



An engineering predictive approach of fatigue crack growth behavior: The case of the lug-type joint



Ahmed Bahloul, Amal Ben Ahmed*, Chokri Bouraoui

Laboratoire de mécanique de Sousse, École nationale d'ingénieurs de Sousse, Université de Sousse, BP 264, Erriadh, 4023, Sousse, Tunisia

ARTICLE INFO

Article history:

Received 3 September 2017

Accepted 21 October 2017

Available online 7 November 2017

Keywords:

XFEM

FCG reliability

Iso-probabilistic FCG life curves

MCS

Fatigue life prediction

ABSTRACT

The present attempt proposes a predictive approach of the fatigue crack growth (FCG) behavior of a lug-type joint used in an aeronautic context. The crack tip residual stress distribution and material dispersions are considered. The developed approach was implemented by coupling the Extended Finite Element Method (XFEM), the Residual Corrected Stress Intensity Factor (RC-SIF), developed by the authors, and the Monte Carlo simulation (MCS) method. The Lemaitre–Chaboche model, developed upon the ABAQUS commercial code, was considered for characterizing material behavior. The developed approach treats FCG life by considering the stochastic behavior of material parameters and the crack tip residual stress field during propagation. Comparing with experimental data, the proposed approach exhibits a good ability in evaluating the FCG reliability of a cracked lug-type joint subjected to different loading conditions. The iso-probabilistic P - a - N curves can be used as an efficient tool for ensuring the safety behavior of cracked components.

© 2017 Académie des sciences. Published by Elsevier Masson SAS. All rights reserved.

1. Introduction

Fatigue failure is among the most detrimental failure modes in several industrial fields such as the automotive, marine, and aeronautical ones. During cyclic loading, micro-cracks can be initiated and propagate in mechanical structures, causing catastrophic consequences, even when the ultimate strength of the material is much higher than the applied stress level. Because of several aircraft crashes, whether in military or in civilian aircraft, all aerospace components were designed in agreement with the damage tolerance design philosophy. This design principle is detailed in the Damage Tolerant Design Handbook and in the Joint Services Structural Guidelines JSSG 2006.

When cracks occur, the remaining fatigue life of these components is considered as the main important factor that should be predicted to ensure correctly the safety behavior of such components. In the literature, several models have been proposed to investigate the FCG life evaluation of cracked structures [1–9]. Looking for a model engineering approach that could evaluate the FCG life of mechanical structures in a reliable way, still remains a challenging topic in various industrial sectors. The Lug-type joint (also called attachment lug) is considered as the primary structural component in the aeronautical industry owing to its widespread use in different airframe structures. It is used generally to connect components with other mechanical structures such as wings to fuselage, engines to pylons, and spoilers to wings. During its service life,

* Corresponding author.

E-mail address: bahloulahmad1@outlook.fr (A.B. Ahmed).

Nomenclature

| | | | | |
|------------|-------------------------------------|-----|-----------------------|---|
| a | Crack size | mm | R | Stress ratio |
| E | Modulus of elasticity | MPa | RC-SIF | Residual corrected stress intensity factor..... |
| $I(\{x\})$ | Indicator failure function | | | MPa·m ^{1/2} |
| EPFM | Elastic Plastic Fracture Mechanics | | S | Strength function |
| FCG | Fatigue Crack Growth | | SIF | Stress intensity factor |
| L | Load function | | t | Thickness of the specimen..... |
| MCS | Monte Carlo simulation | | | mm |
| MTS | Maximum Tangential Stress criterion | | σ_{\max} | Maximum applied stress |
| N_{MC} | Total number of MCS methods | | σ_{res} | Residual stress |
| PDF | Probability density function | | σ_y | Yield strength..... |
| P_f | Failure probability | | XFEM | Extended finite element method |

a growing crack may occur near the hole edge of the attachment lug, leading to disastrous consequences. Due to the risks incurred in the aeronautical field, several researchers [60–67] have been interested in the FCG modelling of these structures.

Based on experimental data, James and Anderson [10] studied the SIF for lugs containing through-the-thickness cracks. Later, an empirical expression for the SIF evaluation was proposed by Liu and Kan [11], based on their experimental works. Aberson and Anderson [12] studied the calculation of the stress intensity factor for nonsymmetrical cracked lugs. They used a special crack-tip singularity element for this purpose. Using the boundary element method, Rigby and Aliabadi [13] evaluated the stress intensity factor for straight lugs subjected to both cosine and uniform pressure distribution.

Based on vast experimental and analytical investigations undertaken to predict the FCG life of the lug-type joint, Kim et al. [14] proved that the residual FCG life of the lug-type joint is affected by the increase in the clipping load level. Mikhchevskiy et al. [15] used the Unigrow model to estimate the FCG life of a cracked lug under cyclic loading. They implemented the weight function technique for computing SIFs. Baljanovic and Maksimovic [16] developed a computational procedure to evaluate the FCG lives of cracked lugs. They used the quarter-point (Q-P) singular finite elements for extracting the SIFs near the crack tip. Residual fatigue lives were estimated, under constant amplitude loading, using the two-parameter driving force model [3,4]. Recently, Naderi and Iyyer [17] used the XFEM [18] to evaluate SIFs. The Walker equation was introduced to estimate the FCG life of the aircraft lug under cyclic loading. More recently, the authors [19] proposed the Residual-Corrected SIF parameter to consider the effect of crack-tip compressive residual stress filed for predicting FCG lives of cracked lugs. Even though appreciable studies have been proposed for modelling the FCG of cracked lug-type joints, all of them have been conducted with deterministic models. However, fatigue life is random in nature [20].

The present paper aims at developing a predictive procedure able to evaluate the residual FCG life of a 7075-T6 aluminum alloy cracked lug component, considering the stochastic behavior of the material parameters and the impact of the crack-tip residual stress distribution on the FCG rate. The proposed approach was implemented by coupling the XFEM, the RC-SIF, and the Monte Carlo simulation (MCS) methods. The XFEM embedded in the commercial code ABAQUS was implemented to simulate the crack growth path. Different attachment lug configurations subjected to different load ratios are considered. A comparison between the suggested approach and the available experimental data is performed.

2. Computational engineering approach

2.1. Procedure for FCG life estimation

It is generally admitted that the crack tip stress/strain fields are of paramount importance for controlling the FCG mechanism. When elastic–plastic behavior is assumed, the evaluation of this crack tip stress/strain field is absolutely dependent on the power of analytical and numerical tools. In the case of metal materials, a plastic zone is taking place close to the crack region during crack propagation. This crack tip plastic zone has a significant effect on the FCG process. It was showed [8,21,22] that the relationship between the SIFs and the crack tip stress/strain fields is often affected by residual stresses generated by reversed plastic deformations. Since fracture mechanics is principally based on the SIF determination for predicting crack growth path, FCG rate and FCG life, there is an imperative need to quantify the impact of these residual stresses in terms of SIF. Therefore, the weight function technique [23] can be used to convert the obtained residual stresses in terms of SIF known as residual stress intensity factor K_{res} :

$$K_{\text{res}} = \int_{x=0}^{x=a} \sigma_{\text{res}} m(x, a) dx \quad (1)$$

where $m(x, a)$ and σ_{res} represent, respectively, the weight function expression [24–26] and the residual stress field surrounding the crack tip:

$$m(x, a) = \frac{2}{\sqrt{2\pi(a-x)}} \left[1 + M_1 \left(1 - \frac{x}{a}\right)^{1/2} + M_2 \left(1 - \frac{x}{a}\right)^1 + M_3 \left(1 - \frac{x}{a}\right)^{3/2} \right] \quad (2)$$

It was shown that the geometrical terms can be neglected when the zone of the compressive residual stress is smaller than the crack length. Therefore, the new expression of the universal weight function can be written as follows:

$$m(x, a) = \frac{2}{\sqrt{2\pi(a-x)}} \quad (3)$$

Noroozi et al. [8,21] suggested in their studies concerning FCG analysis that SIFs are affected by the residual stress field developed close to the crack tip. They assumed that only the maximum SIF is affected by the crack-tip residual stress distribution for positive stress ratios, without significant changes in the value of SIF at the minimum applied load [8,21]. Accordingly, the mechanical driving parameter known as the Residual-Corrected SIF parameter can be expressed as follows:

$$\Delta K_{rc} = \Delta K_{el} + K_{res} \quad (4)$$

The improved model, describing the impact of crack tip residual stress field, can be written as follows:

$$\frac{da}{dN} = C(\Delta K_{rc})^m \quad (5)$$

Hence, using the (RC-SIF) range coupled with the Paris law, the FCG life of cracked component having a single crack is evaluated for each step as follows:

$$\Delta N_i = \frac{\Delta a}{C \Delta K_{rc}^m} \quad (6)$$

Finally, the remaining FCG life of the lug-type joint is evaluated as follows:

$$N_i = N_{i-1} + \Delta N_i \quad (7)$$

2.2. FE modelling

A 2D finite element model was implemented upon the ABAQUS code. The lug-type joint known as attachment lug is considered to predict FCG life under different loading conditions ratios. The lug dimensions are presented in Fig. 1, with lug length $L = 200$ mm, two different widths $w = 85.72$ mm and $w = 114.3$ mm, lug thickness $t = 12.7$ mm, and hole diameter $D = 38.1$ mm. The attachment lug configuration, containing an initial crack of size $a = 0.635$ mm, is subjected to cyclic loadings $\sigma_{0\max} = 41.38$ MPa as shown in Fig. 1, in a fashion similar to the experimental conditions.

A very fine structured mesh with element size equal to 0.05 mm, as illustrated in Fig. 2, was implemented in the crack tip region to capture the high stress/strain gradient in this specified zone. The XFEM is used in this study for FCG modeling.

Using isotropic hardening rules [27,28], Bauschinger effect, mean stress relaxation and ratcheting cannot be treated during FE analysis. In this context, the Lemaitre–Chaboche model, developed upon the ABAQUS commercial code, was considered for characterizing material behavior. More details concerning the basic equations of this advanced plasticity model have been given elsewhere [19,29].

During FE analysis, an iterative procedure for FCG simulation is performed using XFEM. The crack-tip residual stress field is determined for each crack increment. Therefore, using the weight function expression, these residual stresses will be converted to stress intensity factors. Table 1 summarizes the mechanical material parameters of the 7075-T6 aluminum alloy. The cyclic fatigue properties considered in the present study [30] are given in Table 2.

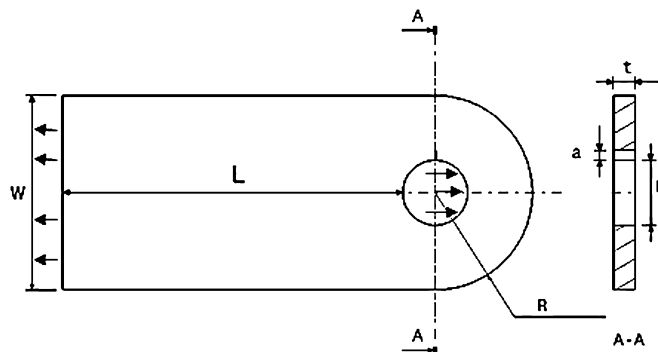


Fig. 1. Attachment lug configuration with initial crack size.

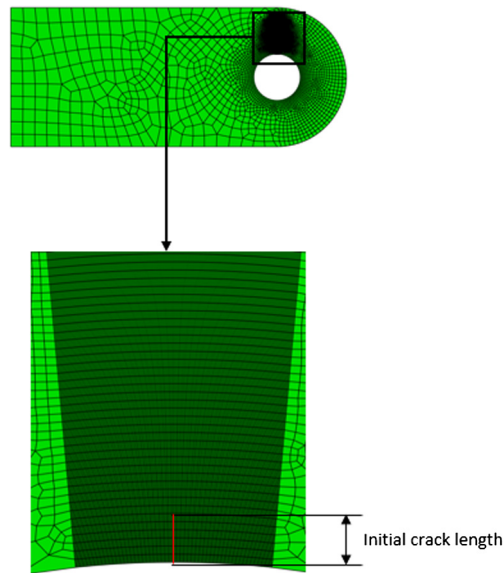


Fig. 2. Finite element mesh in the vicinity of the crack tip.

Table 1

Al 7075-T6 mechanical properties.

| | |
|---------------------|-------|
| Young modulus (MPa) | 70940 |
| Yield stress (MPa) | 420 |
| Poisson's ratio | 0.33 |

Table 2

Al 7075-T6 Cyclic fatigue parameters.

| C^2 (MPa) | C^2 (MPa) | γ^1 | γ^2 | Q (MPa) | b |
|-------------|-------------|------------|------------|-----------|-----|
| 175000 | 9000 | 3500 | 180 | 140 | 40 |

2.3. Reliability analysis of the fatigue crack growth behavior

2.3.1. Computation of FCG reliability for the attachment lug

In this part, the FCG lives of cracked lug-type joints, subjected to cyclic loadings, are investigated using a probabilistic procedure. Fig. 3 presents the main steps for the development of an engineering probabilistic approach.

- (i) In the first stage, a FE model is developed upon the ABAQUS commercial code. The XFEM is used for FCG modelling. Therefore, a numerical FCG code, developed by an iterative procedure within the framework of Python script, is proposed as illustrated in Fig. 4. The Lemaitre–Chaboche model was considered for characterizing material behavior. The obtained residual stress field was extracted in each crack size and converted to SIF using the weight function expression.
- (ii) The FCG lives of the lug-type joints are determined using the RC-SIF, as discussed in section 2.1.
- (iii) Due to the stochastic behavior of the FCG life, the developed procedure was implemented out by considering the effect of crack tip compressive residual stress field and material dispersions. Generating N_{MC} (i.e. the number of MC simulation events) of FCG life curves, three different zones can be observed: (i) a safety zone, (ii) an uncertainly zone, and (iii) a zone of absolute failure, as shown in Fig. 5.
- (iv) Using the MCS method, the reliability is computed for each loading representative point. More details concerning the implementation of the MCS method have been given elsewhere [31–36].
- (v) The iso-probabilistic $a-N$ curves of the cracked lug-type joint are simulated at 5%, 50% and 95% of reliability.

2.3.2. Reliability computation methodology

Using the MC simulation method, two functions should be defined: (i) the resistance function S , and (ii) the load function L . In our case, the resistance function is defined by the scattered FCG life curves and the load function is represented by the loading representative point defined by a given crack size a and a given number of cycles N_0 .

The computational procedure steps for evaluating the fatigue crack growth reliability, using the MCS method, can be summarized as follows (Fig. 6).

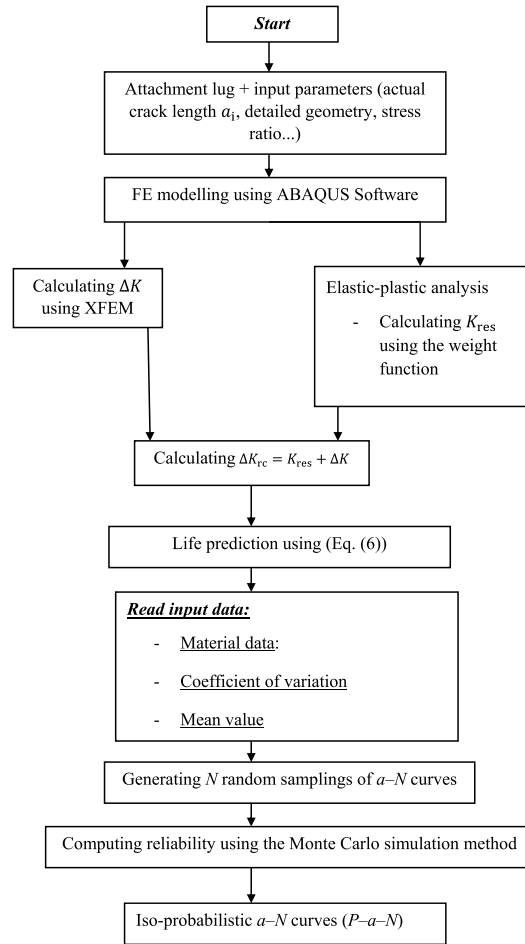


Fig. 3. Flowchart adopted for the engineering probabilistic approach.

- (i) The first step consists in defining the material data (i.e. the coefficient of variation Cov and the mean value of each parameter), their probability density function (PDF), the number of MC simulation events N_{MC} and the loading representative point in which the fatigue reliability will be evaluated.
- (ii) The second step consists in simulating N_{MC} random sampling of fatigue crack growth life curves.
- (iii) The following step consists in extracting the $N_r(i)$ values that present the intersections between a_c and the beam of $a-N$ curves.
- (iv) Then, a comparison between N_0 and $N_r(i)$ was performed to compute the number of safe events (i.e. $N_0 < N_r(i)$).
- (v) End of iteration. If the requirement of convergence is satisfied, the fatigue reliability is obtained using the Monte Carlo approximation:

$$R(\%) = \left(\sum_{i=1}^{N_{MC}} \frac{\text{safe events}(N_0 < N_r(i))}{N_{MC}} \right) \times 100 \quad (8)$$

2.3.3. Iso-probabilistic FCG life curves

In this section, the iso-probabilistic fatigue crack propagation curves are determined using an iterative computational procedure. The proposed methodology allows us to simulate the $a-N$ curve with its corresponding reliability value by introducing the stochastic behavior of FCG material parameters. The developed procedure is based essentially on the following steps, as shown in Fig. 7:

- (i) generating the scattered $a-N$ curves as described in the previous section;
- (ii) initializing the loading representative point, having 100% of reliability, defined by initial crack size a_0 and the number of cycles N_0 ;

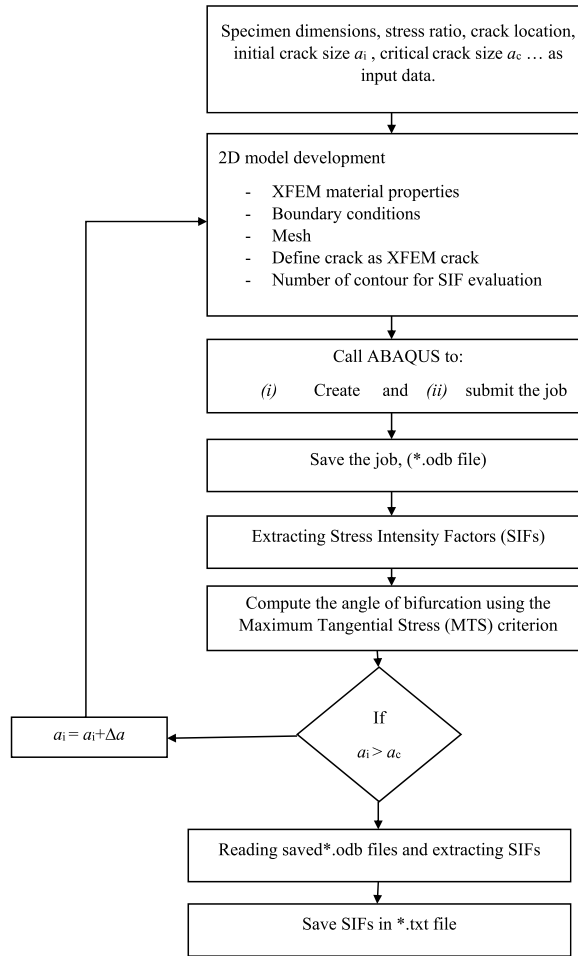


Fig. 4. Flowchart of the Python code for: (i) simulating FCG path, and (ii) evaluating SIFs using XFEM.

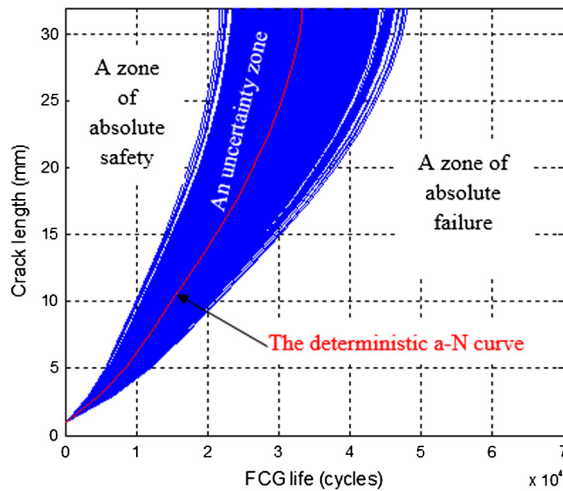


Fig. 5. Material dispersion in the probabilistic a - N curves.

- (iii) computing the fatigue reliability R using the previous flowchart (Fig. 6), as described in section 2.3.2;
- (iv) determining the loading point, which corresponds to the desired reliability value for each crack size increment;
- (v) simulating the iso-probabilistic fatigue crack propagation curves by fitting the iso-reliability recorded points.

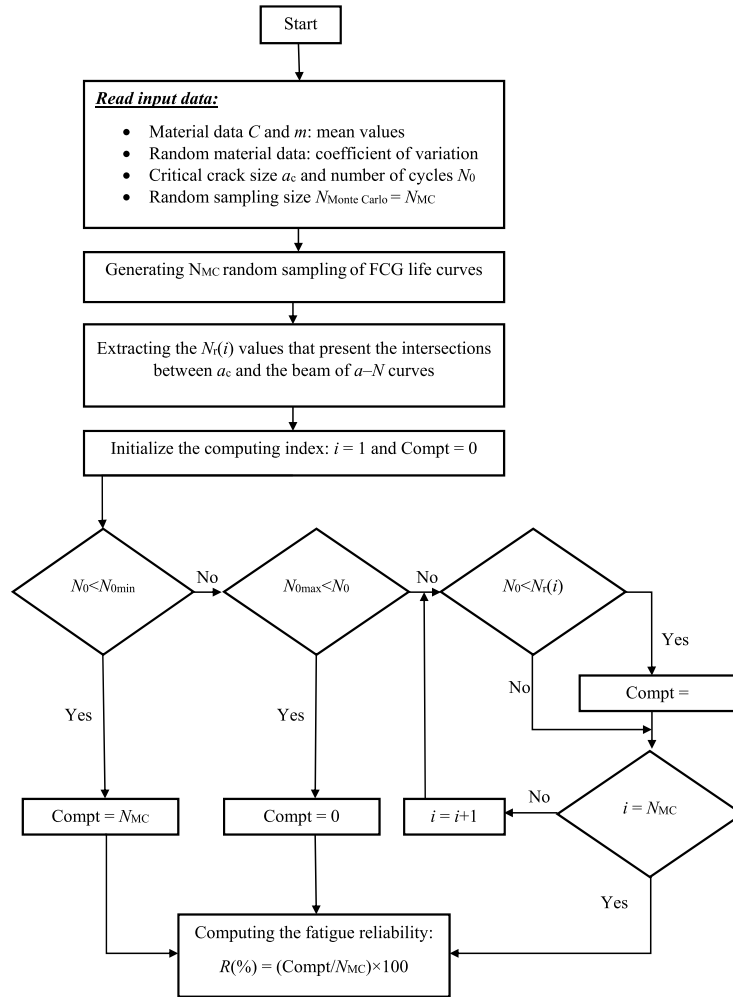


Fig. 6. Flowchart for the fatigue reliability computation.

3. Results and discussion

- (i) In the current work, FCG simulations and SIF computations are performed. The Maximum Tangential Stress (MTS) criterion is used to compute the angle of bifurcation at each step. In XFEM analysis, five contour integrals are implemented for evaluating the average SIF. To validate the numerical iterative procedure developed within the framework of Python script code using XFEM, the fatigue crack growth paths of different specimen configurations were simulated. Different mode mixities were considered. The SIF values were determined numerically for each step and were implemented as input data in the next step. Fig. 8 shows the FCG trajectory at different mode mixities using the XFEM. The numerically simulated paths are in good agreement with those published in the literature [5,37,38].
- (ii) In the case of aircraft component, a good agreement is found between the XFEM and analytical solutions [17] for computing SIF as shown in Fig. 9a. Fig. 9b illustrates the FCG path of the cracked lug-type joint, under cyclic loading.
- (iii) Fig. 10 shows the crack tip Von Mises stress distribution during FCG simulation at maximum applied stress $\sigma_{0 \max} = 41.38$ MPa and at load ratio equals to 0.1, in which the residual stress field along the crack line is extracted at each crack size.
- (iv) Fig. 11 shows the evolution of the crack tip residual stress filed at different crack lengths. It was observed that, at some extension around the crack tip, compressive residual stresses appear, which aim at reducing the tensile stress effect during FCG. The compressive part of the residual stresses increases in absolute value as the crack length increases. The distribution form of the obtained stresses shows good consistency with those obtained by Correia et al. [22].
- (v) To quantify the impact of the crack tip residual stress filed on the SIF range, the weight function expression is implemented to convert the crack tip residual stress distribution in terms of SIF, as discussed in section 2. Table 3 shows the evolution of $\Delta K_{rc}/\Delta K$ versus the crack size for the lug-type joint at two different values of the stress load. It was observed that an increase in the crack length is followed by a decrease in the $\Delta K_{rc}/\Delta K$ ratio. The difference between

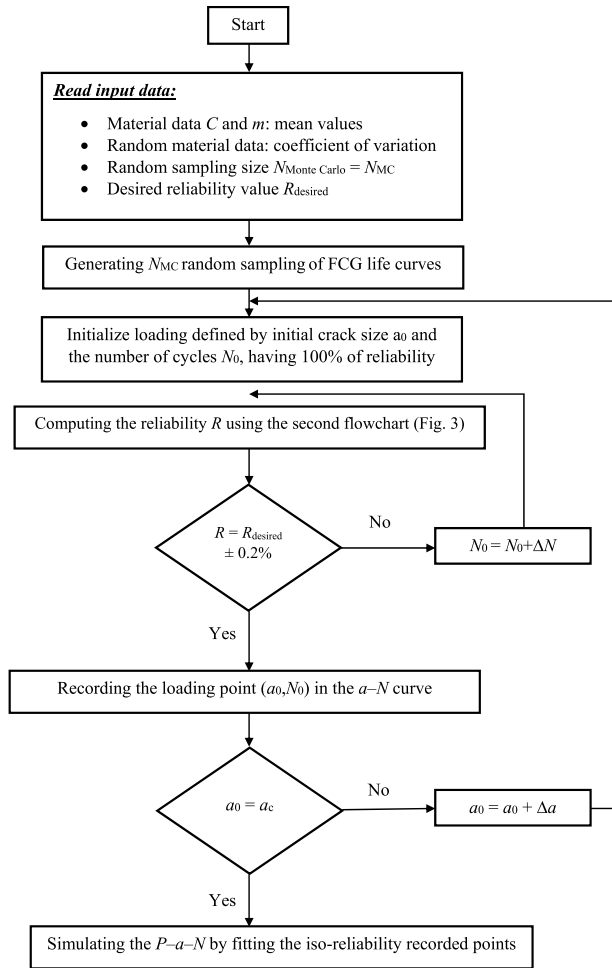


Fig. 7. Flowchart for the iso-probabilistic a - N curves simulation.

Table 3
Evolution of $\Delta K_{rc}/\Delta K$ versus crack length.

| a (mm) | $\Delta K_{rc}/\Delta K$ | |
|----------|--------------------------|-----------|
| | $R = 0.5$ | $R = 0.1$ |
| 3 | 0.957 | 0.923 |
| 5 | 0.939 | 0.891 |
| 7.5 | 0.933 | 0.879 |
| 10 | 0.918 | 0.853 |
| 12.5 | 0.914 | 0.845 |
| 15 | 0.897 | 0.816 |
| 17.5 | 0.903 | 0.826 |

the residual corrected SIF ΔK_{rc} and the elastic SIF ΔK can reach 20% in the case of the attachment lug. In fact, the impact of these residual stresses can be neglected for short crack sizes as is often admitted in automotive components. However, the impact of these residual stresses becomes more important for long crack sizes. Therefore, it should be considered to predict the FCG lives of cracked mechanical parts, especially for aerospace applications.

- (vi) In the authors' work [19], the RC-SIF parameter was developed to predict the FCG life of cracked attachment lugs. The proposed model evaluates FCG life within the framework of EPFM by considering the crack-tip compressive residual stress field. Comparing with experimental results, the improved model allows a better accuracy in the evaluation of FCG life. However, the suggested approach still limited. This limitation is due essentially to the stochastic behavior of the fatigue phenomenon. In fact, a notable scatter was observed between two similarly tested lugs for all cases. This dispersion in FCG life is attributed essentially to the variation of material parameters. The purpose of the present study

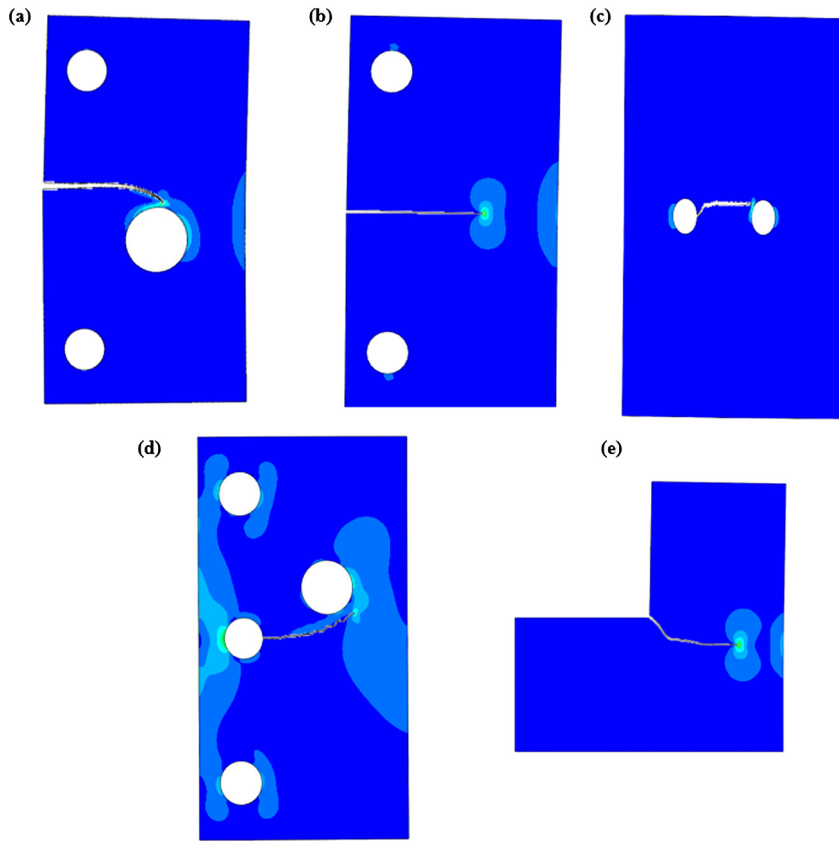


Fig. 8. Fatigue crack growth path simulation for different mode mixities.

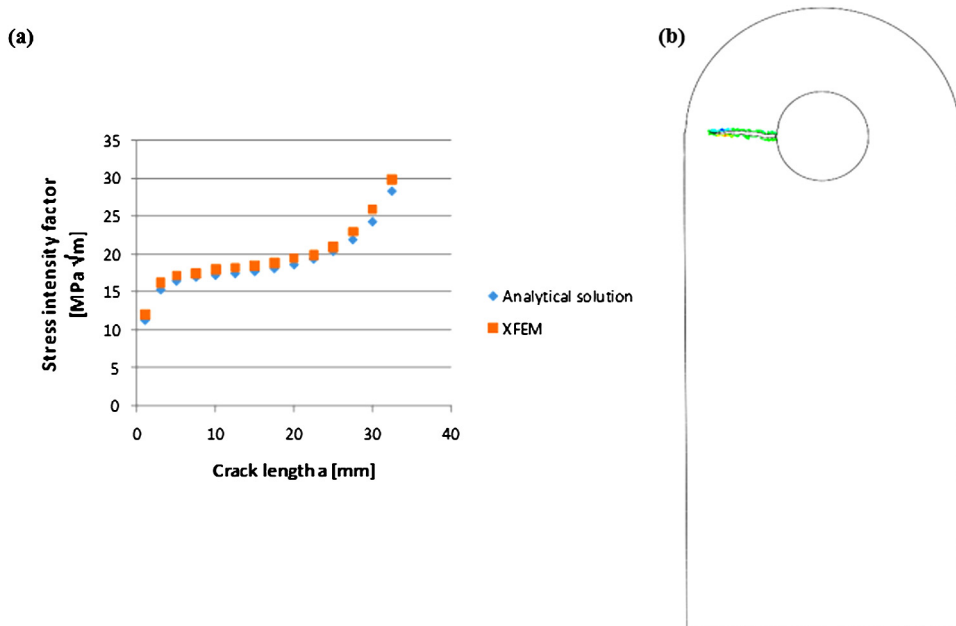


Fig. 9. (a) Stress intensity factor calculation, (b) crack growth path.

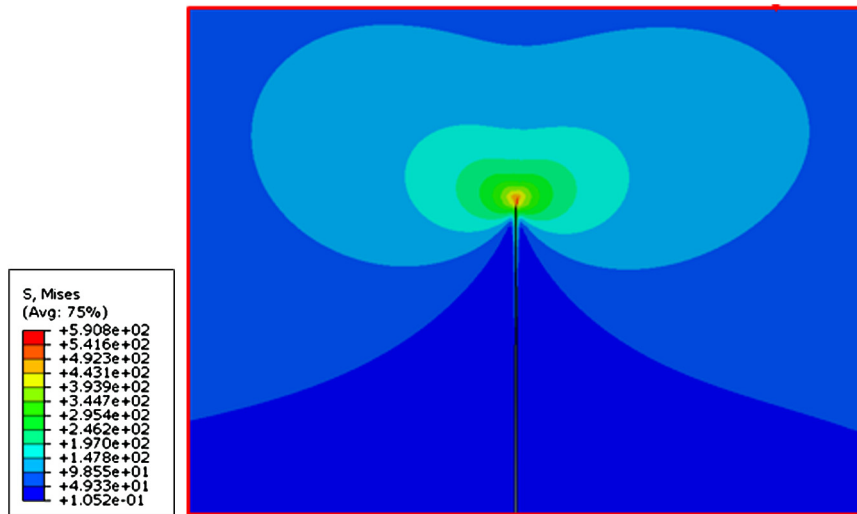


Fig. 10. Von Misses stress distribution near the crack tip at $R = 0.1$ and $W/D = 3$.

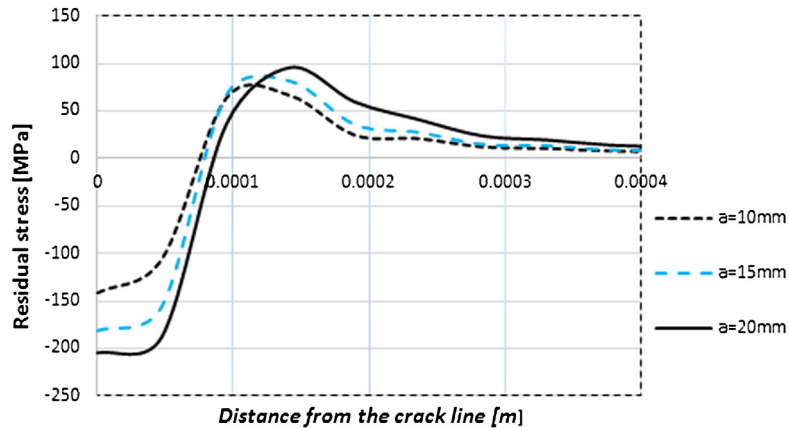


Fig. 11. Residual stress distribution near the crack tip.

aims at improving the deterministic model by considering the stochastic distribution of the material parameters and the crack-tip compressive residual stress field for predicting the FCG lives of the cracked attachment lugs.

- (vii) To validate the ability of the proposed approach for evaluating the remaining FCG life of a lug-type joint, different lug geometries subjected to different load ratios are considered. The RC-SIF parameter is used to introduce the residual stress distribution's effect on the FCG rate as previously mentioned. A normal distribution was assumed for the stochastic behavior of material parameters, with $Cov = 1.75\%$ as the coefficient of variance. A normal distribution, with a coefficient of variance of 1.75%, was assumed for the random behavior of the material parameters. The material parameters considered in this application are: $C = 2.1 \text{ E-}08$, $m = 3.86$ and $C = 11.3 \text{ E-}08$, $m = 3.7$, respectively, for $R = 0.1$ and $R = 0.5$. Due to the random aspect in fatigue life data for cracked lugs, the suggested method consists in determining the iso-probabilistic FCG life curves of the cracked type lug joint at 5%, 50% and 95% of fatigue reliability. Figs. 12a–d show a comparison between the predictive probabilistic procedure and the available experimental results. Different attachment lug geometries subjected to various loading conditions were analyzed. The suggested approach exhibits a good ability in predicting the fatigue crack growth life of the cracked attachment lugs comparing it with experimental results [39]. The obtained iso-probabilistic $a-N$ curves are very useful in engineering applications for evaluating FCG life with an acceptable confidence level. These curves can be used as a practical tool for an optimal inspection strategy of the cracked aircraft components. The proposed engineering approach consists in improving the deterministic models by considering the residual stress field induced by plastic deformation in the crack tip region and material dispersions, allowing one to predict the residual FCG life in a more efficient and reliable way.

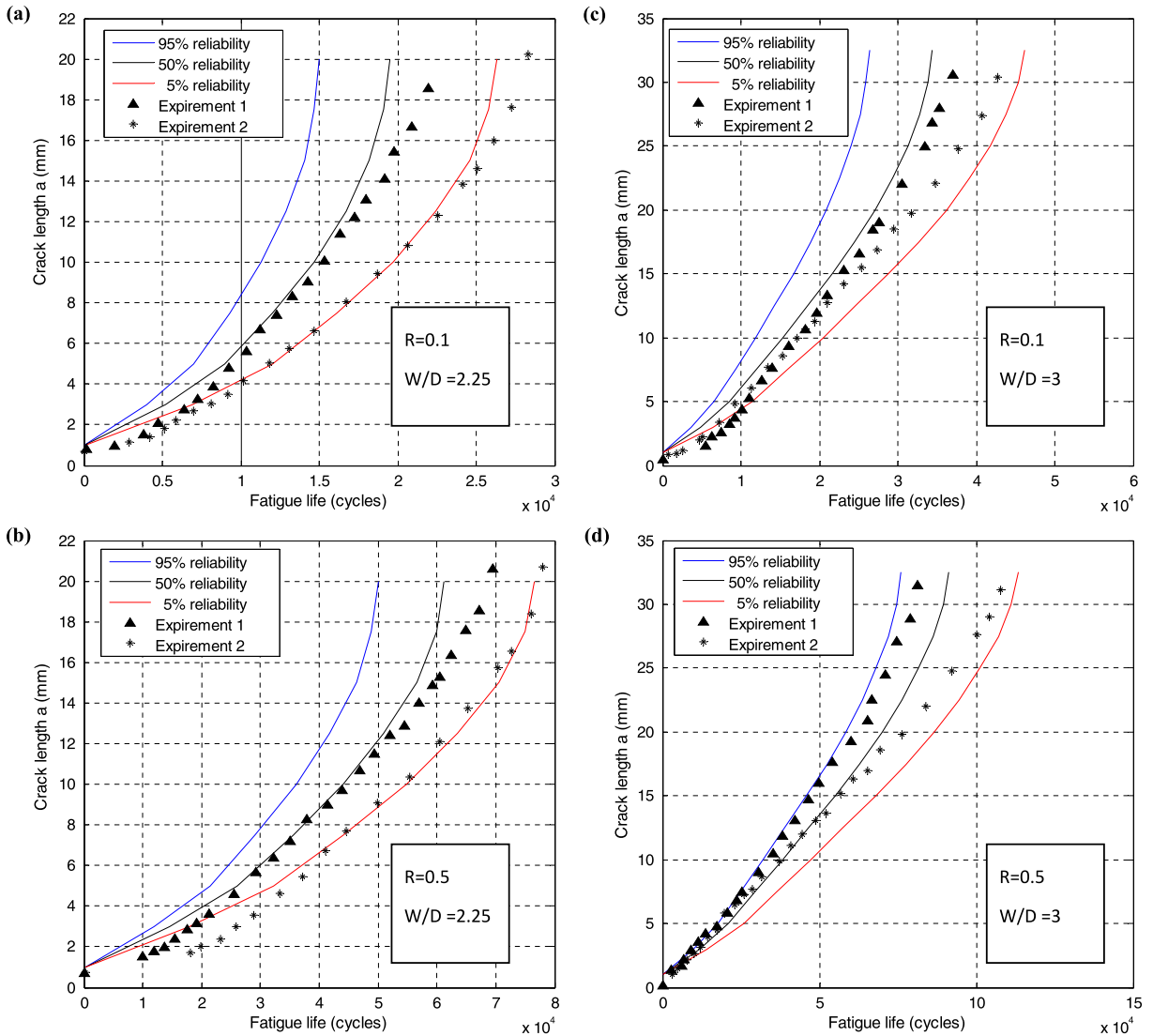


Fig. 12. Iso-probabilistic $a-N$ curves for different stress ratios and different configurations.

4. Conclusions

This attempt addresses the prediction of FCG behavior of a cracked aircraft structure (i.e. lug-type joint). A probabilistic predictive approach was specially developed using XFEM, RC-SIF, and MCS. A particular focus was put on considering the impact of the crack tip residual stress field and material dispersions for evaluating the remaining FCG life. According to the findings, the following points can be mentioned:

- (i) XFEM is an efficient and powerful tool for the modelling of fatigue crack growth. A remeshing technique is not requested for simulating fatigue crack propagation and extracting stress intensity factors;
- (ii) fatigue life data is random in nature and deterministic models seem to be unable of evaluating the remaining fatigue life of cracked structures due to material dispersions, which confirms the need for an engineering approach that takes into account this random aspect;
- (iii) comparing with the available experimental results, the developed probabilistic procedure enables a good and accurate estimation of FCG life in the case of a cracked lug-type joint, for different load ratios and different geometry configurations. The analysis carried out in this work indicates that the stochastic behavior of the material parameters and the residual stress field, generated in the crack tip zone, have a significant effect on predicting FCG life;
- (iv) the iso-probabilistic FCG life curves at different fatigue reliability values have been recorded. The obtained curves allow engineers to provide a significant maintenance planning of the cracked structures, by evaluating the remaining FCG lives in a more safe and efficient way;

- (v) the computational procedure exhibits a good ability in improving the deterministic FCG life evaluation by considering the material parameters' stochastic distribution and the crack-tip compressive residual stress field.

References

- [1] P.C. Paris, F.A. Erdogan, A critical analysis of crack propagation laws, *J. Basic Eng. Trans. ASME, Ser. D* 85 (4) (1963) 528–534.
- [2] W. Elber, The significance of fatigue crack closure, in: M.S. Rosenfeld (Ed.), *Damage Tolerance in Fatigue Aircraft Structures*, Toronto, Ontario, Canada, 21–26 June 1970, in: STP, vol. 486, 1971, pp. 230–242.
- [3] F. Erdogan, R. Roberts, A comparative study of crack propagation in plates under extension and bending, in: T. Yokobori, T. Kawasaki, J.L. Swedlow (Eds.), *Proc. Int. Conf on Fracture, Sendai, Japan, 12–17 September 1965*, Japanese Society for Strength and Fracture of Materials, 1966.
- [4] E.K. Walker, The effect of stress ratio during crack propagation and fatigue for 2024-T3 and 7076-T6 aluminium, in: *Effect of Environment and Complex Load History on Fatigue Life*. ASTM STR 462, American Society for Testing and Materials, Philadelphia, 1970, pp. 1–4.
- [5] S. Boljanovic, S. Maksimovic, Mixed mode crack growth simulation with/without overloads, *Int. J. Fatigue* 67 (2014) 183–190.
- [6] M.S. Toyosada, K. Gotoh, T. Niwa, Fatigue crack propagation for a through thickness crack: a crack propagation law considering cyclic plasticity near crack tip, *Int. J. Fatigue* 26 (2004) 983–992.
- [7] AFGROW, *Fracture mechanics and fatigue crack growth analysis software tool*. Ver 4.12.15.0, LexTech Inc., 2008.
- [8] A.H. Noroozi, G. Glinka, S. Lambert, A two-parameter driving force for fatigue crack growth analysis, *Int. J. Fatigue* 27 (2005) 1277–1296.
- [9] P. Dai, Z. Li, A plasticity-corrected stress intensity factor for fatigue crack growth in ductile materials, *Acta Mater.* 61 (2013) 5988–5995.
- [10] L.A. James, W.E. Anderson, A simple experimental procedure for stress intensity factor calibration, *Eng. Fract. Mech.* 1 (1969) 565–568.
- [11] A.F. Liu, H.P. Kan, Test and analysis of cracked lugs, in: *Fracture*, vol. 3, ICF4, Waterloo, Canada, 19–24 June, Springer, Berlin, 1977, pp. 657–664.
- [12] J.A. Abersson, J.M. Anderson, Cracked finite-elements proposed for NASTRAN, in: *Third NASTRAN Users' Colloquium*, NASA TMX-2893, 1973, pp. 531–550.
- [13] R. Rigby, M.H. Aliabadi, Stress intensity factors for cracks of attachment lugs, *Eng. Fail. Anal.* 4 (1997) 133–146.
- [14] J.H. Kim, S.B. Lee, S.G. Hong, Fatigue crack growth behavior of Al7050-T7451 attachment lugs under flight spectrum variation, *Int. J. Fatigue* 40 (2003) 135–144.
- [15] S. Mikheevskiy, G. Glinka, D. Algera, Analysis of fatigue crack growth in an attachment lug based on the weight function technique and the UniGrow fatigue crack growth model, *Int. J. Fatigue* 42 (2012) 88–94.
- [16] S. Boljanovic, S. Maksimovic, Fatigue crack growth modeling of attachment lugs, *Int. J. Fatigue* 58 (2014) 66–74.
- [17] M. Naderi, N. Iyyer, Fatigue life prediction of cracked attachment lugs using XFEM, *Int. J. Fatigue* 77 (2015) 186–193.
- [18] N. Sukumar, N. Moës, B. Moran, T. Belytschko, Extended finite element method for three-dimensional crack modelling, *Int. J. Numer. Methods Eng.* 48 (2000) 1549–1570.
- [19] A. Bahloul, C.H. Bouraoui, T. Boukharouba, Prediction of fatigue life by crack growth analysis, *Int. J. Adv. Manuf. Technol.* (2017), <https://doi.org/10.1007/s00170-017-0069-8>.
- [20] H. Ghonem, S. Dore, Experimental study of the constant probability crack growth curves under constant amplitude loading, *Eng. Fract. Mech.* 27 (1987) 1–25.
- [21] A.H. Noroozi, G. Glinka, S. Lambert, A study of the stress ratio effects on fatigue crack growth using the unified two-parameter fatigue crack growth driving force, *Int. J. Fatigue* 29 (2007) 1616–1633.
- [22] J.A.F.O. Correia, A.M.P. De Jesus, A. Fernandez-Canteli, R.A.B. Calçada, Modelling probabilistic fatigue crack propagation rates for a mild structural steel, *Frattura Integr. Strutt.* 31 (2015) 80–96.
- [23] H.F. Bueckner, A novel principle for the computation of stress intensity factors, *Z. Angew. Math. Mech.* 50 (1970) 529–546.
- [24] G. Glinka, G. Shen, Universal features of weight functions for cracks in mode I, *Eng. Fract. Mech.* 40 (6) (1991) 1135–1146.
- [25] G. Shen, G. Glinka, Determination of weight functions from reference stress intensity factor, *Theor. Appl. Fract. Mech.* 15 (1991) 237–245.
- [26] X. Wang, S. Lambert, G. Glinka, Approximate weight functions for embedded elliptical cracks, *Eng. Fract. Mech.* 59 (1998) 381–392.
- [27] Y. Jiang, J. Zhang, Benchmark experiments and characteristic cyclic plasticity deformation, *Int. J. Plast.* 24 (9) (2008) 1481–1515.
- [28] S.K. Paul, S. Sivaprasad, S. Dhar, M. Tarafder, S. Tarafder, Simulation of cyclic plastic deformation response in SA333 C–Mn steel by a kinematic hardening model, *Comput. Mater. Sci.* 48 (3) (2010) 662–671.
- [29] J. Lemaitre, R. Desmorat, *Engineering Damage Mechanics*, Springer, Berlin, ISBN 3-540-21503-4, 2005.
- [30] M. Naderi, S.H. Hoseini, M.M. Khonsari, Probabilistic simulation of fatigue damage and life scatter of metallic components, *Int. J. Plast.* 43 (2013) 101–115.
- [31] C. Bouraoui, R. Ben Sghaier, R. Fathallah, An engineering predictive design approach of high cycle fatigue reliability of shotpeened metallic parts, *Mater. Des.* 30 (2009) 475–486.
- [32] A. Nasr, C. Bouraoui, R. Fathallah, Y. Nadot, Probabilistic high cycle fatigue behaviour of nodular cast iron containing casting defects, *Fatigue Fract. Eng. Mater. Struct.* 32 (2009) 292–309.
- [33] W. He, J. Liu, D. Xie, Probabilistic life assessment on fatigue crack growth in mixed-mode by coupling of Kriging model and finite element analysis, *Eng. Fract. Mech.* 139 (2015) 56–77.
- [34] R.F. Sanches, A.M.P. De Jesus, J.A.F.O. Correia, A.L.L. da Silva, A.A. Fernandes, A probabilistic fatigue approach for riveted joints using Monte Carlo simulation, *J. Constr. Steel Res.* 110 (2015) 149–162.
- [35] Y.G. Zhao, T. Ono, Moment methods for structural reliability, *Struct. Saf.* 23 (1) (2001) 47–75.
- [36] A. Bahloul, A. Ben Ahmed, M.M. Mhala, C. Bouraoui, Probabilistic approach for predicting fatigue life improvement of cracked structure repaired by high interference fit bushing, *Int. J. Adv. Manuf. Technol.* 91 (5–8) (2017) 2161–2173.
- [37] M.R. Ayatollahi, S.M. Javad Razavi, M.Y. Yahya, Mixed mode fatigue crack initiation and growth in a CT specimen repaired by stop hole technique, *Eng. Fract. Mech.* 145 (2015) 115–127.
- [38] M. Eltaief, A. Chateaufneuf, C. Bouraoui, T. Hassine, Dynamic approach for optimal inspection planning of fatigue cracked components, *J. Constr. Steel Res.* 115 (2015) 263–275.
- [39] K. Kathiresan, T.M. Hsu, *Advanced life analysis methods-crack growth, analysis methods for attachment lugs*, AFWAL 1984-TR-84-3080, vol. II, 1984.