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Iheb Chouikh and Chokri Bouraoui

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Short paper / *Note*

A fatigue-reliability approach using ultrasonic non-destructive inspection

Iheb Chouikh *, [a] **and Chokri Bouraoui** [a]

[a] Mechanical Laboratory of Sousse LMS/ENISo, University of Sousse, BP 264 Sousse Erriadh 4023, Tunisia

E-mails: Chouikh.iheb@hotmail.fr (I. Chouikh), Chokri.bouraoui2@gmail.com (C. Bouraoui)

Abstract. Fatigue crack propagation can considerably reduce the life of components, leading to sudden failures. This paper provides a method for fatigue life prediction based on ultrasonic non-destructive inspection applied on Al 2024 T3 material.

A new crack quantification model based on ultrasonic waves features is developed. To analyse the performance and efficacity of the model, the probability of detection is determined using the "signal response" technique.

The Paris model is used to predict the fatigue life taking into consideration the initial crack distributions, the dispersion of the parameters underlined by the Least-squares method and Monte-Carlo simulations.

Reliability evaluation is discussed later for two cases: Detection and No-detection case.

If no indication is presented, an inspection detection threshold is determined and optimized. This proposed indicator will be helpful for industrial environments whenever the inspection machine does not have any indication.

Considering the ultrasonic inspection data, an updating reliability via the Bayesian approach is suggested. The results of this approach can lead to a gain in the life span or a gain of the costs generated by the failure of the part.

Keywords. Ultrasonic inspection, POD, Reliability, Bayesian approach, Detection threshold.

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1. Introduction

Fatigue crack growth (FCG) is a dangerous phenomenon that cause unexpected failure of structures and mechanical components. To guarantee the survival of the mechanical components, it is necessary to predict their lifetimes using reliability-based approaches and to control the crack evolution with structural health monitoring (SHM) methods [1–4].

Health monitoring or conditional maintenance can be used to anticipate the failure, evaluate a residual duration and inspection frequency, optimize maintenance and find the most suitable solutions.

* Corresponding author.

The SHM has the same role as the non-destructive evaluation (NDE) but in addition, it uses in-situ monitoring integrating sensors and actuators inside the structure.

Non-destructive evaluation (NDE) techniques include testing methods that are exploited to test material without damaging it. These techniques are commonly used to inspect material for defects or flaws without destroying the sample.

The management of fatigue-affected components requires the use of NDT methods in the right way. Each NDT method has limits concerning its application and level of accuracy which depends on the evaluation procedure, the personal qualification, the evaluated materials, the environment etc…

Many techniques for NDT are accessible, such as magnetic particle inspection (MT), radiographic inspection (RT), ultrasonic inspection (UT), acoustic emission techniques (AE) etc… [5–8].

The SHM has been an effective measure to guarantee the safety and evaluate the reliability of these structural systems. The uncertainties associated to modelling and predicting the performance of the structures can be handled and reduced by including the information collected during inspections and monitoring.

Particularly, ultrasonic inspection technique has shown great capacities for crack detection, analysis and prediction in SHM [9–15]. This method provides an opportunity to detect material discontinuities and obtain information allowing the fatigue life prediction and risk management. The fundamental idea behind Lamb ultrasonic wave technique consists as those discontinuities such as cracks will modify characteristics of the signal such as amplitude, phase, velocity…

Ultrasonic inspection technique has been widely investigated as a non-destructive method for detecting fatigue cracks and contributing to fatigue life prediction.

Wang *et al.* [10] developed multiple Lamb wave models depending on damage-sensitive features. In their work, the model assessment and the impact of model choice on fatigue life prediction are performed using the data of coupon testing with artificial cracks and realistic lap joint testing with naturally developed cracks [10]. Lamb wave limited experimental data was reported by He *et al.* [11]. The precision of this model will directly impact the structural life prediction. Lee and Staszewski [13] studied fatigue crack detection with Lamb waves using the local interaction simulation approach. Lamb wave method of crack identification using Ansys ADPL software and finite element simulation is approved in Mishra *et al.* work [14]. Guan *et al.* [15] studied fatigue reliability assessment integrating automated ultrasonic non-destructive inspections on a steam turbine rotor. They have emphasized the advantage when using a probabilistic POD model in which the uncertainty will affect the POF results.

All these studies have dealt with fatigue life prediction and reliability determination without recourse to information and results derived from inspections, however other authors have arrived at a model of fatigue crack growth prediction where updating reliability was considered. Based on this concept, data from inspections are very essential to update reliability in real time.

The most interesting approach to this issue has been proposed by Eltaief *et al.* [16] who has determined an updating inspection time for random fatigue crack growth.

Zhang and Mahadevan [17] have also suggested a Bayesian procedure updating in reliability-based inspection for a fatigue reliability problem. The used Bayesian approach is a combination of multiple mechanical and statistical models. Moreover, Zareei and Iranmanesh [18] have used the Bayesian updating concept along with Markov Chain Monte Carlo (MCMC) method and Metropolis–Hasting algorithm to update material parameters and enhance fatigue life prediction.

These outcomes will be helpful in order to determine optimal inspection schedule and maintenance operations, that's what several researchers are working on these years [19–22].

In this paper, we explore all these studies and recommend the possibility of using ultrasonic non-destructive inspection data to update reliability using the Bayesian approach and taking into consideration uncertainty of parameters and probability of detection.

The advantage of this work is to use the ultrasonic inspection within a probabilistic framework to predict the remaining life of components and update reliability via the Bayesian approach, taking into account the value of the crack detection limit.

Considering the ultrasonic inspection output, a model that relates the growth of the crack with the characteristics of the inspection method is suggested.

Indeed, the crack can be modeled as a function of the amplitude and phase of the signal from the ultrasonic wave. This model will lead us to a probabilistic approach used to estimate the fatigue life. To evaluate the reliability of the method, we treat the case of detection and non-detection of the crack. If no indication is shown, we will determine an optimal detection threshold. The optimization of this indicator is proposed based on the convergence and safety of the model, and it will serve as an important initial parameter when no indication is available from the inspection machine. Consequently, it can help industries to have an idea of when the machine can detect the crack without any indication. Therefore, they can use different types of machines and may have the same results. Finally, taking into account the information from the inspection itself and for both cases, an update of the reliability is proposed using the Bayesian approach.

The remainder of the paper is arranged as follows:

First, the experimental procedure is presented. Coupon testing data are extracted to recognize damage-sensitive features. Using these data, a new Lamb wave-based crack quantification model is developed. Afterward, the probability of detection (POD) is determined and probability density functions (PDF) of the initial crack are reached. Then, fatigue life of the plate is evaluated via the Paris model using the least-squares method and Monte-Carlo simulations to estimate the statistical crack growth parameters in the two cases: No crack detection (without indication) and crack detection (with indication). After that, a comparison between the two cases and through experimental data is done to validate results and investigate the difference. Next, optimization of the crack detection limit is performed in case of unavailability of an indication based on safety evaluation. Finally, a fatigue crack growth reliability assessment is presented in which failure probability is determined in the two cases using MCS, inspection time is estimated, and updated failure probability considering ultrasonic inspection information is performed using the Bayesian approach.

2. Experimental procedure

In the experiment, 2024-T3 aluminum alloy plates were used with an artificial central crack introduced by electric discharge machining (EDM) and two piezoelectric transducers (PZT) were located on each side of the crack for the emission and the reception of ultrasonic waves.

The PZT elements are SM412 Ceramic discs which are arranged regarding to their pitch catch configuration. The plate thickness is 2 mm and the exciting frequency is 0.16 MHz. It should be limited to a small value, to avoid mode superposition.

The propagation modes are the symmetrical mode $\mathbf{S_0}$ which are more sensitive to cracks than the $\mathbf{A_0}$ mode.

The geometry information of the specimen can be shown in Figure 1 and the mechanical properties are presented in Table 1.

The baseline signal data of six specimens are acquired before applying EDM. The crack size is measured by optical microscopy techniques. The crack size varies from 0 to 20 mm with an increment of 3 mm. After that, the size varies from 20 to 30 mm with 5 mm incrementation.

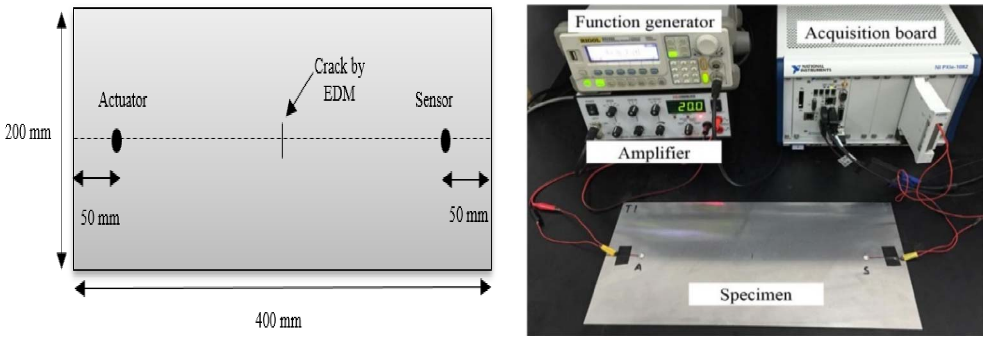

Figure 1. Coupon test geometry and data acquisition system.

Table 1. Mechanical properties of Al alloy 2024-T351

E (GPa)	$\sigma_{0.2}$ (MPa)	Tensile strength (MPa)	A (%)
74	363	363	12.5

For each crack increment, the fatigue test is paused and the ultrasonic wave data acquisition is performed. The signals from damaged specimens for different crack lengths can be collected using a digital oscilloscope connected to a computer. To extract useful data, the signal was filtered with a band-pass filter. The critical problem is to select appropriate damage features from the corresponding time window. For more details, see [10, 11, 15].

To quantify crack size, two damage characteristics were selected: normalized amplitude and phase change. Figure 2 presents a systematic flowchart for the inspection method coupled with the fatigue cycling system.

3. Adopted methodology

In our approach, we developed a crack quantification model based on damage characteristics issued from ultrasonic non-destructive inspection.

Because of the multiple uncertainty factors affecting NDE methods, such as the capability of the machine, the variability of material properties, the environment, personnel, etc. we can't have precise results. That's why we used a probabilistic analysis in which uncertainties are taken into consideration to produce reliable results.

The random nature of the material system has grave consequences on the material properties caused by undesirable and unpredicted cracks. It can lead to an unreliable analysis in case of uncertainty element will not be taken into consideration. Then, current life estimation needs to be improved, where the dispersion and the uncertainty of the parameters are considered. Also, this probabilistic approach is useful to minimize the number and the time of experimental tests, therefore we speak about cost reduction.

The accuracy and the reliability of the developed model are verified by the POD concept. The probability of detection "POD" gives the aptitude of the inspection method to detect flaws.

The initial crack is responsible for severe accidents when it reaches a catastrophic failure. It is primordial to analyze this subject with more attention to the variability in the flaws and fatigue material properties. The probability density functions PDF of the initial crack size are derived from results of the probability of detection POD. The initial flaw size distribution serves as an essential element in risk investigation for damage-tolerance analysis.

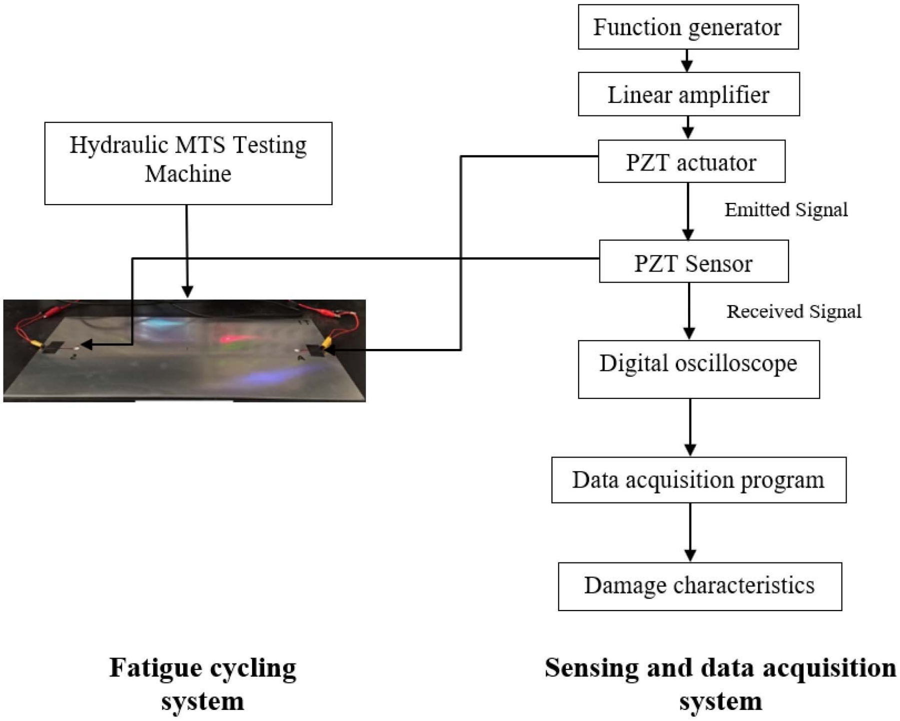

Figure 2. Systematic flowchart for SHM for test coupon.

This initial analysis was accomplished in order to predict the remaining fatigue life. Comparison between detection and no-detection cases was performed. The validity of the proposed model is ensured by the comparison with experimental results. The prediction of the remaining fatigue life serves as an important factor to prepare a maintenance plan and prevent structures from dangerous accidents. A reliability assessment is followed in order to determine the difference between detection and non-detection cases and underline the relationship between reliability and inspections. The results will be helpful in order to develop a decision tool and maintenance strategy connecting reliability, inspection, repair, and replacement notions.

This described methodology can be easy to perceive in the following flowchart in Figure 3.

4. Model construction

Figure 4 represents the measurement data for six simple plates. We notice that the phase change increases as the crack length increases, however, the normalized amplitude decreases with the increasing crack length. Using previous data, a regression model was introduced.

This model expresses the relationship between the signal features and the crack length, and it is written as:

$$a = a_1 + a_2 x + a_3 y + a_4 xy \tag{1}$$

where a is the crack length, a_1, a_2, a_3 and a_4 are the regression parameters, x is the normalized amplitude which is the ratio between the damage signal and the reference signal and y is the phase change.

We have used a bilinear model because of its simplicity and easy interpretation. Also, to quantify the crack length, we need to develop a model using the damage sensitive features as

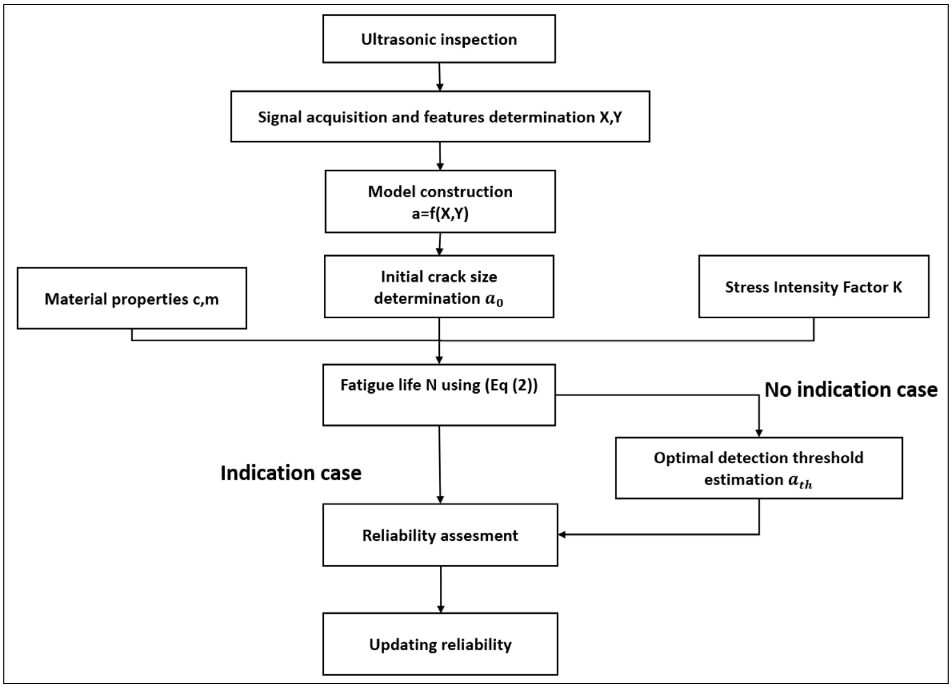

Figure 3. Flowchart adopted for the planned methodology.

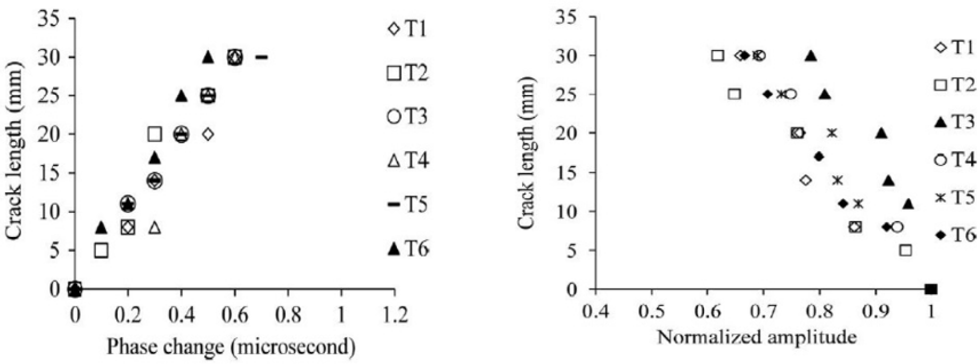

Figure 4. Crack length vs. phase change/normalized amplitude for all six specimens.

independent variables. It is about a response surface model including the normalized amplitude, phase change and the interaction between them.

A regression analysis allows us to estimate the model parameters using 8 observations (see Table 2) which present the relation between the experimental results of selected features and the crack length.

The model established is expressed as:

$$a = 104.13 - 103.33x - 16.21y + 13.14xy \quad (2)$$

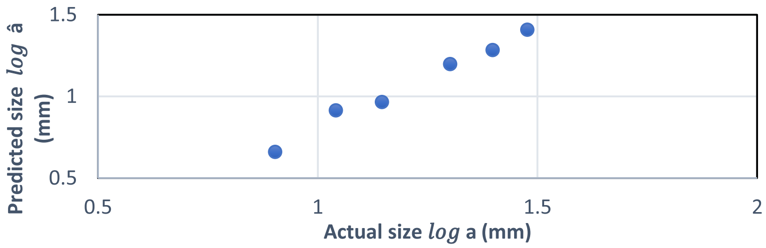

Figure 5. The predicted crack size vs. the actual crack size.

Table 2. Values of selected features in relation with the crack size change (Specimen T1)

a	x	y
0	1	0
5	0.96	0.1
8	0.9	0.67
11	0.91	0.2
14	0.84	0.3
20	0.8	0.39
25	0.73	0.47
30	0.68	0.59

Table 3. Statistical characteristics of the crack size

$\overline{\text{Log}(\hat{a})}$	$\overline{\text{Log}(a)}$	α	β	σ_ε	σ_d	R
1.14	1.13	−0.05	1.04	0.25	0.18	0.98

5. Results and discussions

5.1. *Probability of detection*

The knowledge of reliably detected crack size using NDT techniques can facilitate the prediction of the remaining fatigue life that the component can survive. The Probability of Detection is a statistical parameter that consists of the ability to detect defects by NDT techniques [23–25]. The POD is determined in our study, by the "$\hat{a}$ vs. a" method or "signal response", and it is used to analyse the performance of the model as per MIL-HDBK 1823A [26] standard. The predicted crack size and the actual crack size can be correlated according to [27] as follows:

$$\text{Log}(\hat{a}) = \alpha + \beta \log(a) + \varepsilon \tag{3}$$

where α and β are fitting parameters, ε is a normal random variable with zero mean and standard deviation σ_ε and R is the correlation coefficient.

Statistical characteristics of a (mean, standard deviation) are determined using least squares and the overall results are summarized in Table 3.

Figure 5 shows the relationship between the actual size and the predicted one on a logarithmic scale. A predefined threshold $\hat{a}_{\text{th}}$ is assumed according to the inspection tool parameters, the measurement noise, and other dispersion sources.

The POD model is presented as the probability that the crack predicted size $\hat{a}$ exceeds the detection threshold value $\hat{a}_{\text{th}}$ and it is expressed in this way:

$$\text{POD}(a) = \Pr(\log \hat{a} > \log \hat{a}_{\text{th}}) = \Phi\left(\frac{\beta \log(a) + \alpha - \log(\hat{a}_{\text{th}})}{\sigma_e}\right) \tag{4}$$

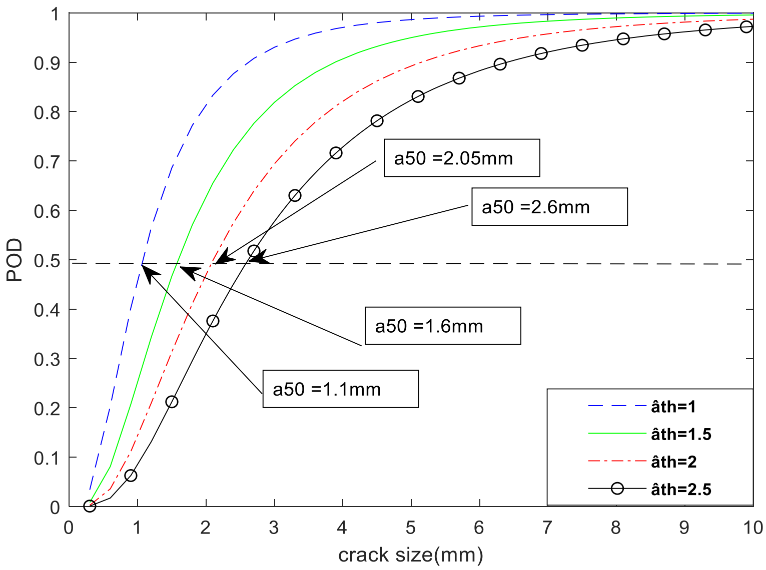

Figure 6. POD curves with threshold values of 1 mm, 1.5 mm, 2 mm and 2.5 mm.

where $\hat{a}_{th}$ is the detection threshold, $\sigma_e = \sqrt{\sigma_\varepsilon^2 + \sigma_{th}^2}$ is the standard deviation and $\Phi()$ correspond to the standard normal cumulative distribution function.

The resulting POD curves for different $\hat{a}_{th}$ are shown in Figure 6 where the values of a_{50} (crack size with 50% probability of detection) for different thresholds are mentioned.

According to [14], the verification of the validity of the plotted POD curve can be checked by the comparison between the observed a_{50} value and the threshold value $\hat{a}_{th}$. In fact, when we have the same results as what we see in this study, it can be concluded that the plotted POD curve is correct.

5.2. *Initial crack size distribution*

The initial flaw size distribution helps to predict the fatigue life and serves in the damage-tolerance analysis. It ensures a better assessment of crack growth propagation and better support for the decision-making process. The probability density function PDF of the initial crack size derivates from the results of the probability of detection POD.

Initial flaw size distributions are developed according to the following equations and shown in Figures 7 and 8.

During inspections, two cases can be reported: the crack is either detected or not detected. When the crack is detected, the inspection tool or machine has an indication about the exact crack size that can be detected. The other case is non-detection, where we do not have any indication about the crack size. Therefore, many people think that no indication means the absence of a crack or a defect in the structure, but it is not true. In fact, this can be due to uncertainties coming from the machine, the materials or the environment. Our following results will be processed for these two cases.

• No indication case:

A non-destructive inspection without any indication does not mean that the structure is empty from defects due to uncertainties coming from the machine, environment, materials,

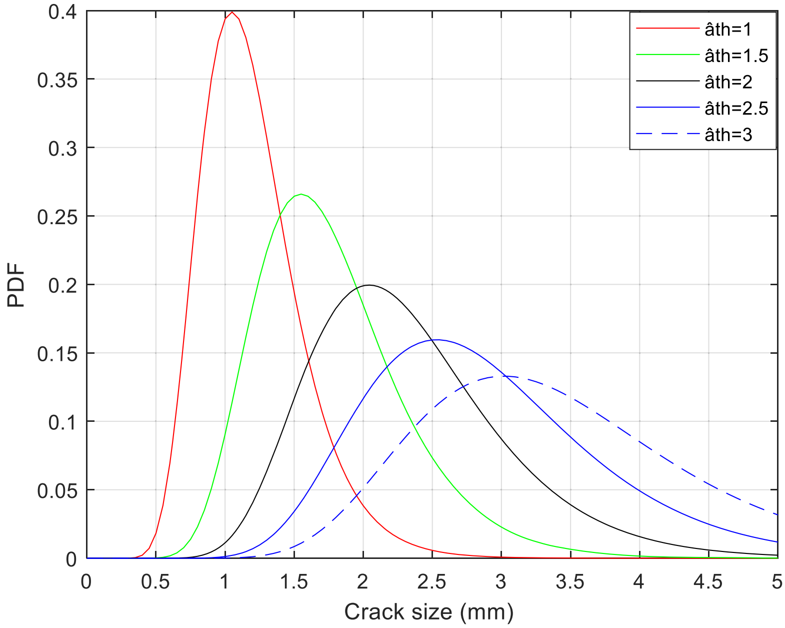

Figure 7. PDF of actual crack sizes with no indication.

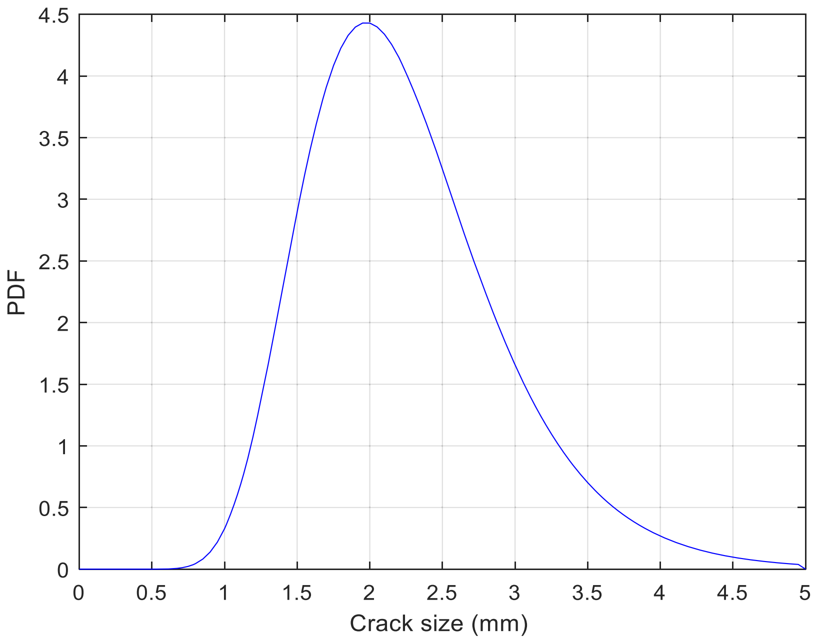

Figure 8. PDF of actual crack size with crack indication $a_d = 1.76$.

operation, etc. The distribution of crack length a, in this case, depends on the inspection threshold $\hat{a}_{\text{th}}$ and can be expressed using **Bayes**' theorem [28] as:

$$P(A|\overline{D}) = \frac{P(\overline{D}|A)P(A)}{P(\overline{D})} \tag{5}$$

$$P(A|\overline{D}) = \frac{P(a < \hat{a}_{\text{th}}|A)P(A)}{P(a < \hat{a}_{\text{th}})} \tag{6}$$

$$\text{Otherwise; } \mathrm{POD}(a) = P(a \geq \hat{a}_{\text{th}})$$

Then, the probability distribution of a crack without any indication from NDE inspection is determined via Equation (7) and represented in Figure 7.

$$f_{a\backslash\overline{D}}(a) = P(A|\overline{D}) = \frac{(1-\mathrm{POD}(a))\cdot f_0(a)}{\int_0^{\infty}(1-\mathrm{POD}(a))\cdot f_0(a)\cdot \mathrm{d}a} \tag{7}$$

• Indication case:

D represents the event that a flaw is detected ($a > a_d$), and based on Equation (4), we obtained the following equations, which determine the PDF of a crack in the detection case. Figure 8 shows the shape of the normal distribution in this case.

$$P(\log A \leq \log a|D) = \Phi\left(\frac{\log(a)-(\beta\log(\hat{a})-\alpha)}{\sigma_e}\right) \tag{8}$$

$$f_{a\backslash D}(a) = P(a|D) = \frac{\partial}{\partial \log a}(P(\log A \leq \log a|D)) \tag{9}$$

$$f_{a\backslash D}(a) = \frac{1}{a\sigma_e}\text{Ø}\left(\frac{\beta\log(a)+\alpha-\log(a_d)}{\sigma_e}\right) \tag{10}$$

where Ø (·) is the standard normal PDF and a_d is flaw indication ($a_d = 1.76$ mm in our case).

5.3. *Fatigue life evaluation*

The evaluation of fatigue life using fracture mechanics implicates the knowledge of various information, such as the material properties, the initial crack size, and fatigue loads.

The PDF of the initial crack size from ultrasonic testing is examined in previous section.

The Paris' equation as described in Equation (11) is employed as the fatigue crack propagation model with considering uncertainties of material parameters due to the stochastic nature of the fatigue crack propagation process.

$$\frac{\mathrm{d}a}{\mathrm{d}N} = C(\Delta K)^m \tag{11}$$

The stress intensity K is reported from [29] as:

$$K = \frac{P}{B\sqrt{W}} f\left(\frac{a}{W}\right) \tag{12}$$

where P is the applied load and $f\ (a/W)$ is a dimensionless geometry function. The dimensions, B, W, and a are defined according to the specimen configuration. The model parameters (log C, m) are normally distributed and estimated using MCS and the least square method which are identified from fatigue testing data. Mean and standard deviations are expressed respectively, as $m = (4.48, 0.26)$ and $\log C = (-8.78, 0.53)$.

Then, the fatigue life is computed using Equation (13), and results are drawn in Figures 9 and 10.

$$N = \int_{a_0}^{a_c} \frac{1}{C(\Delta\sigma\sqrt{\pi a Y})^m}\mathrm{d}a \tag{13}$$

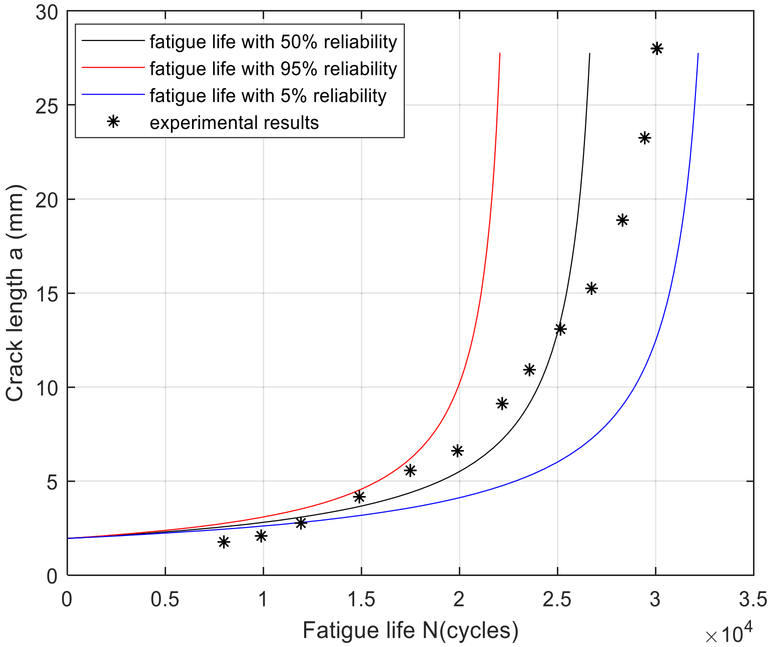

Figure 9. Iso-probabilistic *a*–*N* curves in case of indication.

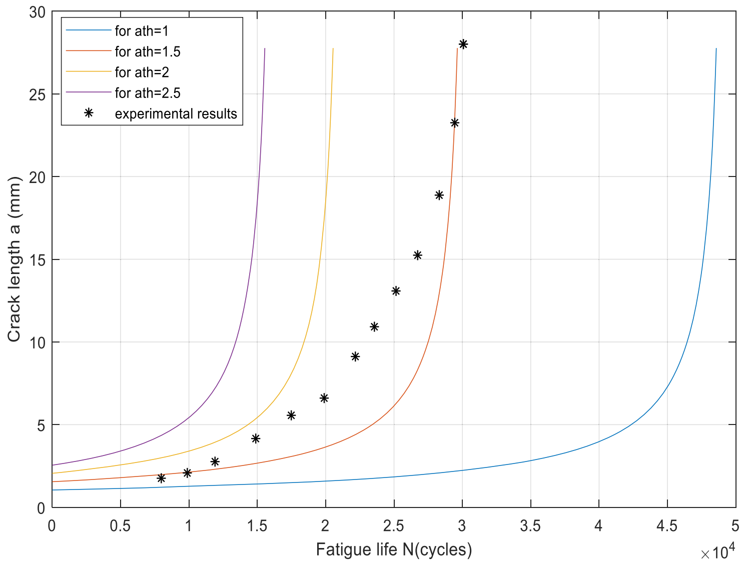

Figure 10. *a*–*N* curves in case of no-indication, for different detection thresholds $\hat{a}_{\text{th}} = 1$, 1.5, 2 and 2.5 mm.

• Indication case:

When an indication is available (in this situation $a_d = 1.76$), there is a good match between the obtained results and the experimental data. This is illustrated in Figure 9 where the fatigue life with 50% reliability and the experimental points are in good agreement (see Table 4).

This result allows us to deduce that the 95% confidence level can provide a good safety margin for decision-making, thus our model is conservative.

Table 4. Fatigue life in detection case for different confidence levels

a_d (mm)	$N_{5\%}$	$N_{50\%}$	$N_{95\%}$	N_{exp}
1.76	32,170	26,640	22,060	30,000

Table 5. Fatigue life in non-detection case for different detection thresholds

$\hat{a}_{\text{th}}$ (mm)	1	1.5	2	2.5
$N_{95\%}$ (cycles)	48,570	29,630	20,540	15,550

• No indication case:

When no crack is detected, we used a detection threshold concept. The crucial element in the POD analysis of the signal response data is the definition of the decision limit. Usually, this indicator influences both the minimum detected size and the probability of false-positive detections. When the flaw size is the parameter used for a signal-response based POD analysis, the selection of a decision threshold requires a criterion [30] based on the precision of the inspection. We have simulated the fatigue life results for different proposed detection limits (see Figure 10 and Table 5).

For a_{th} = 1 mm, the performance of the probabilistic fatigue life prediction is not suitable and represents a big danger for the component even using the lower 95% confidence level.

For a_{th} = 1.5 mm, the model has the best convergence results and can provide conservative results from 29,000 cycles. When using a_{th} = 2 mm, the model ensures more safe results for 95% reliability with acceptable convergence to the experience.

On the contrary, for a_{th} = 2.5 mm, we have total security but the model diverges from the real results. Compared with the detection case, the fatigue life increases when no indication is accessible. A poor inspection quality in this situation that is represented by the unavailability of crack indication leads to uncertain results. The fact that a chosen or estimated decision limit can considerably affect the final results forces us to pay special attention to the determination of the threshold value.

5.4. *Detection threshold optimization*

Analysing the possibility that there is no indication about the size of the detectable crack on the inspection machines, an optimal detection threshold has been determined, which can serve as a very important initial parameter in the fatigue study. The idea here is to determine an optimal detection threshold that ensures safe outcomes despite the absence of an indication.

For this, the research interval was minimized between 1.5 mm and 2 mm since this is where the results converge better and the model is more conservative (see Figure 11).

As shown in Figure 11, for a_{th} = 1.6 mm, the number of cycles before the fracture is about N_C = 30,500 cycles, which corresponds to the experimental results; but we are not in the safe zone. For a_{th} = 1.7 mm, it is obvious that we are in safety from 26,000 cycles, and the critical fatigue life is N_C = 27,300 cycles. Figure 11 illustrates the fact that for a_{th} = 1.8 mm we have security from 25,000 cycles and a fatigue life equal to 26,400 cycles. Considering a_{th} = 1.9 mm, we have total security from 22,000 cycles, but the plate can live only about 24,600 cycles.

The optimization problem, in this case, contains two major parameters: the safety of the component and the convergence of the model to experimental results.

The safety indicator corresponds to the fact that the model is conservative and has no risks of component failure. It is computed by the experimental points on the right of the model.

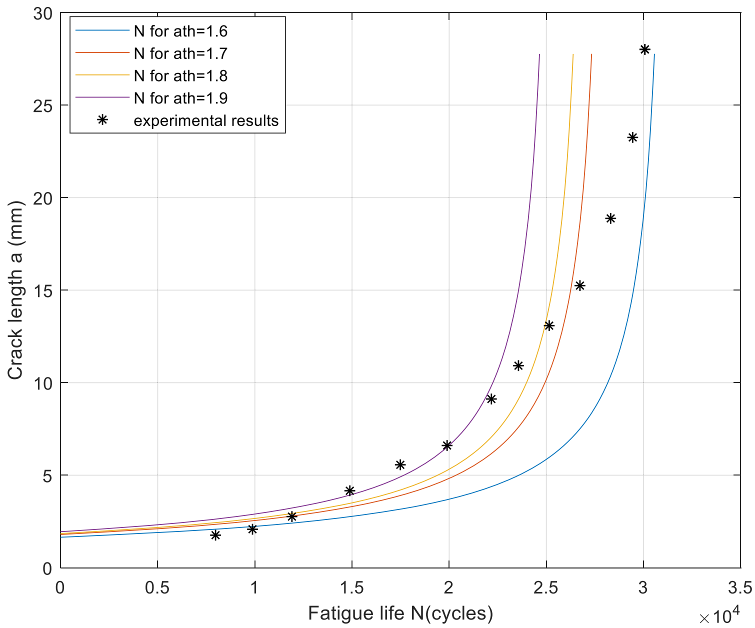

Figure 11. Optimization of the detection threshold.

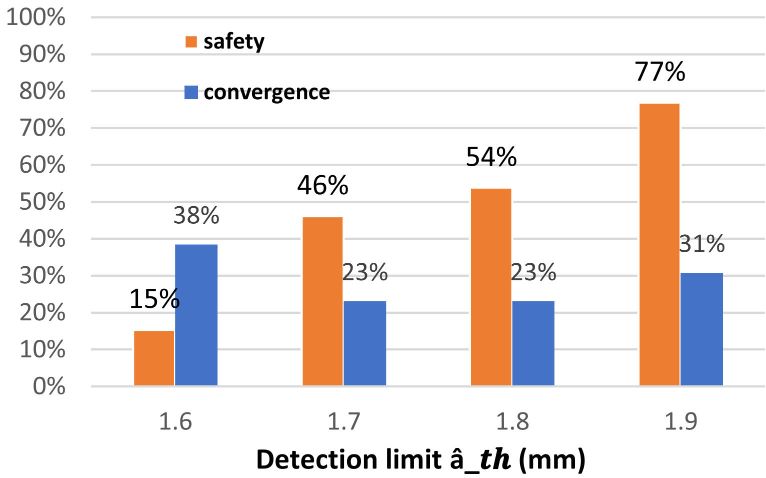

Figure 12. Percentage of safety and convergence of the model for different detection limit.

The more the model tends to the left of the experimental results, the more we can say that our model is conservative and the safety indicator is high.

The convergence is the adherence of the model to the experience. It is calculated by the number of experimental points closer to the model. For each experimental result, we determine the fatigue life N_{exp} and N_{model} and compute the difference $(|N_{\text{exp}} - N_{\text{model}}|)$. Certainly, the model that have the slightest deviation has the best convergence percentage.

To find the best detection limit that guarantees the safety of the component and the adherence to the experience simultaneously, we have created this histogram (see Figure 12). Based on Figure 11, the safety indicator is modeled by the number of experimental points to the right of the model. Furthermore, the adherence or convergence with the experiment is manifested by the number of experimental points closer to the model linked to the corresponding detection limit, compared with the other models.

We notice then that the desired results are corresponding to $a_{\text{th}} = 1.9$ mm, so we can assume that the optimal value of a_{th} that guarantees the safety of the part and the coincidence with the experimental results is equal to 1.9 mm.

The method is an effective way to find the detection limit of the crack in case of unavailability of indication. It helps industrials know the first detectable crack size that can ensure their inspection machine without having any indication.

5.5. *Fatigue reliability assessment*

Specific attention is paid to the reliability assessment of the plate during its service life. In this section, we will try to determine the relationship between inspections and reliability, and examine the difference between the fact that a crack is detected or not.

Moreover, this analysis helps to determine the best time to inspect, repair and replace the component if needed, so a maintenance plan can be proposed.

5.5.1. *Probability of failure and inspection time determination*

In the first step, we calculated the failure probability P_f using Monte-Carlo simulation (MCS).

This method is used to evaluate the probability of different results due to the existence of random variables. It is applied when the performance function G is defined over a vector of more than two random variables and when it is hard to determine the joint probability density function of X.

This procedure is simple but it requires a large number of runs to obtain an accurate result [31,32]. In this case, the probability failure is given by the following relationship:

$$P_f = \lim_{N\to\infty} \frac{N_f(G(x_i) < 0)}{N} \tag{14}$$

where $N_f(G(x_i) < 0)$ is the number of failure events and N is the number of cycles. For a cycle's number N less than 2000, P_f is equal to zero, we can explain this by the reality that the crack length is always much lower than the critical length. For a cycle's number N higher than 2000 cycles, P_f increase significatively (Figure 13).

Figure 14 presents the probability of failure with the confidence intervals 5% and 95%.

Consequently, the inspections operations have to be planned to control (repair, replacement) the evolution of the crack length and avoid the sudden failure of the element. In general, inspection time is based on the target level for reliability or probability of failure. Numerous methods may be applied to establish the target level. The following approaches will be discussed herein [33]:

- The implicit safety or risk level.
- The experienced likelihood of fatalities, environmental damage, or property loss.
- Cost-benefit criteria

In our case, we used a target design P_f equal to 10^{-3} which correspond to an inspection time $t_i = 8750$ cycles (see Figure 12).

5.5.2. *Updating failure probability considering inspection data*

In the previous sections, we have been able to show the influence of dispersions of the parameters affecting the crack propagation process. A reliability analysis is necessary to determine the failure probability P_f.

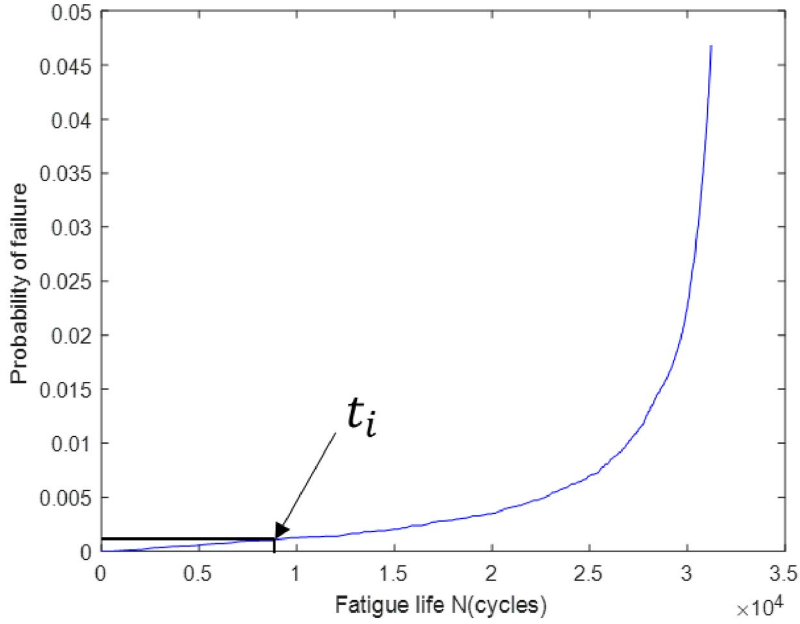

Figure 13. Increase in failure probability according to the number of cycles.

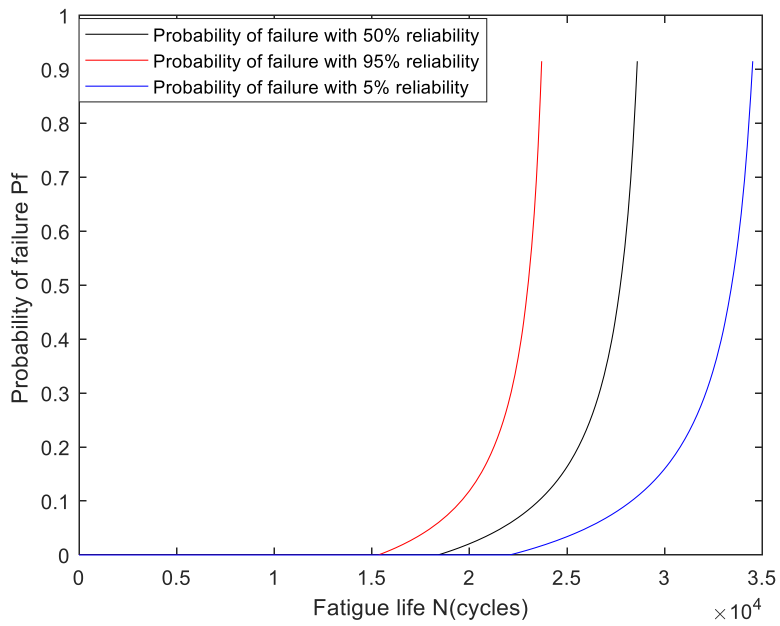

Figure 14. The probability of failure with the confidence intervals.

As a result of this analysis, we have shown, on the one hand, the need for inspections to control the evolution and the propagation speed of the crack to avoid an unexpected failure. On the other hand, we determined the inspection time t_i for a threshold failure probability. As a result of the inspection events performed at t_i, we will then need to update the reliability parameters and

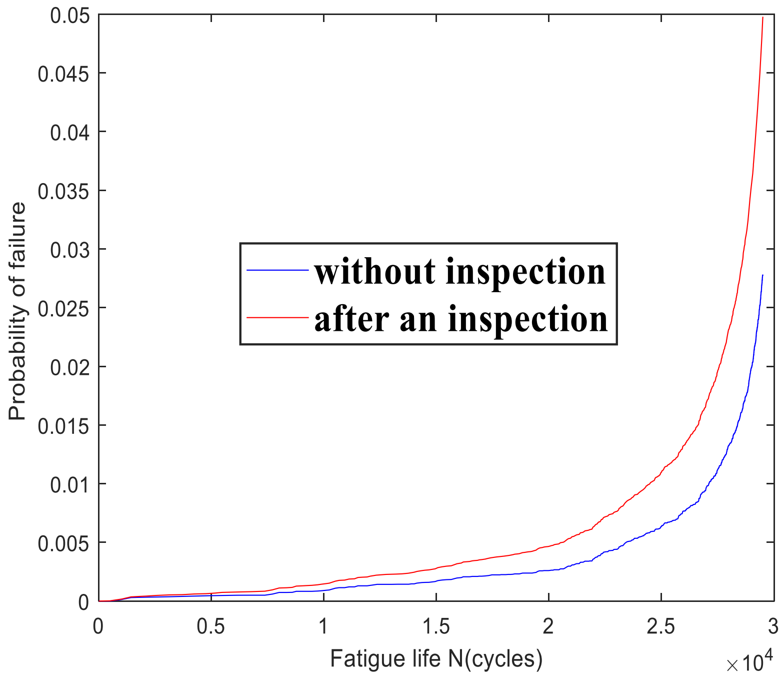

Figure 15. Updated failure probability—Detection case.

do so based on the inspection outcome. In the following, a re-evaluation calculation of P_f is performed for the different cases of crack detection.

The failure probability P_f can be updated by considering the additional data obtained from an inspection. It is based on Bayes' Theorem which describes the probability of an event occurrence using the previous conditions related to this event.

The Bayesian updating algorithm has been exposed to make use of condition monitoring data to improve the models of predictions and integrate the effects of different types of uncertainties. The uncertainty limits for life prediction are controlled by updating the parameter distribution using the detected crack length through periodic measurements [34]. In this paper, the Bayesian updating method has been used for updating failure probability in cases of detection or non-detection of a crack under uncertainties for a coupon test. Equations (15) and (16) define the updated failure probability $P_{f,\mathrm{up}}$ in each case: [20–22]

- Indication case:

$$P_{f,\mathrm{up}\boldsymbol{d}} = P(G(X \le 0)|D \le 0) = \frac{P(G(X) \le 0 \cap D \le 0)}{P(D \le 0)} \tag{15}$$

- No indication case:

$$P_{f,\mathrm{up}\boldsymbol{nd}} = P(G(X \le 0)|\overline{D} \le 0) = \frac{P(G(X) \le 0) - P(G(X \le 0)|D \le 0) \cdot P(D \le 0)}{1 - P(D \le 0)} \tag{16}$$

Figure 15 shows the growth of $P_{f,\mathrm{up}}$ depending on the cycle's number after an inspection in the case of crack detection. From this figure, it can be seen that the crack detection accelerates the increase in P_f. Therefore, the model is profitable here because it can predict the cost of a disaster that would have happened due to that failure. Figure 16 illustrates how no crack detection can delay the increase in P_f, so an extra operational life cycle (about 10^4 cycles) is gained.

As it can be seen in Figures 15 and 16, we have shown that the variation between the probability of failure in the design phase (i.e., without inspection) and the updated probability of failure

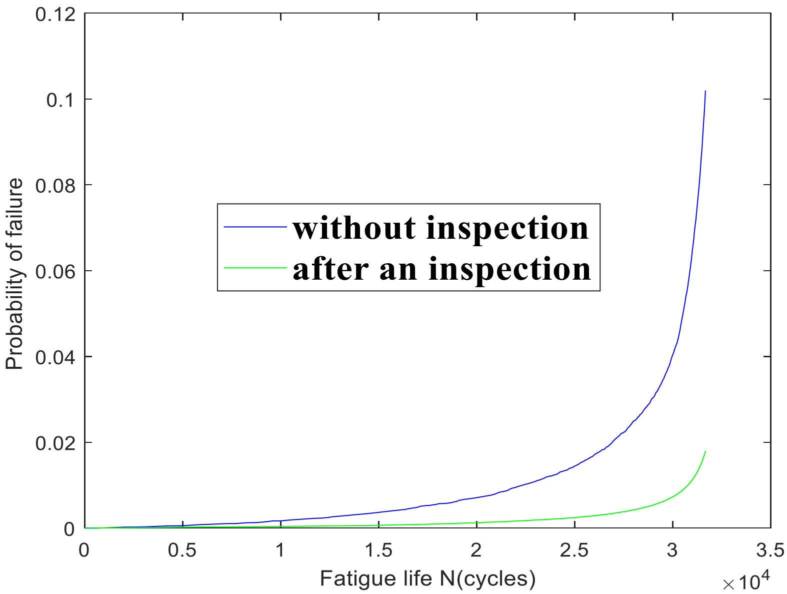

Figure 16. Updated failure probability—No Crack detection case.

after the integration of the inspection results is significant and can thus improve the decisions taken concerning the planning of maintenance operations. We wonder whether when we should perform inspection operations to guarantee a good functioning of the component, thus avoiding an unpredictable failure.

6. Conclusions

In this paper, a fatigue-reliability-SHM coupling is presented. A crack quantification model based on ultrasonic wave features is proposed. The performance of the model is evaluated from a reliability point of view using POD modeling. The PDF of the actual crack size is derived considering two typical cases of inspection data: With no detected crack and with detected crack. The fatigue life of the coupon test is predicted taking into consideration the uncertainty of geometrical and material parameters. Reliability assessment is realized via Monte-Carlo simulation and reliability updating is performed using the Bayesian approach.

From the outcome of our investigation, it is possible to draw the following highlights:

- Method and quality of inspection have an important effect on fatigue life prediction. In fact, a poor inspection quality represented by non-detection of the crack produces unsafe results. However, the availability of an indication can lead to the convergence of results.
- The determination of an optimal detection limit is very helpful in case of the absence of an indication from the inspection system.
- The Bayesian approach is very useful for reliability updating. Two cases are engendered:
  - In a detection case, the updated failure probability $P_{f,\mathrm{up}}$ increases. Consequently, a gain of the cost of a catastrophe can be obtained due to the failure.
  - The no-crack detection case postpones the increase in P_f, so a gain of the operational life cycle is acquired.

6.1. *Perspectives*

- The development of a maintenance plan based on cost optimization in which we determine the optimal time to realize an inspection, repair the structure or replace it if needed.
- The application of this plan on different structures in order to design a general strategy.

Nomenclature

NDE	Non-Destructive Evaluation
SHM	Structural Health Monitoring
POD	Probability of Detection
PDF	Probability Density Function
MCS	Monte-Carlo Simulation
E	Modulus of elasticity
$\sigma_{0.2}$	Yield strength
A (%)	Percent elongation
r	Correlation coefficient
t_i	Inspection time
$\hat{a}$	Predicted crack size
a_d	Crack indication
a_0	Initial crack size
a_c	Critical crack size
A	Crack size random variable
C, m	material parameters of Paris model
P	applied load
B	Thickness of the plate
W	Width of the plate
N	Number of cycles
N_{model}	The theoretical fatigue life
N_{exp}	The experimental fatigue life
ΔK	Stress intensity factor range
$f(\frac{a}{W})$	Geometry factor
$\hat{a}_{\text{th}}$	Detection threshold
σ_{th}	Standard deviation of the detection threshold
$\Phi(\cdot)$	Standard normal cumulative distribution function
$\varnothing(\cdot)$	Standard normal probability density function
D	Event of detection of a crack
$\overline{D}$	Event of non-detection
$P(A)$	Prior probability distribution of crack a, $P(A) = f_0(a)$
$f_{a\backslash D}(a)$	Probability distribution of crack with detection case
$f_{a\backslash \overline{D}}(a)$	Probability distribution of crack without indication
$P_{f,\text{up}\boldsymbol{d}}$	Updated failure probability in case of detection
$P_{f,\text{up}\boldsymbol{nd}}$	Updated failure probability in non-detection case
$P_{f,\text{up}}$	Updated failure probability
P_f	Failure probability
$G(X)$	Performance function
$L(\{X\})$	load function
$S(\{X\})$	Strength function

Conflicts of interest

Authors have no conflict of interest to declare.

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