



Force–displacement relationship in micro-metric pantographs: Experiments and numerical simulations



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ABSTRACT

In this paper, we reveal that the mathematical discrete model of Hencky type, introduced in [1], is appropriate for describing the mechanical behavior of micro-metric pantographic elementary modules. This behavior does not differ remarkably from what has been observed for milli-metric modules, as we prove with suitably designed experiments. Therefore, we conclude that the concept of pantographic microstructure seems feasible for micro-metrically architected microstructured (meta)materials as well. These results are particularly indicative of the possibility of fabricating materials that can have an underlying pantographic microstructure at micrometric scale, so that its unique behavior can be exploited in a larger range of technological applications.

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1. Introduction

One of the frontiers of modern mechanics is the design of the so-called metamaterials or materials with architected properties. The scientific challenges are formidable, as competencies in a variety of fields of mechanical sciences are required. First, the mathematical model that is the most convenient for the designed metamaterial must be established. The mechanical properties of potentially conceivable metamaterials can, then, be determined thoroughly. The established *a priori* mathematical model can, thus, be used to conceive the metamaterial with the desired mechanical properties. The conceived metamaterials, with the *a priori* selected evolution equations, can then be fabricated by using either 3D printing or stereolithography. In other words, this stage includes the solution to the so-called *synthesis problem*; see some details in [2–5]. Finally, experiments must be designed to measure the relevant quantities and establish relationships that have been predicted theoretically.

In this paper, we consider a metamaterial that preserves its elastic properties under relatively large deformations. We hypothesize that the mechanical behavior of such a metamaterial can be described by the discrete Hencky-type model intro-

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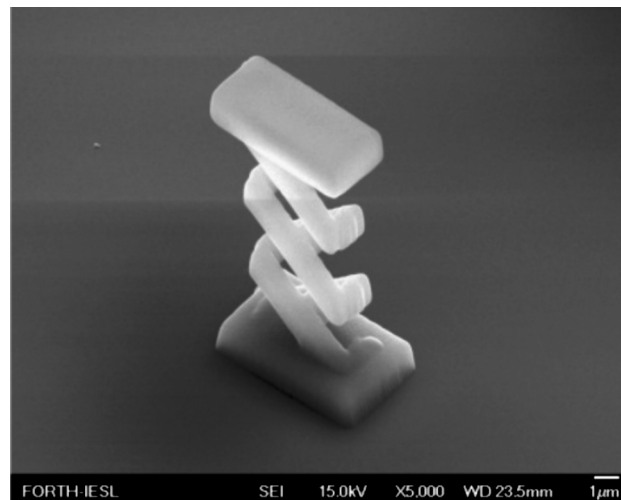


Fig. 1. Micro-pantograph image obtained by a scanning electron microscope.

duced in [1] for pantographic modules. The numerical code developed in [6] was also adapted to serve as a guidance to the design of the experimental procedure in the presented study. A key point to consider relates to the smallest length scale at which the pantographic effect could be utilized. The question that needs to be elucidated is whether these geometries can be used as elementary substructures in 3D printed metamaterials. Through comparison of experimental measurements, we show here that micro-metric pantographic elementary modules exhibit nearly the same pantographic effect as that shown by milli-metric modules. Images were captured using a Scanning Electron Microscope to reveal the deformation mechanism of the structure during the nanoindentation experiments. Comparison of these with similar images for milli-metric pantograph provide further evidence of the congruity in the behavior at these disparate scales. What is even more remarkable is that the micro-metric elementary modules have been fabricated using a completely different 3D printing method in contrast to the milli-metric modules. We also show that the experimentally measured behavior of the micro-metric elementary modules can be efficiently described using the Hencky-type model.

On the basis of the presented results, we can conclude that *the concept of pantographic micro-structure, which allows for the design of metamaterials undergoing large elastic deformations, is also feasible to micro-structures having characteristic length of 50 micrometers*. Therefore, we are convinced that these elementary modules can be used for the microscopic architecture of novel metamaterials. The obtained results can undoubtedly pave the way to some interesting fields of research having relevant technological applications.

In the following sections, we briefly describe the 3D printing process of micro-pantographs and their mechanical tests, see Sec. 2; then, in Sec. 3, we discuss the numerical results obtained from the Hencky-type numerical model introduced in [1,6], comparing its results with those obtained from the corresponding experimental test. Finally, we discuss the key findings and their implications as part of the concluding remarks and future challenges in Sec. 4.

2. 3D printing process of micro-pantographs

Micro-pantographs can be built by using direct laser writing using the multi-photon polymerization technology.¹ Fig. 1 shows an image of the fabricated micro-metric pantograph elementary module obtained with a scanning electron microscope. With direct laser writing, it is possible to print a 3D micro-pantograph using a beam of an ultrafast laser focused into the volume of a photo-sensitive material. A high-resolution 3D structure is obtained by moving the focus of the beam. After this printing, a proper solvent is used to dissolve the unscanned and the unpolymerized area, revealing the 3D printed structure, see [8] for a detailed description of the complete process. All the chemicals used in this work were obtained from Sigma-Aldrich. The material used for the fabrication of the 3D structures is based on the organic-inorganic hybrid described in [9]. It has been produced by the addition of zirconium prop-oxide to methacryloxy-propyl trimethoxysilane. The monomer 2-(dimethylamino)ethyl methacrylate has been added as a quencher, and DMAEMAs were used as the organic photopolymerizable monomers, while ZPO and the alkoxy-silane groups of MAPTMS served as the inorganic network forming moieties.

The experimental setup employed for 3D structure fabrication has been further described in [10,11]. A FemtoFiber pro NIR laser (Ultrafast fiber laser, 78 nm, 80 MHz, <100 fs) was focused into the photopolymerizable composite using an objective lens (100, N.A. = 1.4, Zeiss, Plan Apochromat). Sample movement was achieved using piezoelectric and linear

¹ See [7] for a prime on 3D printing technology of mill-metric printing.

Table 1
Mechanical and geometrical parameters of the printed micro-pantograph (E in GPa, f , b , s , d , and h in μm).

E	ν	f	b	s	d	h
0.5	0.45	7.49	0.72	1.7	2	0.8

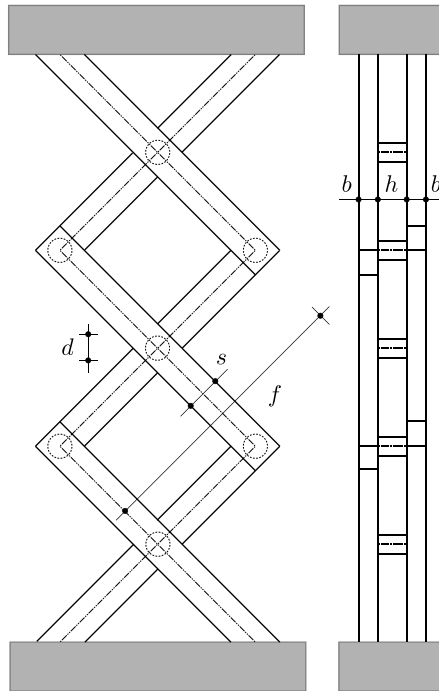


Fig. 2. Schematic representation of the micro-pantograph geometry (frontal view on the left; and lateral view on the right).

stages, for fine and step movement, respectively (Physik Instrumente). The whole setup was computer-controlled using the customized software of A. Lemonis of IESL/FORTH. The average power used for the fabrication of the high-resolution structures was 2 mW, measured before the objective. The scanning speed was always set to 10 $\mu\text{m/s}$. To our knowledge, only the 2-photon polymerization technology can reach the resolutions needed to print the micro-metric structure shown in Fig. 1. In general, the fabrication of such micro-scale structures is not a trivial exercise, although a number of printing approaches have been proposed, some of which are even commercially available. Nevertheless, the effort to realize structures such as those presented here amply demonstrates the feasibility of the approach.

The estimated mechanical and geometrical parameters of 3D-printed micro-pantographs is reported in Table 1 (see also Fig. 2).

The printed micro-pantographs were subjected to compression loading using the Hysitron TI 950 TriboIndenter nano-indenter (Bruker²).

The compression experiments were performed using displacement-control for a specific deformation profile. It was used so that any creep effects will be avoided while conducting the tests, see [14]. The deformation profile that was used is shown in Fig. 3. Force–displacement curves obtained testing two identical printed specimens are reported in Fig. 4.

3. Discrete model and numerical simulations

In a series of papers, see [1,15–18], we have shown that a very simple Lagrangian model can be used to predict accurately, and also with a very low computational cost in terms of memory engagement and computing time, the nonlinear mechanical behavior of pantographic structures.³ The same model was used to predict the first failure of specimen under

² There is an alternative way to perform nano-indentation experiments by using an in-situ micro-indentation and a scanning electron microscope, see [12,13].

³ This is not the only practicable road, for example see that proposed in [19,20].

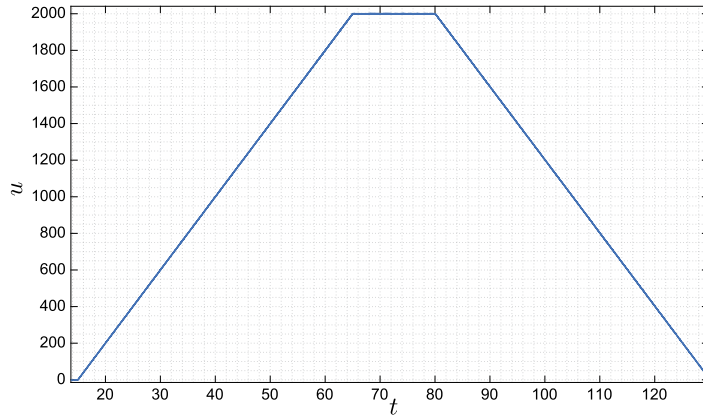


Fig. 3. Displacement, u in μm , vs. time, t in s, plot.

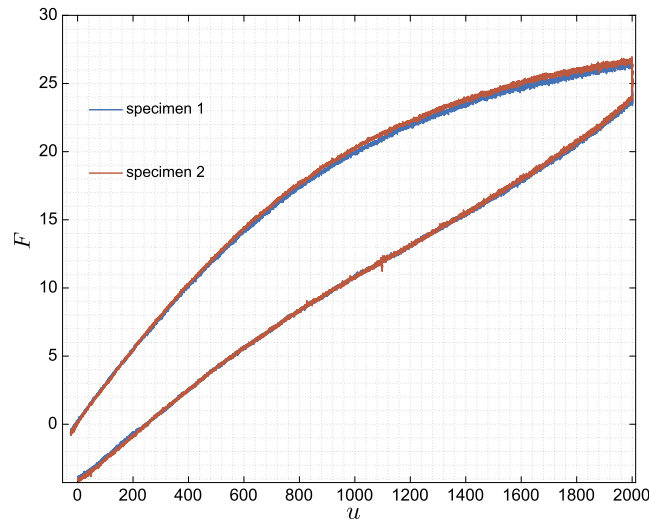


Fig. 4. Force F in the direction of the longer side of the pantograph, in μN , vs. displacement u , in nm, for the compression test: experimental results for a pair of identical specimens.

shear test [21]⁴ and also to prove a relative insensitivity of the model to geometrical and mechanical imperfections, see [23].

We refer to [1] for a precise description of the Hencky-type model. Here we report only its key points: *i*) the pantograph is modeled as a plane lattice of beams connected by means of cylindrical pivots; *ii*) each beam is modeled as an extensible Euler beam by means of rigid links and elastic joints, extensional and rotational; *iii*) pivots are modeled as rotational elastic joints connecting the two sets of orthogonal beams. The ideas briefly sketched in the three key points above completely define the strain energy of each one elastic joint, and therefore of the whole pantographic lattice. Denoting by a and b the extensional and flexural rigidity of the beams, respectively, and c the torsional rigidity of the pivots strain energies are assumed as follows:

$$\begin{aligned} E_a &= \frac{1}{2}a(\ell - \ell_0)^2, \\ E_b &= b(\cos \beta + 1), \\ E_c &= \frac{1}{2}c\left(\gamma - \frac{\pi}{2}\right)^2 \end{aligned} \quad (1)$$

where ℓ and ℓ_0 are the current and reference length of the beam section in-between two consecutive elastic joints, respectively, β is the current angle between two consecutive rigid links of the same beam, and γ is the current angle between two orthogonal beams that share the same pivot. This Hencky-type model was improved in [6,24], refining the flexural part

⁴ See also the recent proposal concerning [22] for brittle materials.

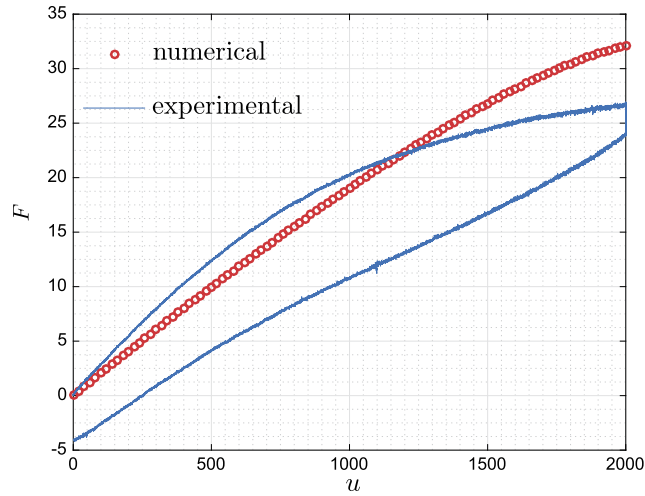


Fig. 5. Force F in the direction of the longer side of the pantograph, in μN , vs. displacement u , in nm , for the compression test: numerical (in red) and experimental (in blue) results.

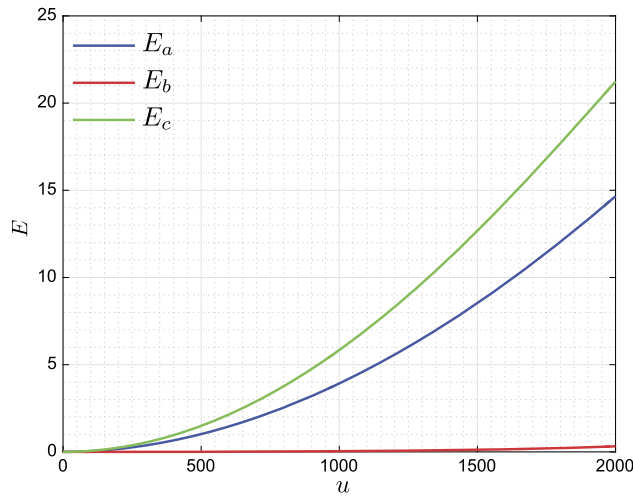


Fig. 6. Strain energy, in $\mu\text{N}\cdot\mu\text{m}$, vs. the displacement u , in nm , for the compression test split in extensional E_a , bending E_b , and shearing E_c parts.

of the strain energy by simply adding an intermediate elastic joint for each beam section in-between two consecutive pivots. In [6], we compare the results of the numerical simulations with the experiments on milli-metric pantographs proving that, after an accurate identification of the rigidities, the results deriving from experiments and numerical simulations are closer, both in terms of deformations and predicted force relative to the imposed displacements. Naturally, to follow out-of-plane buckling phenomena, a different kind of model is necessary, see, e.g., the proposal reported in [25] for a continuum model or the 3D Hencky-type beam model [26] based on the suggestion reported in [27].

The simplistic, but very effective, model derived based upon the conjecture discussed in the preceding paragraphs along with an analysis strategy for solving the nonlinear equilibrium equations, see, e.g., [6], has to be completed with a set of mechanical parameters, that is the rigidities of elastic joints, which have to be estimated by means of some physical or conceptual experiment. This identification has been performed in [6] for a millimetric pantographic lattice following the suggestions reported in [28,29] using the results of an elongation test.

Here we use exactly the same methodology described in [6] to identify the rigidities of elastic joints used in the Lagrangian model of the pantographic lattice starting from the available data (the displacement–force plot reported in Fig. 4) obtaining a quick, surely improvable, guess of the elastic joint rigidities. Using the triplet $a = 300 \mu\text{N}/\mu\text{m}$, $b = 1 \mu\text{N}\cdot\mu\text{m}$ and $c = 100 \mu\text{N}\cdot\mu\text{m}$, the plot reported in Fig. 5 for the numerically obtained loading part of the displacement–force curve has been obtained (in order to make the comparison simple, we have reported also the whole experimental curve for specimen 2).

We remark that experimental and numerical curves, see Fig. 5, are closer, at least until values of displacement u less than 1500 nm , proving the quality of the simple Hencky-type model. In addition, we observe that the numerical curve could

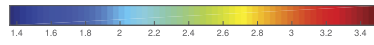
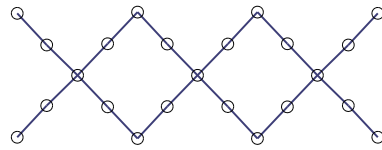
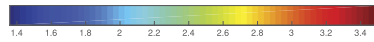
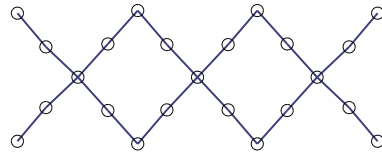
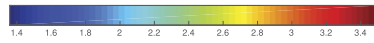
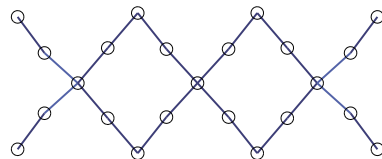
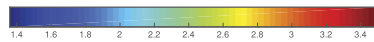
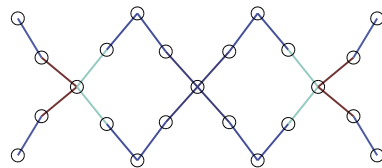
(a) $u = 500$ (b) $u = 1000$ (c) $u = 1500$ (d) $u = 2000$

Fig. 7. Compression test: synopsis of the deformation history for $u = 500, 1000, 1500,$ and 2000 nm along with the strain energy density plotted using a color scale.

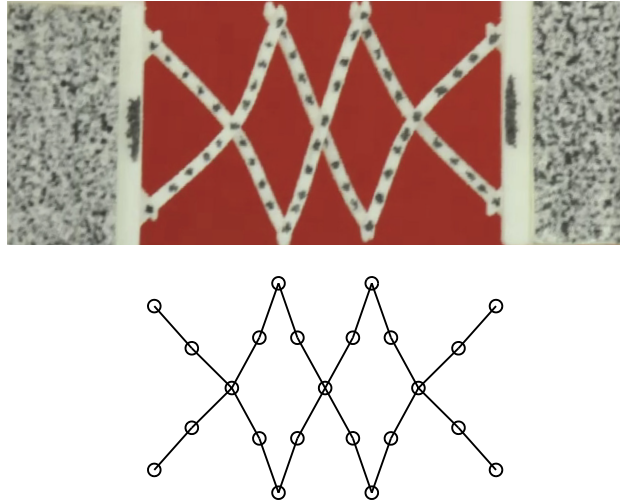


Fig. 8. Compression test for a mini-pantograph (dimensions 13 mm \times 13 mm): comparison between experimental (on the top) and numerical (on the bottom) deformations for an imposed displacement $u = 11.89$ mm.

be much closer to the experimental one by changing the strain energy law (the law chosen for the Hencky-type model used to perform the calculation presented here is quadratic).

Fig. 6 reports a plot of the strain energy as the imposed displacement u increases. The energy is split in extensional E_a , bending E_b , and shearing E_c (that referred to the torsion of pivots) parts. It is worth noticing that, for this case, the extensional part E_a and the shearing part E_c are similar. Conversely, the bending part E_b , which is referred to the beams, is negligible. This suggests, observing that the total volume of the beams is much less than the total volume of the pivots, that the energy density has its own peaks in the pivots.

Fig. 7 reports the synopsis of the deformation history, that is the deformation corresponding to $u = 500, 1000, 1500,$ and 2000 , along with the strain energy density plotted using a color scale on the beams.⁵ The energy density shows clearly the parts of the mini-pantograph that adsorb high levels of strain energy.

At this stage, we are unable to compare the deformation history obtained numerically with the experimental results due to the non-availability of the measured deformation history. It is notable, however, that such a comparison may provide a further confirmation of the quality of the numerical results, besides providing useful information for improving the identification of the rigidity parameters. Based upon our previous experience, we are confident that the results produced by numerical simulations obtained by our very simple model agree with the experimental ones. Our trust is supported by the comparison between the experimental result on the milli-metric pantographic lattice (dimensions 13 mm \times 13 mm); the complete data are reported in the case of an elongation test in [6] under compression test, together with its numerical simulation. For reference, we report here the comparison between the experimental and the numerically obtained deformations at the end of the loading process, see Fig. 8.

4. Concluding remarks and future challenges

The concept of pantographic structures as fundamental units of novel metamaterials has been conceived in [30] from a purely theoretical point of view. The main features of the conceived basic structures are the following: 1) at a lower length scale, there are some elastic elements (*i.e.* the internal pivots, see Fig. 2), which offer an extremely weak resistance to some deformation, while the other elastic elements are capable to support external loads (*i.e.* the beams constituting the lattice whose bending and extensional stiffnesses are relatively stronger); 2) the lower scale structure allows for some macro-scale deformations at the expense of a relatively small deformation energy; 3) the most suitable macroscopic continuum model that is suitable for describing the novel metamaterial must include second gradient effects (see [31,32]).

In [33], it has been proven that this concept has some potentialities as the specimens printed with 3D printing technology have shown a behavior that is very close to the one forecast by the theoretical design analysis. However, the lower length scale that has been considered up to now amounts to 1 millimetre and it has been questioned if smaller length scales could be exploited by keeping the desired macro-effects.

The results presented in this paper used a novel 3D printing technology that allowed us to produce pantographic specimens having a length scale of one micrometer. The experimental apparatus that has been used is at the frontier of the present state of the art, and the obtained measurements were performed in a very precise and effective manner. In this

⁵ The energy referred to each pivot is split in four equal contributions assigning each one to the beams sharing the pivot.

paper, it has been proven that micro-metric and milli-metric pantographic modules have a very similar behavior. This is the result of the comparison among: *i*) numerical simulations with a Hencky-type model; *ii*) experimental measurements with millimetres pantographs, and *iii*) and experimental results obtained by using micrometric pantographs. As a consequence, we believe that the design and construction of pantographic metamaterials having a microstructure whose length scale is 1 micrometer is a technologically feasible and offers avenues for novel and interesting engineering applications.

Going forward, the following research questions can be immediately recognized:

- (1) to get experimental measurements of the deformation patterns of micro-pantographs and to compare the obtained measures with the available numerical predictions;
- (2) to conceive a campaign of measurements in which the micrometric pantographs have different geometrical properties; in particular the possibility to have internal pivots whose dimensions are different from the interconnected beams seems of a great relevance;
- (3) to start building larger specimens constituted by many micrometric pantographic modules.

More long-term perspectives are very ambitious: to build a metamaterial undergoing large elastic deformations that has a lower scale structure with characteristic dimensions of a few micrometers.

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