C. R. Mecanique 346 (2018) 351–365

Contents lists available at ScienceDirect

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# Integration of topological modification within the modeling of multi-physics systems: Application to a Pogo-stick

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ARTICLE INFO

*Article history:*
Received 13 July 2017
Accepted 23 January 2018
Available online 21 February 2018

*Keywords:*
Topological approach
Topological modification
Connectivity
Pogo-stick
MGS language

ABSTRACT

The present work tackles the modeling of multi-physics systems applying a topological approach while proceeding with a new methodology using a topological modification to the structure of systems. Then the comparison with the Magos' methodology is made. Their common ground is the use of connectivity within systems. The comparison and analysis of the different types of modeling show the importance of the topological methodology through the integration of the topological modification to the topological structure of a multi-physics system. In order to validate this methodology, the case of Pogo-stick is studied. The first step consists in generating a topological graph of the system. Then the connectivity step takes into account the contact with the ground. During the last step of this research; the MGS language (Modeling of General System) is used to model the system through equations. Finally, the results are compared to those obtained by MODELICA. Therefore, this proposed methodology may be generalized to model multi-physics systems that can be considered as a set of local elements.



## 1. Introduction

A multi-physics system is a system containing components whose behavior involves various physics (or physical domains). The modeling of such a system is challenging, since it needs a high coordination between different types of modeling dedicated to the different physics involved. Consequently, the links and combination between different physics in the same system are handled in topological modeling of multi-physics systems.

The purpose of the present study is to find a unified tool to model multi-physics systems. The topology-based modeling is used to reach this objective. The idea of applying a topological approach for the modeling of mechanical systems was initiated by the graph theory. As shown in the works of Plateaux et al. [1] and of Björke [2], all mechatronics or multi-physics systems can be broken down into sub-systems belonging to different fields of mechatronics (i.e. electronics, mechanics, etc.). Then each field can be characterized by its topological structure and behavioral laws. Previous developments have been performed following this paradigm.

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https://doi.org/10.1016/j.crme.2018.01.004

In 2007, Plateaux et al. [1] applied the KBR topological graph (named KBR in reference to its creators Kron, Branin and Roth) to model the mechatronic systems. Indeed, the KBR topological graph allows one to obtain relationships between its elements whatever the unknown parameters and specifications of the studied systems. For more details, this graph relates the topological structures of complexes of chains and co-chains. The behavior tensor transforms the variables associated with 1-cochain in variables associated with 1-chain. Moreover, the KBR topological graph permits the distinction between the topological structure of the system and the associated physics.

Miladi in [3] made possible the separation of the topology and the physics in order to have generic local models that allow the optimization of the system behavior. She applied a topological approach for modeling mechatronic systems. With the topological approach, any model can be characterized by local relations between its elements thus making it possible to dissociate the topology from the physics. Miladi used the MGS language to implement the local behavior laws. This language is characterized by its unifying view on several computational mechanisms. MGS embeds the idea of topological collections and their transformations within the framework of a simple dynamically typed functional language [4,5]. It was initially devoted to the simulation of biological processes [6,7]. The basic MGS data structures are presented in the works of Cohen [5] and Spicher [4]. Topological collections can be defined as a set of positions, filled with values, and organized by a topology defining the neighborhood of each element in the collection. These transformations are functions acting on collections and defined by a specific syntax.

The present work aims at enhancing the previously mentioned methodology to enable the modeling of multi-physics systems with dynamically variable structure. In order to improve the modeling of multi-physics systems, a unified and a parameterized tool using the MGS language is created.

The originality of this work lies in the following aspects. First, in using analogies and the concept of changing graphs all along the methodology to validate the modeling of systems with a variable topology. Secondly, in using the MGS language to model multi-physics systems applying mechanical–electrical analogy. Finally, in the integration of the connectivity within the topological structure for multi-physics systems modeling.

Then this paper is organized as follows. Section 2 exposes a state of the art on topological approach (graph theory) and then presents the topological approach that will be used in our work. The proposed methodology consists in integrating the topological modification. Section 3 presents the case study of a Pogo-stick and the results obtained by the MGS language are compared to those obtained by MODELICA. Section 5 covers a discussion. Finally, Section 5 deals with a conclusion.

## 2. Topological modification

In this section, a brief overview is given on the topological graph, and then on the topological approach and its various applications in mechanical systems modeling. Then, our proposed methodology based on modeling the topological modification through a topological graph is presented.

### 2.1. State of the art

Given the intricacy of the suggested methodology, we start by outlining the evolution of the topological graph from the graph theory to the topological structure. Then, the topological graph approach applied in this work is detailed.

*Topological graph*

In 1741, the first known application of the graph theory was performed by Euler, who integrated the topological graph for the resolution of the physics of “The Bridges of Konigsberg” [8]. Later, Kron applied the topology and graph theory through a systematic methodology of decomposition of a physical system named “Diakoptic” [9]. Then, in 1955, Roth validated the work of Kron by applying algebraic topology on networks and electric machines [10]. This work was generalized by Branin [11] through the inverse analogy of Firestone [12]. Based on the Trent [13] and Kron works, Paynter established a special oriented graph, hence the creation of the “bond graph” or “link graphs” technique. Still in 1955, Trent implemented the representation of a topological structure with a linear graph.

In 1995, Björke advanced the philosophy that he had followed for 25 years consisting in the identification of scientifically based tools related to the concept of connection. In fact, he used the interconnection graph in his work while introducing the notion of a topological structure for each system [2].

In 2003, Tonti created a nodal diagram [14]. This diagram is a summary of properties and objects, and he implemented it within the framework of his first network branches and nodes [15].

Maurice dedicated his research to mathematical methods based on oriented graphs such as the topological structure [16].

In 2011, Plateaux heavily relied on the works of Kron, Branin and Roth applying the KBR topological graph to model multi-physics systems. Moreover, he used the MODELICA language that led him to apply this graph. He applied the topological graph to model multi-physics systems and had a particular interest in modeling truss structures [1].

Van der Schaft’s work [17] was devoted to the representation of Port-Hamiltonian systems through Kernel representation and Dirac structure, particularly for the Port-Hamiltonian Differential–Algebraic systems. Then in 2006, considering the overall physical system as the interconnection of simple subsystems, mutually influencing each other via energy flows, Van der Schaft established his research on the network modeling (power-based), which is the basic starting point of the

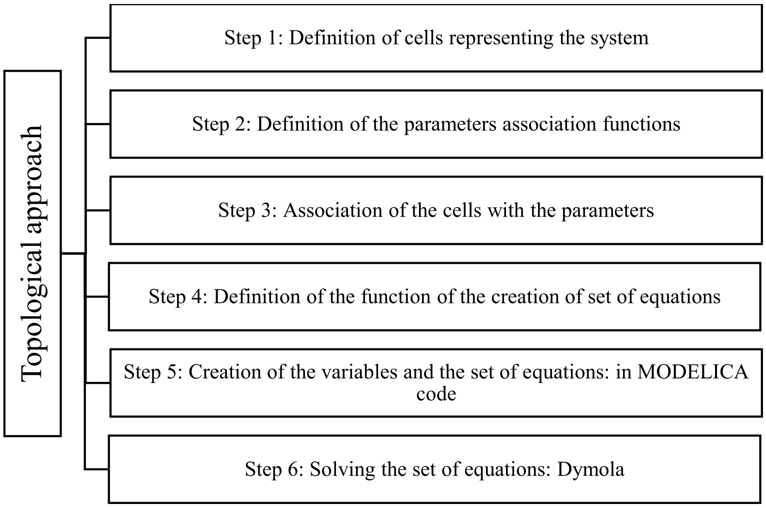



**Fig. 1.** The topological approach steps.

Port-Hamiltonian systems theory. In many cases, it is useful to model fast transitions in physical systems as instantaneous switches. For that purpose, Van der Schaft was interested in Port-Hamiltonian systems with a variable topology [18].

Relying on the works of Van der Schaft [17] and Paynter [19], Magos generalized the representation based on the energetic concept and Graph theory [20]. He applied the energetic approach by creating a structure modeling.

A modular approach based on the energetic concept was introduced by Magos [20]. He was interested in modeling and analyzing systems with a variable topology, whose dynamics depend on discrete variables. In addition, he used the study of modification for the dynamic graph and the change of fundamental matrix to integrate the notions of disconnection and connection. Then, in 2007, Valentin collaborated with Magos to extend a generic method to design a port-Hamiltonian formulation modeling all geometric interconnection structures of a physical switching system with varying constraints [21].

*Topological approach*

In our work, the topological approach proposed by Miladi [22] is used. Thanks to this topological approach, any structure can be characterized by local relations between its elements, which makes it possible to dissociate its topology and its physics. Miladi explained the main idea of her topological approach based on the concepts of topological collections and transformations [23]. The topological structure was studied from a local point of view and not from a global point of view. In Fig. 1, we present the six steps of the topological approach. Interested readers can refer to Miladi [3] for further information.

Miladi applied her topological approach to different systems; beginning with the frame structures (2D and a 3D frame), then the piezoelectric stack (structure) and finally she validated it with the example of a spring with an external load F and with the example of a DC motor.

Thus, the application of this topological approach in the suggested methodology takes shape. The latter uses interconnection graphs, topological graphs, and the notion of connectivity within a topological structure.

This methodology will demonstrate and ensure the internalization of the topological modification within a topological structure via a mechanical–electrical analogy while using the notion of duality.

## 2.2. Proposed methodology

In this section, a methodology for modeling the topological modification in the structure of a multi-physics system is proposed in order to have a single model that represents the corresponding system.

It is worth noting that this methodology uses in modeling systems a variable topology with the MGS language, boosts the topological approach created by Miladi [3], and preserves the dissociation of the physic and topology of the same model.

In the following paragraph, we present the different steps of the proposed methodology to emphasize the topological modification using connectivity.

We introduce the methodology with a full connection diagram as shown in Fig. 2.

Fig. 3 illustrates a brief description of the methodology steps. Then, the details of all the steps are represented.

*Details of the methodology steps*
*Step 1. Description of "Hybrid Dynamic Model"*

The first step consists in describing a system by using three types of graphs namely primal and dual graphs, interconnection graph, and dynamic graph. According to Harary in [24], a dynamic graph can be viewed as a discrete sequence of static

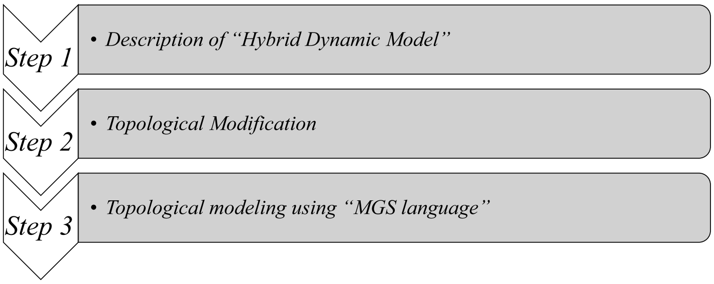



**Fig. 2.** Diagram of the proposed methodology steps.

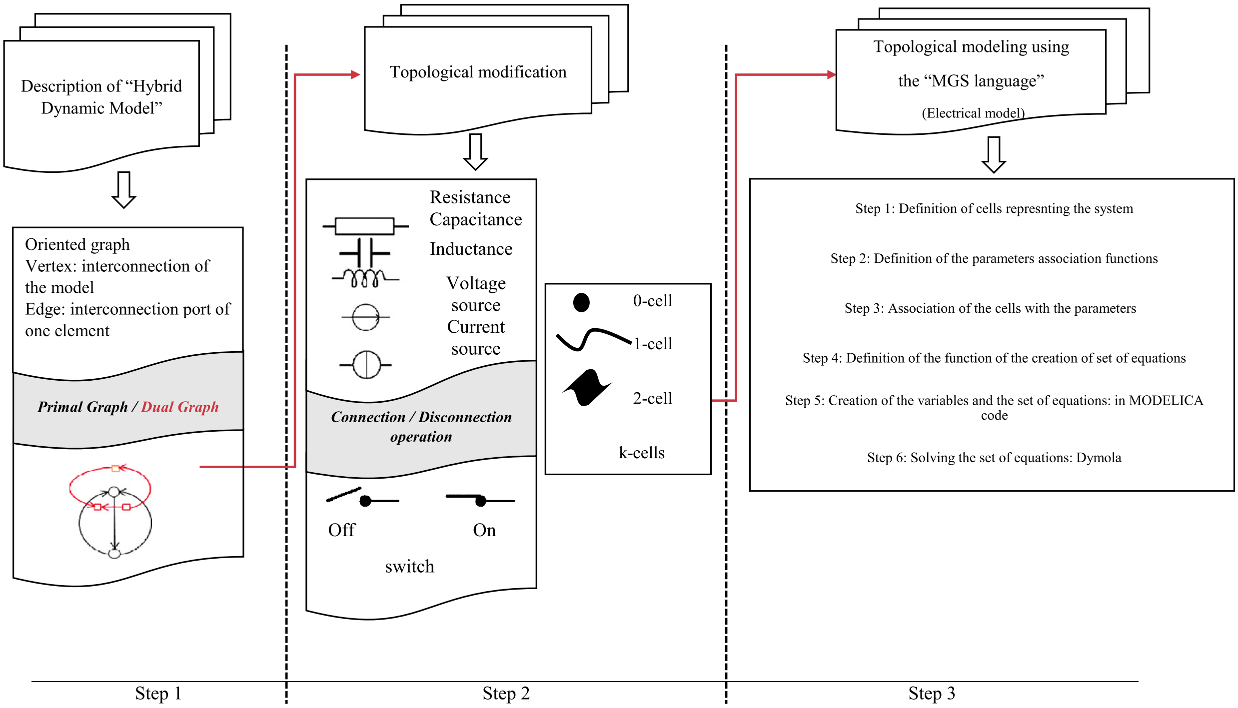



**Fig. 3.** Brief illustration of the steps of the proposed methodology.

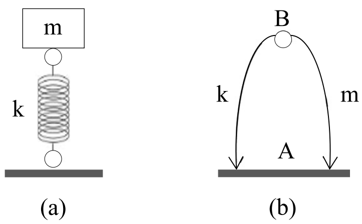



**Fig. 4.** Mass-spring example: (a) physical system, (b) interconnection graph.

graphs. The dynamic (not oriented) graph is used in this methodology to show the different static states of the system. Furthermore, it can also be used as a flexible modeling concept in order to study structural changes in a broad range of systems as diverse as population communities and arms race, compartmental systems and multi-agent formations, chemical processes and multi-controller configurations for reliable control of complex systems [25].

The second graph, named the interconnection graph, represents the system as an oriented graph. This interconnection graph is made up of vertices and edges that respectively describe the interconnection of the model and the interconnection port of one element [26,27]. Interconnection graphs are used for the physical transformation of systems in several domains [2]. The ports interconnection graph is written as an ordered pair $G = (V, E)$, where $V$ is a set of non-valid $n_v$ points called nodes ($n_x \in V$) and $E$ is a set of $n_e$ and a couple of $V$ elements called branches ($n_i \in E/e_i = \{v_x, v_y\}$). The convention of the flux of the sign of variables for the orientation of branches defines the flooding current of an electrical circuit or the

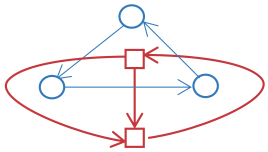


**Fig. 5.** Interconnection graph (mechanical system) and the associated dual graph (electrical system).

**Table 1**
Verification method of Hodge theory.

| | Vertices | Edges | Faces |
|---|---|---|---|
| Primal graph | $n_v = v$ | $n_e = e$ | $n_e - n_v + 2 = f$ |
| Dual graph | $n_v^* = f$ | $n_e^* = e$ | $v$ |

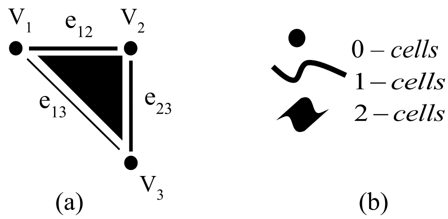



**Fig. 6.** (a) Representation as a cellular complex, (b) $k$-cells.

direction of movement in a mechanical system. Then $v_x$ is the starting point and $v_y$ is the arrival point. The following example shows a simple physical system (mass-spring) with its interconnection graph, which is illustrated in Fig. 4 [2].

The transition from the interconnection graph of the mechanical system to the network graph of the analog electrical system will be achieved through the dual graph, as shown in Fig. 5. It shows the transition from the primal graph to the dual graph (simple illustration of the concept).

The interconnection graph of the mechanical system is represented by the blue graph and the interconnection graph of the analogue circuit of the mechanical system by the red one.

The following step requires the representation of a mechanical system as an electrical network. To solve certain mechanical problems more adequately, a direct electrical analogy has been used.

Poincaré generalized the Euler number given a topological invariant for CW-complexes. As Poincaré stated, this invariant can be determined by the homology group with the alternating sum of Betti numbers [28,29]. He also defined the duality of Poincaré that linked the homology group to the cohomology group where the relations between the dimensions of primal and dual topological spaces are defined. The determined dual graph gives the structure of the analogous elements. Then, direct analogy involves a similar set of equations and the topological duality named Hodge duality [30] ensures the transition from the primal graph to the dual graph. If $G = (V, E)$ is a planar graph, we can associate a dual graph $G^* = (V^*, E^*)$. $V^*$ is a set of $n_v^* = n_e - n_v + 2$ dual vertices $v_x^* \in V^*$ and $E^*$ is a set of $n_e$ dual edges who cross the branches of $G$ in such a way that the dual branch $e_i^*$ cut only the branch $e_i$ of $G$.

Thus, the Hodge theory has a verification method, as shown in Table 1, where $f$ is the number of faces in the interconnection graph, $e$ the number of edges in the interconnection graph, and $v$ the number of vertices in the interconnection graph.

*Step 2. Topological modification*

In case of a mechanical system, the dynamic of change is linked to the connection or disconnection of the elements in the system. Every operating mode corresponds to a different interconnection of the elements of the system. The variable-topology systems are hybrid systems whose topological interconnection of its elements is variable [20].

In the second step of this new methodology, we move from the interconnection graph of analog circuits to the topological graph. Afterwards, the notions of connectivity (disconnection and connection) are integrated into the topological structures of multi-physical models with variable topology. As aforementioned, the connectivity can be modeled either through connection or disconnection or the modification of the demanding boundary condition. This paper highlights the topological modification through the change of states of the system (two states are represented by the switches states (off or on)).

Fig. 6a shows an example of the topological graph. It is made up of three 0-cells ($V_1$, $V_2$, $V_3$) and three 1-cells ($e_{12}$, $e_{13}$, $e_{23}$), which are the components of the system. Fig. 6b then shows examples of $k$-cells, 0-cells representing vertices, 1-cells representing edges, and 2-cells representing faces.

In order to proceed with the following step, a topological graph of the analog circuit of the mechanical system should be illustrated and adjustable.

*Step 3. Topological modeling using "MGS language"*

During this stage, we proceed by applying the steps of the topological approach [20], which employs the topological collections and their transformations. The latter consists in depicting the studied system by a cellular complex with which the variables of interest are associated. Via the transformations, the local behavior law and equilibrium equations of the different components of the studied model are defined. Lastly, the generation of the system of equations is accomplished through sweeping all the cells representing the system.

*Step 3.1. Definition of k-cells representing the analog circuit*

Only 0-cells and 1-cells are employed: 0-cells represent the nodes noted $V_i$ $(i = 1 \ldots N_\mathrm{n})$; 1-cells stand for the component noted $e_{ij}$ $(i = 1 \ldots N_{n-1},\ j = 2 \ldots N_n)$, the electrical mass noted $e_{\mathrm{m}i}$ $(i = 1 \ldots N_\mathrm{m})$, where $N_\mathrm{n}$ and $N_\mathrm{m}$ exemplify, respectively, the number of nodes and electrical mass. What distinguishes the 1-cells representing the components from those representing the frames, the forces, and the electrical mass is the fact that they are bounded by two 0-cells.

*Step 3.2. Definition of the functions of the association of the electrical parameters*

Functions that associate the electrical parameters are prescribed (current, voltage ...) with the corresponding *k*-cells representing the analog structure.

*Step 3.3. Association of the electrical parameters with the k-cells*

After defining these functions, the electrical parameters are associated with the corresponding *k*-cells.

*Step 3.4. Definition of the function of the creation of the system equations*

The set of equations of the analog structure which is written in MODELICA code is generated. Two functions are distinct namely (1) the declaration of the variables of the analog structure, and (2) the creation of the equations by sweeping all the cells representing the analog structure.

*Step 3.5. Resolution*

The equations are written in MODELICA code (applying MGS language) and DYMOLA is used as a solver [28]. When dealing with the components of the analog circuit, the independence of the local law behavior and the equilibrium through transformations, from the topological structure declared through the topological collections, is conspicuous.

## 3. Case study

The case study is made up of three steps. First, the Pogo-stick model is utilized (the three aforementioned steps). The latter is a mechanical system with 1 degree of freedom. Secondly, in order to further endorse the current methodology (Pogo-stick), the modular approach introduced by Magos is applied. In fact, the latter is employed to compare our results using this methodology (topological modeling) to the one used in this study to validate the connection and disconnection modeling notions. It is noteworthy that Magos was interested in the modeling and analyzing of systems of a variable topology that is based on an energetic concept and fundamental matrix. Moreover, one must bear in mind that the two methodologies share similarities namely that of the use of the connectivity notion on the systems of variable topology [8].

In order to validate the proposed methodology, the results obtained by the MGS language are compared with those obtained by MODELICA are compared.

### *3.1. Modeling of the Pogo-stick using the new methodology*

In the following section, we apply the three aforementioned steps of the proposed methodology to a Pogo-stick model.

*Step 1. Description of the "Pogo-stick"*

In order to validate the methodology, a simple mechanical system is studied. Taking into consideration that the solid 1 $(s_1,\ m_1)$ is attached to one end of a spring (vertical) with unconstraint length $x_0$ and a damper (vertical) whose other ends are anchored in a solid 2 $(s_2,\ m_2)$. During this study, it is dictated that $s_2 \ll s_1$, therefore $m_2$ is neglected. Spring (s) and damper (d) are in parallel positions (Fig. 7). The equilibrium state of the system named state 1 corresponds to the contact between "Pogo-stick" and the floor. The other state of the system (state 2) corresponds to the situation in which the main set of a "Pogo-stick" is not in contact with the floor. The solid 1 $(s_1)$ is attached to Hook's law spring recall $F_\mathrm{s} = -kx$ and to Hook's law damper recall $F_\mathrm{d} = \mathrm{d}\,\dot{x}$. From Newton's law $\sum F = ma$, we obtain: $\ddot{x} = -mg - \frac{d}{m}\dot{x} + k(x_0 - x)$.

Subsequently, the following step represents a mechanical system as an electrical network. To solve certain mechanical problems more easily, a direct electrical analogy has been used.

Firstly, it must be recognized that, while the mechanical elements are connected in series, the analogous electrical elements must be connected in parallel. Our hybrid system "Pogo-stick" is composed of three elements; mass, damper, and spring. Then, a mass is displayed through an inductance and a spring that must transmit all the force from one part of the system to another which will be the condenser across the grounded line of a single side [12]. While a damper which was

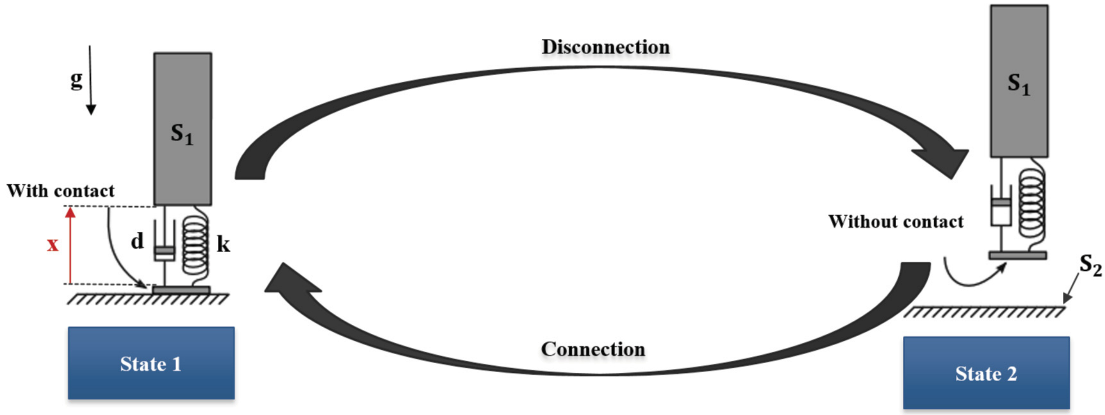



**Fig. 7.** Dynamic graph of a Pogo-stick in the two modes.

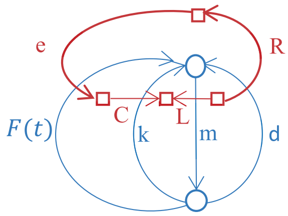
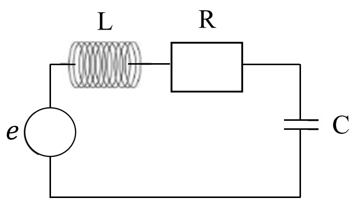



**Fig. 8.** Interconnection graph of the mechanical system.

| Mechanical component | Electrical component |
|---|---|
| d | R |
| m | L |
| k | 1 / C |
| F | e |

(a)




(b)

| Pogo-stick | RLC |
|---|---|
| d $\dot{x}$ | R i(t) |
| m$\ddot{x}$ | L $\frac{di}{dt}$ |
| k $(x_0 - x)$ | $\int$ i dt /C |

(c)

**Fig. 9.** (a) Analogous components with direct analogy, (b) analog circuit, (c) analogous behavior laws.

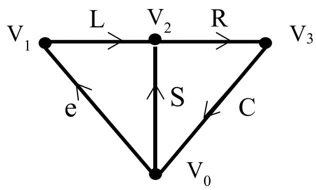



**Fig. 10.** Interconnection graph G1 of the analog circuit.

designed to dissipate the Kinetic Energy of the mechanical system (or used to suppress vibrations) will be represented by a resistance, the excitation force will be shown as a voltage source. The transition from the interconnection graph of the mechanical system to the network graph of the analog electrical system will be done through the dual graph as shown in Fig. 8. The dual graph was defined by the duality of Poincaré [28].

Fig. 9 illustrates the direct analogy of mechanical components with their electrical analogous components (Fig. 9a). The analog circuit shown in Fig. 9b clarifies the serial connected analogous elements. Furthermore, the analogy of the different components' behavior is demonstrated in Fig. 9c. Thus, Fig. 10 represents the graph of an interconnection of the analog circuit. This graph is obtained by superposing different interconnection graphs at both states of connection and disconnection.

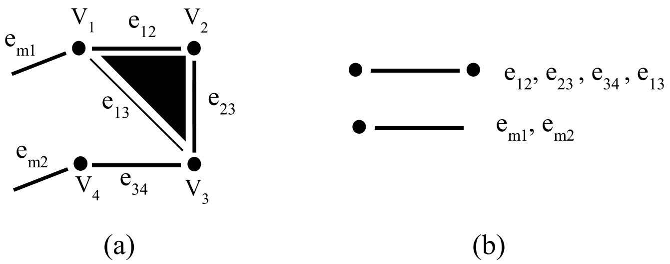



**Fig. 11.** (a) Presentation as a cellular complex, (b) two possibilities of a 1-cells of our analog circuit of a Pogo-stick model.

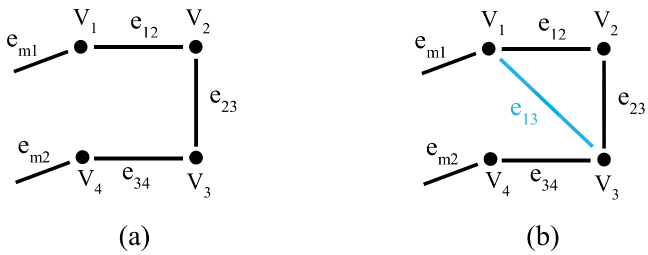



**Fig. 12.** (a) Connection state, (b) disconnection state of our analog circuit of a Pogo-stick model.

*Step 2. Topological modification*

At this point, the connection operation was referred to an open switch and a closed switch presented the disconnection operation. On the one hand, we have to move from the interconnection graph of analog circuit to the topological graph.

Fig. 11a shows the topological graph of our model. It is made up of four 0-cells ($V_1, V_2, V_3, V_4$), four 1-cells ($e_{12}$, $e_{23}$, $e_{34}$, $e_{13}$). For more details, $e_{12}$, $e_{23}$, $e_{34}$, $e_{13}$ represent respectively the elements of resistor, capacitor, inductor and closed switch as illustrated in Fig. 10, where $e_{\mathrm{m}1}$ and $e_{\mathrm{m}2}$ represent the electrical mass (Fig. 11b).

Figs. 12a and 12b depict respectively the connection state with an open switch and the disconnection state in which the edge $e_{13}$ refers to a closed switch.

These two states represent the topological modification of our model (analog circuit).

*Step 3. Modeling using the topological approach: "MGS language"*

The third stage of our present study consists in incorporating the novel notion of "topological modification" into the topological modeling of multi-physics systems with varied topology. See Fig. 13.

### *3.2. Modeling a Pogo-stick using the methodology of MAGOS [20]*

In this section, the methodology of MAGOS [20] is applied to the Pogo-stick model. A modular approach based on the energetic concept was introduced by Magos [20]. He was interested in modeling and analyzing systems with a variable topology, whose dynamics depend on the discrete variables. Added to that, he used the energetic approach by creating a structure modeling in which he integrated the notions of disconnection and connection. These latter are the pillars of the study of modification for the dynamic graph and the change of fundamental matrix. On the one hand, this approach is useful to the current study, as it helps us validate the notions of connectivity in the topological structure and the incorporation of the topological alteration within the structure. On the other hand, by applying the same model, i.e. the Pogo-stick and it analogous circuit, the results of the MAGOS study will be compared and contrasted to the results of the present research. The following section will deal with the following tasks: first, the incidence matrix of the primal and the dual graphs is distinguished via the interconnection graph of our analogous circuit in the mechanical model. Afterwards, the Kirchhoff's equations (1) obtained from the incidence matrices is determined and a family of Dirac's structure is defined. This is done in order to prove that the switch configuration validates Kirchhoff's law:

$$D(G_w, w) = \left\{ (p_f, p_e) \in \mathbb{R}^{n_e} \times \mathbb{R}^{n_e} \Big/ \begin{bmatrix} IM(G_w^*, w) \\ 0_{n_v \times n_e} \end{bmatrix} \cdot p_e + \begin{bmatrix} 0_{n_v \times n_e} \\ IM(G_w, w) \end{bmatrix} \cdot p_f \right\} = 0 \qquad (1)$$

For that, we determinate the parameterized incidence matrix $IM(G_w, w)$ based on Eq. (2).

$$IM(G_w, w) = M_T(G_w) \cdot IM(G_r) \qquad (2)$$

$IM(G_r)$ is the incidence matrix of graph reference presented the matrix whose all switches are open.

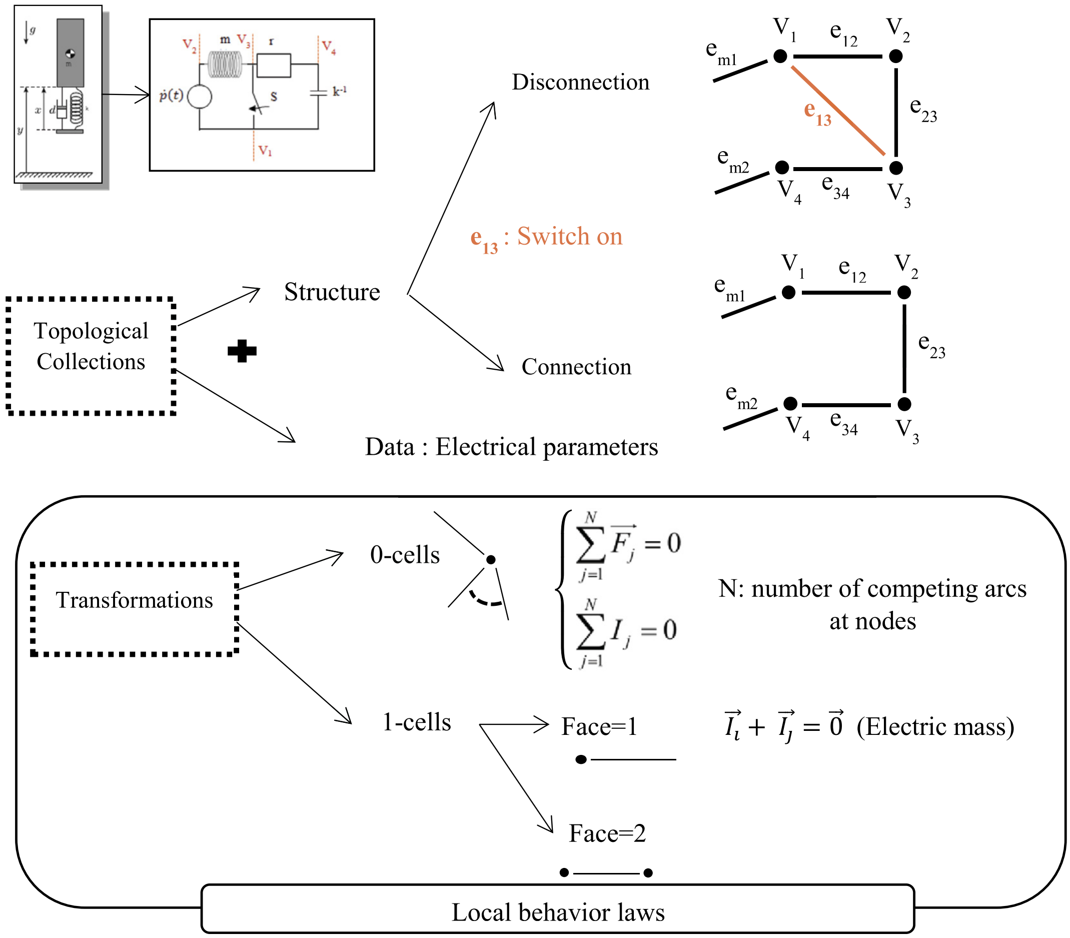



**Fig. 13.** Modeling using topological collections and transformations where each cell is studied in relation with its dimension to apply the right law and to define the right set of variables.

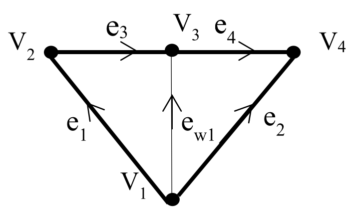



**Fig. 14.** Primal interconnection graph $G$.

$M_T(G_w)$ is the transformation matrix.
$IM(G_w, w)$ one of switches should be closed in the circuit.
Primal interconnection graph $G$ (Fig. 14).

Determination of $IM(G_w, w)$
$IM(G_r)$: with $S_{w1}$ open

$$IM(G_r) = \begin{bmatrix} 1 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & -1 & 0 & -1 \end{bmatrix}, \qquad M_T(w) = \begin{bmatrix} 1-w_1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ w_1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

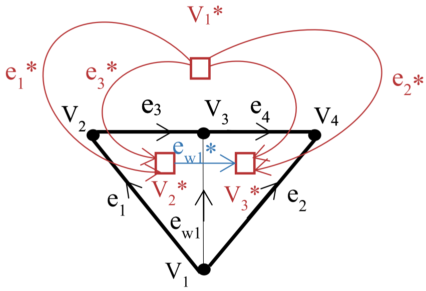



**Fig. 15.** Dual interconnection graph $G^*$.

Application of Eq. (2)

$$IM(G_w, w) = \begin{bmatrix} 1-w_1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ w_1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & -1 & 0 & -1 \end{bmatrix}$$

Then, $IM(G_w, w)$ is obtained:

$$IM(G_w, w) = \begin{bmatrix} 1-w_1 & 1-w_1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ w_1 & w_1 & -1 & 0 \\ 0 & -1 & 0 & -1 \end{bmatrix}$$

Dual interconnection graph $G^*$ (Fig. 15).

$$IM(G_w^*, w) = M_T(G_w^*) \cdot IM(G_r^*) \tag{3}$$

Determination of $IM(G_w^*, w)$

$IM(G_r^*)$: with $S_{w1}^*$ open

$$IM\left(G_r^*\right) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & 0 & -1 & 0 \\ 0 & -1 & 0 & -1 \end{bmatrix}$$

and

$$M_T\left(G_w^*\right) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1-w_1 & 0 \\ 0 & w_1 & 1 \end{bmatrix}$$

Application of Eq. (3)

$$IM\left(G_w^*, w\right) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1-w_1 & 0 \\ 0 & w_1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & 0 & -1 & 0 \\ 0 & -1 & 0 & -1 \end{bmatrix}$$

$$IM\left(G_w^*, w\right) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ w_1-1 & 0 & w_1-1 & 0 \\ -w_1 & -1 & -w_1 & -1 \end{bmatrix}$$

We apply Eq. (2) in our analog circuit to obtain Eq. (1):

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ w_1-1 & 0 & w_1-1 & 1 \\ -w_1 & -1 & -w_1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \cdot p_e + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1-w_1 & 1-w_1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ w_1 & w_1 & -1 & 1 \\ 0 & -1 & 0 & -1 \end{bmatrix} \cdot p_f = 0$$

At the end of the application of Magos' methodology, the Kirchhoff's equations are obtained from parameterized incidence matrices. This expression summarizes the different dynamics that a system can present using the notions of disconnection and connection.

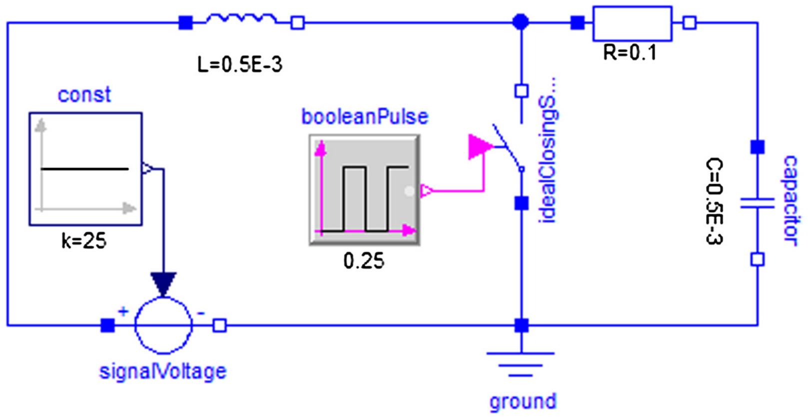



**Fig. 16.** Current of resistor, capacitor (2 states).

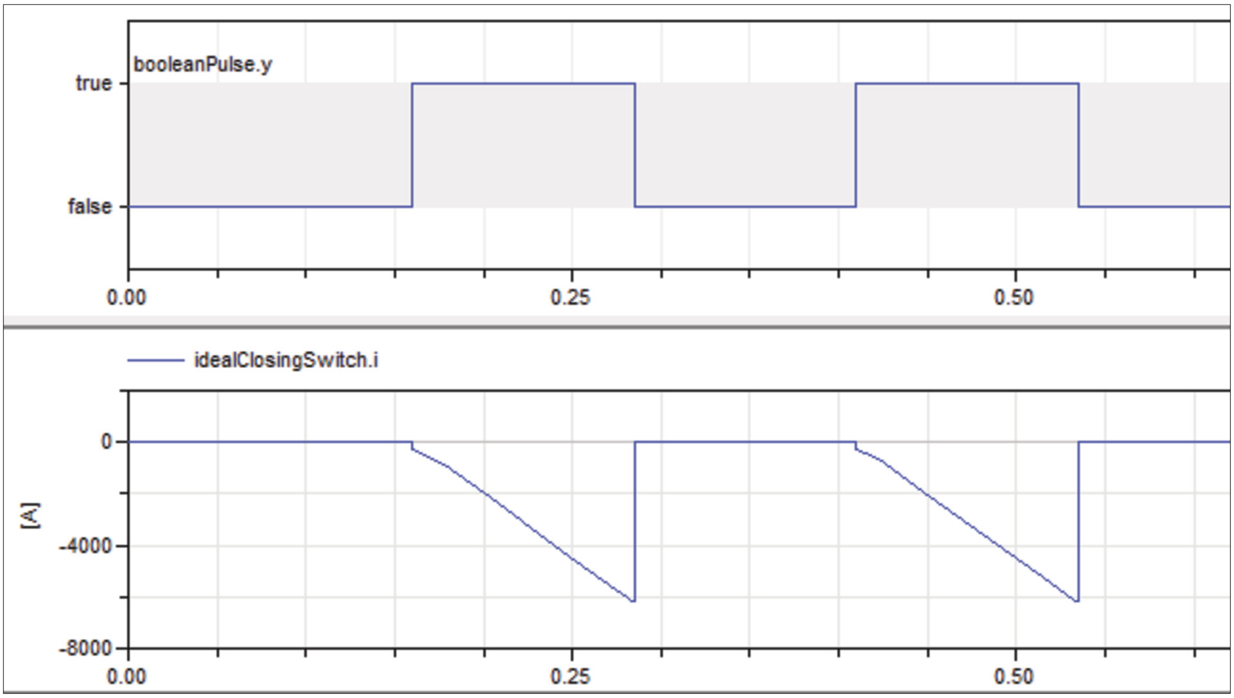



**Fig. 17.** Boolean pulse and current of the ideal closing switch (2 states).

### *3.3. Modeling the Pogo-stick model using MODELICA*

In order to validate the topological modeling of the Pogo-stick system, the results obtained using the MGS language and the results obtained by the MODELICA language [31] are compared. For MODELICA, we started by defining the main components that exist in the MODELICA library (resistor, capacitor, inductor, and the constant voltage), and also the ideal closing switch component connected by a Boolean pulse source. Then we assembled all the components of the analog circuit using the connectors and we switched to the simulation state which allows us to visualize the different parameters and variables of the problem. Fig. 16 shows the MODELICA model of both states: the connection state when we have a switch off and disconnection state when we have a switch on.

Fig. 17 shows the curve shape of the Boolean pulse and the curve shape of the ideal closing switch.

Figs. 18 and 19 just show the expected response of the different electrical elements of our analog circuit such as the resistor, the capacitor, and the inductor.

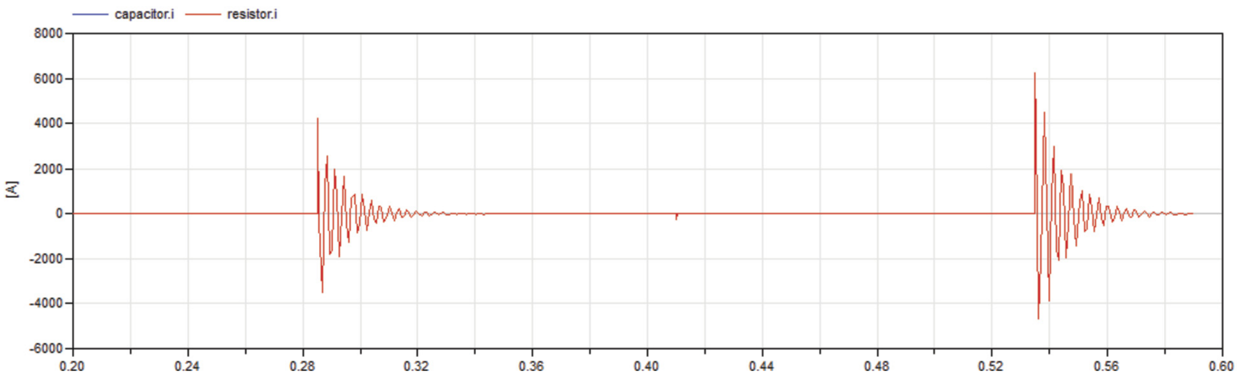



**Fig. 18.** Current of resistor, capacitor (2 states).

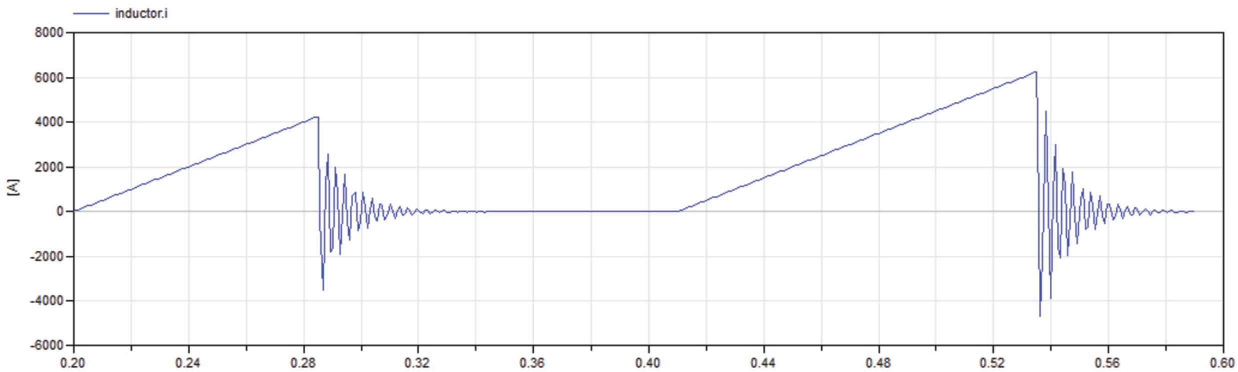



**Fig. 19.** Current of inductor (2 states).

### 3.4. Results and validation

In this section, we present the results after simulation the MGS code in the two modes of connectivity; connection, and disconnection. For a reminder, the connection state of our analog system presents the circuit with the switch off and the disconnection state presents the circuit with the switch on.

*Connection/disconnection states*

In this part, the modeling of disconnection and connection states in MODELICA is presented. The validation consists in comparing the resistor, capacitor, and inductor current obtained by the MGS code to these obtained by MODELICA, as shown in Fig. 20.

The superposition of the resistor, the capacitor, and the inductor, the current obtained by the MGS code is compared to those obtained by MODELICA. The results obtained by the MGS language based on topological collection and transformations are very close to those obtained by the Dymola environment (Fig. 20).

## 4. Discussion

We show in this section the importance of modeling of multi-physics systems using the MGS language, as we have done in this work. Thus, we are going to discuss the difference between our methodology, the Magos' approach and the modeling using MODELICA in Table 2.

## 5. Conclusion

In this paper, we are interested in a Pogo-stick model with a new methodology using the topological approach. The MGS language is used in this work.

Modeling using this language considers the complex system as a simple one at the simulation phase unlike MODELICA, which considers it as a complex system. We implemented a unique code connection/disconnection in order to integrate the topological modification within the structure. A complex system has been modeled with a variable topology by following a proposed methodology such as the one mentioned in this paper.

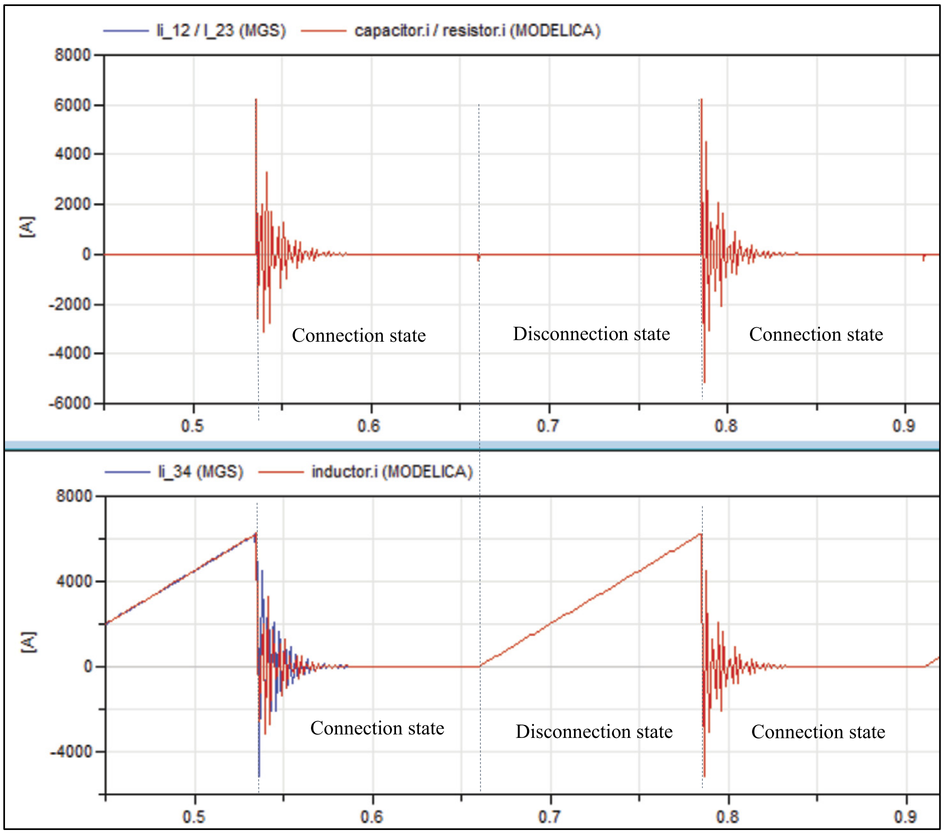
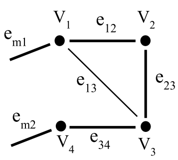



**Fig. 20.** Connection/disconnection states (MGS vs MODELICA).

**Table 2**
Comparing and analyzing the three different types of modeling.

| Type of modeling | Methodology |
|---|---|
| Modeling using MGS language | The modeling using the MGS language has a flexible code. The topological graph and especially the topological approach are used in this methodology.   So, modeling any example is feasible using this language and without remodeling everything (we have a bibliography), the bloc is reused. Then, to consider any mode (connection/disconnection) we only have to change parameters. The code of the case study in this paper is presented by Fig. 21. |
| Modeling using Magos' approach [20] | In this type of modeling, incidence matrices, interconnection graphs and the relation between Kirchhoff's equations are used in order to determinate a parameterized matrix that defines different modes. A parameterized expression for incidence matrices of dynamic graph and its corresponding dual graph is determined using connectivity notion (connection and disconnection operators). It summarizes the different dynamic cases that the model can represent. $$\begin{bmatrix} 1 & 1 & 1 & 1 \\ w_1-1 & 0 & w_1-1 & 0 \\ -w_1 & -1 & -w_1 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \cdot p_e + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1-w_1 & 1-w_1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ w_1 & w_1 & -1 & 1 \\ 0 & -1 & 0 & -1 \end{bmatrix} \cdot p_f = 0$$ |
| Modeling using MODELICA | Modeling using MODELICA is used to simulate and to validate the results obtained by the MGS language. In this type of modeling, an ideal switch is applied to present the connection/disconnection operators. The disadvantage of modeling using MODELICA is represented by the association of the physic of the model and its behavior. |

```
U12 = (Pv1-Pv2)
//U13=(Pv1-Pv3);
U23 = (Pv2-Pv3)
U34 = (Pv3-Pv4)
Pv4 = 10
Pv1 = 0
//Noeuds (V1, V2, V3, V4)
      Ii12+Ij23 = 0
Ii23+Ij34+Ij13 = 0
Ii34+Ijm1+Ii13 = 0
      Ij12+Iim2 = 0
//Arcs(e12, e13, e23, e34, em1, em2)
Ii12+Ij12 = 0
Ii13+Ij13 = 0
     Ii12 = C12·dU12/dt
Ii34+Ij34 = 0
dIi34/dt = (1/L34)U34
Ii23+Ij23 = 0
     Ii23 = (1/R23)U23
Iim1+Ijm1 = 0
Iim2+Ijm2 = 0

if time<0.25 then
        Ii13 = 0
else
   Pv3-Pv1 = 0
end if
```

**Fig. 21.** Unique code using the MGS language of the model of the case study (analog circuit).

In order to validate our work, the Magos' approach and MODELICA modeling have been used. At the end of this paper the difference between these three methodologies of multi-physics systems is discussed. The advantage of applying our methodology is in the flexibility of the modeling; that is, it offers us the possibility to pass from a state to another one without remodeling the system.

This methodology that is based on a topological approach opens up very large perspectives for the modeling of complex systems, because these systems can be considered as a set of local elements connected by a neighborhood relationship. This methodology can be also used as a unification basis for the modeling of mechatronic systems, and this is particularly applied when a same topological structure can support different physics.

The integration of the topological modification within the structure through a study of the changes of a topological structure has already been achieved through the elimination of bars applied to a 2D piezoelectric truss structure [32].

The application of our proposed methodology can be extended to more complex multi-physics systems such as mechatronic systems. Thus, the case of mechatronic systems can constitute a suitable topic for future research.

In order to achieve our objectives in this research work, we have also begun another part concerning the integration of control and command in the topological structure for complex mechatronic systems such as the Wind-Turbine system [33].

## Acknowledgements

The present research work has been supported by (a) QUARTZ Laboratory, the High Institute of Mechanic of Paris (SUPMECA) and by (b) the Laboratory of Mechanics, Modeling and Manufacturing (LA2MP), the National Engineering School of Sfax (ENIS).

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