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Is Universe a big pattern? Comment on Newell and Venkataramani's essay

L'univers est-il un grand damier ? Commentaire sur l'essai de Newell et Venkataramani

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A R T I C L E I N F O

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A B S T R A C T

This note is motivated by the contribution of Alan Newell to this thematic issue of *C. R. Mécanique* honoring Pierre Coullet. The point of this comment is to show that some ideas in Newell and Venkataramani's essay are already there in the present theory of the ultimate structure of our Universe, although formulated in a completely different way. In particular, the relationship between the structure of the Universe at small scale and the nagging question of the missing mass is present in both theories, whereas a precise link between the observations and either theory is yet to be found.

R É S U M É

Cette note a été suscitée par la contribution d'Alan Newell à ce numéro thématique des *C. R. Mécanique* en l'honneur de Pierre Coullet. Nous voudrions présenter dans ce commentaire l'idée selon laquelle certaines des conceptions exposées par Newell et Venkataramani sont déjà présentes dans la théorie actuelle sur la structure ultime de l'Univers, quand bien même elles sont formulées de façon évidemment différente. La question centrale posée par Newell et Venkataramani est celle de l'existence d'une structure sous-jacente envahissant en quelque sorte l'espace géométrique à toutes les échelles. Cette structure est la conséquence des lois de la mécanique quantique, qui imposent l'existence de fluctuations de point zéro pour le rayonnement électromagnétique et celle de la mer de Dirac pour les fermions. Ce « remplissage » a des conséquences bien connues et vérifiées expérimentalement, dont la force de Casimir pour les fluctuations de point zéro et l'écrantage des interactions électrostatiques à courte distance par la polarisation du « vide » constitué par la mer de Dirac, écrantage qui contribue au décalage de Lamb entre niveaux de l'atome d'hydrogène. Je montre que l'existence de la mer de Dirac conduit à un effet d'écran pour l'interaction gravitationnelle, qui se traduit par l'existence d'une échelle de longueur d'écran du même ordre de grandeur que la taille des galaxies si on prend la mer

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de Dirac des neutrinos, ce qui pourrait peut-être expliquer ce que l'on appelle la masse manquante en astrophysique.

In a bold attempt to understand how our Universe is made, Newell and Venkataramani suggest that what we see is a kind of large-scale "amplitude limit" of a small-scale pattern [1]. There are many examples of such changes of scale in science. Part of the nineteenth-century science was devoted to the understanding of macroscopic physics as a result of what happens at the scale of atoms and molecules. One early attempt in this direction was made by Cauchy when he derived the equations of elasticity by assuming an elastic material made of beads and springs and looked at the elastic properties of such a regular lattice on scales much bigger than the lattice spacing. This yields the correct equations for elastic materials with coefficients depending formally on microscopic parameters like molecular forces between the atoms or molecules at the sites of the lattice. Newell and Venkataramani propose a generalization of the same nature by assuming an Universe made like a regular pattern at small scale. What we observe at large scales would be described by the amplitude equation for this pattern, this being inspired by the amplitude equations designed to describe large-scale features of the roll structure generated by Rayleigh–Bénard thermal convection between two parallel horizontal plates [2]. As he notices, General Relativity equations could be seen as a kind of amplitude equation, because their right-hand side depends on the global properties of matter, through its contribution to the energy–momentum tensor.

One of the point of his essay is an explanation of dark matter. As I understand it, in the "pattern theory", dark matter should be a manifestation of geometrical defects in the underlying pattern. Every singularity or defect would contribute like a point mass to the gravity potential. This explanation is interesting because it gives another meaning to what we call particles and their mass. As it is, this theory is not a theory in the usual meaning, because there does not seem to be a possibility to show it is wrong, unless we had access one way or another to the scale of the pattern making the Universe. Nevertheless, it makes the interesting suggestion that the "missing mass" in astrophysics is not the manifestation of yet unobserved particles, but a more fundamental effect linked to the fabric of the Universe, explainable without adding one more kind of particles in an already crowded set. However interesting it is, the idea of a "pattern Universe" meets difficulties when confronted with what we know of the small-scale structure of our Universe. There is little doubt that, at small scales, we have a fairly complete understanding of it: quantum electrodynamics yields an unbelievably accurate picture of the physics down to scales linked one way or another to atomic physics. For sure, when one goes down to scales of high-energy physics, the situation is different, even though almost everything is fairly well understood and there does not seem to be much room for a revolution in this field. At this step, it is important to notice that the world as seen by present-day fundamental physics is not that different of the pattern universe. In particular, it implies that there is "something" at small scales besides an empty 4D Minkowskian Universe. This "something" has a fundamentally quantum origin, and the physics behind it is not that recent. At the very beginning of quantum theory, Planck noticed that a harmonic oscillator in its lowest energy state is not at rest in the classical sense: its energy is half the quantum of energy $h\nu$ he had discovered, h being Planck's constant and ν the classical frequency of the oscillator. If one applies this idea to the oscillation modes of the electromagnetic (EM) field in a box with conducting boundaries, one finds that the EM field has an *infinite* energy, even in the absence of any excitation of this field by sources, just because there are infinitely many oscillations modes of the EM field with a zero-point energy growing at increasing frequencies. This so-called zero-point energy has measurable consequences. It yields a pressure between two parallel conducting plates, as was discovered by Casimir and Polder, and has been measured since with an accuracy of about one percent. This zero-point energy of vacuum is of course different from what I understand of the pattern Universe, but it is a consequence of quantum theory (even long before this theory reached its present state) and is measurable.

Another consequence of quantum theory with the result of filling what we call vacuum is the existence, predicted by Dirac, of the Dirac sea. This is a consequence of the quantization of the relativistic dynamical equation of a particle of mass m (an electron in Dirac's theory). The relativistic energy E of an electron of linear three-momentum p is such that

$$E^2 = c^2 p^2 + m^2 c^4$$

c being the speed of light and m the electron mass. By quantizing the equation of motion of this particle, one finds the same relation between energy and momentum, which yields a possible negative value of E. Such negative values are unbounded from below, which leads to the possibility of getting infinite energy from nothing! Something obviously contrary to everyday experience. This riddle was solved by Dirac, who noticed that, for fermions, if the negative energy states are all filled, then there is no possible transition from one negative energy state to a deeper energy state that is already occupied, and therefore no way of getting energy from the negative energy states. The existence of this Dirac sea of filled negative energy states was proved indirectly by the observation of holes in the negative energy states that make a new kind of particle, called now the positrons.

The existence of this microstructure of the Universe made by the quantum fluctuations has the straightforward consequence to yield strongly divergent terms in Einstein's equations of General Relativity [3]. Those equations read:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = \frac{8\,\pi\,G}{c^2} T_{\mu\nu} \tag{1}$$

The indices μ and ν can take four different values, three for the space components and one for the time component. This last one (the time index) will be called 0 below, as usually done. The left-hand side is made with the curvature tensor $R_{\mu\nu}$, R without indices being Ricci's curvature. The curvatures $R_{\mu\nu}$ and R are related by nonlinear equations to the metric tensor $g_{\mu\nu}$. The right-hand side depends on the energy–momentum tensor $T_{\mu\nu}$, related to the properties of matter. An important quantity in this equation is the cosmological term $\Lambda g_{\mu\nu}$ on the left-hand side. It depends on the so-called cosmological constant Λ. Lastly, G is Newton's constant.

The T_{00} component of $T_{\mu\nu}$ is the energy density, which includes the rest mass of particles. Consider its contribution to T_{00} coming from either the zero-point fluctuations of QED vacuum or from Dirac sea, those contributions diverge just because they correspond to structures of wavelength of unbounded smallness. This yields either an infinite energy (in the case of the EM waves) or infinite mass and energy (in the case of Dirac sea). The pattern Universe would yield likely also a diverging contribution to T_{00}. This divergence of the contributions of quantum fluctuations to T_{00} is linked to the need to have theories invariant under Lorentz transform. This is particularly clear with Dirac's sea: this one is invariant under Lorentz transform, which brings infinitely small scales if the velocity in the Lorentz transform is close to the speed of light. The same has to be true for a small-scale pattern contributing to the fabric of space-time: otherwise it seems to be impossible to keep the Lorentz invariance of the fundamental equations of physics. Said in another way, a pattern that is steady in a certain frame of reference and not uniform (say with a given period in space and time like a finite-amplitude wave) would break a fundamental symmetry in a way that could be measurable by experiments. The consequence of the Lorentz invariance of the quantum vacuum are that this vacuum is made of the superposition of all states (a superposition in the quantum sense) derived from each other by all Lorentz transformations. This leads to an infinite energy of this quantum ground state and therefore to an infinite contribution to T_{00}.

This divergence of T_{00} is "cured" traditionally by assuming that, on the left-hand side of Einstein's equations, the cosmological constant Λ is infinite and balances exactly the divergent part of T_{00}. Once this is done, this gives the impression that the contribution of the quantum fluctuations to gravity is set to zero once for all and should not be worried about anymore, but this is not correct: the exact cancellation of the quantum fluctuations is not universal, whereas we look for the effect of the quantum fluctuations on the law of attraction of masses at very large distance, namely in an Universe with curvature. This means that we have to look at the changes in the Dirac sea induced by a large-scale gravity field: this field makes non-flat the metric and so changes the dynamics of particles or photons with respect to the case of a flat metric. This adds (or subtracts) a mass to vacuum compared to the situation without gravitational field. Such a change of density takes place classically if particles with a mass are scattered by the gravitational field. This change can be estimated by assuming that particles impinge the gravitational field coming from infinity and are then scattered by the (relatively very weak) gravitational field. Thanks to that, a change of density with respect to the density of the straight line motion follows. This change of density contributes to the gravitational field because, as just explained, it is not compensated by the cosmological term of Einstein's equations. Because of its smallness, it can be computed at linear order with respect to the local gravitational field. Putting this term into Poisson's equation for the gravity potential, one finds a contribution to the mass coming from this polarization of the quantum vacuum (see below for an explanation of what is meant by "polarization" here). Because of the linearity of the equation with respect to the local value of the gravitation potential, the equation so derived looks like Debye–Huckel [4] equation for the screening of a charge in a medium with electric charges. But contrary to the case studied by Debye and Huckel, the charges there are masses interacting gravitationally and there is no compensation between charges of different signs.

This introduces a novel feature compared to the pattern Universe of Newell: there the missing mass is linked to the defects of the underlying pattern Universe and those defects condense near Galaxies or clusters of Galaxies. What is missing in this argument are two things: first, the typical scale on which those defects behave and then what kind of defect is considered? On this last point one may wonder what are the typical defects of a pattern existing in a four-dimensional Minkowski space, reference being made mostly to defects in roll systems in 2D ordinary space. In amplitude equations, there are long-range forces because the pattern deformed by defects tends to restore its symmetry by eliminating the defects, usually by attraction of such defects of opposite signs ending up by merging and disappearance. A large-scale motion of the population of defects is possible, this being triggered by the gravitation field of large-scale objects like Galaxies. A weakness of this chain of arguments is that the contribution to the attraction due to the defects of the pattern has to be negligible at the scale of the Solar system and pertinent at the much bigger space scale of the Galaxy. This seems to be hard to explain, because the gradient of the gravitational potentials would be much bigger in our Solar system than in the Galaxy and so have much bigger effect locally (say at scales of a Cavendish balance) than on the big scale. This riddle can be solved only by introducing an intrinsic large scale where the gravitation changes over scales about the size of the Galaxy, as sketched below.

As this is a comment, no detailed derivation will be given, only the main features, the message being that a large length scale can be derived from the existence of the Dirac sea and from its effect on gravitation. As said before, the exact cancellation of the quantum fluctuations does not eliminate all possible effects of the Dirac sea and of QED vacuum on gravitation. We look for the effect of the quantum fluctuations on the law of attraction of masses at very large distance, namely in an Universe with small but non-zero curvature. This means that we have to look at changes in the Dirac sea induced by a large-scale gravity field: this field makes the metric non-flat and so changes the dynamics of particles or photons with respect to the case of a flat metric. We look for a polarization effect of the vacuum quite similar to the one considered by Dirac [5] and Heisenberg [6] when they computed the change of the "bare" charge of an electron due to the polarization of the Dirac sea around it. Those famous pieces of theoretical physics introduced also the fundamental idea of renormalization, necessary to get at the end an effect that is finite and not given by diverging integrals. We shall use the word "polarization" in a slightly extended meaning. Usually this refers, as in the works of Dirac and of Heisenberg, to changes in the electric potential by the perturbation of the density of charges induced by the field itself. We shall use the same word "polarization" to denote changes in the distribution of masses and energies induced by a gravity potential. Indeed, there are some formal similarities between the two cases (gravitation and electrostatic attraction), both being long ranged. For instance, the Jeans instability of an uniform distribution of point masses is described by equations very similar to the ones of the Langmuir oscillations in plasmas, but for a crucial sign difference making one oscillating in time (Langmuir oscillations) and the other just plainly unstable. Moreover, there is also a fundamental difference because there are no difference of sign in the gravitational attraction: all masses attract each other (whence the Jeans instability).

The derivation sketched below makes appear a coefficient of the polarization term in Poisson's equation with the physical dimension of an inverse square length. This length scale depends on the mass of the particles of the Dirac sea contributing to polarization. If one considers that those particles are electrons, the length scale is far too short compared to what happens in the solar system, for instance. Therefore, this polarization of the Dirac sea of electrons cannot be observable, supposing that its derivation is correct. This is because it is made by assuming that the electrons of the Dirac sea do not interact, which is obviously not true: electrons interact by the EM field, including when in negative energy states. This is taken into account in the renormalization effect of the electric charges by the vacuum fluctuations. Therefore, on large scales, one cannot assume that the particles are free particles. A simple guess is to assume that those charged particles interact in such a way that they make a kind of fluid with short-range correlations, which is far less sensitive to external forces like gravitation than free particles, although this is to be verified by calculations yet to be done.

Therefore, it is natural to turn to other fermions, interacting very weakly. Among the known particles neutrinos make a fair choice: they are fermions, with a very small mass and they interact very weakly with other particles. Therefore, we shall assume that the Dirac sea of neutrinos makes a possible contribution to the polarization by the gravity field. Because of their very small mass, the length scale introduced by the polarization is very large, in the same range as the size of Galaxies, which is precisely the length scales for which the missing mass is observed.

We shall derive now the Debye-like equation for the polarization of the Dirac sea. Let $\Phi(r)$ be the gravity potential generated by a point mass M located at $r = 0$, this mass being very large, the one of a star for instance. Because the gravitation potential is also partly due to particles in Dirac sea, this potential has to be the solution to a Debye-like equation including this polarization. With this mass M only, the gravity potential (a quantity with the physical dimension of a velocity square, or an energy per unit mass) is a solution to Poisson's equation:

$$\nabla^2 \Phi = 4\,\pi\, G\, M\, \delta(\mathbf{r}) \tag{2}$$

In this equation, $\delta(\mathbf{r})$ is the three-dimensional Dirac singular function, boldface being for vectors in 3D. Now we shall add to the right-hand side of this equation a contribution coming from the changes of density of particles in the Dirac sea by their interaction with the gravity potential. This change of density is due to the displacement of particles scattered by the gravity potential Φ due either to the mass M at $r = 0$ or to the polarization induced by the field. Let us estimate first the number density of the Dirac sea. From obvious scaling laws, this number density is the cube of an inverse length. The only length scale there is the Compton wavelength $\frac{\hbar}{m\,c}$, m being the mass of the particles in Dirac sea (even though we keep the same symbol m as for the electron mass, it does not imply that we are dealing with electrons), c the speed of light, and $\hbar = 2\,\pi$ times Planck's constant. Therefore the number density is estimated as the inverse cube of this wavelength. Of course, this should not be understood stricto sensu: the true number density of the Dirac sea is infinite because all negative energy states are filled. Nevertheless, when computing the quantity we would like to obtain, one should introduce such a number density as a coefficient of an integral over the momentum of particles in the Dirac sea. Because we are trying to estimate a quantity found by adding all contributions from particles in the Dirac sea, we have to worry concerning the convergence of this integral at large momenta. We shall come to this question in a future publication and take $\left(\frac{\hbar}{m\,c}\right)^{-3}$ as the number density of particles in the Dirac sea.

The order of magnitude of the change of number density induced by the scattering can be estimated in two different ways. One can assume first that it is proportional to the perturbation brought by the potential Φ and also to the "number density" in the Dirac sea times the mass of each particle there. This amounts to add to the right-hand side of equation (2) a term like $m\Phi\left(\frac{\hbar}{m\,c}\right)^{-3}$ times Newton's constant G. Another derivation amounts to estimate for an average trajectory the change of radial position induced by the scattering by the gravitational potential and to estimate next the total change by adding contributions from all particles in Dirac's sea. Each state is perturbed by the gravity potential in such a way that the trajectory is shifted radially by an amount $\delta r \sim \frac{1}{c^2}\int_r^{\infty} \mathrm{d}r\, \Phi(r)$. The sum of contributions of all particles, as far as orders of magnitude are concerned, is the same quantity derived with respect to r (which comes from the Jacobian from the state without gravity potential to the state with gravity potential) times the number density $\left(\frac{\hbar}{m\,c}\right)^{-3}$. The result is that the change of density is of order $\frac{\Phi(r)}{m\,c}\left(\frac{\hbar}{m\,c}\right)^{-3}$, as already estimated.

Therefore, the Poisson equation including the polarization of the Dirac sea reads:

$$\nabla^2 \Phi = 4\pi G \left(\rho(\mathbf{r}) + C\, m \frac{\Phi(r)}{c^2} \left(\frac{m\, c}{\hbar} \right)^3 \right) \tag{3}$$

an equation formally similar to the Debye–Huckel equation for the local polarization in electrolytes and plasmas, C being a dimensionless constant of order 1. Moreover, the quantity $\rho(\mathbf{r})$ is the mass density inside the astrophysical object one considers, which may include stars, molecular clouds, etc. The derivation just made does not tell if C is positive or negative. This equation makes appear a length scale somewhat analogous, at least mathematically, to the Debye screening length in electrolytes and plasmas. This is the length $\lambda = \left(\frac{\hbar^3}{G\, m^4\, c} \right)^{1/2}$. Interestingly, this length is within astronomical scales. Because of the way it depends on the mass m, it is the largest for the smallest mass. The derivation of this length assumes that particles in the Dirac sea interact only gravitationally, which is obviously not the case for many fermions, which could be candidate for contributing to the polarization by gravitation. To give an example of the order of magnitudes involved, one can take first the electrons, of mass approximately 10^{-30} kg. This yields a length scale λ of order 10^{10} m, smaller than the typical size of galaxies, about 10^{20} m for the Milky way. If one takes instead an estimate of the rest mass of neutrinos are in the range of 0.1 eV/c^2 (this is actually an upper bound for this mass, which seems otherwise unknown), one finds a λ, of order 10^{22} m, much larger than for the electron, but larger than the size of Galaxies by a factor of one hundred. This factor of one hundred could perhaps be balanced by other factors in a theory incorporating the renormalization of the gravitational mass in a more detailed (and perhaps more convincing!) way.

I do not believe that it makes much sense to end this comment by a more or less formal set of conclusions. As Newell and Venkataramanil's paper, it is an open-ended piece aiming mostly to trigger a debate on fundamental issues coming from the development of our understanding of the world as it is. Our main point is that the ultimate word in this field has yet to be said, and perhaps will never be. Lessons have still to be drawn from the development of quantum theory and gravitation to get a fully coherent understanding of how our Universe is made.

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