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A class of fast diffusion p-Laplace equation with arbitrarily high initial energy ☆

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ABSTRACT

In this paper, the authors investigate a class of fast-diffusion p-Laplace equation, which was considered by Li, Han and Li (2016) [1], where, among other things, blow-up in finite time of solutions was proved for positive but suitably small initial energy. Their results will be complemented in this paper in the sense that the existence of finite time blow-up solutions for arbitrarily high initial energy will be proved. Moreover, an abstract criterion for the existence of global solutions that vanish at infinity will also be provided for high initial energy.

1. Introduction

In this paper, we investigate the following fast diffusion p-Laplace equation coupled with the homogeneous Neumann boundary condition

$$\begin{cases} u_t = \operatorname{div}(|\nabla u|^{p-2}\nabla u) + |u|^{q-1}u - \frac{1}{|\Omega|}\int_\Omega |u|^{q-1}u\,dx, & x \in \Omega,\ t > 0 \\ |\nabla u|^{p-2}\dfrac{\partial u}{\partial n} = 0, & x \in \partial\Omega,\ t > 0 \\ u(x,0) = u_0(x), & x \in \Omega \end{cases} \tag{1}$$

where Ω is a bounded domain in $\mathbb{R}^N (N \geq 2)$ with smooth boundary $\partial\Omega$, $\dfrac{2N}{N+2} < p < 2$, $2 < q+1 \leq p^* = \dfrac{Np}{N-p}$, n is the unit outward normal on $\partial\Omega$, and the initial datum $u_0(x)$ satisfies

$$0 \not\equiv u_0(x) \in L^\infty(\Omega) \cap W^{1,p}(\Omega), \qquad \int_\Omega u_0(x)\,dx = 0 \tag{2}$$

It is easily seen from the structure of the nonlinearity and the homogeneous Neumann boundary condition that the integral of the solution $u(x,t)$ to problem (1) with respect to x is conserved, that is $\int_\Omega u(x,t)\,dx = \int_\Omega u_0(x)\,dx = 0$ as long as $u(x,t)$ exists.

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In recent years, parabolic equations with nonlocal reaction term like $|u|^{q-1}u - \frac{1}{|\Omega|}\int_\Omega |u|^{q-1}u\,\mathrm{d}x$ have been investigated extensively because of their wide applications in many applied sciences. Interested readers may refer to [2–7,1,8–12] and the references therein for the motivation to study problems like (1) and for some results on the existence of local and (or) global solutions as well as solutions that blow up in finite time. In particular, the authors in [1] studied finite-time blow-up and extinction of solutions to problem (1). To state the blow-up results in [1] a little more precisely, we first recall some notations and definitions of functionals in that paper. Denote by $\|u\|_r$ the $L^r(\Omega)$ norm of a Lebesgue function $u \in L^r(\Omega)$ for $1 \le r \le \infty$ and let $W_*^{1,p}(\Omega)$ be the subspace of $W^{1,p}(\Omega)$, whose element u satisfies $\int_\Omega u\,\mathrm{d}x = 0$. $W_*^{1,p}(\Omega)$ will be equipped with the norm $\|u\|_{W_*^{1,p}(\Omega)} = (\int_\Omega |\nabla u|^p \mathrm{d}x)^{1/p}$, which, by recalling Poincaré's inequality, is equivalent to the classical one equipped with $W^{1,p}(\Omega)$. Let $B > 0$ be the optimal constant of the embedding inequality

$$\|u\|_{q+1} \le B\|\nabla u\|_p, \quad u \in W_*^{1,p}(\Omega) \tag{3}$$

where $1 < q \le (Np - N + p)/(N - p)$, and set

$$\alpha_1 = B^{-\frac{q+1}{q-p+1}}, \quad J_1 = \Big(\frac{1}{p} - \frac{1}{q+1}\Big)B^{-\frac{p(q+1)}{q-p+1}} > 0 \tag{4}$$

Moreover, define the energy functional

$$J(t) \triangleq J(u(x,t)) = \int_\Omega \Big[\frac{1}{p}|\nabla u(x,t)|^p - \frac{1}{q+1}|u(x,t)|^{q+1}\Big]\mathrm{d}x \tag{5}$$

Then the main blow-up result in [1] is the following.

Theorem 1.1. *(Blow-up with positive initial energy) Assume that* $\max\{1, \frac{2N}{N+2}\} < p < 2$, $1 < q \le (Np - N + p)/(N - p)$ *and that the initial datum* $u_0(x)$ *is chosen to satisfy* $J(u_0) < J_1$ *and* $\|\nabla u_0\|_p > \alpha_1$, *where* J_1 *and* α_1 *are given in* (4)*. Then the weak solution* $u(x,t)$ *to problem* (1) *blows up in finite time.*

In this paper, we will extend the blow-up result obtained in [1] to the case where the initial energy $J(u_0)$ is bigger than J_1. We will show, for any $M > J_1$, that there exists a u_0 such that $J(u_0) > M$, and that the solution to problem (1) with u_0 as initial datum blows up in finite time. The organization of the remaining of this paper is as follows. In Section 2, we give some notations, definitions, and lemmas concerning the basic properties of the related functionals and sets. The main results will be stated and proved in Sections 3.

Remark 1.1. In a recent paper [8], the authors investigated problem (1) for $p > 2$ and obtained the blow-up results when the initial datum u_0 satisfies $\frac{p(q+1)}{q+1-p}|\Omega|^{\frac{q-1}{2}} J(u_0) < \|u_0\|_2^{q+1}$, by applying Levine's concavity arguments (see [13]). The condition that $p > 2$ plays an essential role in their proof and it seems that the proof cannot be generalized to the case $p < 2$ trivially. It is not difficult to check that the methods used in this paper can also be applied to the case $p > 2$, and our assumption on the initial data is a little weaker than that in [8]. Moreover, we also give an abstract criterion for the existence of global solutions that tend to 0 in $W_*^{1,p}(\Omega)$ as t tends to ∞ (vanish at infinity) in this case.

2. Preliminaries

Since $1 < p < 2$, the equation in (1) is singular at the points where $\nabla u = 0$, and therefore classical solutions may not exist in general. We first give the definition of weak solutions to problem (1).

Definition 2.1. We say that a function $u \in L^\infty(\Omega \times (0,T)) \cap L^p(0,T; W^{1,p}(\Omega))$ with $u_t \in L^2(\Omega \times (0,T))$ is a weak solution to problem (1) if

$$\int_0^t \int_\Omega \Big[u\varphi_s - |\nabla u|^{p-2}\nabla u \cdot \nabla\varphi + \Big(|u|^{q-1}u - \frac{1}{|\Omega|}\int_\Omega |u|^{q-1}u\,\mathrm{d}x\Big)\varphi\Big]\mathrm{d}x\,\mathrm{d}s$$
$$= \int_\Omega u(x,t)\varphi(x,t)\,\mathrm{d}x - \int_\Omega u_0(x)\varphi(x,0)\,\mathrm{d}x \tag{6}$$

holds for all $\varphi \in C^1(\overline{\Omega} \times [0,T])$.

For $u \in W_*^{1,p}(\Omega)$, set

$$J(u) = \frac{1}{p}\|\nabla u\|_p^p - \frac{1}{q+1}\|u\|_{q+1}^{q+1}$$
$$I(u) = \|\nabla u\|_p^p - \|u\|_{q+1}^{q+1}$$

and define the Nehari's manifold

$$N = \{u \in W_*^{1,p}(\Omega) |\ I(u) = 0,\ \|\nabla u\|_p \neq 0\}$$
$$N_+ = \{u \in W_*^{1,p}(\Omega) |\ I(u) > 0\}$$
$$N_- = \{u \in W_*^{1,p}(\Omega) |\ I(u) < 0\}$$

Since $q+1 \le Np/(N-p)$, the functionals J and I are well defined and continuous on $W_*^{1,p}(\Omega)$. Next, we define the potential well (see [14]) and the set outside the potential well respectively by

$$W = \{u \in W_*^{1,p}(\Omega) |\ I(u) > 0,\ J(u) < d\} \cup \{0\}$$
$$V = \{u \in W_*^{1,p}(\Omega) |\ I(u) < 0,\ J(u) < d\}$$

where

$$d = \inf_{0 \neq u \in W_*^{1,p}(\Omega)} \sup_{\lambda \ge 0} J(\lambda u) = \inf_{u \in N} J(u)$$

is the depth of the potential well W. The positivity of d will be given in the next lemma.

Lemma 2.1. *The depth d of the potential well W is positive.*

Proof. Fix $u \in N$. It follows from (3) and the definition of N that

$$\|\nabla u\|_p^p = \|u\|_{q+1}^{q+1} \le B^{q+1}\|\nabla u\|_p^{q+1}$$

which implies $\|\nabla u\|_p \ge \left(\frac{1}{B^{q+1}}\right)^{\frac{1}{q+1-p}}$. Noticing that $q > 1 > p-1$, we have

$$\begin{aligned} J(u) &= \frac{1}{p}\|\nabla u\|_p^p - \frac{1}{q+1}\|\nabla u\|_p^p = \frac{q+1-p}{p\,(q+1)}\|\nabla u\|_p^p \\ &\ge \frac{q+1-p}{p\,(q+1)}\left(\frac{1}{B^{q+1}}\right)^{\frac{p}{q+1-p}} = J_1 \end{aligned} \tag{7}$$

Taking infimum over N, we see that $d \ge J_1 > 0$. The proof is complete. □

Remark 2.1. Since B is the best embedding constant in (3), it is easy to check that $d = J_1$.

For any $s > d$, define the (closed) sublevels of J by

$$J^s = \{u \in W_*^{1,p}(\Omega) |\ J(u) \le s\}$$

By the definition of $J(u)$, N, J^s and d, we see that

$$N_s \triangleq N \cap J^s = \left\{u \in N \middle| \left(\frac{1}{p} - \frac{1}{q+1}\right)\|\nabla u\|_p^p \le s\right\} \neq \emptyset, \quad \forall\, s > d \tag{8}$$

We also define

$$\lambda_s = \inf\{\|u\|_2 \mid u \in N_s\}, \quad \Lambda_s = \sup\{\|u\|_2 \mid u \in N_s\} \tag{9}$$

It is clear that λ_s is nonincreasing with respect to s, and that Λ_s is nondecreasing.

Finally, we introduce the following sets:

$$\mathcal{B} = \{u_0 \in W_*^{1,p}(\Omega) \mid \text{the solution } u = u(t) \text{ of (1) blows up in finite time}\}$$
$$\mathcal{G}_0 = \{u_0 \in W_*^{1,p}(\Omega) \mid \text{the solution } u = u(t) \text{ of (1) tends to 0 in } W_*^{1,p}(\Omega) \text{ as } t \to \infty\}$$

To study the long-time behaviors of solutions to problem (1) with high initial energy, we need the following properties of the functionals and sets defined above.

Lemma 2.2.
(i) *$J(t)$ defined in (5) satisfies $J'(t) = -\int_\Omega u_t^2 \mathrm{d}x$, and therefore is nonincreasing in t.*
(ii) *0 is away from both N and N_-, i.e. $dist(0, N) > 0$, $dist(0, N_-) > 0$.*
(iii) *For any $s > d$, the set $J^s \cap N_+$ is bounded in $W_*^{1,p}(\Omega)$.*

Proof. (i) The proof of (i) is accomplished by taking u_t as a test function for smooth solutions. By approximation, we can see that it also holds for weak solutions.

(ii) For any $u \in N$, by the definition of d and (7) we see that

$$d \le J(u) = \frac{1}{p}\|\nabla u\|_p^p - \frac{1}{q+1}\|\nabla u\|_p^p = \frac{q+1-p}{p\,(q+1)}\|\nabla u\|_p^p$$

which implies that $\mathrm{dist}(0, N) = \inf_{u \in N} \|\nabla u\|_p \ge \left(\frac{p\,d\,(q+1)}{q+1-p}\right)^{1/p}$.

For any $u \in N_-$, we have $\|\nabla u\|_p \neq 0$. Then it follows that

$$\|\nabla u\|_p^p < \|u\|_{q+1}^{q+1} \le B^{q+1}\|\nabla u\|_p^{q+1}$$

which yields

$$\|\nabla u\|_p \ge \left(\frac{1}{B^{q+1}}\right)^{\frac{1}{q+1-p}}$$

Therefore, $\mathrm{dist}(0, N_-) = \inf_{u \in N_-} \|\nabla u\|_p > 0$.

(ii) For any $u \in J^s \cap N_+$, $J(u) \le s$ and $I(u) > 0$. Therefore, we have

$$s \ge J(u) = (\frac{1}{p} - \frac{1}{q+1})\|\nabla u\|_p^p + \frac{1}{q+1}I(u) > \frac{q+1-p}{p\,(q+1)}\|\nabla u\|_p^p$$

and

$$\|\nabla u\|_p^p \le \frac{p\,s\,(q+1)}{q+1-p}$$

The proof is complete. □

Lemma 2.3. *For any $s > d$, λ_s and Λ_s defined in (9) satisfy*

$$0 < \lambda_s \le \Lambda_s < +\infty \tag{10}$$

Proof. To show the positivity of λ_s we need the following Gagliardo–Nirenberg inequality for $u \in W_*^{1,p}(\Omega)$ (see [15] page 241),

$$\|u\|_{q+1}^{q+1} \le C\|\nabla u\|_p^{\alpha(q+1)}\|u\|_2^{(1-\alpha)(q+1)}, \quad \forall\, u \in W_*^{1,p}(\Omega) \tag{11}$$

where α is determined by $(\frac{1}{2} + \frac{1}{N} - \frac{1}{p})\alpha = \frac{1}{2} - \frac{1}{q+1}$, and C is a positive constant depending only on N, p, and q. Since $\frac{2N}{N+2} < p < 2$ and $2 < q+1 \le \frac{Np}{N-p}$, it is easy to check that $\alpha \in (0, 1)$. Therefore, for any $s > d$ and $u \in N_s$, we obtain from (11) that

$$\|\nabla u\|_p^p = \|u\|_{q+1}^{q+1} \le C\|\nabla u\|_p^{\alpha(q+1)}\|u\|_2^{(1-\alpha)(q+1)} \tag{12}$$

which then yields that

$$\|\nabla u\|_p^{p-\alpha(q+1)} \le C\|u\|_2^{(1-\alpha)(q+1)} \tag{13}$$

By Lemma 2.2 (ii) and (8), we see that the left-hand side of (13) is bounded away from 0 no matter what the sign of $p - \alpha(q+1)$ is. This proves $\lambda_s > 0$ by the definition of λ_s. On the other hand, the fact that $\Lambda_s < \infty$ just follows from (8) and the Sobolev embedding inequality $\|u\|_2 \le C_*\|\nabla u\|_p$ since $\frac{2N}{N+2} < p$ is equivalent to $2 < p^* = \frac{Np}{N-p}$. The proof is complete. □

3. Main results

With the help of Lemmas 2.2 and 2.3 and inspired by [16,17], we can give an abstract criterion for the existence of global solutions that tend to 0 as t tends to ∞ or finite-time blow-up solutions in terms of λ_s and Λ_s for supercritical initial energy, i.e. $J(u_0) > d$.

Theorem 3.1. *Assume that $J(u_0) > d$, then the following statements hold:*

(i) *if $u_0 \in N_+$ and $\|u_0\|_2 \le \lambda_{J(u_0)}$, then $u_0 \in \mathcal{G}_0$;*
(ii) *if $u_0 \in N_-$ and $\|u_0\|_2 \ge \Lambda_{J(u_0)}$, then $u_0 \in \mathcal{B}$.*

Proof. Denote by $T(u_0)$ or T the maximal existence time of the solutions to problem (1) with initial datum u_0. When the solution is global, i.e. $T = \infty$, we denote by

$$\omega(u_0) = \bigcap_{t \ge 0} \overline{\{u(s) :\ s \ge t\}}^{W_*^{1,p}(\Omega)}$$

the ω-limit set of u_0.

(i) Assume that $u_0 \in N_+$ with $\|u_0\|_2 \le \lambda_{J(u_0)}$. We first claim that $u(t) \in N_+$ for all $t \in [0, T)$. If not, there would exist a $t_0 \in (0, T)$ such that $u(t) \in N_+$ for $t \in [0, t_0)$ and $u(t_0) \in N$. Recalling Lemma 2.2, (i) we have $J(u(t_0)) \le J(u_0)$, which implies that $u(t_0) \in J^{J(u_0)}$. Therefore, $u(t_0) \in N_{J(u_0)}$. According to the definition of $\lambda_{J(u_0)}$, we have

$$\|u(t_0)\|_2 \ge \lambda_{J(u_0)} \tag{14}$$

On the other hand, it follows by choosing $u\chi_{[t_1,t_2]}$ as a test function in (6) and then using Lebesgue's differentiation theorem that

$$\frac{\mathrm{d}}{\mathrm{d}t}\|u(t)\|_2^2 = -2I(u) \tag{15}$$

Noticing that $I(u(t)) > 0$ for $t \in [0, t_0)$, we then have

$$\|u(t_0)\|_2 < \|u_0\|_2 \le \lambda_{J(u_0)}$$

which contradicts the definition of $\lambda_{J(u_0)}$ and the claim is proved. Lemma 2.2 (iii) shows that the orbit $\{u(t)\}$ remains bounded in $W_*^{1,p}(\Omega)$ for $t \in [0, T)$ so that $T = \infty$. Let ω be an arbitrary element in $\omega(u_0)$, then by (15) and Lemma 2.2 (iii), we have

$$\|\omega\|_2 < \lambda_{J(u_0)}, \quad J(\omega) \le J(u_0)$$

which, recalling the definition of $\lambda_{J(u_0)}$ again, implies $\omega(u_0) \cap N = \emptyset$. Therefore, $\omega(u_0) = \{0\}$, i.e. $u_0 \in \mathcal{G}_0$.

(ii) Assume that $u_0 \in N_-$ with $\|u_0\|_2 \ge \Lambda_{J(u_0)}$. By applying a similar argument as above, we see that $u(t) \in N_-$ for all $t \in [0, T)$. Now suppose, on the contrary, that $T = \infty$, then for every $\omega \in \omega(u_0)$, it follows from (15) and Lemma 2.2 (i) that

$$\|\omega\|_2 > \Lambda_{J(u_0)}, \quad J(\omega) \le J(u_0)$$

By the definition of $\Lambda_{J(u_0)}$ again, we then infer that $\omega(u_0) \cap N = \emptyset$. Therefore, it must hold that $\omega(u_0) = \{0\}$, which is contradictive with the fact that $dist(0, N_-) > 0$. Hence, $T < \infty$ as claimed, and the proof of this theorem is complete. □

Remark 3.1. Since $\lambda_{J(u_0)} > 0$, Theorem 3.1 (i) is nontrivial. Moreover, Theorem 3.1 (ii) implies that there exists a u_0 such that $J(u_0)$ is arbitrarily large, while the corresponding solution $u(x, t)$ to problem (1) with u_0 as initial datum blows up in finite time. To illustrate this, we need the following proposition.

Proposition 3.1. *Let $u_0 \in W_*^{1,p}(\Omega)$ with $J(u_0) > d$. If $\dfrac{p(q+1)}{q+1-p}|\Omega|^{\frac{q-1}{2}} J(u_0) \le \|u_0\|_2^{q+1}$, then $u_0 \in N_- \cap \mathcal{B}$.*

Proof. Since $\dfrac{p(q+1)}{q+1-p}|\Omega|^{\frac{q-1}{2}} J(u_0) \le \|u_0\|_2^{q+1}$, we obtain, by making use of Hölder's inequality, that

$$\frac{p(q+1)}{q+1-p}|\Omega|^{\frac{q-1}{2}} J(u_0) \le \|u_0\|_2^{q+1} < \|u_0\|_{q+1}^{q+1}|\Omega|^{\frac{q-1}{2}} \tag{16}$$

The last inequality in (16) is strict since u_0 is not a constant. Recalling the definition of $J(u_0)$, $I(u_0)$ and noticing (16) we have

$$
\begin{aligned}
J(u_0) &= \frac{1}{p}\|\nabla u_0\|_p^p - \frac{1}{q+1}\|u_0\|_{q+1}^{q+1} = (\frac{1}{p} - \frac{1}{q+1})\|u_0\|_{q+1}^{q+1} + \frac{1}{p}I(u_0) \\
&= \frac{q+1-p}{p\,(q+1)}\|u_0\|_{q+1}^{q+1} + \frac{1}{2}I(u_0) > J(u_0) + \frac{1}{p}I(u_0)
\end{aligned}
$$

which implies that $I(u_0) < 0$, i.e. $u_0 \in N_-$.

To show that $u_0 \in \mathcal{B}$, it remains to prove $\|u_0\|_2 \geq \Lambda_{J(u_0)}$ by Theorem 3.1 (ii). For this, $\forall\, u \in N_{J(u_0)}$, we have

$$
\begin{aligned}
\|u\|_2^{q+1} &\leq |\Omega|^{\frac{q-1}{2}}\|u\|_{q+1}^{q+1} = |\Omega|^{\frac{q-1}{2}}\|\nabla u\|_p^p \\
&= |\Omega|^{\frac{q-1}{2}}\frac{p\,(q+1)}{q+1-p}(\frac{1}{p} - \frac{1}{q+1})\|\nabla u\|_p^p \\
&\leq |\Omega|^{\frac{q-1}{2}}\frac{p\,(q+1)}{q+1-p}J(u_0)
\end{aligned}
$$

The last inequality holds because of (8). Taking supremum over $N_{J(u_0)}$ on the left-hand side of the above inequality and recalling (16), we obtain

$$
\Lambda_{J(u_0)}^{q+1} \leq |\Omega|^{\frac{q-1}{2}}\frac{2\,(q+1)}{q-1}J(u_0) \leq \|u_0\|_2^{q+1}
$$

i.e. $\|u_0\|_2 \geq \Lambda_{J(u_0)}$. Therefore, $u_0 \in N_- \cap \mathcal{B}$, and this completes the proof. □

Theorem 3.2. *For any $M > d$, there exists a $u_M \in N_- \cap \mathcal{B}$ such that $J(u_M) \geq M$.*

Proof. Similar treatments have been used in [16,17]. We repeat the proof here for the convenience of the readers. For any $M > d$, let Ω_1 and Ω_2 be two arbitrary disjoint open subdomains of Ω, and assume that $v \in W_*^{1,p}(\Omega_1)$ is an arbitrary nontrivial function. Since $q > p-1$, we can choose $\alpha > 0$ large enough that $J(\alpha v) \leq 0$ and $\|\alpha v\|_2^{q+1} > |\Omega|^{\frac{q-1}{2}}\dfrac{p\,(q+1)}{q+1-p}M$.

Fix α and choose a function $w \in W_*^{1,p}(\Omega_2)$ such that $J(w) + J(\alpha v) = M$. Extend v and w to be 0 in $\Omega \setminus \Omega_1$ and $\Omega \setminus \Omega_2$, respectively, and set $u_M = \alpha v + w$. Then $J(u_M) = J(\alpha v) + J(w) = M$, and

$$
\|u_M\|_2^{q+1} \geq \|\alpha v\|_2^{q+1} > |\Omega|^{\frac{q-1}{2}}\frac{p\,(q+1)}{q+1-p}J(u_M)
$$

By Proposition 3.1, it is seen that $u_M \in N_- \cap \mathcal{B}$. The proof is complete. □

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