **I had not doubted but that appearance of eddies would be almost simultaneous with the condition of instability.** *I had not, therefore, considered the disturbances except to try and diminish them as much as possible. I had expected to see the eddies make their appearance as the velocity increased, at first in a slow or feeble manner, indicating that the water was but slightly unstable. And it was a matter of surprise to me to see the sudden force with which the eddies sprang into existence, showing a highly unstable condition to have existed at the time the steady motion broke down.*

Thus, in 1895, he proposed that the flow in round tubes was stable for $\frac{\rho V d}{\mu} < 1900$ and unstable when it is greater than 2000 [21]. But the research for a "critical Reynolds number" characterizing the transition from laminar to turbulent flow was made difficult by the dependences of the transition on initial disturbances.

Maurice Couette (1858–1943) investigated whether the two terms in Darcy's law were due to different phenomena or not. If he discovered Reynolds' results just before the publication of his paper (he thank Boussinesq for having showed him Reynolds' results), there is no mention of Hagen's works. Couette used three different experiments to check that there is a discontinuity between the two (laminar and turbulent) regimes [22]: i) he measured the friction applied to the walls of a cylinder by a fluid entrained by the rotation of a second coaxial cylinder, ii) he measured the head loss produced by the flow of a liquid through a pipe and iii) he observed the jet coming out from a long tube. He showed that for a flow Q less than a threshold flow Q_0, the jet was always smooth; contrary to this, when the flow was greater than $Q_1 > Q_0$, the jet was always rough. When the flow was such $Q_0 < Q < Q_1$, he observed a jet with "*sudden variations, without apparent regularity, in its shape as well as in its amplitude, the latter being always longer for the smooth jet than for the rough jet.*"

In his book published in 1907, Marcel Brillouin (1854–1948) made a quite comprehensive description of Hagen's observations [23]: when the temperature is increased, the viscosity decreases, and irregular motions are no longer "*sufficiently damped*" for allowing water motions to remain almost rectilinear. Brillouin used "*Poiseuille regime*" for designating the laminar regime and "*hydraulic regime*" for the turbulent regime. We systematically replaced his terminology by the "modern" one between brackets. Brillouin also distinguished "*ondulatory motion*" from turbulent motion, a distinction which was not later followed: we will therefore use turbulent motion. So, for Brillouin, turbulent motion "*appear in the liquid, amplify the*
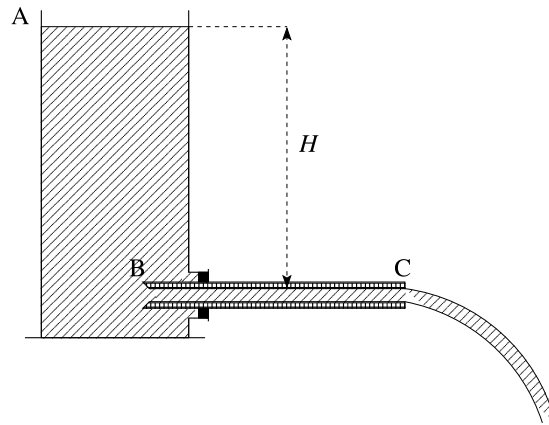
Fig. 3. A capillary tube BC is adapted horizontally to a measuring tube A filled with mercury. The end B of the capillary tube has a funnel shape to obtain more easily a laminar regime. The jet flowing out in C is observed while the height *H* of mercury is decreasing.

Table 1
Observations made by Brillouin with mercury flowing in a capillary tube ($l = 18.9$ cm and $d = 1.0$ mm). The Reynolds numbers (not provided by Brillouin) were here computed according to Poiseuille's law.

Mercury height	Regime	Reynolds number
$H > 160$ mm	[Turbulent] regime	$Re > 11131$
$H = 136$ mm	Small oscillations	$Re = 9461$
$H = 126$ mm	Large oscillations	$Re = 8766$
$73.9 < H < 122$ mm	Huge oscillations	$5141 < Re < 8487$
$H < 73.9$ mm	[Laminar] regime dominates with few sudden loss in amplitude (sudden returns to turbulent regime)	$Re = 8766$
$H = 68.1$ mm	End of the blurred regime. Beginning of the stationary laminar regime.	$Re = 4737$

inequalities in velocity and damp proportionally more energy than [laminar] *regime, for a given flow*". Brillouin added that "*when the temperature is still increased,* [turbulence] *is first, more and more important, and then becomes stationary; the regime is more regular and the flow increases again.*" Brillouin understood that what is observed at a given pressure when the temperature is varied is the image of what occurs when the temperature is kept constant and the pressure is progressively increased.

In order to evidence this phase where the laminar regime becomes turbulent, Brillouin used a very simple experiment (Fig. 3) he presented in his lecture on 4 February 1899. Brillouin describes his observations as follows [23]:

At first, the flow produced under a high pressure, and the [turbulent] *regime is observed: the jet flowing out is regular but not smooth. Since mercury is flowing, the pressure decreases; at a given time, the "blurred phase" starts; the* [laminar] *regime (smooth jet) starts to occur by intermittencies; each time it appears, the jet is longer, showing that the flow increases and that the resistance decreases; but the* [turbulent] *regime restarts quickly, the jet is once more time suddenly blurred and, simultaneously, the jet is more curved and its amplitude decreases.*

The jet thus jerks, from one amplitude to the other. First, these oscillations are rare, the smallest amplitude being the most observed; the [turbulent] *regime dominates. Then, the height yet decreasing, the oscillations are more frequent, before being rare again, but the large amplitude being now the most often observed; the* [laminar] *regime dominates. Finally, below a certain value of the driving pressure, the sole* [laminar] *regime remains, the jet is smooth and the oscillations disappeared.*

With a given tube ($l = 18.9$ cm and $d = 1.0$ mm, Brillouin provided a detailed description of his observations (Table 1). He repeated this experiment with other tubes and with water, and got similar observations. He was thus able to conclude that "*for a given glass tube, the transition from one regime to the other does not occur suddenly, at a threshold velocity, but there is a* blurred phase *during which the two regimes are possible and alternate with a more or less large frequency*". Intermittencies were thus already clearly identified in 1907! It seems that these works by Hagen, Couette and Brillouin remained quite unknown...

2.2. The friction coefficients

In his book, Julius Weisbach (1806–1871) proposed a law for the head loss *h* per unit of length as [24] (quoted by Flamant [25, p. 143])

$$\frac{dh}{4} = \left(a + \frac{b}{\sqrt{V}}\right) V^2 \tag{3}$$

where d is the pipe diameter and V the mean velocity of the water. The parameter values are $a = 0.0007336$ and $b = 0.0004828$. Few years later, in his experimental researches on water flow in pipes, Henri Darcy (1803–1858) expressed the head loss h per unit of length as [17, p. 14] (as written by Flamant [25, p. 142])

$$\frac{\mathrm{d}h}{4} = \left(\alpha + \frac{\beta}{d} \right) V^2 \tag{4}$$

where the parameter values are $\alpha = 0.000507$ and $\beta = 0.00001294$ [17]. To this background, Jean–Louis–Marie Poiseuille's (1797–1869) contribution [26] should be added. He obtained a resistance proportional to the velocity [26] under the form

$$Q = k'' \frac{Pd^4}{l} \tag{5}$$

where d is the pipe's diameter, l its length and k'' a constant coefficient for the same temperature and the same gravitational acceleration. Poiseuille designated by Q what he named the "*product corresponding to pressures*" and what is named today the flow. Reynolds rewrote it as [19, p. 972]

$$\alpha' \frac{d^3}{\mu^2} h = \beta' \frac{d}{\mu} V \tag{6}$$

where we expressed the pressure h in terms of height of water per unit of pipe length. Introducing explicitly the pressure loss ΔP and the length l of the pipe, Reynolds' equation (6) can be rewritten as

$$\Delta P = \gamma' \frac{\mu l}{d^2} V \tag{7}$$

that is, under the form proposed by Eduard Hagenbach-Bischoff (1833–1910) who used the mean velocity V and not the flow Q [27]: the resistance to the flow is thus proportional to the velocity. A law similar to Poiseuille's one was also obtained by Hagen [28]. Hagenbach-Bischoff, who was aware of these two different contributions, choose to designate equation (7) as Poiseuille's law.

Reynolds was thus aware that [19, p. 995]

> *the relations between the resistance encountered by, and the velocity of, a solid body moving steadily through a fluid in which it is completely immersed, or of water moving through a tube, present themselves mostly in one or other of two simple forms. The resistance is generally proportional to the square of the velocity, and when this is not the case it takes a simpler form and is proportional to the velocity.*

Nevertheless, his experiments convinced him that rather than using the law $h \propto V^2$ or $h \propto aV + bV^2$ as "*propounded by any of the previous experimenters*", no other law than $h \propto V^{1.723}$ should be used [19, p. 975], a law that he designated as the "*law of pressures*". Reynolds thus proposed a law for the pressure loss as

$$\alpha \frac{d^3}{\mu^2} h = \left(\frac{\beta d V}{\mu} \right)^{1.723} \tag{8}$$

which is here rewritten using our notations, h being the pressure loss (expressed as a height of water) between the two ends of one unit of length of the considered pipe, μ being the dynamic viscosity. For making simpler the comparisons with later results that we will discuss below, let us rewrite this expression as

$$h = 0.00078 \frac{V^{1.723}}{d^{1.277}} \tag{9}$$

where the numerical coefficient is obtained from the numerical values provided by Reynolds.

Slightly later, Alfred Flamant (1839–1915) made a detailed review of the different laws proposed for the resistance head in pipes and proposed to use [25, p. 149]

$$\mathrm{d}h = 0.00092 \sqrt[4]{\frac{V^7}{d}} \quad \Leftrightarrow \quad h = 0.00092 \frac{V^{1.75}}{d^{1.25}} \tag{10}$$

which is therefore not too different from equation (9) proposed by Reynolds (when rewritten in a similar form).

In 1901, Augustus V. Saph and Ernest W. Schoder started a joint doctoral thesis to perform — between February 24, 1902, and February 25, 1903 — approximately 800 experiments, testing 23 kinds of pipe and hoses, covering a 200-fold range of velocities [29]. They obtained for the head loss [30]

$$h = 0.00054 \frac{V^{1.75}}{d^{1.25}} \tag{11}$$
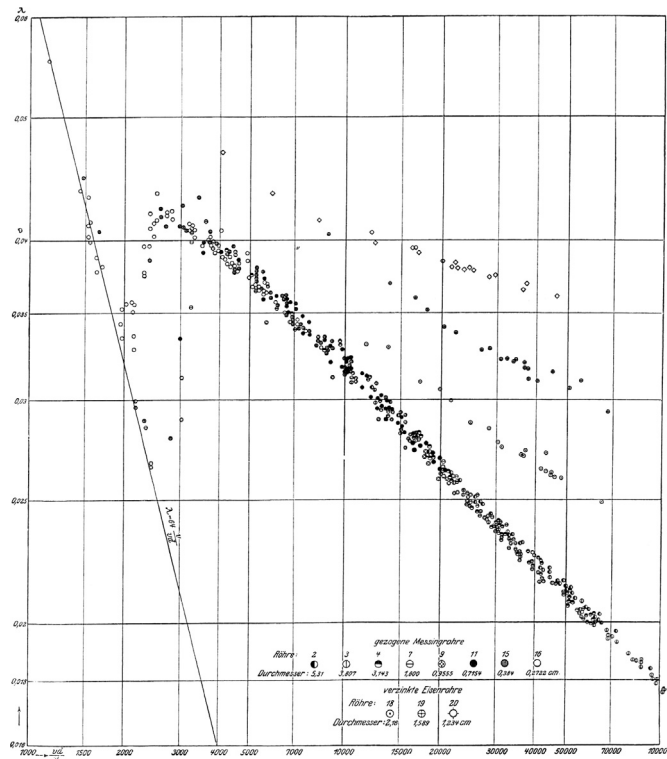
Fig. 4. Saph & Schoder's measurements on the resistance of flow in pipes as plotted by Blasius. The three upper sets of points correspond to galvanized pipes ($d \in \{2.00, 1.589, 1.234\}$ in cm), the other being performed with brass pipes ($d \in \{5.31, 3.807, 3.143, 1.000, 0.9555, 0.7154, 0.384, 0.2722\}$ in cm). From Blasius (1912).

which is not too different from Reynolds' and Flament's laws. Saph and Schoder were still following this long series of experiments for determining the head loss per unit of length of the considered pipe.

Heinrich Blasius (1883–1970) was looking for a law of similarity in friction processes [31]. He started from the fact that the head loss — that he expressed using the head loss H for a pipe of length l — in a pipe was clearly dependent on the ratio $\frac{l}{d}$, the square of the speed, and on the velocity head $\frac{V^2}{2g}$. He thus exhibited a function λ occurring in the head loss as

$$H = \lambda \frac{l}{d} \frac{V^2}{2g} \tag{12}$$

The introduction of the now so-called friction coefficient λ is a first crucial step introduced by Blasius. His next step was to determine this coefficient λ. For Blasius, this coefficient obviously depends on the nature of the flow since the head loss was proportional to V for laminar flow (Poiseuille) and to V^n for turbulent flows ($n = 2$ for Weisbach and Darcy, $n \approx 1.75$ for Reynolds, Flament, Saph and Schoder). The obvious quantity to distinguish these two types of flow was therefore the Reynolds number. Blasius therefore looked for a dependence of the function λ on the Reynolds number. In order to determine how λ could depend on the Reynolds number Re, he used the very accurate measurements performed by Saph and Schoder [30] and got the diagram shown in Fig. 4. The great difference between Saph and Schoder is that, rather than plotting $\log h$ versus $\log V$, Blasius plotted its friction coefficient λ versus the Reynolds number Re, an obvious way to determine how λ was depending on Re.
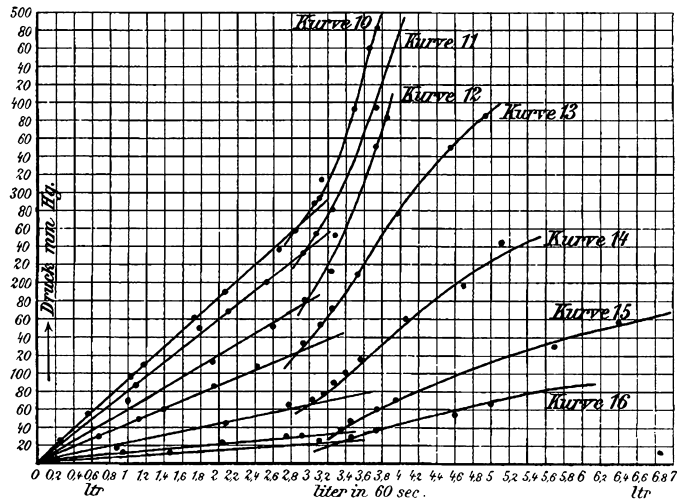
According to Hager [32], Blasius was the first to propose a law between the friction coefficient λ and the Reynolds number Re. He started by expressing λ for laminar flow (Poiseuille's law) and got

$$\lambda = \frac{64}{Re} \tag{13}$$

that matched to Saph and Schoder's measurements for low Reynolds number (Fig. 4). By interpolating the data obtained for $Re > 2500$, he got

$$\lambda = \frac{0.3164}{\sqrt[4]{Re}} \tag{14}$$

Fig. 5. Head loss H versus the airflow Q in glass capillary tubes. Diameter $d = 2$ mm. Curve 10: $l = 15.30$ m; curve 11: $l = 12.15$ m; curve 12: $l = 9.17$ m; curve 13: $l = 6.08$ m; curve 14: $l = 2.88$ m; curve 15: $l = 1.55$ m; and curve 16: $l = 1.03$ m. For an airflow $Q = 3$ l min^{-1} ($V = 15.92$ m s^{-1}), the critical Reynolds number is equal to 2300.

Note that, starting from Saph and Schoder's law (11) multiplied by l to obtain the head loss H for a pipe of length l, then equating the obtained relation to Blasius' equation (12), we get

$$H = 0.00054\, l\, \frac{V^{1.75}}{d^{1.25}} = 0.00054\, l\, \sqrt[4]{\frac{V^7}{d^5}} = \lambda\, \frac{l}{d}\, \frac{V^2}{2g} \tag{15}$$

The friction coefficient can thus be expressed as

$$\lambda = 0.00054\, 2g \sqrt[4]{\frac{\rho}{\mu}}\, \sqrt[4]{\frac{1}{Re}} \tag{16}$$

Using $\rho = 10^3$ kg^{-3} and $\mu = 10^{-3}$ Pa s, and Saph and Schoder's numerical coefficients, we obtain $\lambda = 0.335 \sqrt[4]{\frac{1}{Re}}$, that is, an expression very close to Blasius' one. Blasius' contribution is therefore not for a better law for the head loss, but rather for the form he wrote it, introducing the friction coefficient λ. Moreover, his diagram (Fig. 4) suggests a clear switch from Poiseuille's law to Blasius one — a switch which is not so obvious when $\log h$ is plotted *versus* $\log V$ —, thus suggesting the reality of a critical Reynolds number. Blasius who contributed to promote the Reynolds number and the law of hydrodynamic similitude as introduced by Reynolds [19,21] (for a review on the use of similitude in the 19th century fluid mechanics, see [33]).

2.3. The critical Reynolds number

If the Reynolds number became clearly the right concept for investigating the emergence of turbulence (as evidenced by Blasius [31]), the challenging problems were i) to determine its critical value at which the transition from laminar regime to turbulence occurs and, ii) to understand through analytical works the mechanism underlying the destabilization of the laminar flow. We will mostly focus on the former problem.

In 1908, W. Ruckes checked that the critical Reynolds number was at about 2300 for air flows in capillary tubes (Fig. 5) [34]. But in an iron capillary tube he found a critical Reynolds number equal to 439 (when recomputed with modern values for the air viscosity $\mu_{air} = 18 \times 10^{-6}$ Pa s, volumic mass $\rho_{air} = 1.3$ kg m^{-3}, and with his experimental values, $V = 16.46$ m s^{-1}, $d = 0.4$ mm, the critical value is 476). In 1911, Vagn Walfrid Ekman (1874–1954) obtained a Reynolds number up to 40,000 [35]. In 1921, Ludwig Schiller (1882–1961) showed that the transition strongly depends on the perturbations imposed by two plates placed at the entrance of a tube with a diameter $d = 0.8$ mm (Fig. 6) and can appear at a Reynolds number up to 10,000 [36]. In 1954, the problem was not yet fully solved, and E. Rune Lindgren was questioning the reality of a critical Reynolds number to characterize the transition between laminar and turbulent flow [37]:

> It is believed that these conditions are fulfilled for flow in smooth cylindrical tubes [...] According to Schiller [36], the value of the critical Reynolds number should be $R_k \approx 2300$. Strong objections have been raised against this concept...
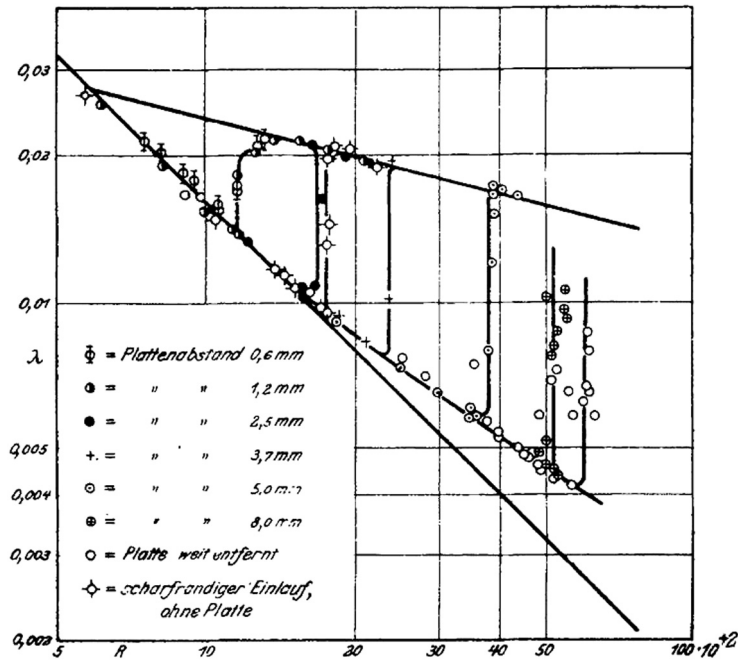
Fig. 6. Friction coefficient *versus* the Reynolds number for a water flow in a pipe with a diameter $d = 0.8$ mm: perturbations were imposed by two plates at its entrance. The distance between the plates was varied. The Reynolds number used by Schiller was defined using the half-diameter: the values of the abscissa must therefore be multiplied by 2 to recover the common values for the Reynolds number.

3. Toward a characterization of intermittencies

According to Reynolds, these intermittent behaviors result from two opposite mechanisms [19]: one destabilizes the laminar flow, and one damps eddies. This approach was followed by H.T. Barnes and E.G. Coker who also considered two antagonist mechanisms [38]:

> We may observe the critical velocity in two ways: either by observing the velocity at which the stream-lines break up into eddies, or by obtaining the velocity at which the eddies from initially disturbed water do not become smoothed out into stream-lines in a long uniform pipe. The first change may be at any velocity within certain limits (depending on the initial steadiness of the inflowing, water, while in, the second, the change can take place at only one velocity. It therefore depends on whether we start with initially quiet water or disturbed water what value will be attained.

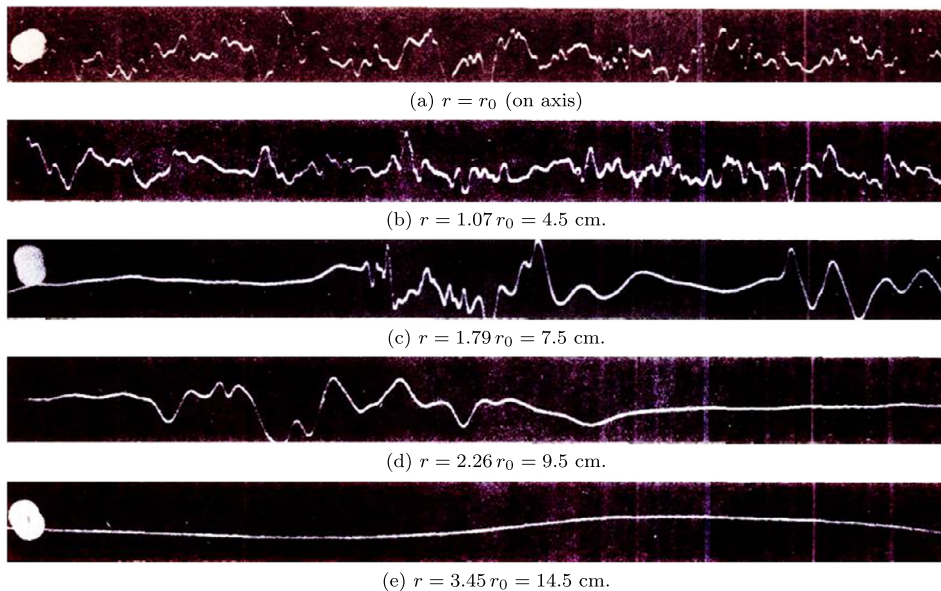They also observed the intermittent behaviors already described by Reynolds:

> If we tried the experiments too soon after filling the tank, before the water had become perfectly steady, the upper limit came much lower, and, as the water became more settled, a higher limit was indicated by a tendency to form stream-lines, followed by a breaking-up, giving rise to the phenomenon described by Reynolds as "flashing." The flashes became less frequent until, finally, the water being perfectly steady, the higher limit was reached. It was possible to obtain the higher limit as soon as the flashes started, since it was only necessary to increase the flow until the flashing ceased. The flashes showed themselves by an oscillation on the thread of the thermometer. [...] flashing was taken to indicate a tendency to form stream-lines, and if they appeared, the upper limit was obtained by increasing the flow until the permanent reading on the thermometer indicated that eddy motion was the steady flow.

Barnes and Coker clearly observed intermittencies, used them to have a better measurement of the upper Reynolds number at which instabilities occur, but did not investigate them as a dynamical behavior to characterize.

In 1943, Stanley Corrsin (1920–1986) explicitly showed that turbulent flow is progressively mixed with laminar flow by measurements in a heated air jet issued from a convergent-straight nozzle with a mouth diameter of 1 inch [39]. His "oscillograms" (Fig. 7) were described as follows.

> *Fig. 7 is a series of oscillograms of the axial velocity fluctuations at various radial positions on a section 20 diameters from the nozzle mouth. This series shows that the flow in a "turbulent jet" is fully turbulent only out to approximately $r = r_0$. For $r > r_0$, there exists first an annular "transition region" in which the flow alternates between the turbulent and the laminar regimes. The fraction of the total time during which the turbulent state prevails decreases as the radial distance is increased.*

(a) $r = r_0$ (on axis)

(b) $r = 1.07\, r_0 = 4.5$ cm.

(c) $r = 1.79\, r_0 = 7.5$ cm.

(d) $r = 2.26\, r_0 = 9.5$ cm.

(e) $r = 3.45\, r_0 = 14.5$ cm.

Fig. 7. Velocity fluctuations measured with a hot-wire experiments in a heater air jet at $x = 30d$ by Stanley Corrsin (1943) at the Graduate Aeronautical Engineering Laboratories of the California Institute of Technology. The nozzle had a diameter $d = 2.54$ cm (1 inch). From Corrsin [39].

From various studies to characterize (not to explain) the transition between laminar and turbulent regimes, the contribution by Louis Sackmann (1905–1990) from the "Université Louis-Pasteur" in Strasbourg is certainly one of the most decisive ones (although his works mostly reproduce what Couette and Brillouin observed 40 years earlier, using Blasius' approach). In 1947, he wrote [40]

> *These phenomena present the character common to the reversible and alternated transitions between laminar and turbulent states of the boundary layer: they occur in the neighborhood of a critical Reynolds number Re_c, which separates the two domains of stability of the regime subject to these transitions; for this critical value, the two alternating regimes are equally probable and are alternating with an unpredictable rhythm; when the Reynolds number becomes different from this critical value, by increasing or decreasing values, the predominance of one or of the other of the two possible regimes increases up to the emergence of a unique regime, definitively stable for greater or smaller Reynolds numbers.*

From his first investigation, he was able to evidence that "*for a given flow in a transitional regime, the alternations of the free jet, irregular in durations, but constant in reach, define two extreme instantaneous velocities, and lead to a splitting in two of the point representative of the mean head loss deduced from the flow*" [40, p. 795]. The same year, with François Codaccioni, he distinguished three Reynolds numbers, namely the superior Re_s, critical Re_c and inferior Re_i ones (Fig. 8), the transition between laminar and turbulent regimes occurring for $Re_s > Re > Re_i$ [41]. For distinguishing the two regimes, Sackmann and Codaccioni used Blasius' friction coefficients for laminar regime — equation (13) — and for turbulent regime — equation (14).

They understood that [41]:

(i) to any driving head H from the transition domain $Re_s - Re_c - Re_i$ correspond two points characteristic of extreme instantaneous regimes, with $Re_T > Re_L$;
(ii) extreme instantaneous regimes are either turbulent or laminar;
(iii) lifetime of these regimes evolves when the transition domain $Re_s - Re_c - Re_i$ is crossed by varying the mean Reynolds number: for $Re_s < Re < Re_c$, the turbulent regime is dominating; it is definitely stable for $Re = Re_i \neq 2800$;[1] for $Re = Re_c = 2376$, the two regimes are equally probable; their equal mean lifetime are at about 1 s;
(iv) the alternating regimes are metastable.

They were thus able to conclude that "*the phenomenon of real and instantaneous transition is displayed by two distinct branches, homologue segments from Poiseuille's and Blasius' lines, alternatively visited*" (Fig. 8). They detailed the transition between laminar and turbulent regimes as reported in Table 2. One year later, Sackmann investigated the probabilities p_{lam} to observe the laminar and $(1 - p_{lam})$ to observe the turbulent regime [42]. If he confirmed that the turbulent regime was indeed made mostly (94%) of turbulent flow, this was not the case for the laminar flow only made of 60% of laminar regime. He also

---

[1] Here the Reynolds numbers are given using the diameter, that is, Sackmann and Codaccioni's values are multiplied by 2.

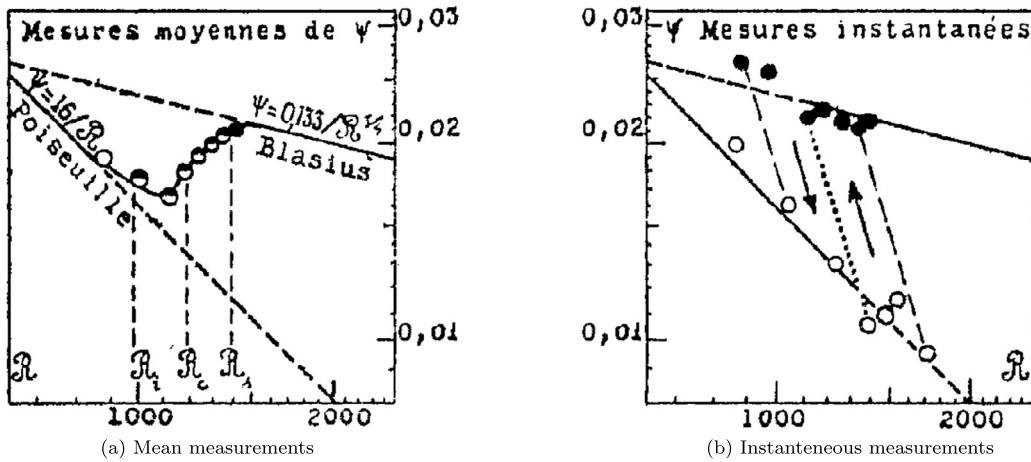(a) Mean measurements

(b) Instanteneous measurements

Fig. 8. Regime changes of a water flow in pipe ($d = 4.214$ mm and $l = 89.1$ cm). According to Blasius' diagram, the friction coefficient (symbolized by Ψ in Sackmann and Codaccioni's work) is plotted *versus* the Reynolds number (the radius is used to compute the Reynolds number: the values must be multiplied by two to allow comparison with other works). From Sackmann and Codaccioni [41].

Table 2
Different behaviors of a water flow observed by Sackmann and Codaccioni in a pipe ($d = 4.214$ mm and $l = 89.1$ cm). From [41]. The probability p_{lam} of observing a laminar regime [42] is also reported. Measurements collected in 1948 are reported in bold.

ΔP (cmH$_2$O)	Re	Qualitative observations	p_{lam}
23.90	2814	Turbulent regime 100%	**0.06**
22.25	2720	Discontinuities	**0.08**
20.20	2606	Increasing dropping out	**0.13**
18.05	2488	Turbulent preponderence	**0.21**
15.95	2376	Isochronous transitions	**0.35**
14.90	**2320**	–	**0.50**
13.90	2266	Laminar preponderence	**0.57**
11.90	2026	Discontinuities	**0.58**
9.90	1776	Laminar regime 100%	**0.60**

(a) From laminar to turbulent regime

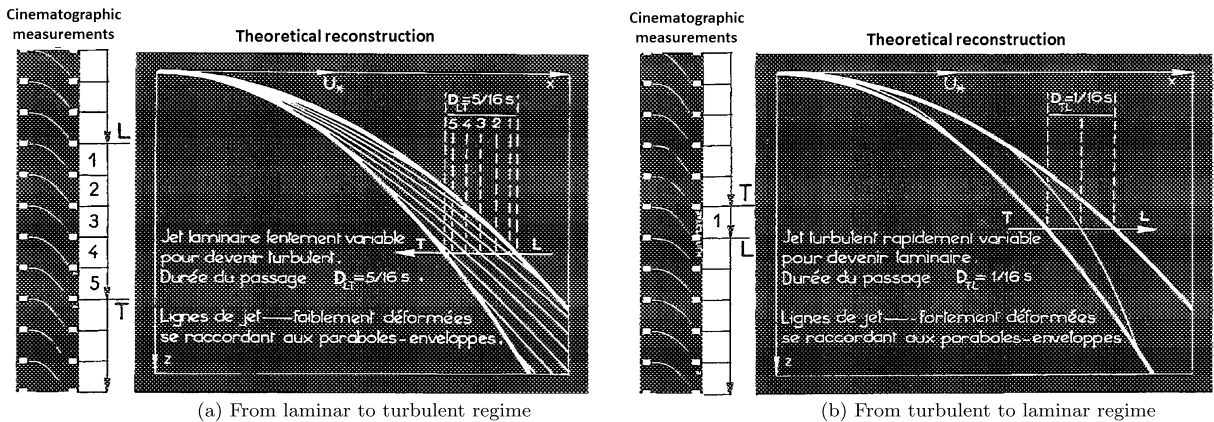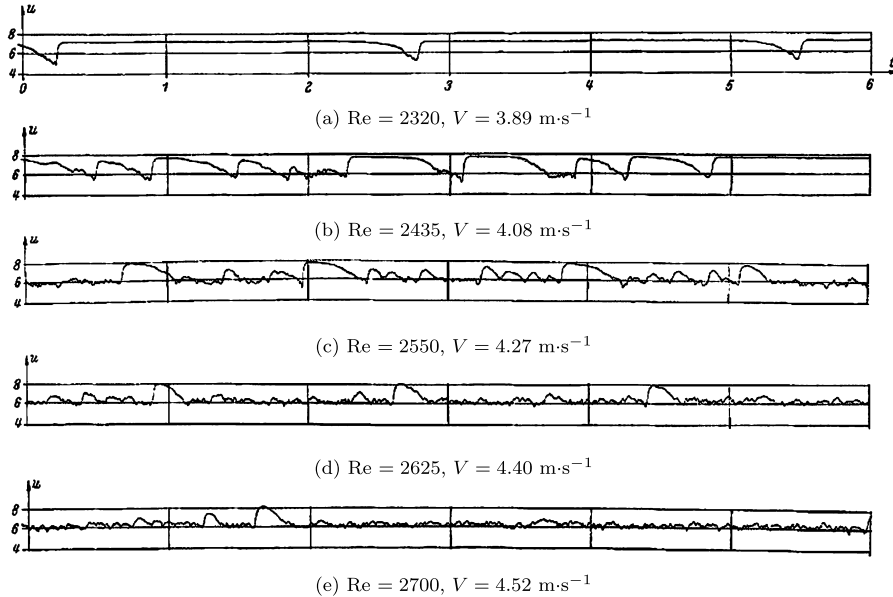(b) From turbulent to laminar regime

Fig. 9. Cinematographic measurements of the jet falling out from the pipe and theoretical reconstruction of the transitions. From [43].

corrected the value at which he observed with Coddaccini "*isochronous transitions*". Later, Sackmann investigated the duration of these transitions, from turbulent to laminar and *vice versa* [43]. He found that the flow was switching quite quickly, in 5/16 s (1/16 s), from laminar to turbulent (from turbulent to laminar) regimes by using cinematographic measurements thus sampled at 1/16 s (Fig. 9): he never observed a stable intermediate regime [43].

With Sackmann's works as a background (eight of his papers are quoted, confirming a strong influence by many aspects), Julius Rotta (1912–2005) investigated in 1956 the intermittent nature of the transition between laminar and turbulent flows [44]. Rotta started by quoting Oskar Tietjens (1894–1944) in his book written from the lectures of Ludwig Prandtl (1875–1953) for a description of the phenomenon observed at the transition [45]
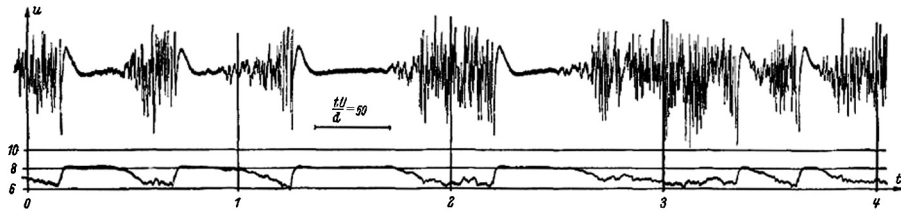
Fig. 10. Registered velocity in a pipe at $Re = 2550$. Upper curve: instantaneous fluctuations of the velocity. Lower curve: mean velocity. The mean velocity during turbulent bursts is less than during laminar phases. From Rotta, Fig. 6, 1956.

(a) Re = 2320, $V = 3.89$ m·s^{-1}

(b) Re = 2435, $V = 4.08$ m·s^{-1}

(c) Re = 2550, $V = 4.27$ m·s^{-1}

(d) Re = 2625, $V = 4.40$ m·s^{-1}

(e) Re = 2700, $V = 4.52$ m·s^{-1}

Fig. 11. Velocity V of the airflow measured in a pipe ($d = 8.30$ mm) for different values of the Reynolds number. From Rotta, 1956.

Here the maximum values of the velocities always correspond to those moments where the turbulence suddenly begins within a certain section of the pipe. When the mass of fluid is turbulently flowing through the pipe, the flow resistance in the pipe is larger, and the discharge speed decreases. Once this starts, the resistance decreases in the pipe, inducing again an increase in speed up to the point where partial turbulence starts again, and the process described above is repeated.

In order to improve the probabilities of observing laminar regimes as used by Sackmann, Rotta introduced the intermittent factor and the intermittent number to quantify intermittencies during the transition. He computed these quantities from measuring velocities with a hot-wire technique. He checked that the mean velocity during turbulent bursts was less than the one measured during laminar phases (Fig. 10), as already observed by Sackmann. The great improvement compared to Sackmann's results is the quality of the time series Rotta obtained.

Rotta was thus able to show that the turbulent bursts become more frequent when the Reynolds number increases (Fig. 11). In order to quantify the progressive occurrence of turbulent bursts, Rotta used the intermittency factor

$$\gamma = \frac{\text{Mean duration of turbulent flow}}{\text{Total duration}} = \frac{\overline{\tau}_{\text{turbulent}}}{\tau_{\text{tot}}} \tag{17}$$

and the intermittency number $\bar{n}$ defined as the mean number of transitions per unit of times, that is,

$$\bar{n} = \frac{N_{\text{transition}}}{\tau_{\text{tot}}} \tag{18}$$

For Rotta, a transition means one laminar phase followed by one turbulent burst, up to a new laminar phase. The transition from laminar to turbulent flows is thus characterized by an intermittency factor γ with a sigmoid shape (Fig. 12a), while the intermittency number $\bar{n}$ presents a maximum (Fig. 12b) for a Reynolds number of about 2500. This measurements clearly show that there is a transition between laminar and turbulent flows by some intermittencies where the two types of flows alternate.

According to Hans Liepmann (1914–2009) in his introduction to the session about free turbulence at the colloquium organized by Alexandre Favre (1911–2005) on the occasion of the inauguration of the "Institut de mécanique statistique de
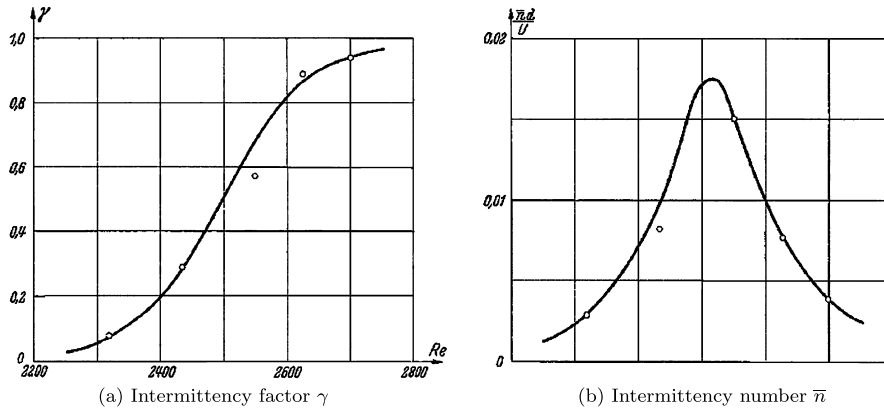
Fig. 12. Intermittency factor γ and intermittency number $\bar{n}$ *versus* the Reynolds number. Measurement were performed with air at a distance $\frac{x}{d} = 322$ from the entrance of a pipe whose diameter was $d = 8.3$ mm. From Rotta [44].
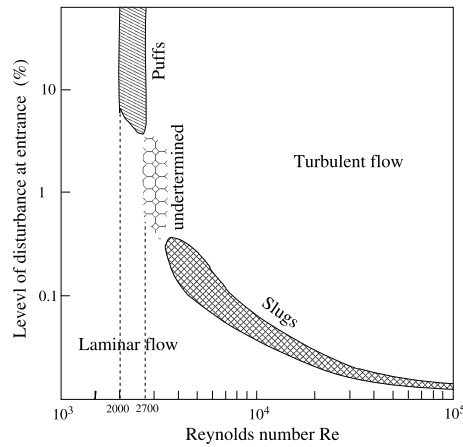

Fig. 13. Parameter space (level of disturbance at entrance *versus* Reynolds number *Re*) for the regime of pipe flow. From Wygnanski and Champagne [49].

la turbulence" (Marseille) en 1961 [46], these alternations between laminar and turbulent regimes were called *intermitten-cies* by Alfred Townsend [47]. Donald Coles (1924–2013) stated that "*intermittency implies the existence of definite interfaces separating regions of laminar and turbulent flow*" [48]. The transition range, in terms of Reynolds number, can be identified by measurements performed far from the "*disturbed entry conditions*". In such a case, "*for Re < 2000 (approximately), turbulent regions tend to decay and disappear as the fluid proceeds downstream; for Re > 2800 (approximately), turbulent regions tend to spread into laminar ones and to fill the pipe completely. In an intermediate range* 2200 < Re < 2600*, however, there apparently exists a regime of mixed laminar-turbulent flow which is statistically stationary far downstream.*" In order to show this, Coles used the intermittency factor γ introduced by Rotta. Coles considered that intermittency is a random phenomenon by nature. More interestingly, he proposed to investigate intermittency "*by supposing that turbulent flow is the normal state, while laminar flow is abnormal*" [48, p. 244].

In the early 1970s, Israel Wygnanski and Francis Champagne [49] started by distinguishing *slugs* — caused by the instability of the boundary layer to small instabilities in the inlet region of the pipe, corresponding to the transition from laminar to turbulent states (for $Re \geqslant 3200$) — from *puffs* — produced by large disturbances at the inlet and associated with an incomplete relaminarization (for $2000 \leqslant Re \leqslant 2700$). These two phenomena reflect the two processes already identified by Reynolds [19]. The problem of the transition from laminar to turbulent regimes is thus dependent on two parameters: the level of disturbance at entrance and the Reynolds number (Fig. 13). A very clear review about the dynamics of "puffs" and "slugs" is provided by Barkley [16].

Quite recently, a critical Reynolds number was defined by using the mean lifetime of turbulent bursts (Fig. 14), distinguishing the "*time scale for turbulence to spread* [from] *the time scale for turbulence to decay*" [50]: the puff lifetime was determined as the duration between its occurrence and its splitting (thus increasing the intermittency number), and as the duration after which the flow is relaminarized, respectively. Typically, the flow is said to corresponds to "*sustained turbulence in the thermodynamic limit*" when the lifetime of the spreading turbulence is greater than the lifetime of the decaying turbulence. It remains that these results have to be related to spatio-temporal intermittencies and the percolation theory as developed by Pomeau [51]. Spatio-temporal intermittencies were introduced by Kunihiko Taneko [52], and by Hugues
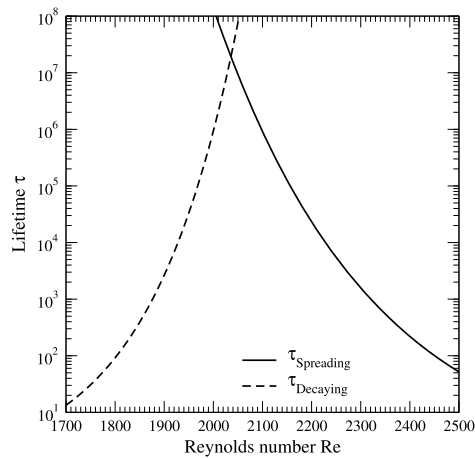
Fig. 14. Mean lifetime of spreading and decaying puffs. When the lifetime of the spreading puffs becomes longer than the lifetime of the decaying puffs, turbulence can be qualified as being sustained. From Avila et al. [50].

Chaté and Manneville [53]. They were able to describe some features as those experimentally observed by Pierre Bergé and Monique Dubois [54]. In these spatio-temporal intermittencies, laminar and turbulent domains coexist and evolve in space and in time. Such a description has also a long history sketched by Itiro Tani [55]. These intermittencies could provide a direct link with the percolation theory as developed by Pomeau [51].

## 4. Approach from the dynamical systems theory

Although intermittencies were experimentally investigated since Reynolds' experiments, this phenomenon was not sufficiently well understood from the theoretical point of view to contradict Landau's scenario requiring an infinite number of degrees of freedom to produce a turbulent behavior. One of the theory that seemed plausible was due to the independent works of Lev Landau (1908–1968) [57] and Eberhard Hopf (1902–1983) [58]. According to this theory, a fluid makes a transition from a laminar regime to a turbulent one by the appearance of successive degrees of freedom. This theory of turbulence is based on the assumption that a hydrodynamical system could be considered as a set of weakly coupled oscillators in which each oscillator is associated with a characteristic frequency. In this framework, turbulence results from interactions among a large number of oscillators, required for providing a large spectrum of frequencies.

By the end of the 1960s David Ruelle, from the "Institut des hautes études scientifiques" in Bures-sur-Yvette, France, had many discussions with his colleague, the mathematician René Thom, who founded the theory of catastrophes, and with Stephen Smale, who proposed the so-called "horseshoe map", one of the fundamental models for chaotic behavior [59]. With this background, Ruelle suspected that a small set of weakly coupled "oscillators" could account for the behavior of turbulent fluids. Since real fluids are viscous, the different oscillators used for explaining the frequencies observed in the spectra of turbulent flows would have to interact strongly, viscosity being responsible for this coupling. Moreover, Ruelle was aware that simple equations with few degrees of freedom could produce complicated behavior as already shown by Smale. Consequently, Ruelle and Floris Takens, a Dutch mathematician, proposed a new theory for the development of turbulence in 1971 [60]. The main idea is that, due to the nonlinear couplings between oscillators, there is no need for an infinite number of degrees of freedom to obtain complicated behavior (four oscillators are sufficient), a few degrees of freedom being sufficient. Ruelle and Takens thus described "turbulent dynamics" in terms of a low-dimensional "strange" attractor. When he discovered Lorenz's paper about a three-dimensional system as a simplified model for the Rayleigh–Bénard convection [15], Ruelle realized that the attractor that became the "Lorenz attractor" had characteristics similar to those of the "strange attractor" involved in his scenario. The initial title for the 1963 Lorenz's paper was *Deterministic turbulence*, but the editor, P. Philipp, considered that his model did not conform to the accepted idea of turbulence and pushed to replace the title by the less explicit *Deterministic nonperiodic flows* [61].

Still today, there is neither strong objection to Ruelle–Takens' scenario, nor conclusive experiment to validate it. Some experiments were conducted by Jerry Gollub and Harry Swinney in a Taylor–Couette flow [62]: results clearly contradicts Landau–Hopf's scenario but they are only in agreement with Ruelle–Takens' scenario, they are not conclusive. At this time, emerged the "chaos theory" with contributions by Robert May [63], Otto Rössler [64], and John Guckenheimer and co-workers [65]. The Lorenz system [15]

$$\begin{cases} \dot{x} = \sigma(y - x) \\ \dot{y} = Rx - y - xz \\ \dot{z} = -bz + xy \end{cases} \tag{19}$$
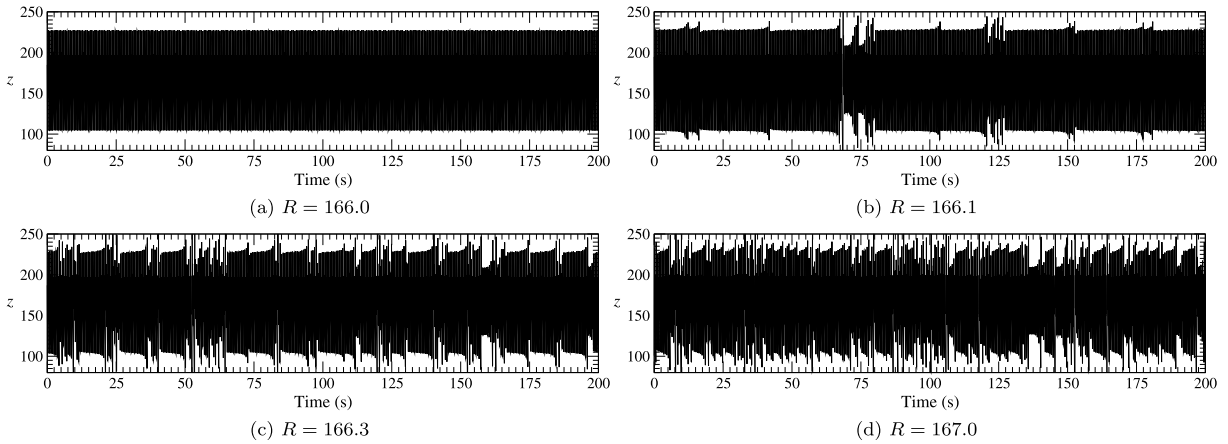
Fig. 15. Intermittencies as observed in the Lorenz model by Manneville and Pomeau. Parameter values: $b = 8/3$ and $\sigma = 10$.

thus became widely studied. In this context, Yves Pomeau was performing "*systematic studies of the Lorenz set of equations with the help of an analog computer*" [66] for investigating "*the way the Lorenz model was switching to turbulence, Ruelle–Takens' scenario being excluded* [in this model] *due to its too small dimension of the state space*".[2] Paul Manneville was then using a HP 9825 to develop the detailed analysis. They thus discovered a route to chaos by intermittencies [67].

Using the Lorenz system (19) with parameter values $b = 8/3$ and $\sigma = 10$, and varying R, Manneville and Pomeau observed that [67]

"*a limit cycle stable for R < 166.07 is (apparently) randomly interrupted by turbulent bursts. The duration of the laminar phases being shorter and shorter as R is increased.*"

Simulations similar to those provided by Manneville and Pomeau are shown in Fig. 15. The key point was that intermittencies were found in a low-dimensional deterministic system without any stochastic fluctuations. Although very different from Ruelle and Takens' scenario, these simulations were supporting the idea that complex behaviors as turbulence could be produced by simple systems: an infinite dimensional state space was not necessary to explain turbulent behaviors.

As developed in the first studies Pomeau conducted with Jose-Luis Ibañez on the attractor observed in the Lorenz system after a period-doubling cascade [68], the "natural" first step to understand these intermittencies was to "*find a rational explanation of this remarkable phenomenon* [...] *by looking iterations of one-dimensional maps*" [66]. Manneville and Pomeau thus started by considering a first-return map to a neighborhood of the limit cycle ($R = 166.0$) in a surface of section. The key point was to choose correctly the surface of section (which was not the commonly used one, defined by $z = R - 1$). The first-return map they obtained looks like the one shown in Fig. 16. When the first-return map does no longer touch the first bisecting line, the trajectory remains in the small "channel" thus formed: the time during which the trajectory is in it corresponds to a "laminar" phase. When the trajectory leaves the channel, a turbulent burst is produced. The origin of the intermittency between nearly-periodic orbit (laminar regime) and chaotic attractor (turbulent regime) is therefore a dynamical phenomenon. It should be noted that intermittencies were also observed by N. Morioka and T. Shimizu (Fig. 16b), but very poorly described, mostly because they did not construct properly the first-return map to a surface of section and missed the tangency between the map and the first bisecting line [69].

Pomeau and Manneville started from the knowledge that "*the laminar structure is destroyed at the onset of turbulence by bursts*", explicitly mentioning that, "*at the onset of turbulence,* [the intermittency factor] γ *goes to zero*" [70]. They were aware that "*intermittency* [...] *is a well-known phenomenon in fluid flows*" from Wygnanski and Champagne's paper[3] [49], although they quoted David Tritton's book [71] in which Rotta (1956) is quoted as well as Wygnanski and Champagne's paper (1973), and in which Rotta's intermittency factor is introduced. Moreover, by this time, Albert Libchaber and Jean Maurer just completed experiments in a Rayleigh–Bénard convection using liquid ^{4}He, in which they observed a transition to turbulence *via* intermittency between a nearly periodic orbit and a turbulent regime [72], experimental results that Manneville and Pomeau were aware of.
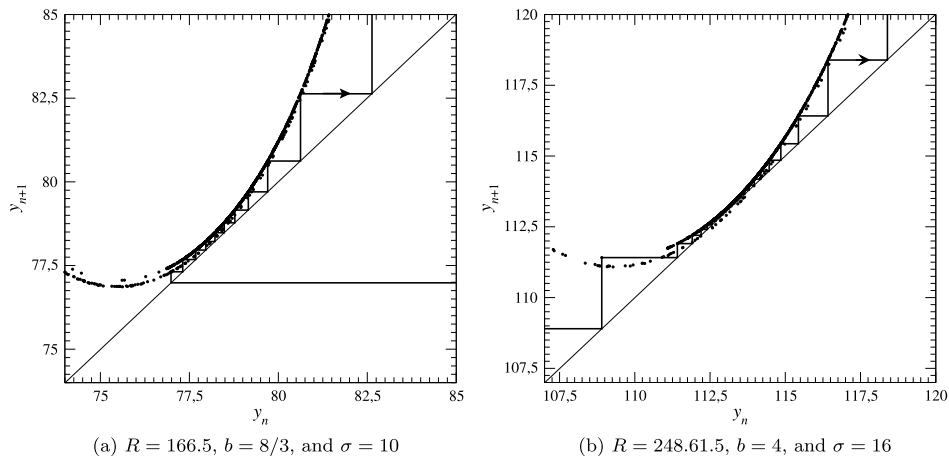
Based on that knowledge, and assuming that a laminar motion can be associated with a limit cycle, that is, a single stable periodic point when a Poincaré map is considered, Pomeau and Manneville proposed a qualitative map as a prototype for producing type-ɪ intermittency [70]. An explicit formulation was provided in [73] under the form

$$\theta \mapsto 2\theta + R\sin 2\pi\theta + 0.1\sin 4\pi\theta \tag{20}$$

(a) $R = 166.5$, $b = 8/3$, and $\sigma = 10$ (b) $R = 248.61.5$, $b = 4$, and $\sigma = 16$

Fig. 16. First-return maps to a neighborhood of a surface of section of the chaotic attractor produced by the Lorenz system for two different sets of parameter values. Pomeau and Manneville (a) built such a map showing the tangency between the map and the first bisecting map, which allowed them to explain the origin of intermittencies. Morioka and Shimizu (b) did not get the tangency − as here shown to check that this is indeed a type-ɪ intermittency (b) − and were not able to explain the phenomenon.
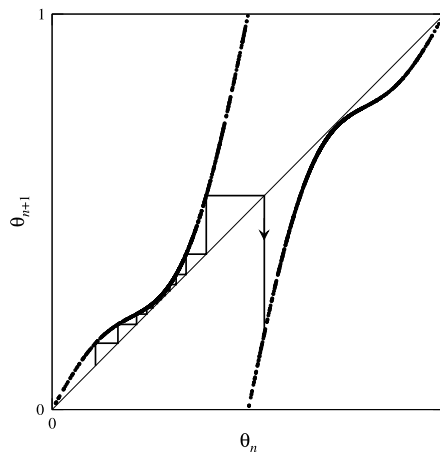
Fig. 17. Type-ɪ intermittency observed in a deformed dyadic map as discussed in [73]. Parameter value: $\epsilon = 0.1$.

The tangent bifurcation responsible for the type-ɪ intermittency is observed for $R_c = -0.24706$. Using $R = R_c + \epsilon$, there is a "channel" between the map and the first bisecting line and intermittencies were observed (Fig. 17). This type-ɪ intermittency results from the loss of a pair of periodic orbits *via* a sub-critical saddle–node bifurcation. A one-dimensional unstable manifold is sufficient to produce this bifurcation: type-ɪ intermittency can therefore be observed in three-dimensional systems, as well-exemplified by the Lorenz system.

Two other types of intermittency were then proposed by Pomeau and Manneville. Type-ɪɪ intermittency corresponds to a subcritical Hopf bifurcation combined with a reinjection mechanism: the Poincaré map has therefore to be at least three-dimensional, two dimensions for the oscillator presenting the subcritical Hopf bifurcation and one dimension for the reinjection mechanism. A map applying the torus T^2 four times on itself was initially proposed by Pomeau and Manneville [73]. The type-ɪɪ intermittency is rather difficult to characterize due to the high dimensionality of the state space in which it takes place. For instance, it was found in the five-dimensional driven system [74]

$$\begin{cases} \dot{x}_1 = z(1 - y_1) - x_1 \\ \dot{x}_2 = \xi z(1 - y_2) - R_1 x_2 \\ \dot{y}_1 = \omega(-y_1 + zx_1) \\ \dot{y}_2 = \omega(-R_2 y_2 + zx_2) \\ \dot{z} = \rho\,[-z + Ax_1 + R_1(1 - C)x_2] + B\cos(\omega_1 t) \end{cases} \tag{21}$$

where ω_1 is the bifurcation parameter. The subcritical Hopf bifurcation is observed on a saddle–node singular point that is associated with a homoclinic orbit − resulting from the combination of an oscillator with a reinjection mechanism − when the system is not excited ($B = 0$). When $B \neq 0$ and $\omega_1 \leq \omega_c \approx 0.153$, there is a period-1 limit cycle that loses its stability at the bifurcation. A type-ɪɪ intermittency is observed when $\omega_1 > \omega_c$. A typical time series is shown in Fig. 18.
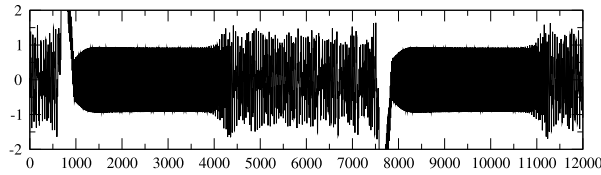
Fig. 18. Typical time series of the type-II intermittency observed in the five-dimensional driven system (21). Parameter values: $\rho = 0.01$, $A = 18.5$, $B = 0.1$, $C = 50$, $R_1 = 0.1$, $R_2 = 1.2$, $\xi = 1.1$, $\omega = 0.001$, and $\omega_1 = 0.159$.

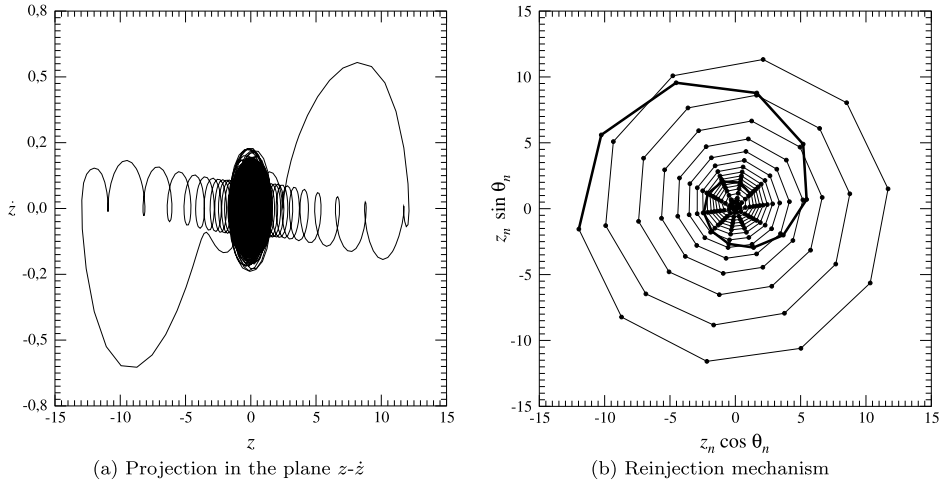(a) Projection in the plane z-$\dot{z}$ (b) Reinjection mechanism

Fig. 19. Type-II intermittency observed in the five-dimensional-driven system (21). The reinjection mechanism combined with the oscillator is evidenced in a plane projection of the state space (a) or in a discretized version of the dynamics (b). Same parameter values as in Fig. 18.

The difficult aspect for characterizing a type-II intermittency is to evidence the reinjection mechanism [75]. A projection of the state space is shown in Fig. 19a. By using a sub-space defined by variables ($z_n \cos \theta_n$, $z_n \sin \theta_n$) where z_n is variable z_n sampled at a frequency $\frac{2\pi}{10\omega_1}$ and angle θ_n is equal to $\omega_1 t$, it is possible to clearly show the combination of oscillations due to the subcritical Hopf bifurcation and the reinjection mechanism (Fig. 19b).

A type-III intermittency was also proposed by Pomeau and Manneville. It corresponds to a sub-critical period-doubling bifurcation and they proposed as a prototype, the map [73, p. 195]

$$\theta \mapsto 1 - 2\theta - \frac{1}{2\pi}(1 - \epsilon) \cos \left[2\pi \left(\theta - \frac{1}{12} \right) \right] \mod(1) \tag{22}$$

of the circle S_1 onto itself. This map is equivariant under an order-2 symmetry. The "channel" can therefore be seen using a second-return map as shown in Fig. 20a. A typical time series is shown in Fig. 20b. Pomeau and Manneville showed that these three types of intermittencies were characterized by different distribution of the laminar phases, more or less in the spirit of the distribution of "*intermissions*" as developed by Benoît Mandelbrot [56].

Due to personal relationships between experimentalists as Albert Libchaber, Pierre Bergé, Monique Dubois and theoreticians (here Pomeau and Manneville), intermittent behaviors were "rediscovered" in fluid experiments [72,76] but also by Jerry Gollub and Harry Swinney [77], without any connection with the traditional experiments performed in fluid mechanics, as exemplified by Maurer and Libchaber [72]: "*let us note that intermittent transitions to turbulence have been observed recently in water [77] and in silicone oil [76]*", but not mentioning the older ones. In spite of this, this second approach reinforced the idea that turbulence could be produced by simple and low-dimensional systems.

5. Conclusion

We showed that intermittencies, discovered by Reynolds in 1883 with a first rather correct explanation, were fully characterized after about one century of experiments. It took time to clearly evidence the two underlying phenomena suggested by Reynolds since this was properly done only by Wygnanski and Champagne in 1973. After Reynolds' contribution and the very comprehensive but underconsidered synthesis provided by Brillouin, a key step was provided by Blasius who rewrote the pressure loss in a pipe in a more theoretically based form and, in particular, using the now so-called friction coefficient and its correct dependence on the Reynolds number. The "Blasius' diagram", obtained by plotting the friction coefficient *versus* the Reynolds number, is more sensitive to the nature of the flow (laminar or turbulent) than the diagram used before his work. Then, step by step, mostly by improving the experimental characterization of the transition between laminar and
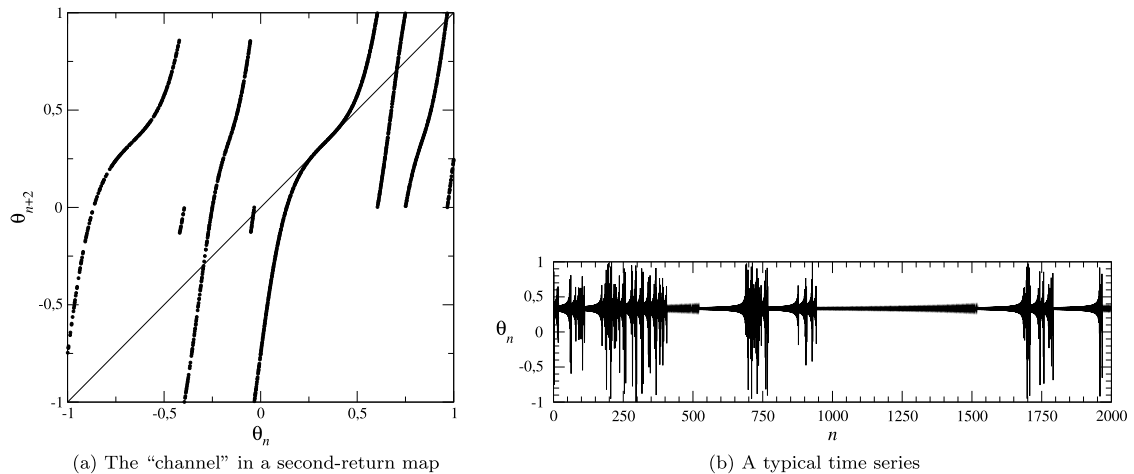
(a) The "channel" in a second-return map

(b) A typical time series

Fig. 20. Type-III intermittency observed in the one-dimensional Poincaré map (22). Parameter values: $\epsilon = 0.001$.

turbulent regimes (Hagen, Reynolds, Couette, Brillouin, and then Sackmann, followed by Rotta), it was possible to better show that the transition was "continuous", for instance, by using the intermittency factor. This long series of experiments led to clearly distinguish "puffs" associated with unstable turbulence from "slugs" corresponding to stable turbulence (Wygnanski and Champagne). Recently, a critical Reynolds number at which the lifetimes for decaying and spreading puffs are equal was evidenced (Avila and co-workers), closing in a certain sense the question concerning the existence of such a critical number.

If some theoretical explanations to these intermittencies were provided in the context of fluid mechanics, a clear picture of the process — purely dynamical — responsible for the long phase in the neighborhood of a "ghost" periodic orbit was only provided in the context of the nonlinear dynamical system theory, although the two discoverers, Pomeau and Manneville, were quite aware about the experimental evidences in fluid mechanics. With their very simple deterministic — and generic — models, they were able to show that intermittency is a phenomenon that can arise in many — if not nearly all — dynamical systems. In fluid mechanics, it reinforced the idea that turbulence does not necessarily result from a high-dimensional system. For physicists, in a very general way, it helped to understand that very carefully conducted experiments can lead to intermittent behaviors. In ecology, intermittent fluctuations of interacting populations can arise without any environmental change, etc. The relevance of their models for the nonlinear dynamical systems theory is non questionable. In that respect, although strongly inspired by experimental observations in fluid mechanics, intermittencies had a greater impact in another field than the one from which they are issued.

Acknowledgements

References

[1] H. Poincaré, Les méthodes nouvelles de la mécanique céleste, Gauthier-Villars, Paris, 1899.
[2] J.E. Barrow-Green, Poincaré and the Three Body Problem, AMS and the London Mathematical Society, 1997.
[3] M. Chapront-Touzé, Progress in the analytical theories for the orbital motion of the Moon, Celest. Mech. Dyn. Astron. 26 (1982) 53–62.
[4] C.-E. Delaunay, Théorie du mouvement de la lune I, Mém. Acad. Sci. 28 (1860) 1–883.
[5] C.-E. Delaunay, Théorie du mouvement de la lune II, Mém. Acad. Sci. 29 (1867) 1–931.
[6] G.W. Hill, Researches on the Lunar theory, Amer. J. Math. 1 (1878) 5–26.
[7] E.W. Brown, On the plans for new tables of the Moon's motion, Mon. Not. R. Astron. Soc. 70 (1909) 148–175.
[8] J. Laskar, Large scale chaos and marginal stability in the solar system, Celest. Mech. Dyn. Astron. 64 (1996) 115–162.
[9] P.S. Laplace, Mémoire sur les solutions particulières des équations différentielles et sur les inégalités séculaires des planètes, 1772, Oeuvres complètes, VIII, 1895, pp. 325–366.
[10] P.S. Laplace, Mémoire sur les inégalités séculaires des planètes et des satellites, 1784, Oeuvres complètes, XI, 1895, pp. 49–92.
[11] P.S. Laplace, Théorie de Jupiter et de Saturne, 1785, Oeuvres complètes, XI, 1895, pp. 95–207.
[12] U.-J.-J. Le Verrier, Recherches astronomiques: extrait des Annales de l'Observatoire de Paris, Tomes 1–14, Mallet-Bachelier, Paris, 1855–1878.
[13] G.D. Birkhoff, Dynamical Systems, American Mathematical Society, 1927.
[14] V.I. Arnold, V.V. Kozlov, A.I. Neishtadt, Mathematical Aspects of Classical and Celestial Mechanics, 2nd ed., Springer-Verlag, New York, 1997.
[15] E.N. Lorenz, Deterministic nonperiodic flow, J. Atmos. Sci. 20 (1963) 130–141.
[16] D. Barkley, Theoretical perspective on the route to turbulence in a pipe, J. Fluid Mech. 803 (2016) P1.
[17] H. Darcy, Recherches expérimentales relatives au mouvement de l'eau dans les tuyaux, Imprimerie impériale, Paris, 1857.
[18] G.H.L. Hagen, Über den Einfluss der Temperatur auf die Bewegung des Wassers in Röhren, Abh. Königl. Akad. Wiss. Berlin (1854) 17–98.
[19] O. Reynolds, An experimental investigation of the circumstances which determine whether the motion of water shall be direct or sinuous, and of the law of resistance in parallel channels, Philos. Trans. R. Soc. Lond. 174 (1883) 935–982.

[20] A. Sommerfeld, Ein Beitrag zur hydronimaschen Erklärung der turbulenten Flüssigkeitsbewegung, in: IV Congresso Internatzionale dei Mathematici, Roma, 6–11 April 1908, vol. 3, 1908, pp. 116–124.

[21] O. Reynolds, On the dynamical theory of incompressible viscous fluids and the determination of the criterion, Philos. Trans. R. Soc. Lond. A 186 (1895) 123–164.

[22] M. Couette, Distinction de deux régimes dans le mouvement des fluides, J. Phys. Théor. Appl. 9 (1890) 414–424.

[23] M. Brillouin, Leçons sur la viscosité des liquides et des gaz, Gauthier-Villars, Paris, 1907.

[24] J. Weisbach, Lehrbuch der Ingenieur- und Maschinen-Mechanik, Braunschweig, 1845.

[25] A. Flamant, Hydraulique, Ch. Béranger, Paris, 1900.

[26] J.-L-M. Poiseuille, Recherches expérimentales sur le mouvement des liquides dans les tubes de très petits diamètres, C. R. Acad. Sci. Paris 11 (1840) 1041–1048.

[27] E. Hagenbach-Bischoff, Über die Bestimmung der Zäkigkeit einer Flüsigkeit durch den Aushluß aus Röhren, Ann. Phys. 185 (1860) 385–426.

[28] G.H.L. Hagen, Über die Bewegung des Wassers in engen cylindrischen Röhren, Ann. Phys. 122 (1839) 423–442.

[29] P. Steen, W. Brutsaert, Saph and Schoder and the friction law of Blasius, Annu. Rev. Fluid Mech. 49 (2017) 575–582.

[30] A.V. Saph, E.W. Schoder, An experimental study of the resistances to the flow of water in pipes, Trans. Amer. Soc. Civ. Eng. 51 (1903) 253–272.

[31] H. Blasius, Das Aehnlichketsgesetz bei Reubungsvorgängen, Z. Ver. Dtsch. Ing. 56 (1912) 639–643.

[32] W.H. Hager, Blasius: a life in research and education, Exp. Fluids 34 (2003) 566–571.

[33] O. Darrigol, Worlds of Flow: A History of Hydrodynamics from the Bernoullis to Prandtl, Springer, New York, 2005.

[34] W. Ruckes, Untersuchungen über den Ausfluß komprimierter Luft aus Kapillaren und die dabei auftretenden Turbulenzerscheinungen, Ann. Phys. 330 (1908) 983–1021.

[35] V.W. Ekman, On the change from steady to turbulent motion of liquids, Ark. Mat. Astron. Fys. 6 (1911) 5.

[36] L. Schiller, Experimentelle Untersuchungen zum Turbulentzproblem, Z. Angew. Math. Mech. 1 (1921) 436–444.

[37] E.R. Lindgren, Note on the flow of liquids in tubes, Appl. Sci. Res. A 4 (1954) 313–316.

[38] H.T. Barnes, E.G. Coker, The flow of water through pipes – Experiments on stream-line motion and the measurement of critical velocity, Proc. R. Soc. Lond. 74 (1904–1905) 341–356.

[39] S. Corrsin, Investigation of Flow in an Axially Symmetrical Heated Jet of Air, NACA Advance Confidential Report 3L23, 1943.

[40] L.A. Sackmann, Sur les changements de régime dans les canalisations: mesures instantanées des caractéristiques, C. R. Acad. Sci. Paris 224 (1947) 793–795.

[41] L.A. Sackmann, F. Codaccioni, Sur les changements de régime dans les canalisations: étude sélective de la perte de charge, C. R. Acad. Sci. Paris 224 (1947) 1326–1328.

[42] L.A. Sackmann, Sur les changements de régime dans les canalisations: étude statistique de la transition, C. R. Acad. Sci. Paris 226 (1948) 1887–1889.

[43] L.A. Sackmann, Sur les changements de régime dans les canalisations: étude cinématographique de la transition, C. R. Acad. Sci. Paris 239 (1954) 220–222.

[44] J. Rotta, Experimenteller Beitrag zur Entstehung turbulenter Strömung im Rohr, Ing.-Arch. 24 (1956) 258–281.

[45] O.K.G. Tietjens, Hydro- und Aeromechanik nach Vorlesungen von L. Prandtl II. Bewegung reibender Flüssigkeiten und technische Anwendungen, Springer, Berlin, 1931.

[46] H.W. Liepmann, Free turbulent flows, in: Mechanics of Turbulence, Marseille, 28 August–2 September 1961, in: Colloques internationaux du CNRS, vol. 108, 1962, pp. 211–227.

[47] A.A. Townsend, The Structure of Turbulent Shear Flow, Cambridge University Press, 1956.

[48] D. Coles, Interfaces and intermittency in turbulent shear flow, in: Mechanics of Turbulence, Marseille, 28 August–2 September 1961, in: Colloques internationaux du CNRS, vol. 108, 1962, pp. 228–250.

[49] I.J. Wygnanski, F.H. Champagne, On transition in a pipe. Part 1. The origin of puffs and slugs and the flow in a turbulent slug, J. Fluid Mech. 59 (1973) 281–335.

[50] K. Avila, D. Moxey, A. de Locar, M. Avila, D. Barkley, B. Hof, The onset of turbulence in pipe flow, Science 333 (2011) 192–196.

[51] Y. Pomeau, Front motion, metastability and subcritical bifurcations in hydrodynamics, Physica D 23 (1986) 3–11.

[52] K. Kaneko, Spatiotemporal intermittency in coupled map lattices, Prog. Theor. Phys. 74 (1985) 1033–1044.

[53] H. Chaté, P. Manneville, Transition to turbulence via spatio-temporal intermittency, Phys. Rev. Lett. 58 (1988) 112.

[54] P. Bergé, M. Dubois, Rayleigh–Bénard convection, Contemp. Phys. 25 (1984) 535–582.

[55] I. Tani, History of boundary layer theory, Annu. Rev. Fluid Mech. 9 (1977) 87–111.

[56] B.B. Mandelbrot, Intermittent turbulence in self-similar cascades: divergence of high moments and dimension of the carrier, J. Fluid Mech. 62 (1974) 331–358.

[57] L.D. Landau, Sur le problème de la turbulence (en russe), Dokl. Akad. Nauk SSSR 44 (1944) 339–342.

[58] E. Hopf, A mathematical example displaying the features of turbulence, Commun. Pure Appl. Math. 1 (1948) 303–322.

[59] S. Smale, Differentiable dynamical systems. I. Diffeomorphisms, Bull. Amer. Math. Soc. 73 (1967) 747–817.

[60] D. Ruelle, F. Takens, On the nature of turbulence, Commun. Math. Phys. 20 (1971) 167–192.

[61] E.N. Lorenz, The Essence of Chaos, University of Washington Press, 1993.

[62] J.P. Gollub, H.L. Swinney, Onset of turbulence in a rotating fluid, Phys. Rev. Lett. 35 (1975) 927–930.

[63] R. May, Deterministic models with chaotic dynamics, Nature 256 (1975) 165–166.

[64] O.E. Rössler, Chaotic behavior in simple reaction system, Z. Naturforsch. A 31 (1976) 259–264.

[65] J. Guckenheimer, G.F. Oster, A. Ipatchki, Periodic solutions of a logistic difference equation, J. Math. Biol. 4 (1977) 101–147.

[66] P. Coullet, Y. Pomeau, History of chaos from a French perspective, in: C. Skiadas (Ed.), The Foundations of Chaos Revisited: From Poincaré to Recent Advancements, Springer-Verlag, 2015, pp. 91–101.

[67] P. Manneville, Y. Pomeau, Intermittency and the Lorenz model, Phys. Lett. A 75 (1979) 1–2.

[68] J.-L. Ibañez, Y. Pomeau, A simple case of non-periodic (strange) attractor, J. Non-Equilib. Thermodyn. 3 (1978) 135–151.

[69] N. Morioka, T. Shimizu, Transition between turbulent and periodic states in the Lorenz model, Phys. Lett. A 66 (1978) 447–449.

[70] Y. Pomeau, P. Manneville, Intermittency: a generic phenomenon at the onset of turbulence, in: G. Laval, D. Gresillon (Eds.), International Workshop on Intrinsic Stochasticity in Plasmas, Cargèse, 17–23 June 1979, 1979, pp. 329–340.

[71] D.J. Tritton, Physical Fluid Dynamics, Van Nostrand Reinhold, New York, 1977.

[72] J. Maurer, A. Libchaber, Effect of the Prandtl number on the onset of turbulence in liquid ^{4}He, J. Phys. Lett.–Paris 41 (1980) 515–518.

[73] Y. Pomeau, P. Manneville, Intermittent transition to turbulence in dissipative dynamical systems, Commun. Math. Phys. 74 (1980) 189–197.

[74] J. San-Martin, J.C. Antoranz, Transition to chaos via type-II intermittency with saturable absorbed externally excited, Prog. Theor. Phys. 94 (1995) 535–542.

[75] J.-Y. Huang, J.-J. Kim, Type-II intermittency in a coupled nonlinear oscillator: experimental observation, Phys. Rev. A 3 (1987) 1495–1497.

[76] P. Bergé, M. Dubois, P. Manneville, Y. Pomeau, Intermittency in Rayleigh–Bénard convection, J. Phys. Lett.–Paris 41 (1980), L-341–345.

[77] J. Gollub, S.V. Benson, Many routes to turbulent convection, J. Fluid Mech. 100 (1980) 449–470.