INSTITUT DE FRANCE
Académie des sciences

Comptes Rendus
Mécanique

Alain Dervieux

To be structured, or unstructured, fifty years of slings and arrows

Published online: 3 October 2022

https://doi.org/10.5802/crmeca.115

Part of Special Issue: More than a half century of Computational Fluid Dynamics
Guest editor: Mohammed El Ganaoui (Professeur des Universités, Spécialiste : Mécanique des Fluides et Transferts de Chaleur et de Masse, Université de Lorraine)

Les Comptes Rendus. Mécanique *sont membres du*
Centre Mersenne pour l'édition scientifique ouverte
www.centre-mersenne.org
e-ISSN : 1873-7234

Comptes Rendus
Mécanique
Published online: 3 October 2022
https://doi.org/10.5802/crmeca.115

More than a half century of Computational Fluid Dynamics / *Plus d'un demi-siècle de mécanique des fluides numérique*

To be structured, or unstructured, fifty years of slings and arrows

Alain Dervieux [a]

[a] Société LEMMA, 2000 route des Lucioles, Sophia-Antipolis, France, and Institut National de Recherche en Informatique et en Automatique (INRIA), 2004 Route des lucioles, F-06902 Sophia-Antipolis, France

E-mail: alain.dervieux@inria.fr

Abstract. This paper is not a review, but narrates the personal experiences (nearly fifty years) of the author concerning unstructured mesh, a well debated theme during these years.

Keywords. Unstructured mesh, Finite difference, Finite element, Finite volume, Computational fluid dynamics.

Published online: 3 October 2022

1. Introduction

In an unstructured mesh, the number $N(i)$ of nodes neighboring a node i is not regularly defined from one node to the other one. Computing structures has led rapidly to consider unstructured meshes. Try to mesh the Eiffel tower (in fact Sidney opera was one of the first famous success of unstructured meshes). The static computations allowed to introduce variational formulation and then the finite element method.

2. In the earlier times

When I started studying CFD/CSM, the two main enemies were finite differences/volumes on my right and finite elements on my left. FD/FV were applied on structured meshes, and FE on unstructured meshes.[1] Advised by Jean Céa and then recruited by Roland Glowinski, I was necessarily in the FE party. The preferred flow model in aeronautics was the full potential model, a mainly elliptic model and Roland's team and friends (involving Olivier Pironneau) in association with the Dassault team (around Jacques Périaux) had succeeded to compute the flow around a complete aircraft with a finite-element method and an unstructured mesh [1].

[1] FD = Finite Difference, FV = Finite Volume, FE = Finite Element.

3. Euler flows

However, in the eigthies, the challenge was to compute with the Euler model, a first step before (compressible) Navier–Stokes. The mathematical model is hyperbolic and its discretization requires the derivation of a sophisticated stabilization. Indeed, for the convection of a scalar field, finite element computations were limited by Peclet effects (unstability when the diffusion term does not exists or is too small). The answer proposed by the FD/FV camp was "upwinding", that is managing in such a way that the concerned derivatives are not computed in a central manner around the node but upward with respect to the convection velocity. An important activity in finite elements developed in order to upwind this method.

A second debate in compressible CFD was related to the building of explicit time advancing having a sufficient stability domain. One-step and two-step advancing were initially preferred, like the two-step method of MacCormack [2], with a predictor step with upwinding in one mesh direction, a corrector step upwinded in the opposite direction. Two main advantages were identified. Firstly, such schemes need only one or two flux assemblies for a stable time step, which was interpreted as a chance for best efficiency, secondly, the schemes were variant of the Lax–Wendroff scheme [3] and enjoyed a built-in dissipation, satisfying the dissipative Kreiss criterion [4]. The Lax–Wendroff FD method was then extended to P_1 FE independantly by three teams [5–7]. Lax–Wendroff type time-advancing took a backseat with the rise of Jameson–Schmidt–Turkel scheme [8], derived first for structured meshes, then for unstructured ones [9].

A third debate concerned the satisfaction of a so-called entropy condition, which is sufficient for ensuring the uniqueness of solution of many nonlinear hyperbolic models. Conversely, the discretization of an hyperbolic model can produce parasitic non-entropic (non-unique) numerical solutions. The entropy issue was observed for full potential models, but it was soon clear that a simple upwinding of the density permitted to ensure that entropic solutions were obtained. For the Euler equations, most time advancing methods were sufficiently dissipative to ensure entropic solutions.

The above events did not close the debate concerning upwinding. In the late fifties, Godunov pointed the interest of using Riemann problems solution for upwinding the approximation of nonlinear hyperbolic problems (he recounts the story of this in [10]). The option was a little complex for routine computing and Roe proposed his approximate Riemann solver [11] and Osher with Chakravarthy, a more sophisticated and robust one [12].

First-order accurate upwind schemes were not sufficiently accurate, and second-order schemes not sufficiently stable for shock capturing (although artificial viscosity methods were not so bad). Significant progress was carried by the introduction of limiters by Boris and Book [13]. van Leer [14] brought a clever synthesis between Godunov upwinding and limiters, introducing the idea of local reconstruction. An illuminating TVD theory was then proposed by Harten [15].

Alongside, the stabilization of finite element for hyperbolic CFD models was addressed by Hughes and Mallet [16].

Many of these ideas (together with implicit time advancing) were integrated in an unstructured method developed for hypersonic flows around the shapes considered in the European project Hermes—Figure 1—[17], project under the direction of Pierre Perrier which gave a favourable impulse to CFD researches in Europe.

4. Supercomputer is coming

Few remarks must be made concerning the computational time. An interesting question is: in the 80's–90's, did computational aerodynamics reduce "wall clock calculation times", thanks to

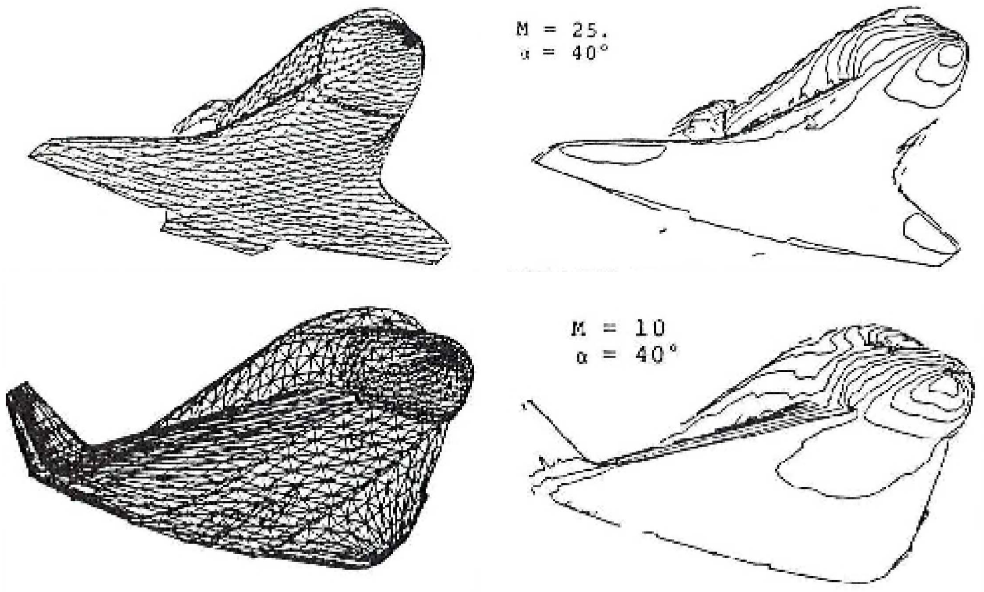

Figure 1. High Mach number high angle of attack Euler computations around the geometry of the spatial shuttle and the geometry of European space aircraft Hermes project. Meshes (skin) and Mach number, from [17].

progress in *algorithms progress* rather than *computer progress*. It is clear that in the 80's, we enjoyed the rise of supercomputers like Cray 1 which was more expensive than mainframes and more powerful. The rise of "micro killers" (IBM RS6000, etc) set the price of flop/s to a much lower level. The answer to the above question (probably in favor to computers) is difficult to establish, as many algorithms were designed for a better use of the pipelined number-crunchers. Meanwhile this evolution was in some manner favourable to structured ("my pipeline prefers structured").

Meanwhile the scientific community was debating on the physical limits to computer speed. Grand priests invoked the light velocity, atom size,... In short we were pushed towards the unique issue, parallelism.

Indeed first parallel computers appeared, taking so many forms (network, hypercubes, farms,...), but not so powerful at the beginning as vector computers continuing their progress and preferred by non academic institutions. Extra noise were brought by the expectation, starting already before pipelines, of the fifth generation of computers, specialized in Artificial Intelligence. A typical example was the development by the Thinking Machine Corporation of the Connection Machine which became unexpectedly a much studied and appreciated number-cruncher.

5. But algorithms?

Let us examine the progress of algorithms, the focus on solution algorithms, in particular. Two communities were particularly active.

The practitioners of multigrid had a yearly office, the Copper Mountain MG conference, also good for skiers. A variant, full multigrid was the graal, since $O(N)$ complexity was attainable, superior to other methods, as far as the problem size N is sufficiently large, going by the advice

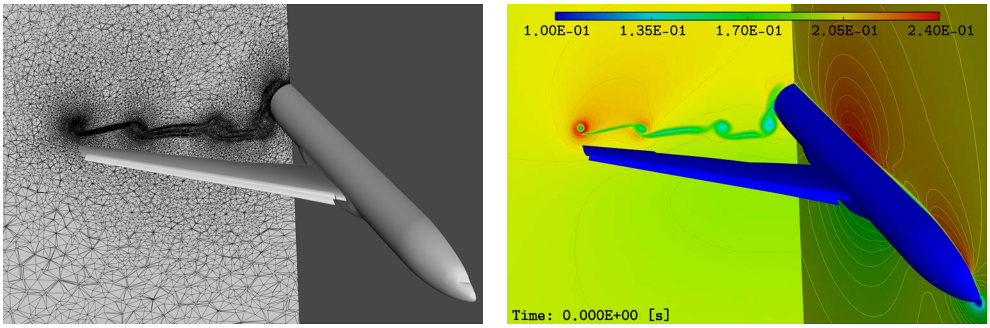

Figure 2. HL-CRM 16° high lift calculation (Courtesy of F. Alauzet). The Mach number values demonstrate the accurate capture of the wake, with a first vortex from wing tip, a second one from the gap between flaps, a third one starting where the wing is cracked, and a fourth one from wing root [23].

of grand priest Achi Brandt. Extension to unstructured meshes was not a piece of cake and could not show the miraculous speed up of certain structured demonstrations.

The practitioners of Domain Decomposition Methods (DDM) were also very active, they produced popular algorithms like Additive-Schwarz (e.g. [18]) or FETI [19], both being definitively adapted to systems coming from unstructured meshes.

Both methodologies give parallel algorithms. A common paradox is the problem posed by the coarse grid. In MG, the coarse grid is too small to be efficiently solved on the many processors reserved by the user for the fine grid problem. In DDM, the use of deflation to accelerate the global coupling also sets the question of an efficient solution of the coarse grid with one degree of freedom by processor [20].

6. Towards high order

In earlier days, there were first-order methods and higher order ones, i.e. second order. High order methods (order >2) are also fascinating. A graal also was identified in Spectral methods with a convergence to continuous, better than polynomial. However, the increasing interest to industrial issues led to concentrated investigations on unstructured high order methods. The Discontinuous Galerkin (DG) for CFD was popularized by Cockburn [21], Bassi and Rebay [22]. It should be noted that for many higher-order approximations, “unstructured” assumes a regular substructure inside each element.

7. Mesh generation and adaptation

The issue of easy mesh generation was the main motivation for choosing the unstructured option. Indeed, after many works, the unstructured mesh generation became fast and robust (at least for tetrahedra). The unstructured option eventually won the competition on preprocessing, in terms of engineer time. But the important capabilities of unstructured mesh *adaptation* was early identified. An active competition in unstructured mesh adaptation started no later than in 1986 at the Reading Conference. Unstructured mesh adaptation took a strong acceleration with the use of mesh metrics, and goal oriented strategies. A recent impressive accomplishment [23] is the use of goal oriented metrics for computing high-lift flows around an aircraft geometry, Figures 2 and 3.

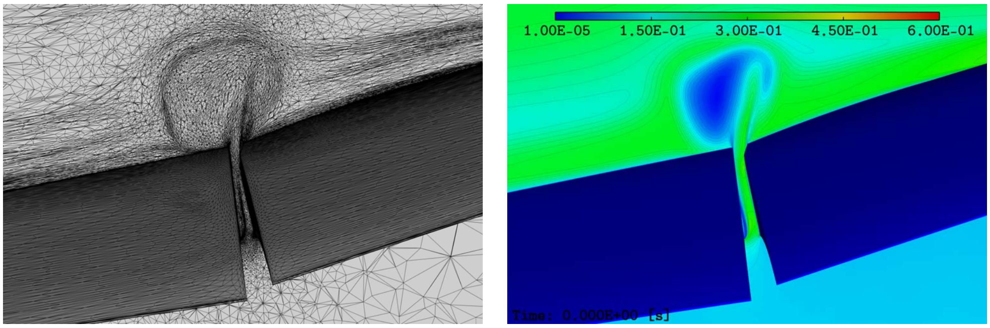

Figure 3. HL-CRM 16° high lift calculation (Courtesy of F. Alauzet). A zoom at flaps gap shows the initial capture of the second vortex [23].

8. Concluding remarks

The debate between unstructured and structured for compressible Navier–Stokes is at least 40 years old. It has been more or less a debate about *when* unstructured would be definitively better than structured. However, examining High-lift or drag reduction workshops, we observe that block-structured codes enjoy still higher performances for the same number of cells, and can be used with higher numbers of cells. Regarding engineering efficiency, however, almost all main software vendors in CFD have migrated towards unstructured.

Conflicts of interest

The author has no conflict of interest to declare.

References

[1] M.-O. Bristeau, R. Glowinski, J. Périaux, P. Perrier, O. Pironneau, "On the numerical solution of nonlinear problems in fluid dynamics by least squares and finite element methods (II). Application to transonic flow simulations", *Comput. Methods Appl. Mech. Eng.* **17/18** (1985), p. 619-657.

[2] R. MacCormack, "The effect of viscosity in hypervelocity impact cratering", in *AIAA Paper 69-364*, ARC, Reston, VA, 1969.

[3] P. Lax, B. Wendroff, "Systems of conservation laws", *Commun. Pure Appl. Math.* **13** (1960), p. 217-237.

[4] A. Lerat, R. Peyret, "Noncentered schemes and shock propagation problems", *Comput. Fluids* **2** (1974), p. 35-52.

[5] F. Angrand, A. Dervieux, "Some explicit triangular finite element schemes for the Euler equations", *Int. J. Numer. Methods Fluids* **4** (1984), p. 749-764.

[6] R. Löhner, K. Morgan, O. Zienkiewicz, "The solution of non-linear hyperbolic equation systems by the finite element method", *Int. J. Numer. Methods Fluids* **4** (1984), p. 1043-1063.

[7] J. Donea, "A Taylor–Galerkin method for convective transport problems", *Int. J. Numer. Methods Eng.* **20** (1984), p. 101-119.

[8] A. Jameson, W. Schmidt, E. Turkel, "Solutions of the Euler equations by finite volume methods using Runge-Kutta time-stepping schemes", in *AIAA Paper 81-1259*, ARC, Reston, VA, 1981, p. 1-17.

[9] A. Jameson, T. Baker, N. Weatherhill, "Calculation of inviscid transonic flow over a complete aircraft", in *AIAA Paper 86-0103*, ARC, Reston, VA, 1986, p. 1-16.

[10] S. K. Godunov, "Reminiscences about numerical schemes", in *INRIA Report*, vol. RR6666, INRIA, Rocquencourt, 2008.

[11] P. Roe, "Approximate Riemann solvers, parameter vectors, and difference schemes", *J. Comput. Phys.* **43** (1981), no. 2, p. 357-372.

[12] S. Osher, S. Chakravarthy, "Upwind difference schemes for hyperbolic systems of conservation laws", *Math. Comput.* **38** (1982), no. 158, p. 339-374.

[13] D. Book, J. Boris, K. Hain, "Flux-corrected transport: II. Generalizations of the method", *J. Comput. Phys.* **18** (1975), p. 248-283.

[14] B. van Leer, "Towards the ultimate conservative difference scheme IV: A new approach to numerical convection", *J. Comput. Phys.* **23** (1977), p. 276-299.

[15] A. Harten, "High resolution method for hyperbolic conservation laws", *J. Comput. Phys.* **49** (1983), p. 357-393.

[16] T. Hughes, M. Mallet, "A new finite element formulations for computational fluid dynamics: III", *Comput. Methods Appl. Mech. Eng.* **58** (1986), p. 329-336.

[17] B. Stoufflet, J. Périaux, L. Fezoui, A. Dervieux., "3-D hypersonic Euler numerical simulation around space vehicles using adapted finite elements", in *25th AIAA Aerospace Meeting, Reno (1987), AIAA Paper 87-0560*, AIAA, Reston, VA, 1987.

[18] X. Cai, M. Sarkis, "A restricted additive Schwarz preconditioner for general sparse linear systems", *SIAM J. Sci. Comput.* **21** (1999), p. 239-247.

[19] C. Farhat, F. X. Roux, "A method of finite element tearing and interconnecting and its parallel solution algorithm", *Int. J. Numer. Methods Eng.* **32** (1991), p. 1205-1227.

[20] H. Alcin, B. Koobus, O. Allain, A. Dervieux, "Efficiency and scalability of a two-level Schwarz algorithm for incompressible and compressible flows", *Int. J. Numer. Methods Fluids* **72** (2013), no. 1, p. 69-89.

[21] B. Cockburn, "Discontinuous Galerkin methods", *ZAMM - J. Appl. Math. Mech./Z. Angew. Math. Mech.* **83** (2003), no. 11, p. 731-754.

[22] F. Bassi, S. Rebay, "High-order accurate discontinuous finite element solution of the 2D Euler equations", *J. Comput. Phys.* **138** (1997), no. 2, p. 251-285.

[23] F. Alauzet, L. Frazza, "Feature-based and goal-oriented anisotropic mesh adaptation for RANS applications in Aeronautic and Aerospace", *J. Comput. Phys.* **439** (2021), article no. 110340.