



# A damage identification technique for beam-like and truss structures based on FRF and Bat Algorithm



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## ABSTRACT

In this paper, a Structural Health Monitoring (SHM) technique for damage identification in beam-like and truss structures using Frequency Response Function (FRF) data coupled with optimization techniques is presented. Genetic Algorithm (GA) and Bat Algorithm (BA) are used to estimate the location and severity of damage. The damage in the structures is simulated by reduction in rigidity of specific members. Both optimization techniques are coupled with modelled structures using Finite Element Method (FEM). The approach is based on minimizing an objective function by comparing measured and calculated FRFs. The results show that better accuracy is obtained using BA than using GA in terms of precision and computational time. Furthermore, it is found that the proposed approach provides faster solution than other approaches in the literature.

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## 1. Introduction

Structural Health Monitoring (SHM) is a damage assessment tool that can be used to prevent structural failure and to avoid serious catastrophic problems. In the last decades, a special attention was paid to prevent sudden failure of a structure by early damage detection. More specifically, many efficient techniques of damage identification with vibration tests based on modal deformation have been proposed [1–5]. Some approaches and theories using inverse problem formulations to identify crack location and severity were presented in Refs. [6,7].

SHM has been extensively applied to plates and beam-like structures in the literature. Damage detection and quantification in thin plates based on vibration data using BA in a beam-like and complex structures, in which damage was represented by a reduction in stiffness, was presented by Khatir et al. [8]. Structural damage detection using vibration-based techniques and transmissibility with elaborating the mathematical interrelationship between Modal Assurance Criterion (MAC) and cosine similarity measure was presented in Ref. [9]. A new approach for crack detection in steel beams by sine-sweep vibration measurements was presented by Dougdag [10]. Waisman et al. [11] combined XFEM with GA for crack identification based on inverse problem. Detection and localization of damage in structures by applying concepts derived

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from the theory of POD were investigated by simulating tests on two beams, which provided promising results found by Galvanetto and Violaris [12]. The three-dimensional Spectral Element Method (SEM) was introduced for propagation problems in plate structures to predict crack location. The Lagrange interpolation function supported by the Gauss–Lobatto–Legendre (GLL) points in conjunction with the GLL integration used by Peng et al. [13]. This application can detect damages in those structures. Crack identification in plate using eXtended IsoGeometric Analysis (XIGA) and PSO is presented in the Ref. [14]. Three objective functions, namely (a) change of natural frequencies, (b) Modal Assurance Criterion (MAC), and (c) MAC natural frequency were used for identification damage using GA [15]. A model reduction based on the POD method was used for damage identification in a beam structure coupled with optimization techniques to estimate the crack's length and its position in a structure using boundary displacements as input data to build data matrix. GA and PSO were then applied for the minimization of the objective function expressed as the difference between the boundary displacements of the actual crack and those of the estimated cracked plate in Refs. [16]. The accuracy of the method was verified through different damage configurations.

Damage detection in composite structures has also been studied by many authors. A multiple damage detection in uni-directional graphite-epoxy composite beams using Particle Swarm Optimization PSO and GA was presented in Ref. [17]. A review of vibration-based structural health monitoring based on composite materials was presented by Montalvao et al. [18]. A hybrid Particle Swarm Optimization Simplex algorithm (PSOS) for structural damage identification was introduced in Ref. [19]. Li et al. [20] used the modal characteristics extracted from vibration tests from an original FE model in an identification approach developed by combining the advantages of two classes of techniques, i.e. eigen-sensitivity and multiple-constraint matrix adjustment. Khatir et al. [21] presented a new approach for damage detection and localization based on model reduction. The problem was formulated as an inverse problem, where an optimization algorithm was used to minimize the cost function expressed as the normalized difference between a frequency vector of the tested structure and its numerical counterpart. The frequency time domain for damage detection and localization has been developed in previous research by Lu and Law [22], proposing a new technique to identify damage effectively in structures. A review of damage detection and health monitoring of mechanical systems based on changes in the measured data of linear and non-linear vibrations was presented by Sinou [23]. A theoretical model describing the behaviour of Carbon Fibre Reinforced Plastic (CFRP) cantilever beams was developed on the basis of previous research, which investigated the free vibration of beams with single and multiple cracks [24] or notches [25,26]. The dynamic response of structural elements was modified due to real damage resulting from defects, loss of integrity and cracking of CFRP material by overloading during service life [27,28]. A FE model updating method using Frequency Response Functions was experimentally validated using a pseudo-linear sensitivity equation as presented by Shadan et al. [29]. Hwang and Kim [30] presented a method to identify the locations and severity of damage in structures using Frequency Response Function (FRF) data. Mohan et al. [31] used a Frequency Response Function (FRF) coupled with the Particle Swarm Optimization (PSO) technique for structural damage detection and quantification. A new fast approach for crack identification using vibration analysis based on model reduction using POD–RBF coupled with the cuckoo search algorithm and GA as an inverse problem was presented by Khatir et al. [32]. A large number of works based on cracks in composite materials can be found in Ref. [33].

Application of SHM to truss structures has been also extensively reported in the literature. A multi-damage identification of large-scale truss structures using a two-step approach presented by Fella et al. [34]. Ernest and Dewangan [35] presented an approach for damage detection in the members of a tower truss using the changes in natural frequency parameter. A Finite Element (FE) technique was used to find the stiffness and the mass matrix for natural frequency evaluation. The distributed mass matrix parameters are considered in the formulation of the mass matrix model. The natural frequency graphs were plotted for various cases. A new technique was presented by Khatir et al. [36], which allowed damage identification of an open crack in beam-like and 2D structures. The proposed approach considered the variation in local flexibility near the position of the crack. The natural frequencies of a cracked beam were determined experimentally and numerically using the FEM of the beam. FE model updating was used to build models for intact and damaged beams. Bureerat and Pholdee [37] presented a technique based on an inverse problem using differential evolution for efficient SHM of trusses. Damage detection in truss bridge joints using Artificial Neural Networks with different scenarios was presented by Mehrjoo et al. [38]. The technique that was employed to overcome the issues associated with many unknown parameters in a large structural system is sub-structural identification. The natural frequencies and mode shapes were used as input parameters to the neural network for damage identification. Modal-parameter identification and vibration-based damage detection of a damaged steel truss bridge was presented by Chang and Kim [39]. Damage assessment in truss structures with limited sensors using a two-stage method and model reduction was reported by Dinh-Cong et al. [40]. This study proposed a practical two-stage approach for damage assessment using noisy modal data collected from a limited number of sensors. A truss topology optimization with static and dynamic constraints using modified subpopulation teaching–learning-based optimization was presented by Savsani et al. [41]. An improved version of the teaching–learning-based optimization (TLBO) algorithm was proposed for truss topology optimization (TTO), with static and dynamic constraints on planar and space trusses. Hosseinzadeh et al. [42] suggested a method to detect and quantify damages in structures such as steel plane truss and frames, with a limited number of sensors by using a democratic particle swarm optimization (DPSO) algorithm. In this study, data was gathered by installing sparse sensors, and a Neumann series expansion-based model reduction approach was employed to condense the structural model.

According to recent works, many damage indicators were used to detect the damage location with older optimization techniques. In this paper, we use new optimization technique, namely BA, which is compared with GA, for damage iden-

tification in beam-like and truss structures using a damage indicator based on FRF as an objective function. This paper is divided into six main sections. After the introduction section, damage detection using FRF is presented in the second section. Numerical examples of a 1-D cantilever beam are illustrated in section 3. In section 4, the results using FEM along with GA and BA are presented for a beam-like structure. In section 5, BA is used for damage localization and quantifications in a truss structure. Finally, conclusion remarks are presented.

## 2. Frequency Response Function (FRF)

Damage will change the dynamic characteristics of a structure. This is characterized by changes in the modal parameters, i.e. modal frequencies, damping ratios, and mode shapes. These changes will affect the mass, damping, stiffness, and flexibility matrices of the structure. The dynamical equations of a structure with  $n$  degrees of freedom can be expressed as [43]:

$$[M]\{\ddot{x}(t)\} + [D]\{\dot{x}(t)\} + [K]\{x(t)\} = f(t) \quad (1)$$

where  $[M]$ ,  $[D]$  and  $[K]$  represent the  $n \times n$  mass, damping, and stiffness matrices, respectively. The external force and displacement can be expressed as  $f(t) = \{F(\omega)\}e^{j\omega t}$  and  $x(t) = \{X(\omega)\}e^{j\omega t}$ , and Eq. (1) becomes:

$$(-\omega^2[M] + j\omega[D] + [K])\{X(\omega)\}e^{j\omega t} = \{F(\omega)\}e^{j\omega t} \quad (2)$$

From the above equation, the FRF matrix  $[H(\omega)]$  is defined as:

$$[H(\omega)] = (-\omega^2[M] + j\omega[D] + [K])^{-1} \quad (3)$$

Then Eq. (2) can be expressed as:

$$\{X(\omega)\} = [H(\omega)]\{F(\omega)\} \quad (4)$$

If damping is neglected from Eq. (3), then we have:

$$[H(\omega)] = [-\omega^2[M] + [K]]^{-1} \quad (5)$$

The mass matrix  $[M]_A$  and the stiffness matrix  $[K]_A$  are estimated by the FE model. Therefore, the simulated FRFs are expressed as:

$$[H(\omega)]_A = [(-\omega^2[M]_A + [K]_A)^{-1}] \quad (6)$$

From Eq. (5), the experimental FRFs of a damaged structure are presented in the following equation:

$$[H(\omega)]_T = [(-\omega^2[M] + [K]_T)^{-1}] \quad (7)$$

where  $[K]_T = [H]_T^{-1} + \omega^2[M]$ .

If we assume that the mass of the structure remains constant before and after the damage, the change in the stiffness as a result of the damage is  $[\Delta K] = [K]_A - [K]_T$ , i.e.:

$$[\Delta K] = [H]_A^{-1} - [H]_T^{-1} \quad (8)$$

When multiplied by  $[H]_T$ , (8) gives:

$$[H]_T[\Delta K] = [H]_T[H]_A^{-1} - [H]_T[H]_T^{-1} \quad (9)$$

The calculated  $\beta$  values for damage location based on degree of freedom and elements location using the first row of the global FRF matrix are defined as:

$$\beta(1, i) = ([H]_T)_{1n} * [H]_A^{-1}_{(i,i)} - [I]_{(1,i)} \quad (10)$$

## 3. Damage detection using optimization techniques

### 3.1. Bat Algorithm

Bat Algorithm (BA) is a bio-inspired algorithm developed by Yang and He [44] and has been found to be very efficient. BA is based on three idealised rules. In the first rule, all bats use echolocation to sense distance, and they also 'know' the difference between food/prey and background barriers in some magical way. In the second rule, bats fly randomly with velocity  $v_i$  at position  $x_i$  with frequency  $freq_{min}$ , varying wavelength  $\lambda$ , and loudness  $A_0$  to search for prey. They automatically adjust the wavelength (or frequency) of their emitted pulses and adjust the rate of pulse emission  $r \in [0, 1]$ ,

depending on the proximity to their target. In the third rule, although the loudness can vary in many ways, it is assumed that the loudness varies from a large (positive)  $A_0$  to a minimum constant value  $A_{\min}$ .

Rules are updated with frequency  $freq$ , position  $x_i$  and velocity  $vel_i$  in a  $d$ -dimensional search space. The corresponding updated solutions for  $S_i^t$  and velocity  $vel_i^t$  at time step  $t$  are represented as:

$$freq_i = freq_{\min} + (freq_{\max} - freq_{\min})U(0, 1) \quad (11)$$

$$vel_i^t = vel_i^{t-1} + (x_i^t - best)freq_i \quad (12)$$

$$S_i^t = S_i^{t-1} + vel_i^t \quad (13)$$

The term 'best' is used for the current global best location, which is calculated by comparing all the solutions among all the bats. The new best solution is updated based on the following equation:

$$S_{new} = S_{old} + \epsilon A^t \quad (14)$$

where  $\epsilon$  is a random number between  $-1$  to  $1$ , and  $A^t$  is used for average loudness of all the bats at time step  $t$ . In this paper, BA is used to determine the best position for damage location and severity.

### 3.2. Genetic Algorithm

Genetic Algorithm (GA) is a general optimization method based on the class of evolutionary algorithms. The objective is to combine the problem with optimizations to find the best solution based on an objective function. The individuals have chromosomes, and then they evolve gradually through the genetic operations such as selection, crossover, and mutation to find the best solutions [45].

GA proposes damage in all elements by the calculation of FRFs and compare them with the measured ones using the objective function in Eq. (12). The searching schematics of a simple GA can be generalized in the following steps:

- step 1, organize the initial population of solutions;
- step 2, evaluate the fitness of all individuals;
- step 3, finish operations when the stop criterion is satisfied;
- step 4, select more fit individuals based on fitness and transform them into new individuals, called 'offspring';
- step 5, if the measured and calculated values are the same,  $\gamma_j$  is the location of damage.

Through a series of damage location tests, the following genetic parameters were chosen based on the accuracy of the results. For GA, a number of generations of 50, a population size of 200, a crossover rate of 0.8, i.e. 800 individuals, were selected for crossover, and the mutation rate was equal to 0.01. In addition, for BA, the number of generations was 50, the population was 200, loudness was 0.5, and the pulse rate was 0.5. This identification method was implemented in MATLAB, on a PC with Intel i5, 3.0 GHz and 16 GB RAM. The methodological approach is presented in a flowchart displayed in Fig. 1.

### 3.3. Objective function

The optimization techniques presented in the previous section are used to find optimal damage location with severity based on an objective function (OBF). FRF is introduced in the new proposed indicator as objective function and integrated into BA and GA. The objective is to minimize the difference between the measured FR and the calculated FR by BA and GA, and is given by:

$$\gamma_j = \left| \sum_{j=1}^{\text{Mode}} \beta(j, i) \right| \quad (15)$$

$$\text{OBF} = \frac{|\gamma_{\text{measured}} - \gamma_j|}{\gamma_{\text{measured}}} \quad (16)$$

For implementation, we used optimisation algorithms from MATLAB toolboxes, and we programmed the FEA and the damage detection algorithm illustrated in Fig. 1.

## 4. Numerical simulations

### 4.1. Damage indicator based on Frequency Response Function

In this section, the damage detection method is applied to a simple cantilever beam structure system without optimization. For the system model shown in Fig. 2, a cantilever beam had a fixed boundary condition at the left end. The beam

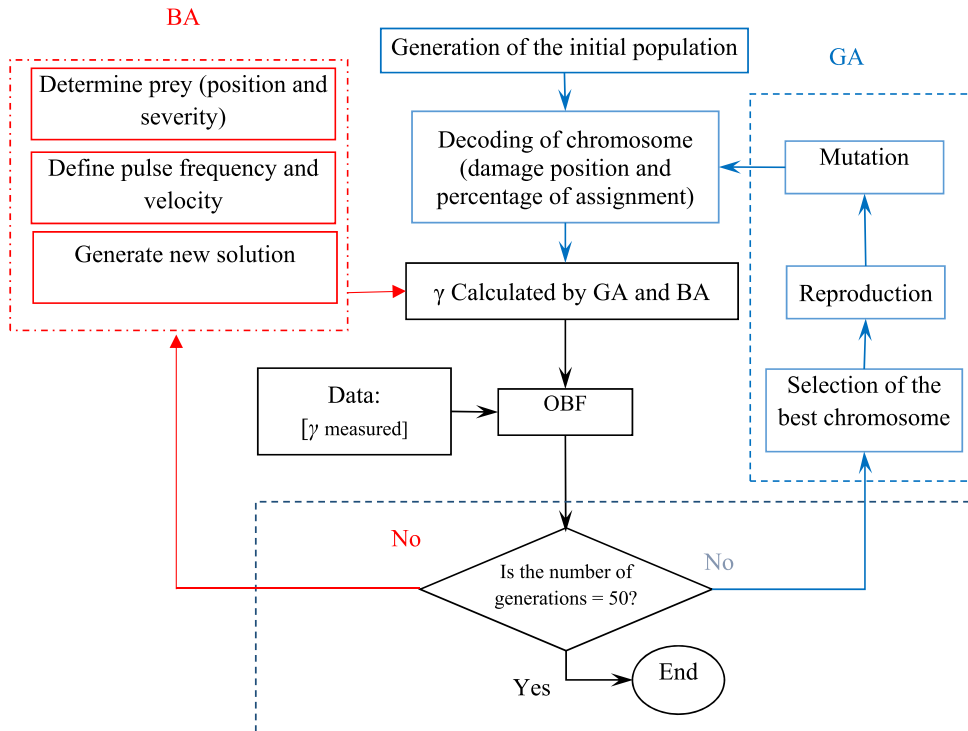


Fig. 1. BA and GA for damage localization and quantification.

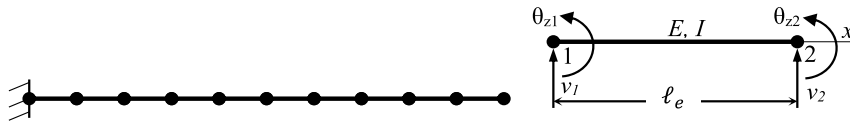


Fig. 2. FE model of a cantilever beam.

Table 1  
Material properties of the beam structure.

Property (unit)	Value
Young modulus $E$ (GPa)	200
$\rho$ , density (kg/m <sup>3</sup> )	7850
Length (m)	0.762
Section (m <sup>2</sup> )	0.0254

is discretized into ten Euler beam elements with two degrees of freedom at each node. The geometrical and mechanical properties are given in Table 1.

In the first damage scenario D1, we assume that the stiffness value is altered from  $E = 206.84$  to  $137.895$  GPa due to damage in the 3rd, 4th, and 9th elements of the FE model. In the second damage scenario D2, we assume that each element has a different amount of damage, i.e. the 3rd and 9th elements' stiffness values are changed from  $E = 206.84$  to  $137.895$  GPa, and the 4th element's stiffness value was changed to  $E = 172.369$  GPa. For FRF, we chose  $[[H(\omega)]_A(1, 4)]$  before damage to compare with  $[[H(\omega)]_T(1, 4)]$  after damage [30]. The results are presented in Fig. 3.

The results show that the change in FRF can be used to detect the damage by comparing  $H(1, 4)$  before and after damage. For the first scenario,  $\beta$  is plotted for each degree of freedom and for each element in the damaged structure in Figs. 4 and 5, respectively. Similarly, for the second scenario,  $\beta$  is plotted for each degree of freedom and for each element in the damaged structure, as presented in Figs. 6 and 7, respectively.

The results from both damage scenarios show that the damage indicator based on FRF can detect and locate the damage. Therefore, we conclude that this technique of damage can be used for structures with multiple damage locations.

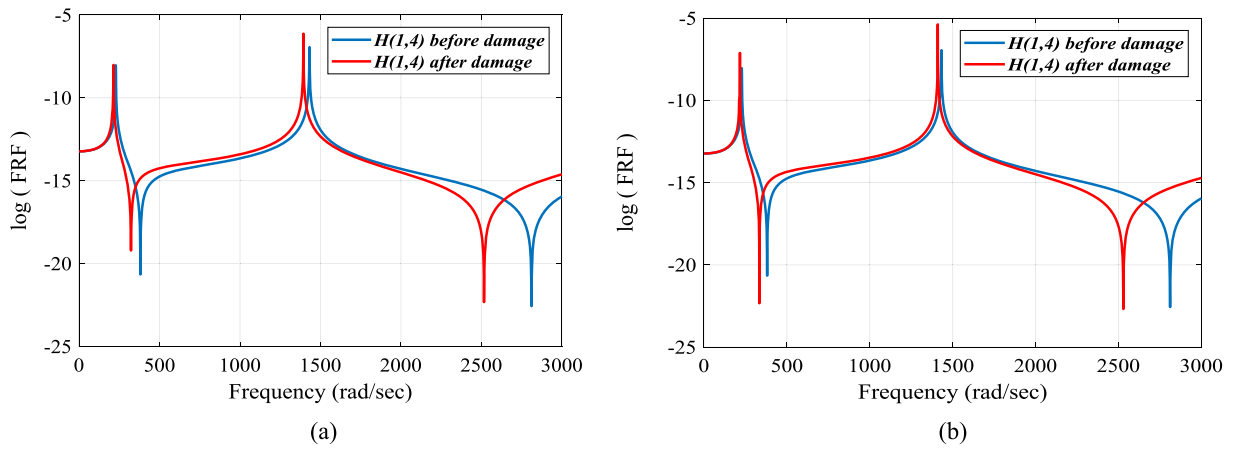


Fig. 3. Comparison of  $H(1,4)$  before and after damage for damage scenarios (a) D1 and (b) D2.

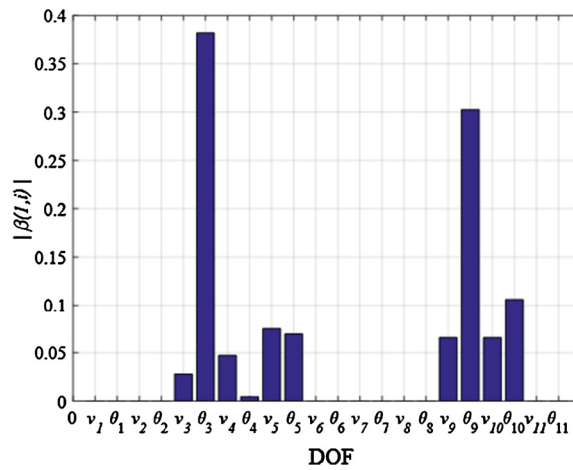


Fig. 4. Damage location based on DOF – D1.

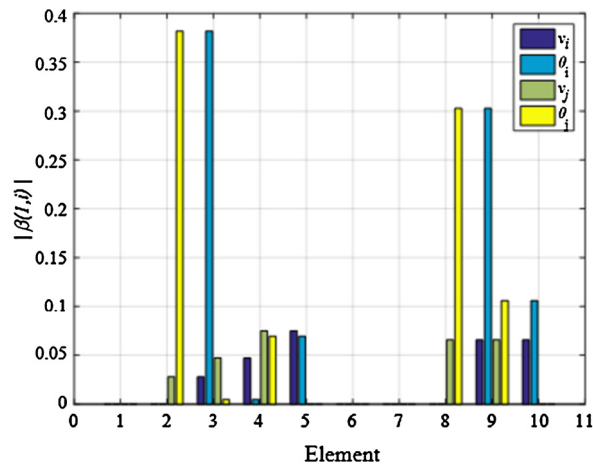


Fig. 5. Damage location based on elements – D1.

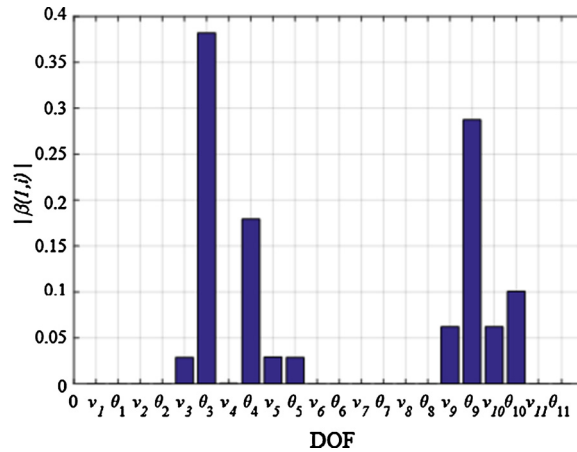


Fig. 6. Damage location based on DOF - D2.

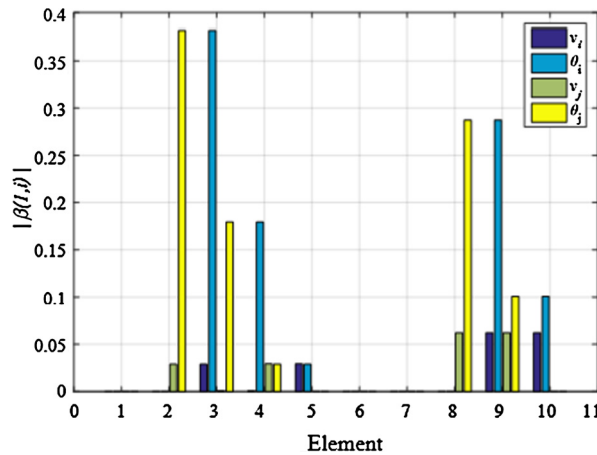


Fig. 7. Damage location based on elements - D2.

4.2. Optimization for damage identification based on frequency response

Both BA and GA are used to evaluate the performance of the proposed approach. We carried out some simulations using position of damage and loss of rigidity similar to what has been presented in the first section using two damages scenarios.

4.2.1. Damage scenario D1

In the first damage scenario, we assumed that the stiffness value was altered from  $E = 206.84$  to  $137.895$  GPa due to damage at the 3rd, 4th, and 9th elements of the model. Both BA and GA are used to diagnose the structure using the objective function defined in Eq. (16). The results are illustrated in Figs. 8 and 9, and show that damage locations using BA are more accurate and faster than those found using GA.

4.2.2. Damage scenario D2

In a second damage scenario, the 3rd and 9th element stiffness values were changed from  $E = 206.84$  to  $137.895$  GPa, and the 4th element stiffness value is changed to  $E = 172.369$  GPa. The results are presented in Figs. 10 and 11. Again, the results show that BA is better than GA from the point of view of computational time and accuracy.

In Table 2, we present a comparative study between BA-FEM-FRF, GA-FEM-FRF and FEM-FRF. The results show that the optimization technique combined with FEM-FR is better than FEM-FR alone without optimization. Again, BA provides more accurate results than GA. The computational time presented in Table 2 shows that BA is faster than GA.

5. Complex structures

In this section, we used two different structures for damage identification using the proposed objective function based on the Frequency Response Function.

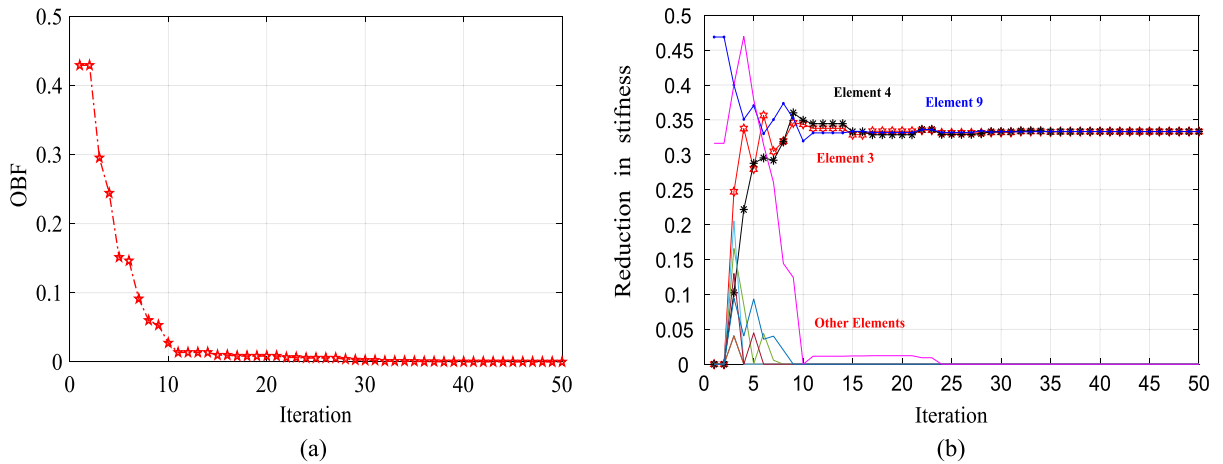


Fig. 8. Damage scenario D1 using GA: (a) iteration process of the objective function and (b) convergence of the damaged elements.

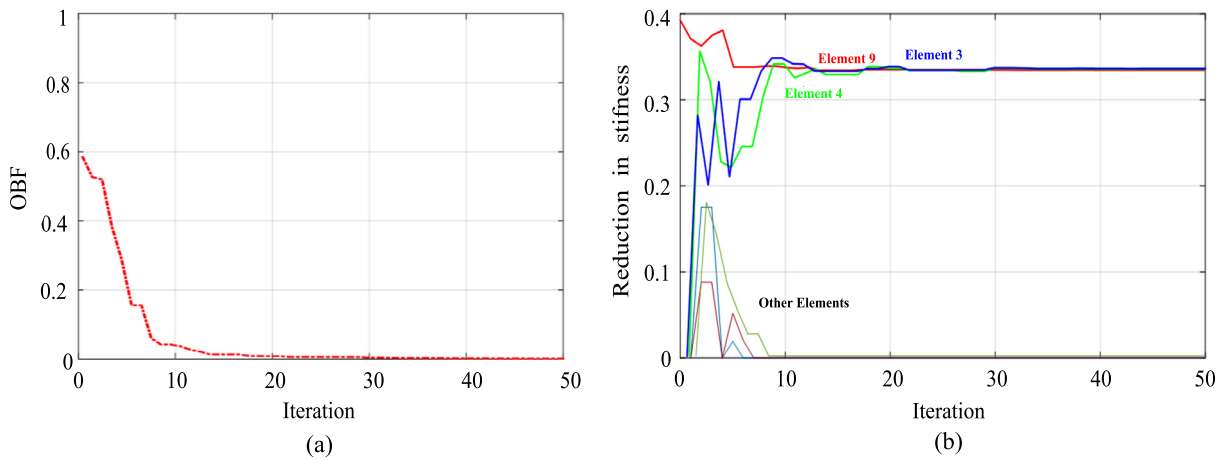


Fig. 9. Damage scenario D1 using BA: (a) iteration process of the objective function and (b) convergence of the damaged elements.

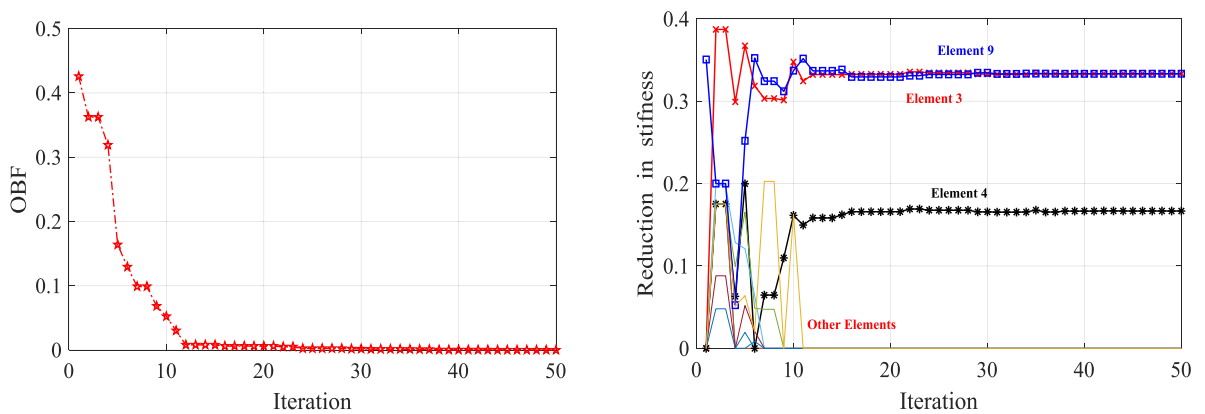


Fig. 10. Damage scenario D2 using GA: (a) iteration process of the objective function and (b) convergence of the damaged element.

5.1. Truss structure with 10 elements

In the first example, a planar truss structure containing ten bars, as shown in Fig. 12, is considered. This structure was used previously by several researchers, e.g., Kaveh and Zolghadr [46], Grandhi and Venkayya [47], and Sedaghati et al. [48]. A non-structural mass of 454 kg is attached to the free nodes. This structure has 8 degrees of freedom and its material properties are presented in Table 3.



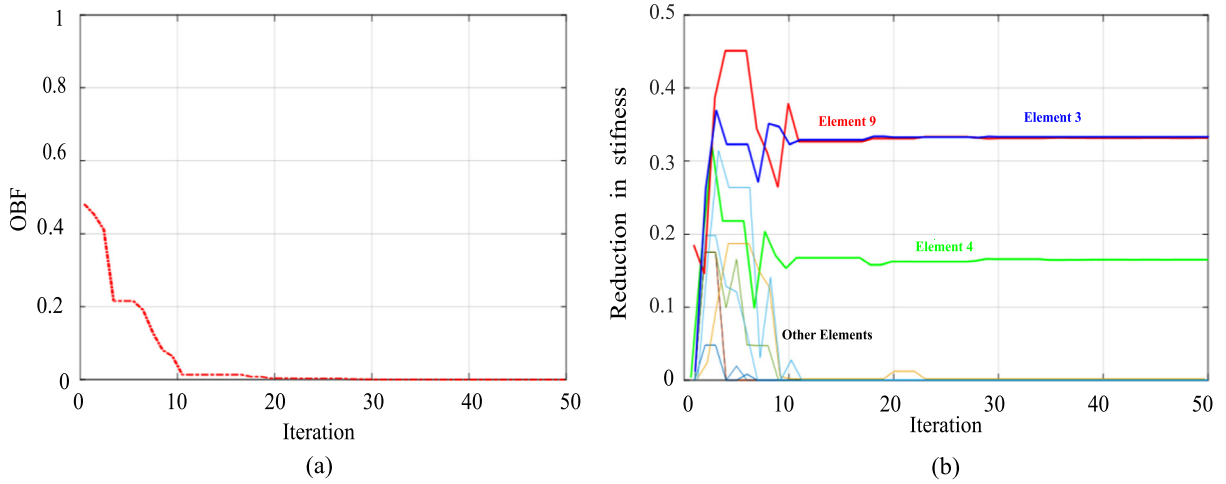


Fig. 11. Damage scenario D2 using BA: (a) iteration process of the objective function and (b) convergence of the damaged element.

Table 2  
Comparison between BA and GA for damage location and severity.

Technique	Scenario	Element 3	Element 4	Element 9	Calculation time (s)
FEM-FRF	Real D1	0.333	0.333	0.333	8.7
	Predicted D1	0.388	0.07	0.309	
	Real D2	0.333	0.166	0.333	9.1
	Predicted D2	0.388	0.177	0.288	
GA-FEM-FRF	Real D1	0.333	0.333	0.333	970
	Predicted D1	0.331	0.331	0.331	
	Real D2	0.333	0.166	0.333	1120
	Predicted D2	0.330	0.168	0.330	
BA-FEM-FRF	Real D1	0.333	0.333	0.333	650
	Predicted D1	0.333	0.333	0.333	
	Real D2	0.333	0.166	0.333	730
	Predicted D2	0.332	0.165	0.332	

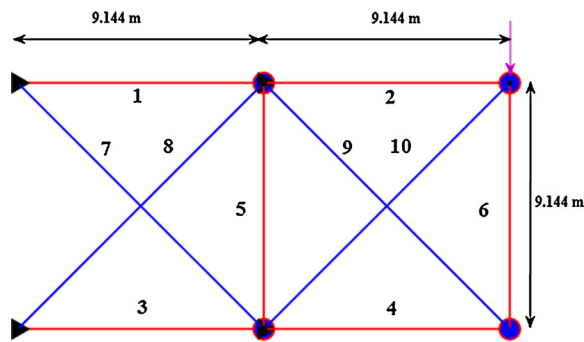
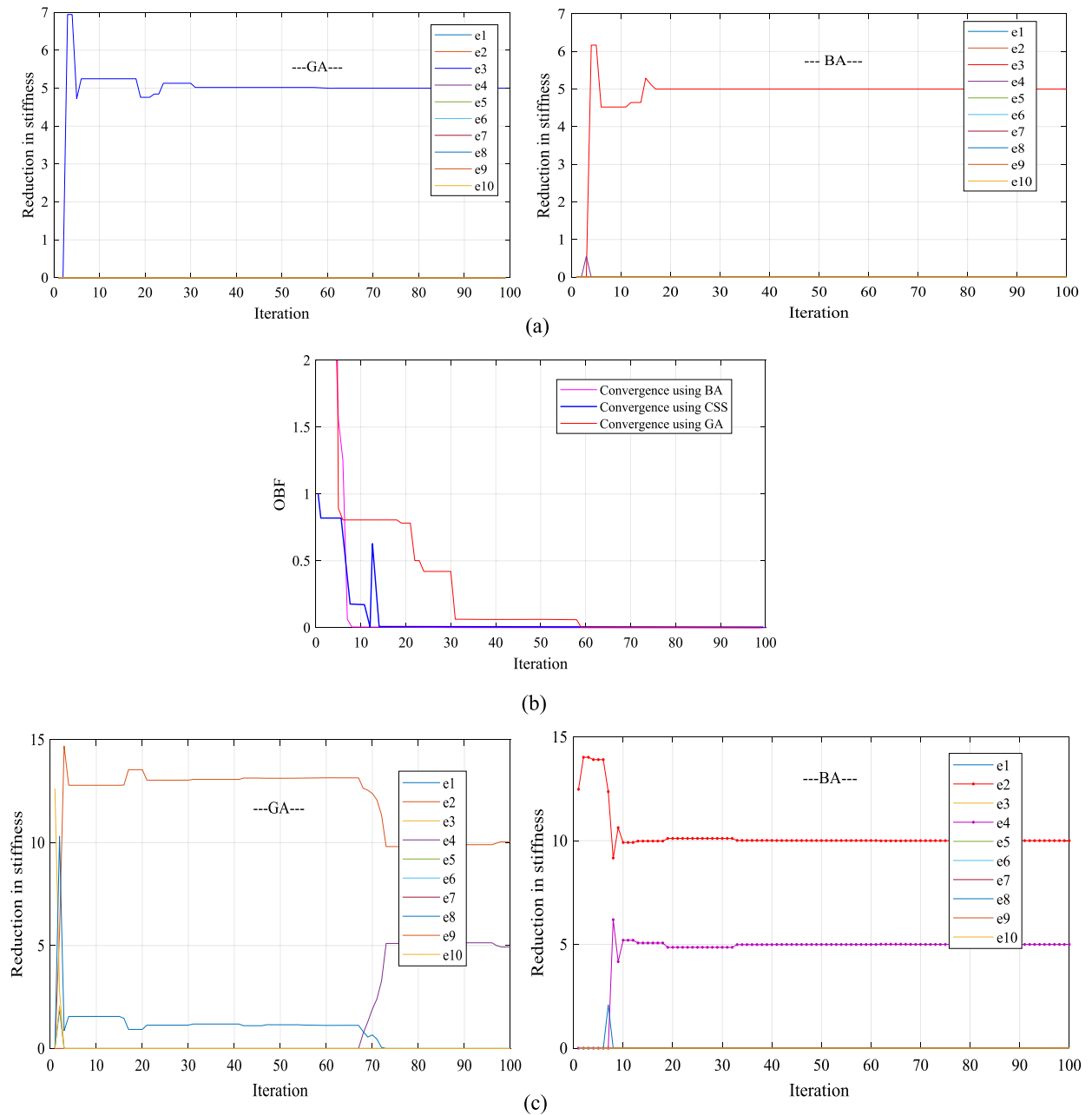


Fig. 12. 10 bar truss structure.

Table 3  
Material properties of the 10-bar planar truss.

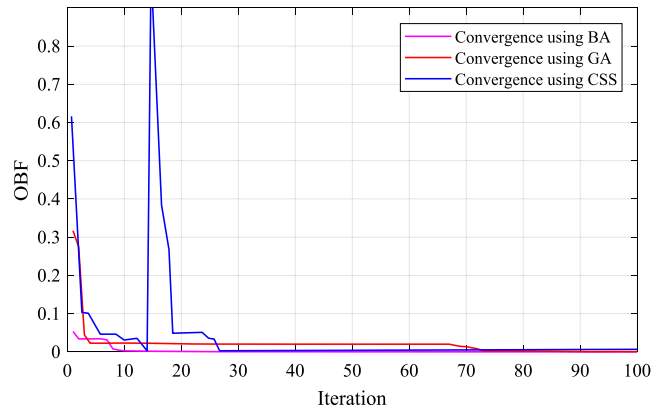
Property (unit)	Value
$E$ , modulus of elasticity ( $N/m^2$ )	$6.98 \times 10^{10}$
$\rho$ , density ( $kg/m^3$ )	2770
Added mass (kg)	454
$L$ , main bar's dimension (m)	9.144
$A$ , cross-sectional area of the members ( $m^2$ )	0.0025



**Fig. 13.** Damage scenarios in truss structure using GA, BA and CSS [46]: (a) reduction in stiffness – D3, (b) convergence – D3, (c) reduction in stiffness – D3 and (d) convergence – D4.

In this example, we use BA for damage identification, and we consider two damage scenarios, e.g., D3 and D4 as used in Ref. [46]. For D3, the location of damage is in element 3, with loss of rigidity of 5% and for D4, the locations of damage are in elements 2 and 4, with losses of rigidity of 10% and 5%, respectively. For the optimization parameters, the number of generations chosen is 100 and the population is 200. The results for both cases are presented in Fig. 13.

For the truss structure, the results show that BA can detect single damage with higher accuracy in the first five iterations for D3. Furthermore, the computational time of BA is faster than that with GA, as it can be seen in Fig. 13. For damage scenario D3, Fig. 13(b), the results obtained by BA are converged after a few iterations, while those obtained by GA are converged after 50 iterations. In damage scenario D4, which represents the multiple damage case, we can see also high accuracy in finding the optimal results for the truss structure at iteration 25. Furthermore, the results obtained by BA are converged faster than those obtained using the improved charged system search (CSS) used in Ref. [46].



(d)

Fig. 13. (continued)

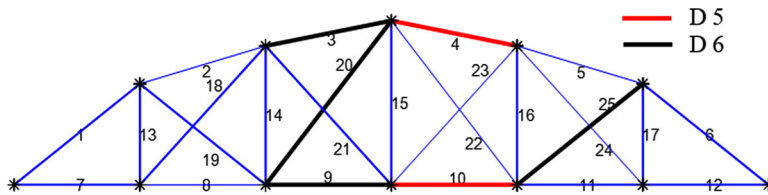


Fig. 14. FE model of planner truss [49].

**Table 4**  
The invariant cross sections of the truss structure.

Elements number	Area (m <sup>2</sup> )
1–6	0.0018
7–12	0.0015
13–17	0.0010
18–25	0.0012

**Table 5**  
Damage cases for the planner truss structure.

Damage scenario	Elements numbers	Stiffness reduction
D5	4	5
	10	7.5
D6	3	5
	9	10
	20	12
	25	15

5.2. Truss structures with 25 elements

In the second example, a more complex planar truss structure containing 25 bars [49], as shown in Fig. 14, is considered. The invariant cross sections of the truss structure are presented in Table 4 and the damage scenarios are presented in Table 5. In this example, we use only BA, as it was proven to perform better than GA in the previous example for damage identification using two scenarios as presented in Ref. [49].

The results for damage elements are presented in Figs. 15 and 16, and compared with those obtained using initial and improved MSE in Ref. [49]. According to the results, we can see that BA can detect damage with high accuracy in complex truss structures. Furthermore, it provides better identification than the initial and improved MSE for scenario D5.

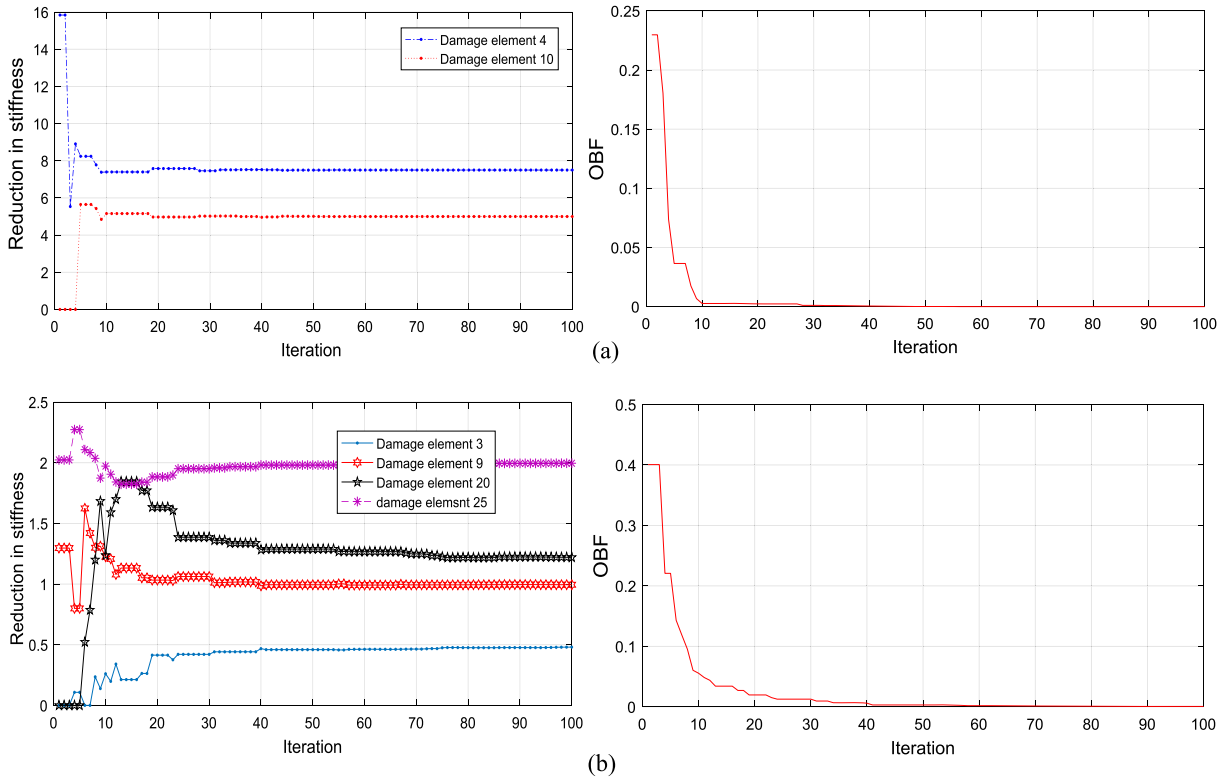


Fig. 15. Damage identification, reduction in stiffness and convergence, of a planner truss structure using BA: (a) D3 and (b) D4.

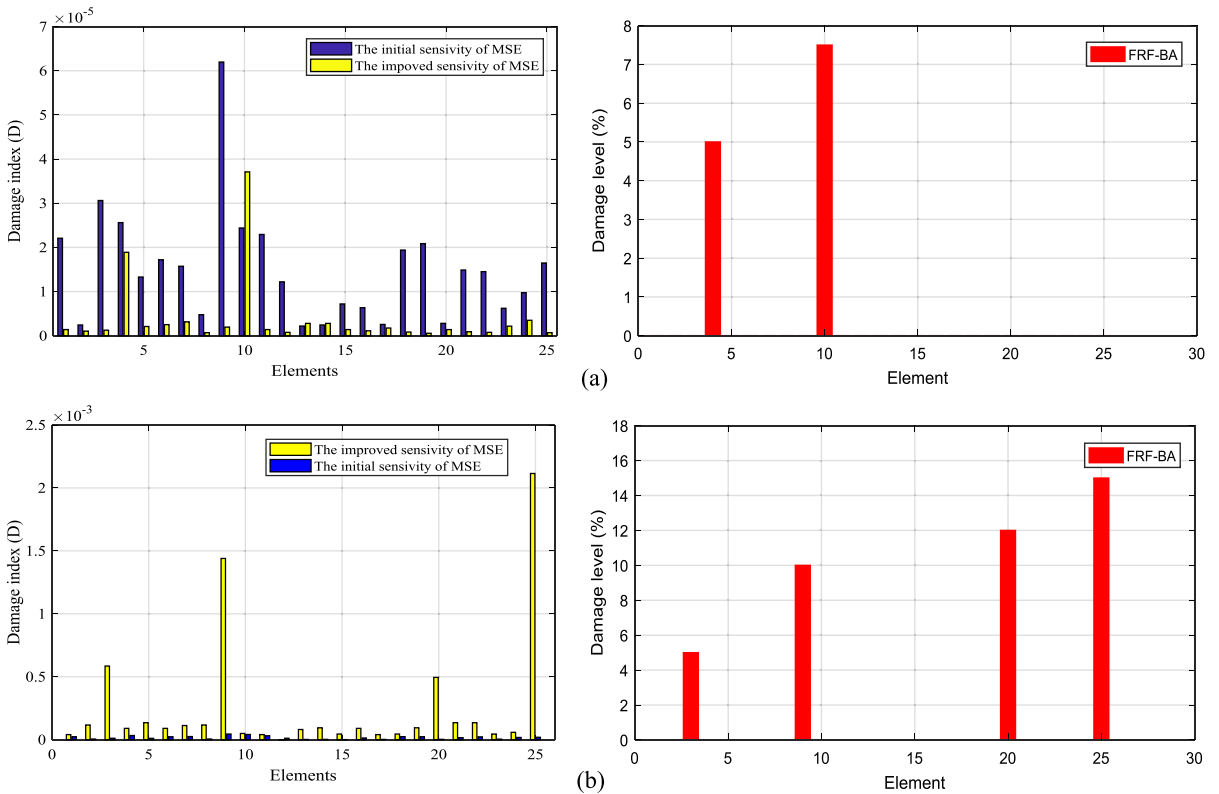


Fig. 16. Damage index using initial and improved MSE and FRF-BA: (a) D5 and (b) D6.

## 6. Conclusion

In this paper, we proposed a new technique based on FRF to find the location and level of damage in beam-like and truss structures using GA and BA. Optimization methods were proven to solve the objective function based on FRF as a damage indicator. Continuous beam-like and truss structures were studied as numerical example using FEM programmed in MATLAB to illustrate the correctness and efficiency of the proposed methodology with different damages locations and loss of rigidity. The results showed that damage detection, localization, and severity assessment with a high precision were possible using both BA and GA. However, BA was found to be more accurate and faster than GA.

Although, in this paper, the proposed damage detection technique is applied to structures modelled with one-dimensional beam and truss elements, it can be applied to 2D and 3D continuum structural models. The limitation in such a case is the large number of degrees of freedom, which in turns leads to high computational costs. As damage will be searched in each element, high computational costs will be required. Therefore, in order to apply the technique to continuum structures, model reduction based on Proper Orthogonal Decomposition (POD) and Radial Basis Function (RBF) should be used. This will be the topic of our future work.

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