Comptes Rendus

Mécanique

Salah Nissabouri, Mhammed El Allami and El Hassan Boutyour

Quantitative evaluation of semi-analytical finite element method for modeling Lamb waves in orthotropic plates

Quantitative evaluation of semi-analytical finite element method for modeling Lamb waves in orthotropic plates

Salah Nissabouri[*, a], **Mhammed El Allami**[a, b] **and El Hassan Boutyour**[c]

[a] Labo MISI, Department of Applied Physics, FST, Settat 26000, Morocco

[b] CRMEF, Settat, Morocco

[c] Labo MISI, FST, Settat 26000, Morocco

E-mails: s.nissabouri@uhp.ac.ma (S. Nissabouri), m.elallami@gmail.com
(M. El Allami), boutyour.elhassan@gmail.com (E. H. Boutyour)

Abstract. A semi-analytical finite element method algorithm was established to plot the dispersion curves of isotropic aluminum and orthotropic plates. The curves obtained are compared with those plotted by the DISPERSE software and with previous experimental work. The results showed that the accuracy of the method depends on the number of elements for meshing. To ensure good precision and speed of the method, the number of elements per plate thickness must be optimized.

Keywords. SAFE, Dispersion curves, Isotropic, Orthotropic, Interpolation functions, Meshing.

Manuscript received 6th February 2020, revised 2nd April 2020 and 3rd May 2020, accepted 4th May 2020.

1. Introduction

Non-destructive testing (NDT) permits us to control the integrity of mechanical components in many domains such as nuclear, medical, and aeronautic industries.

One of the most widely used ultrasonic methods is the control by Lamb waves. This kind of wave is dispersive, and it is also sensitive to small defects. However, the presence of multiple modes in a structure causes, in general, difficulties in terms of signal interpretation. For this reason, it is important to choose an adequate frequency of Lamb mode excitation.

To choose an adequate frequency requires knowing precisely the dispersion curves of a tested structure. These curves enable us to estimate the wavenumber, phase, and group velocities of propagating modes.
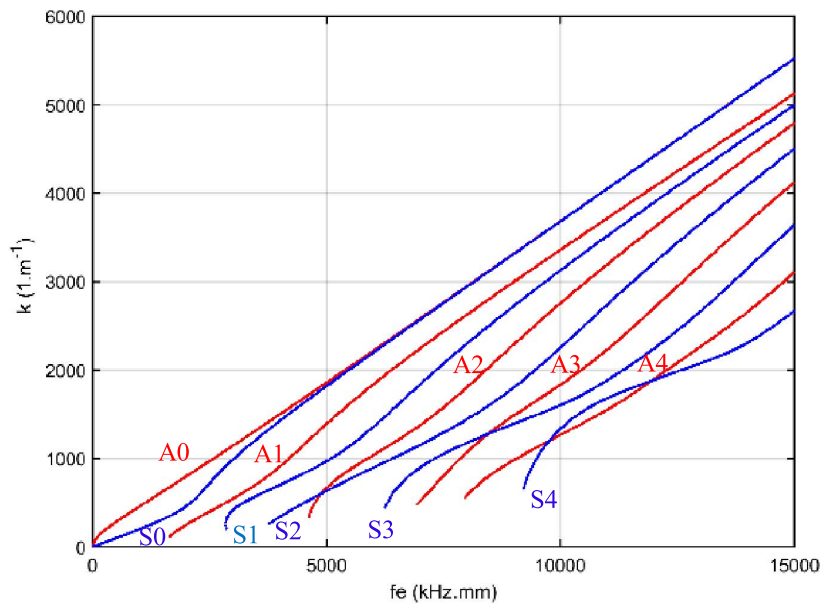
---

[*] Corresponding author.

Figure 1. Dispersion curves plotted by the bisection method [1].

In previous research, the authors have discussed dispersion curves and their multiple applications in NDT. They proposed several techniques to plot the dispersion curves. These methods include the bisection method [1], the Newton–Raphson method [2], the transfer matrix method [3], the spectral method [4], and so on.

Moreover, the dispersion curves plotted by the iterative methods, even if they are very close to analytical curves, present inaccuracy in determining the cutoff frequencies (Figure 1). For this reason, we propose an alternative method, which allows a better estimation of the dispersion curves.

The most robust methods of plotting dispersion curves for the case of isotropic and composite materials are based on finite elements. However, they require significant storage capacities since they involve the meshing of the whole structure.

In this article, we use the semi-analytical finite element (SAFE) method. It is a method combining the semi-analytical and finite elements. The guided modes and their movements in the section are calculated by the finite element method (FEM) and then supplemented by analytical displacement in the propagation direction.

This method is optimal for predicting the propagating modes in a plate. This technique requires discretization only according to the thickness of the plate. Therefore, the calculation times are very short.

Moreover, this method adapts particularly well to the structures of complex geometries discretized by finite elements. In addition, it allows us to characterize the properties of a wave propagating in an invariant section guide.

The SAFE method was first applied by Waas [5] for calculating the surface parameters of multilayer soil. Afterward, it was used by many researchers for the study of Lamb wave propagation in isotropic and composite structures. Ahmad *et al.* [6] applied the method for plotting the dispersion curves of an aluminum plate and a composite-type structure $[0°/45°/90°/-45°]$. Bartoli *et al.* [7] used the SAFE method to model the wave propagation of Lamb waves in waveguides of an arbitrary section. The authors extended the use of the SAFE method
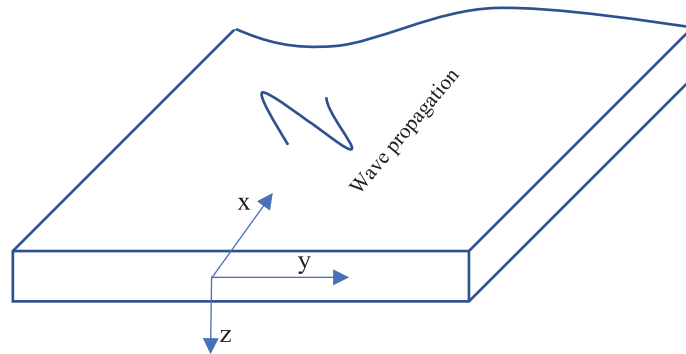
Figure 2. Waveguide with a rectangular cross section.

to viscoelastic materials by considering damping. Takahiro *et al.* [8] simulated the propagation of Lamb waves in cylinders using the SAFE method. Takahiro *et al.* [9] calculated dispersion curves by the SAFE method. The curves obtained were compared with the experimental curves calculated by two-dimensional (2D) fast Fourier transform. Mukdadi *et al.* [10] also studied the propagation of Lamb waves in multilayer composites. Predoi [11] proposed an extension of the SAFE method for periodic structures of infinite width. Recently, Wenbo *et al.* [12] have presented a formulation using the SAFE method and the perfect match layer technique for calculating pipeline dispersion curves immersed in a fluid. Xing *et al.* [13] proposed a defect localization method for rails and plotted rail dispersion curves by the SAFE method. All the authors have studied the method, but no one has conducted a quantitative evaluation of dispersion curves plotted by the SAFE method.

The aims of this article are as follows:

- Establish an algorithm of the SAFE numerical method.
- Calculate the dispersion curves of an isotropic plate and validate the results with the DISPERSE software [14].
- Calculate longitudinal and transverse displacements.
- Calculate the dispersion curves of an orthotropic plate.

For this purpose, we start with a definition of the problem. Then we establish the equation of motion based on Hamilton's principle. The relation obtained is discretized using interpolation functions. Subsequently, the relation is reformulated for the case of a plate in the form of an eigenvalue problem. Solving the equation makes it possible to find the propagating modes in isotropic and orthotropic plates. Finally, we evaluate the curves obtained by the DISPERSE software and by previous experimental work. We observe that the accuracy of the method depends on the parameters that must be optimized.

2. Problem definition and equation of motion

2.1. *Problem definition*

We consider a waveguide with a rectangular cross section as presented in Figure 2.

The wave propagates along the x-axis. Discretization is needed only in the cross section (2D discretization). The wave propagates along the x-axis with a wavenumber k and an angular frequency ω. The harmonic expression of the displacement is written in the form

$$u(x, y, z, t) = U(y, z)\exp^{-\mathrm{i}(kx-\omega t)}. \tag{1}$$

2.2. *Equation of motion*

The equation for the eigenvalue problem is deduced from Hamilton's relation [6]:

$$[K_1 + \mathrm{i}kK_2 + k^2K_3 - \omega^2 M]U = 0. \tag{2}$$

Here,

$$K_1 = \bigcup_{e=1}^{n_{el}} k_1^{(e)}; \quad K_2 = \bigcup_{e=1}^{n_{el}} k_2^{(e)}; \quad K_3 = \bigcup_{e=1}^{n_{el}} k_3^{(e)}; \quad M = \bigcup_{e=1}^{n_{el}} m^{(e)}$$

with

$$k_1^{(e)} = \int_{\Omega_e} [B_1^T C_e B_1]\mathrm{d}\Omega_e, \tag{3}$$

$$k_2^{(e)} = k_{21}^{(e)} - k_{12}^{(e)} = \int_{\Omega_e} [B_2^T C_e B_1]\mathrm{d}\Omega_e - \int_{\Omega_e} [B_1^T C_e B_2]\mathrm{d}\Omega_e = (k_{21}^{(e)})^T - k_{12}^{(e)}, \tag{4}$$

$$k_3^{(e)} = \int_{\Omega_e} [B_2^T C_e B_2]\mathrm{d}\Omega_e, \tag{5}$$

$$m^{(e)} = \int_{\Omega_e} [N^T \rho_e N]\mathrm{d}\Omega_e, \tag{6}$$

where C_e is the elementary elastic matrix, ρ_e is the elementary mass density, n_{el} is the number of elements, and

$$B_1 = L_y N_{,y} + L_z N_{,z}, \tag{7}$$

$$B_2 = L_x N. \tag{8}$$

Here, $N_{,y}$ and $N_{,z}$ are the interpolation function derivatives with respect to variables y and z, respectively, and

$$L_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}; \quad L_y = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}; \quad L_z = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

N(y,z) is the matrix of interpolation functions expressed as

$$N(y, z) = \begin{bmatrix} N_1 & 0 & 0 & N_2 & 0 & 0 & & N_n & 0 & 0 \\ 0 & N_1 & 0 & 0 & N_2 & 0 & \cdots & 0 & N_n & 0 \\ 0 & 0 & N_1 & 0 & 0 & N_2 & & 0 & 0 & N_n \end{bmatrix}.$$

3. SAFE formulation and solution of equation of motion for a plate

3.1. *SAFE formulation*

We consider the case of an infinite plate along the y-axis (Figure 3a). The displacement is then independent of y. Therefore, the plate can be modeled by one-dimensional (1D) elements. The plane strain condition is considered here. Hence $N_{,y} = 0$. Thus, Equations (7) and (8) become $B_1 = L_z N_{,z}$ and $B_2 = L_x N$, respectively.

The integrals given in (3)–(6) are computed numerically using the Gaussian quadrature method with three points. The integration limits need to be changed to -1 to 1. For this reason, the elements are discretized by isoparametric elements (Figure 3b).

The interpolation functions N_i with three nodes for the element are given by

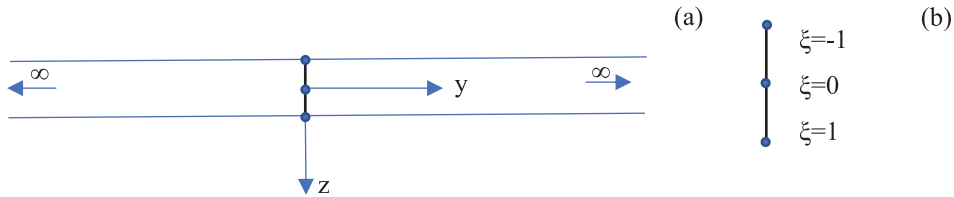$$N_1(\xi) = \frac{\xi^2 - \xi^2}{2}; \quad N_2(\xi) = 1 - \xi^2; \quad N_3(\xi) = \frac{\xi^2 + \xi^2}{2}.$$

Figure 3. (a) Infinite (2D) plate with three nodes per element; (b) 1D three-node isoparametric element.

To calculate $N_{,z}$, we need to use the Jacobian function:

$$\frac{\mathrm{d}N(\xi)}{\mathrm{d}z} = \frac{\mathrm{d}N(\xi)}{\mathrm{d}\xi}\frac{\mathrm{d}\xi}{\mathrm{d}z}$$

$$= N_{,\xi}\frac{1}{J}, \qquad (9)$$

where

$$J = \frac{\mathrm{d}z}{\mathrm{d}\xi} = \left[\, \xi - 1/2 \;\; -2\xi \;\; \xi + 1/2 \,\right]\begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix}.$$

3.2. *Solving the equation of motion*

Equation (2) contains two variables, the wavenumber k and the angular frequency ω. Therefore, the equation can be solved by two approaches:

- Approach 1 : Fix k and solve for the angular frequency ω.
- Approach 2: Fix ω and solve for the wavenumber k.

In the first approach, we fix positive values of wavenumber k and then we consider only the propagating modes. Equation (2) is reformulated as follows [6]:

$$[K(k) - \omega^2 M]U = 0. \qquad (10)$$

4. Algorithm for plotting dispersion curves by SAFE

Figure 4 shows the algorithm for plotting dispersion curves. The first step is to set parameters for the studied model. We define the material parameters: ρ, E, and C. The geometry of the plate is defined by the thickness e.

The second step is the meshing of the cross section. For this purpose, we need to determine the number of elements and the order of interpolation functions. The third step is the calculation of elementary matrix $k_i^{(e)}$ using the Gaussian quadrature method of approximation. Elementary matrices are then assembled into a global matrix. The final step is to compute the eigenvalue problem to find the eigenvalues and their corresponding eigenvectors for each value of the wavenumber k.

5. Plotting dispersion curves (wavenumber) for isotropic plate by SAFE method

A Matlab program is established to compute dispersion curves. The wavenumber and the phase velocity are assumed to be functions of the frequency–thickness product (fe).

The dispersion curves are calculated for an aluminum plate with thickness 1 mm, Young's modulus 69 GPa, Poisson's ratio 0.33, and mass density 2700 kg/m^3.
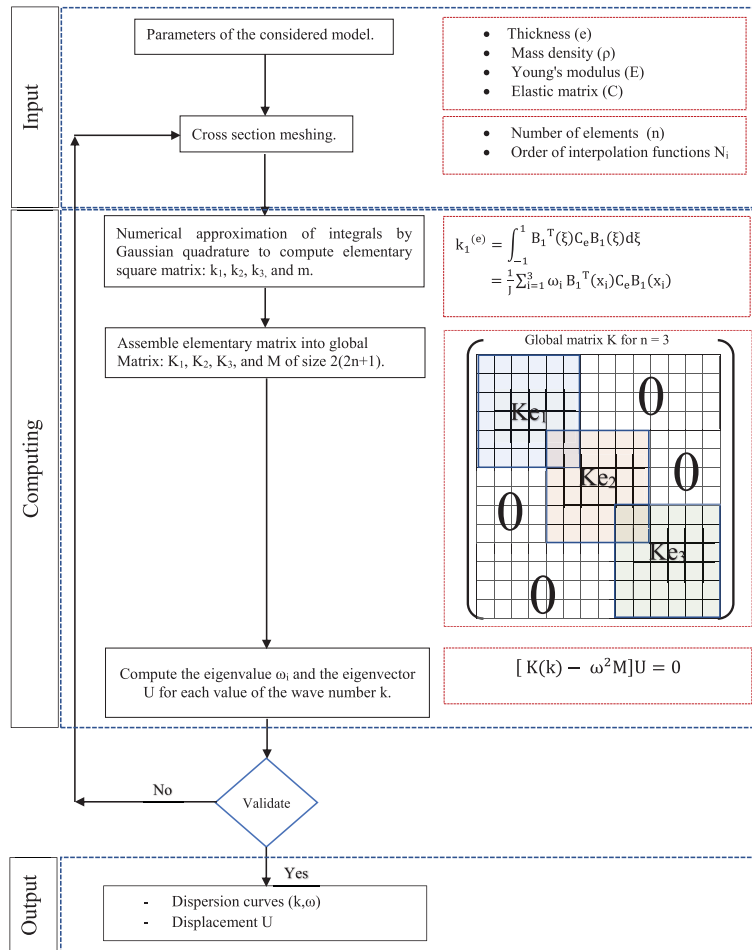
Figure 4. Algorithm for plotting dispersion curves $\omega\,(k)$ by SAFE using approach 1.

To verify the accuracy of the method, we compare the plotted dispersion curves by the SAFE method with those plotted by the DISPERSE program [14]. The software, developed by Imperial College NDT Laboratory, permits us to create dispersion curves. It is based on the global matrix method; the displacements and stresses are described by a material layer matrix. By satisfying the boundary conditions at each interface, the individual layer matrices are assembled.

We have chosen to validate our algorithm with DISPERSE as it is based on analytical equations and yields solutions considered very close to analytical solutions as has been obtained by many authors [15–19].

In Figure 5, we observe that the accuracy of the method depends on the frequency–thickness product. The first two modes A_0 and S_0 are sufficiently accurate in comparison with the other modes. Therefore, we conclude that the error increases for a high value of the frequency–thickness product.

5.1. *Quantitative evaluation of SAFE method*

Similarly to the FEM, the precision of the SAFE method depends on the number of elements and the order of interpolation functions. The more we discretize the section, the greater the precision
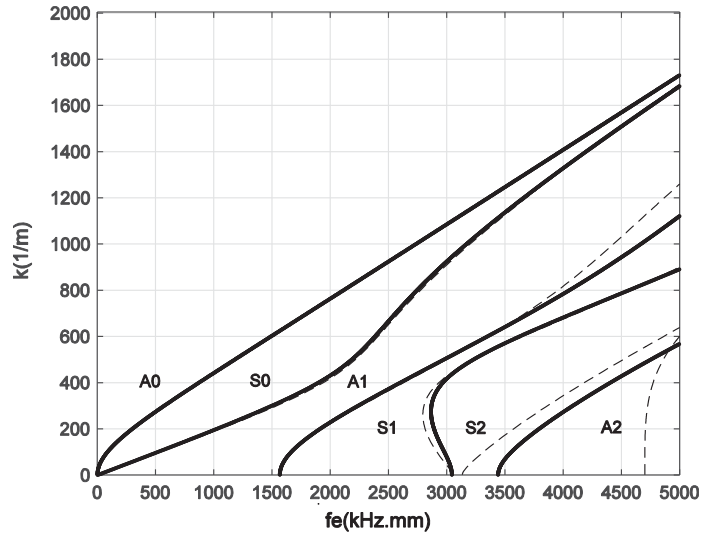
Figure 5. Dispersion curves plotted by SAFE (continuous line) compared with dispersion curves plotted by DISPERSE (dashed line).

Table 1. Relative errors for the calculation of mode A_2 by SAFE as a function of n

Number of elements (n)	Relative error
4	1.46%
5	0.8%
6	0.46%

we obtain. In this case, as presented in Figure 5, we have meshed the section using two elements in one dimension. We have used the quadratic interpolation function as we are interested in a low value of the frequency–thickness product.

5.1.1. *Effect of the number of elements on accuracy of SAFE method*

Figure 6 shows that the accuracy of the method depends on the number of elements. Plots for meshing by three elements are shown in Figure 6a. The method permits us to calculate the first three modes A_0, S_0, and A_1 with good precision. The modes S_1 and S_2 are calculated with an error. To compute the A_2 mode, we need to mesh the plate thickness by a number of elements greater than 4 (Table 1).

Table 1 shows that the error for the calculation of A_2 by SAFE decreases for a number of elements greater than 4.

Considering an isotropic plate with a thickness of 1 mm, if we are interested in frequencies up to 5000 kHz, then meshing by six elements is sufficient to compute the first six modes. However, if we are interested in frequencies greater than 5000 kHz, then we need to increase the number of elements. This is proved by the author in [11]:

$$\frac{\lambda_T}{l} > \beta. \tag{11}$$

Here, l is the length of the element, $\lambda_T = 2\pi V_T/\omega$ is the wavelength of transverse waves propagating at velocity V_T, and $\beta = 4$ in the case of quadratic interpolation functions [20].
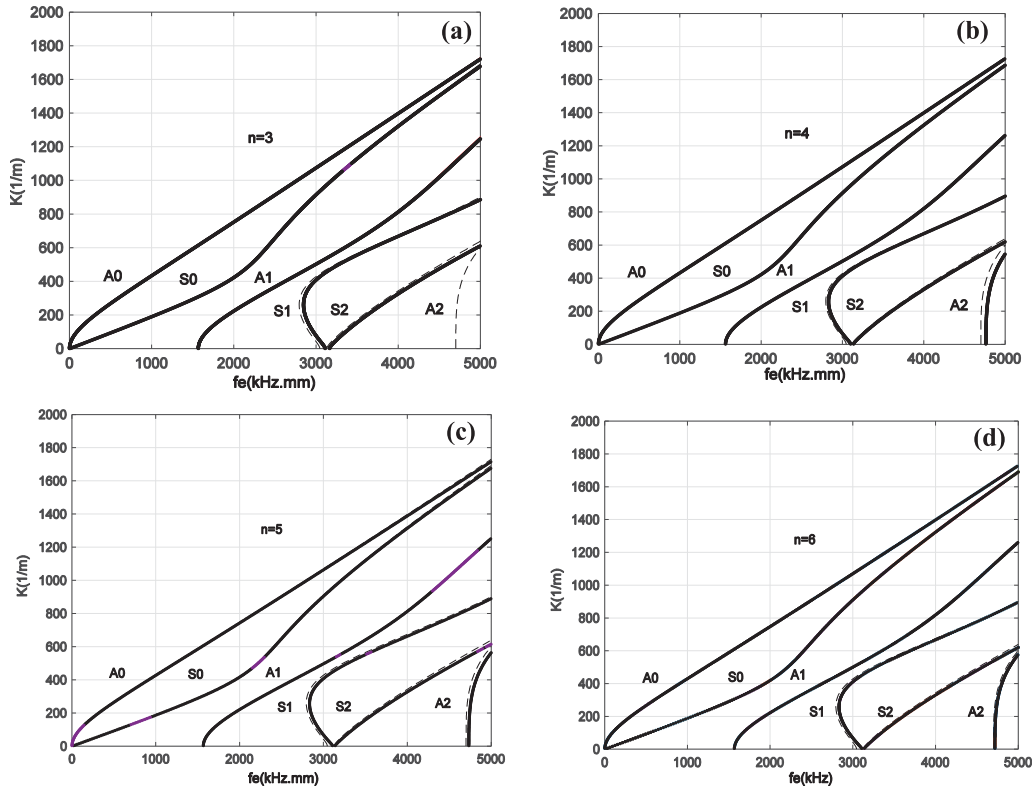
Figure 6. Dispersion curves plotted by SAFE and compared with DISPERSE according to the number of elements: (a) $n = 3$; (b) $n = 4$; (c) $n = 5$; (d) $n = 6$.

We insert the expression of λ_T in (11), and we obtain

$$\frac{2\pi V_T}{\omega l} > \beta. \tag{12}$$

We substitute ω by $2\pi f$ and l by e/n, and we get

$$\frac{V_T n}{\text{fe}} > \beta. \tag{13}$$

Therefore, for a given product "fe", we can determine the minimal number of elements to ensure good accuracy of the method:

$$n_{\min} = \frac{\beta \text{fe}}{V_T}. \tag{14}$$

In this case, we replace V_T by 3040 m/s, β by 4, and "fe" by 5000 kHz·mm. We obtain $n_{\min} = 6.5$. Subsequently, we plot the dispersion curves of an aluminum plate for $n = 7$ (Figure 7).

5.1.2. *Convergence of the proposed algorithm*

To measure the convergence of the method, we have plotted the variation in the relative error against the studied frequencies. The relative error was calculated for the first two modes (A_0 and S_0) as they are the most used in NDT. Figure 8 shows the errors of dispersion curves compared to those of DISPERSE solutions within the studied frequency range. A higher number of elements per plate thickness yields lesser error. Furthermore, we find that the errors increase as the frequency increases.
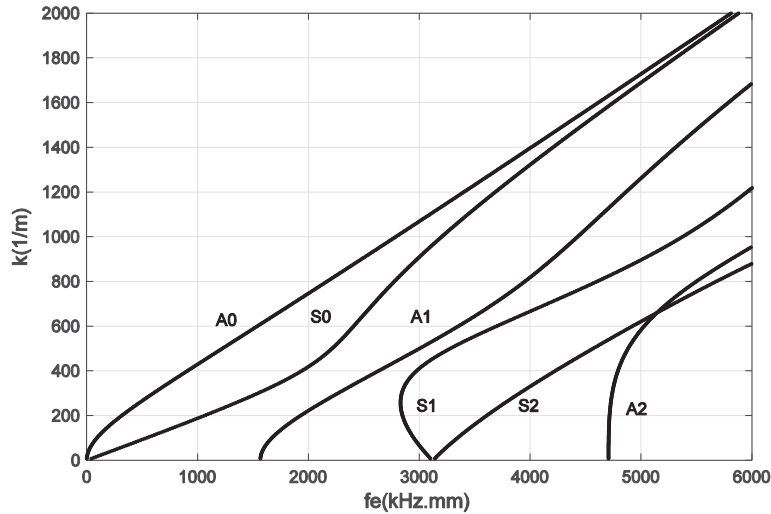
Figure 7. Dispersion curves plotted by SAFE for an aluminum plate ($n = 7$).
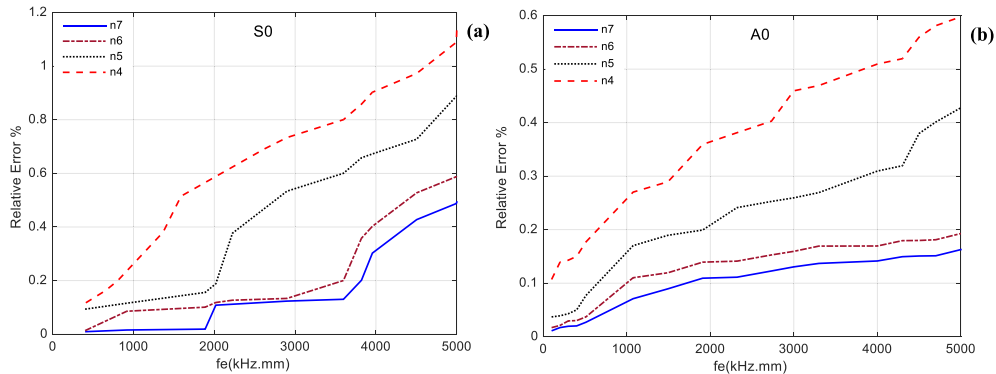

Figure 8. Evolution of relative error between DISPERSE and SAFE using different numbers of quadratic elements per thickness for (a) S_0 mode and (b) A_0 mode.

5.1.3. *Effect of the number of elements on running time*

If we consider a number of elements greater than 7, we will certainly obtain good precision. However, this increases the running time.

To see how the running time changes with respect to the number of elements, we have presented Table 2. The computer used has a processor of 2.6 GHz and a memory (RAM) of 4 Go. The stepwise increment in k is set as 0.5.

Table 2 and Figure 9 show that the running time changes linearly when the number of elements is less than 4. When the number of elements is greater than 4, the process slows down. This is shown by the increase in size of the elementary matrix to be assembled.

The DISPERSE software is based on analytical equations, and it gives accurate dispersion curves in many cases (multilayer and composite structures). We also know that the SAFE method is based on 1D FEM, which means that it depends on the number of elements per thickness.

However, DISPERSE had difficulties in retrieving higher orders of propagating modes. In contrast, our SAFE algorithm is capable of retrieving all the wave modes including shear horizontal modes, which are not considered in this study.
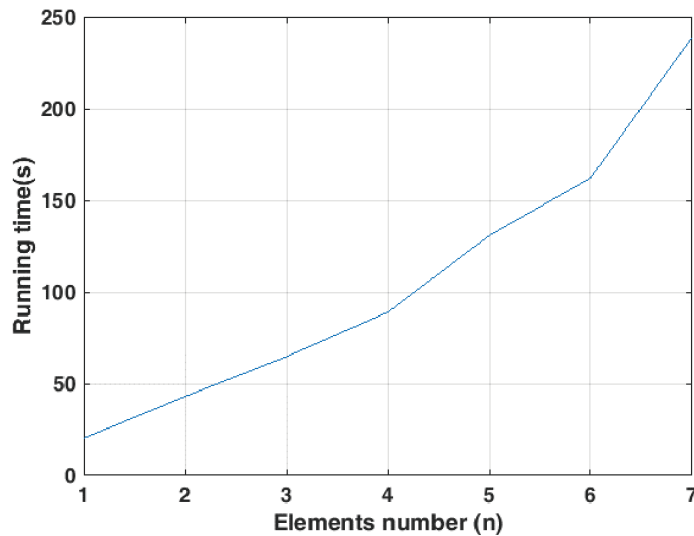
Figure 9. Variation in running time with respect to the number of elements (n).

Table 2. Running time using SAFE as a function of the number of elements (n)

Number of elements	Running time (s)
1	20.28
2	43.04
3	64.77
4	89.057
5	130.86
6	162.25
7	238.86

Figure 10 shows all the higher modes of dispersion curves plotted by SAFE and DISPERSE. It illustrates that the SAFE method (even for $n = 6$ elements) permits us to compute more Lamb modes than does DISPERSE. Moreover, if we mesh the thickness of the studied plate with more elements, the number of plotted modes increases. In addition, the DISPERSE software suffers from another limitation on materials and geometries with viscoelasticity, which is not the case for the SAFE program [21].

5.2. *Dispersion curves: phase velocity of an isotropic plate*

The solution based on the SAFE method for the eigenvalue problem (10) helps in identifying the angular frequency ω for each increment in the wavenumber. Using the relation $V_p = \omega/k$, we can calculate the phase velocity for each value of the product "fe" (Figure 11).

5.3. *Transverse and longitudinal displacements*

The eigenvectors of (10) correspond to the longitudinal (u_x) and transverse (u_z) displacements.
Figure 12 shows the displacement profiles through the plate thickness (1 mm) of an aluminum plate at a frequency of 1000 kHz.
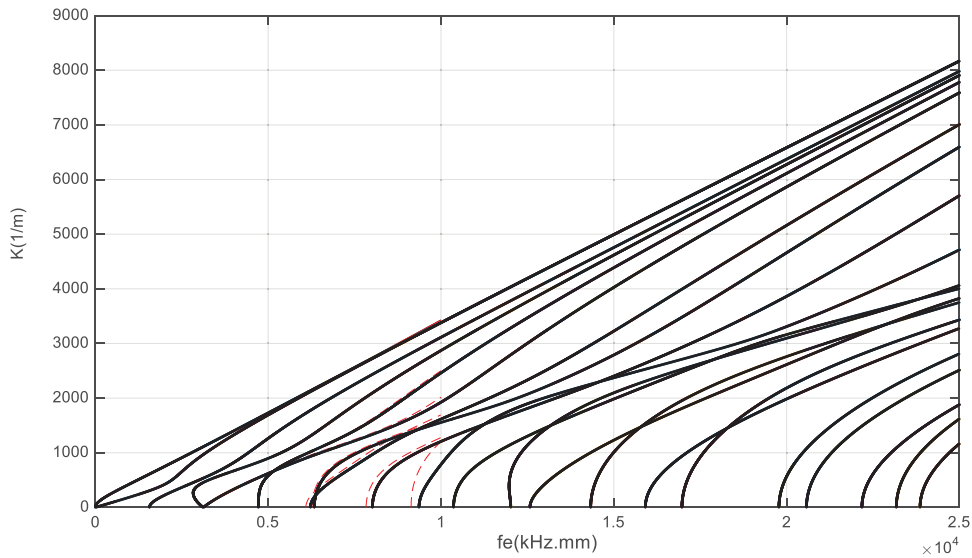
Figure 10. Dispersion curves obtained by SAFE (continuous line) and by DISPERSE (dashed line) for an isotropic aluminum plate ($n = 6$).
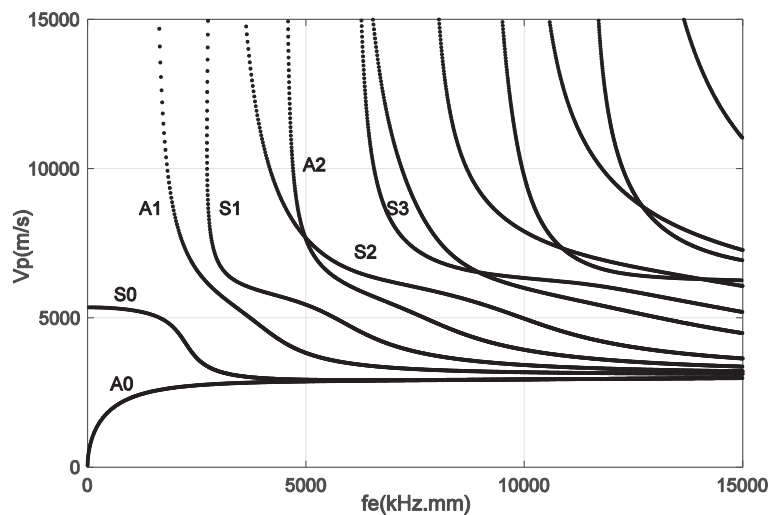

Figure 11. Dispersion curves for an aluminum plate: phase velocity as a function of the product "fe" plotted by SAFE for $n = 7$.

6. Dispersion curves: phase velocity and wavenumber of an orthotropic plate

6.1. *Wavenumber as a function of "fe"*

We consider an orthotropic plate having the properties listed in Table 3. The dispersion curves plotted by the iterative method of Newton–Raphson [2] were validated by experiments (Figure 13). We have chosen these curves as reference to validate the dispersion curves obtained by SAFE.
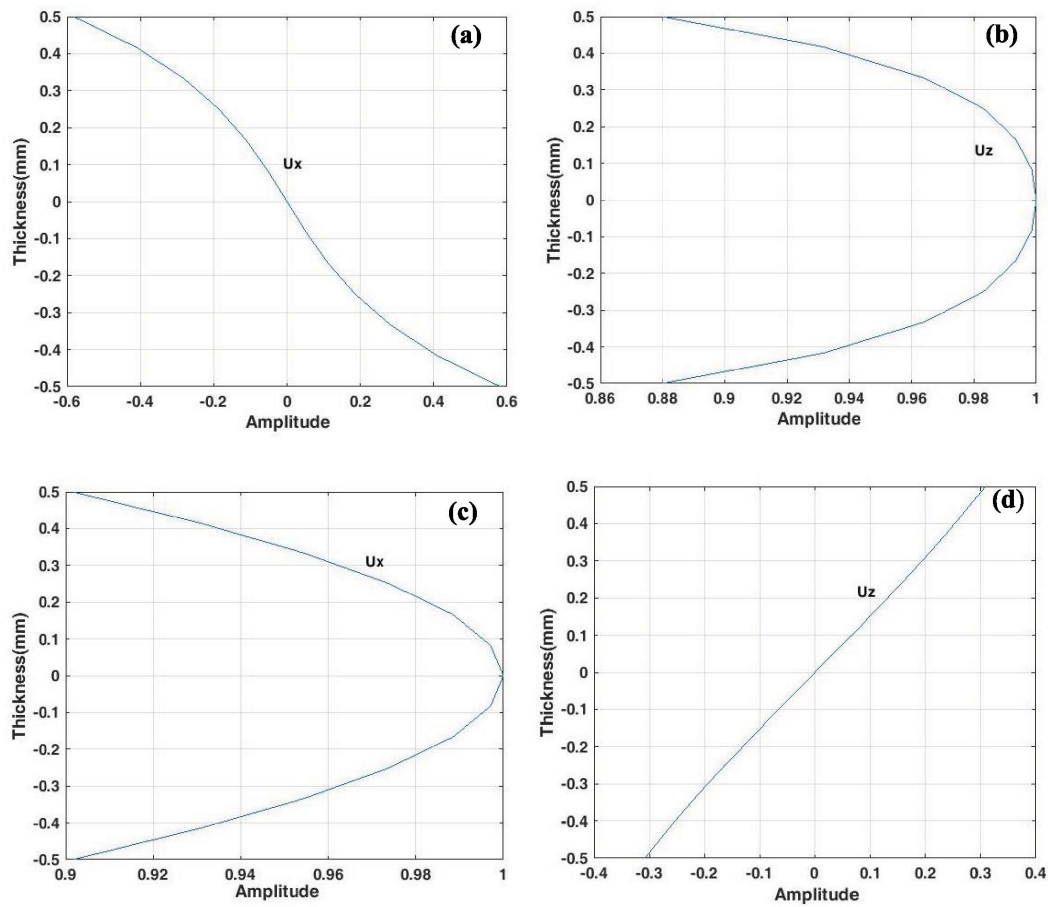
Figure 12. Displacement profiles through the plate thickness (1 mm) of an aluminum plate at a frequency of 1000 kHz: (a) u_x, (b) u_z and (c) u_x, (d) u_z, respectively, for S_0 and A_0.

Table 3. Properties of the orthotropic plate

ρ	1500 (kg/m^3)
e	1.6 mm
C_{11}	57 GPa
$C_{22} = C_{33}$	15 GPa
$C_{13} = C_{12} = C_{23}$	10 GPa
$C_{55} = C_{66}$	4 GPa
C_{44}	2.5 GPa

The same procedure was used as indicated in Figure 4 to plot the dispersion curves with respect to the wavenumber as a function of "fe".

6.2. *Minimal number of elements*

Equation (14) permits us to find the minimal number of elements to mesh the cross section. The relation providing the calculation of V_T is given in [22]: $V_T = \sqrt{G_{12}/\rho}$, where G_{12} is the shear modulus. The relation for G_{12} is given by [23]: $G_{12} = C_{11} - C_{12}/2$.
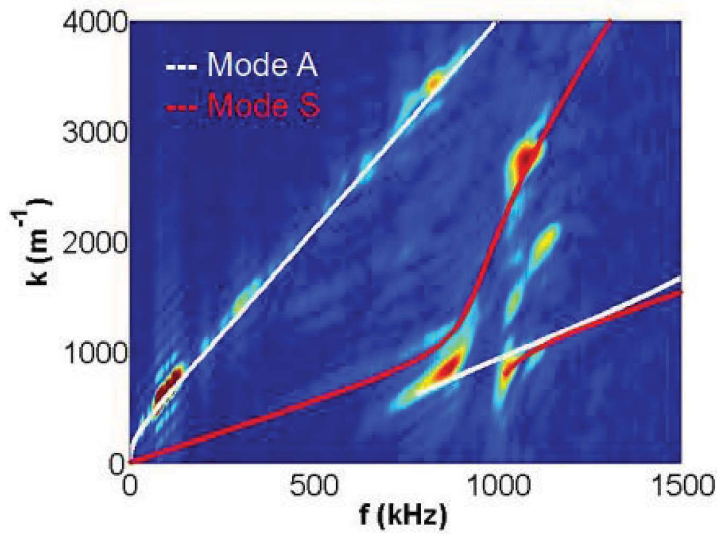
Figure 13. Experimental dispersion curves plotted over numerical curves obtained by the Newton–Raphson method [2].
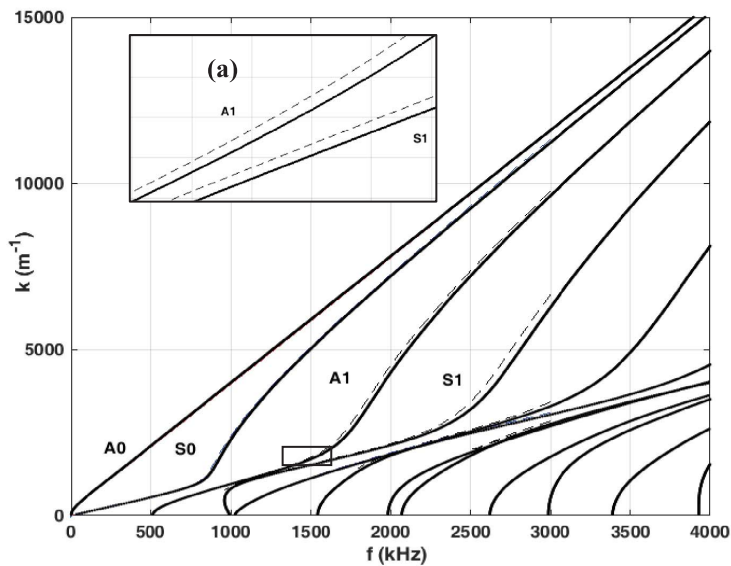

Figure 14. Comparison between dispersion curves plotted by SAFE (continuous line) and dispersion curves plotted by the Newton–Raphson method (dashed line) for $n = 6$: (a) zoomed-in view.

If we are interested in the range of frequencies between 0 and 4000 kHz, then we need to take $n_{\min} = 6.84$.

Figure 14 compares the dispersion curves plotted by the SAFE method and the dispersion curves plotted by the Newton–Raphson method for an orthotropic plate for six elements.

Figure 15 compares the two methods for $n = 7$. The dispersion curves plotted by the SAFE method and those plotted by Newton–Raphson method overlap perfectly ($n = 7$).
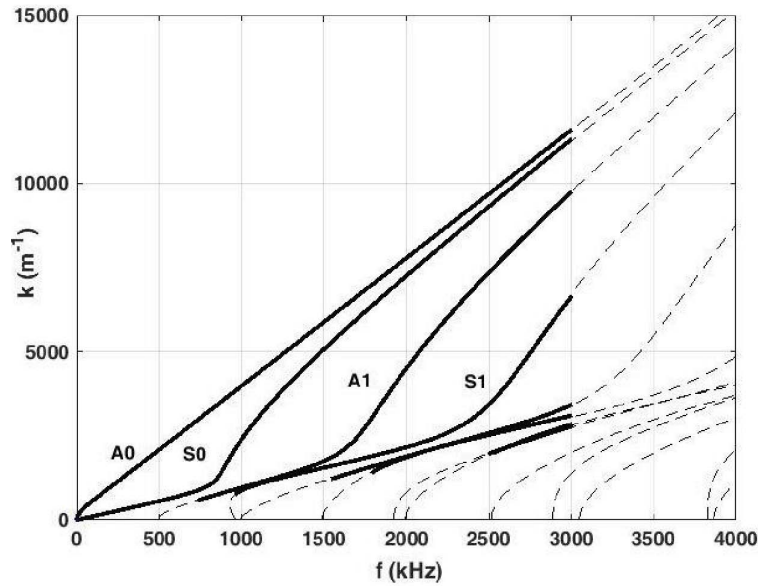
Figure 15. Comparison between dispersion curves plotted by SAFE (dashed line) and dispersion curves plotted by the Newton–Raphson method (continuous line) for $n = 7$.
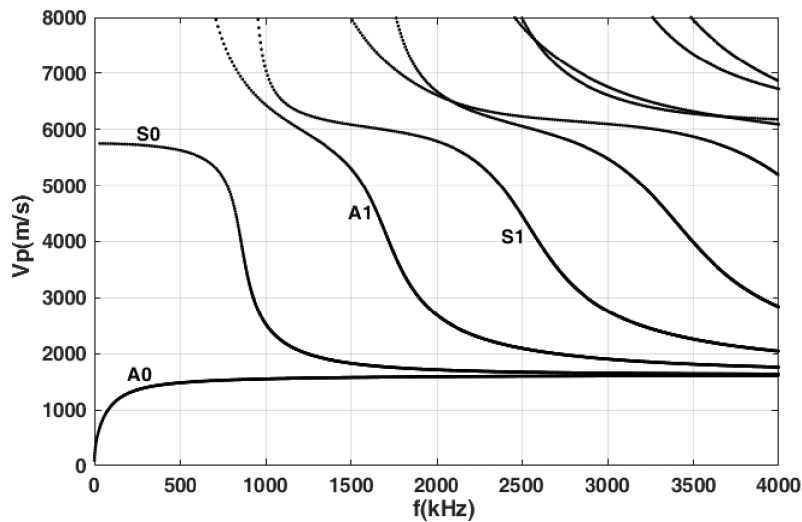

Figure 16. Dispersion curves of orthotropic plate of thickness 1.6 mm with phase velocity as a function of frequency by SAFE ($n = 7$).

6.3. *Phase velocity of orthotropic plate*

Figure 16 presents the dispersion curves of an orthotropic plate with the phase velocity as a function of frequency by the SAFE method for $n = 7$.
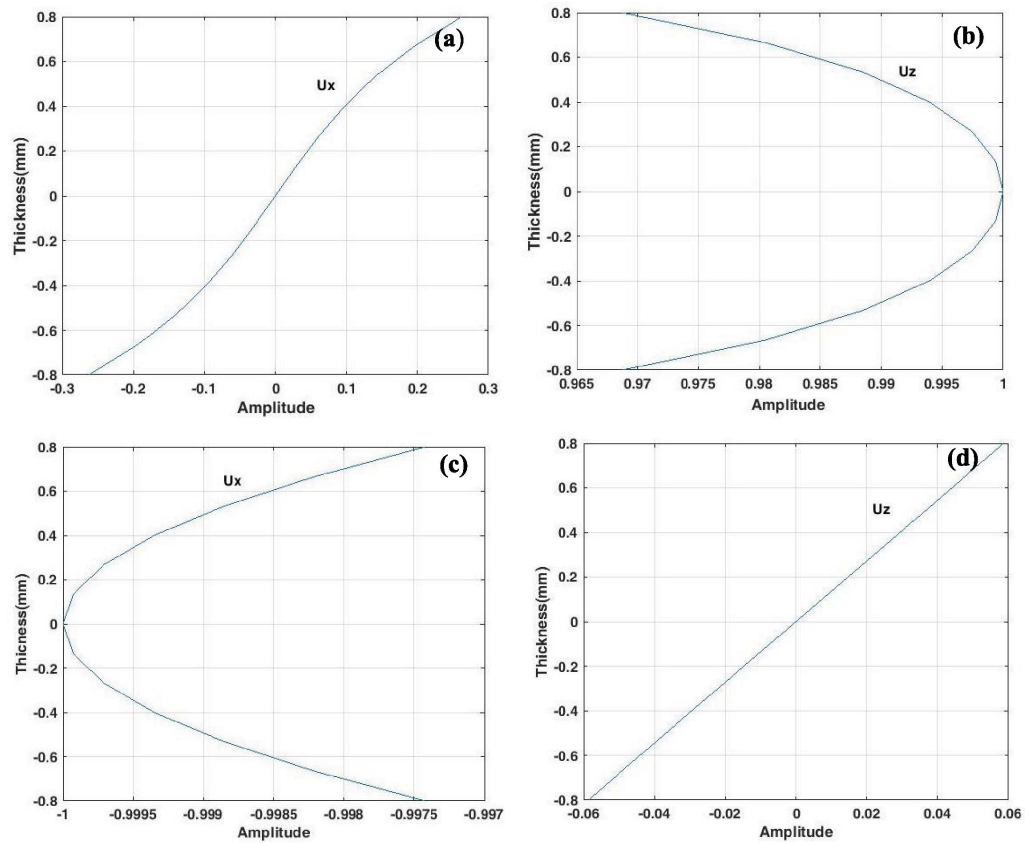
Figure 17. Displacement profiles through the plate thickness (1.6 mm) of an orthotropic plate at a frequency of 100 kHz: (a) u_x, (b) u_z and (c) u_x, (d) u_z, respectively, for S_0 and A_0.

6.4. *Transverse and longitudinal displacements*

Figure 17 shows the displacement profiles through the plate thickness (1.6 mm) of an orthotropic plate at a frequency of 100 kHz for S_0 and A_0 modes.

7. Conclusion

In this article, we have established an algorithm based on the SAFE method to plot the dispersion curves of isotropic and orthotropic plates.

The dispersion curves obtained are compared with those by the DISPERSE software and the Newton–Raphson method. The results have shown that when the frequency increases, the error increases.

To ensure good precision of the method, the number of elements to mesh the cross section must be optimized.

The SAFE method permits us to calculate propagating modes with good precision and does not require the meshing of the whole structure. However, the method is difficult in terms of coding.

Acknowledgment

References

[1] S. Nissabouri, M. El Allami, M. Bakhcha, "Lamb waves propagation plotting the dispersion curves", in *Conference: ICCWCS16Volume: PROCEEDINGSISSN, Morocco*, 2016, p. 153-156.

[2] E. B. Ndiaye, H. Duflo, "Non destructive testing of sandwich composites: adhesion defects evaluation; experimental and finite element method simulation comparison", in *Acoustics 2012 Nantes, Nantes, France*, 2012, p. 2659-2664.

[3] A. Nayfeh, "The general problem of elastic wave propagation in multilayered anisotropic media", *J. Acoust. Soc. Am.* (1991), p. 1521-1531.

[4] B. Sinha, H. P. Valero, F. Karpfinger, A. Bakulin, B. Gurevich, "Spectral-method algorithm for modeling dispersion of acoustic modes in elastic cylindrical structures", *Geophysics* **75** (2010), p. H19-H27.

[5] G. Waas, "Analysis report for footing vibrations through layered media", PhD Thesis, University of California, 1972.

[6] Z. A. B. Ahmad, J. M. Vivar-Perez, U. Gabbert, "Semi-analytical finite element method for modeling of lamb wave propagation", *CEAS Aeronaut. J.* **4** (2013), p. 21-33.

[7] I. Bartoli, M. Alessandro, L. Francesco, V. Erasmo, "Modeling wave propagation in damped waveguides of arbitrary cross-section", *J. Sound Vibrat.* **295** (2006), p. 685-707.

[8] H. Takahiro, K. Koichiro, S. Zongqi, L. Joseph, "Analysis of flexural mode focusing by a semi analytical finite element method", *J. Acoust. Soc. Am.* **113** (2003), p. 1241-1248.

[9] H. Takahiro, S. Won-Joon, L. Joseph Rose, "Guided wave dispersion curves for a bar with an arbitrary cross-section, a rod and rail example", *Ultrasonics* **41** (2003), p. 175-183.

[10] M. Osama, D. S. Mukdadi, "Transient ultrasonic guided waves in layered plates with rectangular cross section", *J. Appl. Phys.* **93** (2003), p. 9360-9363.

[11] P. Mihai Valentin, "Guided waves dispersion equations for orthotropic multilayered pipes solved using standard finite elements code", *Ultrasonics* **54** (2014), p. 1825-1831.

[12] D. Wenbo, K. Ray, "Guided wave propagation in buried and immersed fluid-filled pipes: Application of the semi analytic finite element method", *Comput. Struct.* **212** (2019), p. 236-247.

[13] B. Xing, Y. Zujun, X. Xining, Z. Liqiang, S. Hongmei, "Research on a rail defect location method based on a single mode extraction algorithm", *Appl. Sci.* **9** (2019), p. 1-16.

[14] B. Pavlakovic, M. Lowe, D. Alleyne, P. Cawley, "Disperse: A general purpose program for creating dispersion curves", in *Review of Progress in Quantitative NDE*, Springer, Boston, MA, 2013, p. 185-192.

[15] R. Sanderson, "A closed form solution method for rapid calculation of guided wave dispersion curves for pipes", *Wave Motion* **53** (2015), p. 40-50.

[16] L. Draudvilienė, R. Raišutis, E. Žukauskas, A. Jankauskas, "Validation of dispersion curve reconstruction techniques for the A0 and S0 modes of lamb waves", *Inti J. Struct. Stability Dyn.* **14** (2014), p. 1-11.

[17] B. Hernandez Crespo, C. R. P. Courtney, B. Engineer, "Calculation of guided wave dispersion characteristics using a three-transducer measurement system", *Appl. Sci.* **8** (2018), p. 1253.

[18] B. Hernandez, B. Engineer, C. R. P. Courtney, "Empirical technique for dispersion curve creation for guided wave applications", in *Conference: 8th European Workshop on Structural Health Monitoring, EWSHM, Spain*, 2016.

[19] S. Soua, S. Chan, T.-H. Gan, "Modelling of long range ultrasonic waves in complex structures", in *BINDT Annual Conference, UK*, 2008.

[20] M. Jose, A. R. Galan, "Numerical simulation of Lamb wave scattering in semi-infinite plates", *Int. J. Numer. Meth. Engng* **53** (2002), p. 1145-1173.

[21] F. Zhu, B. Wang, Z. Qian, E. Pan, I. E. Kuznetsova, "Accurate characterization of 3D dispersion curves and mode shapes of waves propagating in generally anisotropic viscoelastic/elastic plates", *Intl J. Solids Struct.* **150** (2018), p. 52-65.

[22] F. G. Lei Wang, "Yuan, Group velocity and characteristic wave curves of Lamb waves in composites: Modeling and experiments", *Composit. Sci. Technol.* **67** (2007), p. 1370-1384.

[23] K. A. Kaw, *Mechanics of Composite Materials*, 2nd ed., CRC Press, 2006.