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The legacy of Jean-Jacques Moreau in Mechanics

The legacy of a deep thinker: Jean-Jacques Moreau

Jean-Jacques Moreau[1] (1923–2014) was a visionary mechanician and mathematician whose contributions to fluid and solid mechanics are of prime importance. Attracted by profound problems he liked to face with no compromise, sometimes building the tools when these were lacking to solve them, he had not a skimpy conception of rigor, but was keener at reaching new sensible results.

Vladimir Arnold once claimed that mathematics is a part of physics [1]; in that sense, the mathematics of Jean-Jacques Moreau was for mechanics as it is for applied mathematicians in the tradition of Newton, primarily a tool to discover, understand, and describe the laws of nature.

Out of the many contributions of Jean-Jacques Moreau, three need to be underlined in particular: the discovery of helicity and its conservation in inviscid fluids, the foundation of Convex Analysis in infinite dimensions, and the so-called Non-Regular, or 'Non-Smooth' Mechanics. Each of these contributions have open the way to flourishing fields still under active development.

Helicity. In addition to several incursions into diverse particular topics like reaction–diffusion systems [2] or suspended particles in fluids [3], the great work of Jean-Jacques Moreau in his early career has been the discovery of the conservation of helicity in inviscid flows [4,5].

It is not, admittedly, given to every scientist to have the chance discovering even once in his lifetime a conservation law in nature. Helicity is vorticity projected on the fluid velocity [4]. Jean-Jacques Moreau showed extremely elegantly that this quantity, integrated on a patch (which he called 'îlot' [5]), is conserved along the Lagrangian path of the patch in the fluid, in the absence of viscous dissipation. This is a very strong remark, in the same vein as Leray's demonstration of the impossibility of a singularity in two-dimensional viscous flows [6], since it constrains the possible outcomes in any particular situation by a rigorous bound. Jean-Jacques Moreau anticipated helicity conservation, which can be viewed as an integral version of Kelvin's circulation theorem, as a possible basis for a Hamiltonian representation of fluid flows in three dimensions, similar to the representation by Onsager [7] of two-dimensional flows by a collection of point vortices. Modern developments, however, although taking advantage of this crucial bound in fluids, and other areas of science, also underline the decisive role played by viscosity in the local dynamics of vortex lines.

From cavitation to Convex Analysis. Among the many problems that Jean-Jacques Moreau addressed in Fluid Mechanics, cavitation, that he had formulated as a unilateral problem [8], provided him with the motivation for addressing more general problems:

> "The study of dynamical problems for systems of finite or infinite freedom with unilateral constraints (e.g., the inception of cavitation in a perfect incompressible fluid) initially motivated the part taken by the author in the development of convexity theory. " [9]

A general mathematical framework for describing very strong nonlinearities, of which unilateral problems are a paradigmatic example, was missing, at least for systems with infinite degrees of freedom. So, instead of following a general tendency that he found regrettable, "traditional physics almost always starts from linear laws as first approximations to which improvements have possibly to be added by taking terms of higher order into account", he rather built by himself all the mathematics necessary to attack unilateral problems with no compromise on nonlinearity. It was the origin of his pioneering contribution to Convex Analysis [10], a domain where his name stands on the same footing as that of Werner Fenchel and Ralph Tyrell Rockafellar. He started in Montpellier the *Séminaire d'analyse unilatérale* in 1968, which then became the

[1] We use the hyphenated version of Jean-Jacques Moreau's first name, as he used to do in his first Notes to the *Comptes rendus*. Later, the hyphenation has become less systematic, so that both abbreviations (J.-J., J.J. and even simply JJ) are commonly encountered.

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Fig. 1. Jean-Jacques Moreau and Michel Jean contemplating a non-smooth landscape in Cape Sounion, Greece (1994). Picture by Mrs L. Moreau.

Séminaire d'analyse convexe in the early seventies. The Fenchel–Moreau theorem (on bipolar functions), and the 'proximal function' (or regularization) of Moreau–Yosida and more generally his celebrated course at the Collège de France [11] that he gave at the invitation of Jean Leray, are a few examples of the profound impact that he had on the subject. The *Journal of Convex Analysis* published recently a special issue [12] in homage to Jean-Jacques Moreau, which gives an idea of the breath of this work essential to optimization, optimal control, and whose applications concern, beyond mechanics, economics as well.

Besides his taste for abstract mathematical constructions, exemplified in his contribution to Convex Analysis, Jean-Jacques Moreau never overlooked mechanics. He actually used to say that mechanics is the very domain in physics where the notion of convexity is not only natural, but necessary. He often recalled at the opening of his talks that a table with N feet all lying on a floor (with unilateral contact of course!) can only be in equilibrium if the projection of its center of gravity lies within the convex envelope of the N feet.

As a direct follow-up of his work on Convex Analysis, he tackled the problem of the quasi-static evolution of elastoplastic bodies, which he formulated in the form of a strongly nonlinear evolution equation, prototypical of the process of 'sweeping by a moving set' [13]. He then forged new mathematical [14] and numerical formalisms [15], obtaining the first proof of existence and uniqueness for the stresses history in an elastoplastic material.

Non-smooth dynamics. From the beginning of the eighties, Jean-Jacques Moreau concentrated on non-smooth dynamics. It was his program to handle properly the non-smoothness in time of the dynamic evolution of mechanical systems. For this purpose, he would write the equations of dynamics in the presence of shocks using velocity fields with bounded variations only, not resorting to accelerations explicitly written as second derivatives in time [16] and this, in a rigorous way without approximating a discontinuous function as the limit of continuous functions (although a discontinuity is an idealization of nature that can, according to the ambivalent presentation of Dirac himself [17], equally be viewed as the limit of a smooth function).

In the mid-eighties, he developed his own numerical codes[2] that he applied to solve various problems involving impacts of rigid bodies, in granular materials in particular, for which he enjoyed a fruitful interaction with physicists. He devised rigorous algorithms for unilateral problems and friction [18] exempt from regularization or penalization artifacts. His pioneering simulations attracted the interest of experimentalists whose experiments were thus reproduced and explained, including the most puzzling ones.

Found of arts in general, and very knowledgeable in many artistic domains, Jean-Jacques Moreau got interested in the upholding of old buildings, those made of a stack of stones, merely assembled by a fragile cement. Fig. 1 shows him with his fellow Michel Jean as they were attending in Greece a European meeting devoted to 'Monuments under seismic actions'.

Jean-Jacques Moreau was all but a career-oriented person. The distance from his provincial city (Montpellier) to the capital shielded him from the Parisian atmosphere and allowed him to do his research and teaching in a quiet environment, in all freedom. A school of thought on Non-Smooth Mechanics and Analysis has spontaneously developed around, and after him, but he did not favor, nor encouraged a tradition of students who would have been the inheritors and protectors of their mentor's spirit. He did not, either, looked for recognition, although he has not been surprised when it came.

As seen from the list of references below, Jean-Jacques Moreau has always first announced his discoveries in the *Comptes Rendus de l'Académie des Sciences* (e.g., Helicity [5], Convex Analysis [10], Non-Smooth Mechanics [16]). It is therefore quite

[2] In 1986 he was awarded the Joanidès prize by the French Academy of Sciences. He used this prize for purchasing a personal computer that allowed him, as he had just retired, to work and play with passion, doing simulations and numerical experiments.

natural to publish this thematic issue celebrating the pioneering contributions of Jean-Jacques Moreau in Mechanics in the present-day version of them, the *Comptes Rendus Mecanique*.

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