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Fitts Law as a Restrained Random Walk

La loi de Fitts comme une marche aléatoire restreinte

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Abstract. Fitts law, one of the rare quantitative relations in psychology, describes the time it takes for a human being to aim at and hit a target of a given size, starting from a given remote position. We provide here a new interpretation of this law, not invoking a discretization of space and time as in the original information theory representation of Fitts, but involving a simple restrained random walk on a continuum, in space and time. We not only predict that the pointing time is proportional to the logarithm of the starting distance relative to the target size (which is Fitts law), but also describe the complete probability of presence of the pointer in its route to destination. In particular, we quantify the pointer large time overlapping efficiency with the target from the comparison of a new length-scale intrinsic to the motion, with the target size.

Résumé. La loi de Fitts, l'une des rares relations quantitatives en psychologie, décrit le temps nécessaire à un être humain pour viser et atteindre une cible d'une taille donnée, à partir d'une position éloignée donnée. Nous proposons ici une nouvelle interprétation de cette loi, qui ne fait pas appel à une discrétisation de l'espace et du temps comme dans la représentation originale de Fitts invoquant une théorie de l'information, mais qui implique une simple marche aléatoire restreinte sur un continuum, dans l'espace et dans le temps. Nous ne nous contentons pas de prédire que le temps de pointage est proportionnel au logarithme de la distance de départ par rapport à la taille de la cible (ce qui est la loi de Fitts), mais nous décrivons également probabilité de présence complète du pointeur sur son trajet jusqu'à la destination. En particulier, nous quantifions l'efficacité de la superposition du pointeur avec la cible à partir de la comparaison d'une nouvelle échelle de longueur intrinsèque au mouvement, avec la taille de la cible.

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1. Fitts Law: reaching a target

Fitts[1] law [1] is one of the rare quantitative relations in psychology [2]. It describes the time it takes for a human being to aim at and hit a target of size w (sometimes called the tolerance, or incertitude), when starting from a given remote position a (sometimes called the amplitude of the movement). The very interesting observation, confirmed experimentally in many different settings [2, 3], is that the pointing time t_p is proportional to the *logarithm* of the ratio between a and w, namely

$$t_p = T \ln\left(\frac{a}{w}\right) \tag{1}$$

where T is a setup dependent characteristic time. The situations where this law has been proved to be valid include pointing with fingers on a screen, aligning objects though a lens or

[1] Paul Fitts (1912-1965) was an American psychologist, professor and US Air Force officer. He is considered as one of the founders of "man-machine interactions" studies.

a microscope, moving a joystick, turning a handle, throwing darts at a target, shooting with a rifle, archery etc... [2].

Along with the measurement of its overall duration t_p, studies have also documented the form of the pointer trajectory in its route towards the target. After a phase of acceleration (starting from rest), the pointer gradually slows down, on average, as it approaches the target while displaying hieratic motions superimposed on the mean trajectory. These rapidly varying, random-like movements result from the many retro-actions between the visual clues perceived but necessarily imperfectly analyzed by the operator, who consequently over/under reacts to adjust the pointer position when commanding to his muscles, which may themselves activate with some delay and/or not the desired strength. This complex chain of operations, regulated by active control through several feedback loops has been identified for a long time, notably the decisive role of vision [4], and is a fascinating topic for psycho-physiologists that we will not attempt to discuss here [2].

The relation in Eq. (1) has received, by Fitts himself, a description within the language of information theory [5]. In that frame, the "index of difficulty" $I = \log_2(a/w)$ is interpreted as a number of bits (hence the logarithm in base 2) the observer holding the pointer has to process through a sequence of successive interrogations of the pointer versus target positions (the target occupies one of the a/w slots of a discretized in intervals w). Obviously, more interrogations are needed when the target ("the bull's eye in rifle shooting" [6]) is finer for a given movement amplitude. The interrogations sequence is punctuated by the clock T and it was assumed that the pointing time t_p is proportional to this interrogation time, multiplied by the number of bits I.

Here, we will opt for a different approach, not based on a discretization of space and time, and will rather retain the only two salient features of the motion mentioned above, that is

(1) The existence of a mean drift velocity $u(x)$, driving the pointer to the target while decaying with the distance x from it [7, 8],
(2) The existence of random fluctuations around the average trajectory [8, 9], whose intensity is measured by a diffusion coefficient D,

to show that they are sufficient to appreciate the physical content of Fitts law in Eq. (1), and make additional predictions.

2. Restrained Random Walk

We conduct, for simplicity, the discussion in one dimension of space (the generalization to two-dimensions, like for a shooter aiming at a circular target [9], or three [7], is straightforward), considering a pointer moving along the x-axis. The target has a width $2w$ around $x = 0$.

2.1. *Random walk with drift*

We are interested in describing the position of a pointer drifting towards the source, along a noisy path. More precisely, we will compute its probability of presence $p(x,t)\mathrm{d}x$ between x and $x+\mathrm{d}x$ at time t. The pointer must be somewhere on the axis at any time, implying that the evolution of $p(x,t)$ must be of a conservative form

$$\partial_t p = -\partial_x j \tag{2}$$

where $j(x,t) = u(x)p - D\partial_x p$ is a flux. The flux is split in two, one part $u(x)p$ being a deterministic drift driving the pointer towards the target, the other $-D\partial_x p$ being a diffusive-like term reflecting the random motions superimposed on the mean drift, with D a diffusion coefficient independent

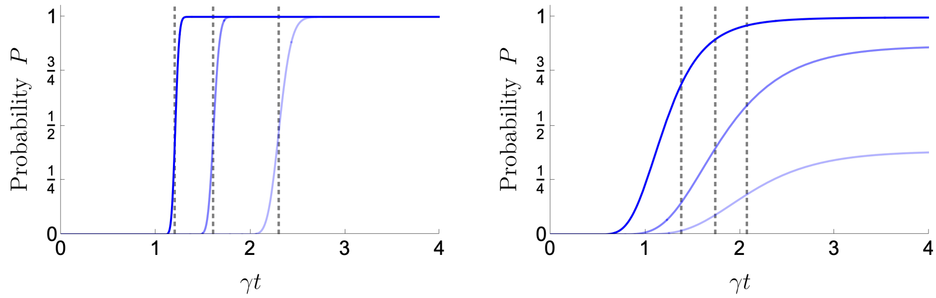

Figure 1. The probability of presence of the pointer in the target P given in Eq. (15), as a function of time. Left: $\ell = 0.01$ and $w = 0.3, 0.2, 0.1$ following the fainter blue colors. The vertical dashed lines are expectations from Eq. (13). Right: $\ell = 0.1$ and $w = 0.3, 0.15, 0.05$ following the fainter blue colors. When $\ell/w > 1$, the efficiency of the shot $E = \lim_{t\to\infty} P < 1$.

of x. The distinction between "regular" and "random" components of the motion is familiar in this context [8, 9]. We thus have

$$\partial_t p + \partial_x \big(u(x)\, p\big) = D\partial_x^2 p. \tag{3}$$

Several observations [7, 8] suggest that, at least in the final period of the motion when the pointer approaches the target, its mean velocity is decaying in proportion to the distance x left to reach the target as

$$u(x) = -\gamma x \tag{4}$$

where γ is a rate, proportional to the inverse of a time.

In order to solve Eq. (3), it is convenient to introduce an auxiliary function $q(x,t) = e^{-\gamma t} p(x,t)$ which now obeys

$$\partial_t q - \gamma x \partial_x q = D\partial_x^2 q \tag{5}$$

and which is solved by mapping it onto a pure diffusion equation

$$\partial_\tau q = \partial_\xi^2 q \tag{6}$$

in the variables $\{\xi, \tau\}$ such that

$$\xi = \frac{x}{s(t)}, \quad \text{and} \quad \tau = D\int_0^t \frac{\mathrm{d}t'}{s(t')^2}, \quad \text{with} \quad s(t) = a e^{-\gamma t}. \tag{7}$$

Let us admit that the pointer is with certitude in $x = a$ at $t = 0$. Therefore, the initial condition for Eq. (3) is $p(x,0) = \delta(x-a)$, which translates into $q(\xi,0) = \delta(\xi - 1)$ and we have from Eq. (6)

$$q(\xi,\tau) = \frac{1}{2\sqrt{\pi\tau}} e^{-\frac{(\xi-1)^2}{4\tau}}. \tag{8}$$

Noting that $q(x,t) = q(\xi,\tau)/a$, we finally find the probability of presence of the pointer at any time on the axis

$$p(x,t) = \frac{1}{\ell\sqrt{2\pi(1-e^{-2\gamma t})}} e^{-\frac{(x-ae^{-\gamma t})^2}{2\ell^2(1-e^{-2\gamma t})}}, \tag{9}$$

where the intrinsic length-scale ℓ is defined from the two parameters involved in the model as

$$\ell = \sqrt{\frac{D}{\gamma}}. \tag{10}$$

Eq. (9) describes a noisy trajectory from $x = a$ to the location of the target in $x = 0$, a location where the pointer finally ends, with however an incertitude whose width is given by ℓ. The short time ($t \ll \gamma^{-1}$) form of $p(x,t)$ features the diffusive broadening of the pointer position over a width $\sim 2\sqrt{Dt}$ while still about its initial position a, and its large time ($t \gg \gamma^{-1}$) form corresponds to the *localization* of the pointer about the target in $x = 0$, with a tolerance ℓ:

$$p(x,t) \xrightarrow[t \ll \gamma^{-1}]{} \frac{1}{2\sqrt{\pi D t}} e^{-\frac{(x-a)^2}{4Dt}}, \quad \text{and} \quad p\,(x,t) \xrightarrow[t \gg \gamma^{-1}]{} \frac{1}{\ell\sqrt{2\pi}} e^{-\frac{x^2}{2\ell^2}}. \tag{11}$$

At large times, the drift $-\gamma x$ prevents the pointer from wandering freely along the x-axis and in that sense, the motion in Eq. (9) can be viewed as a "restrained" random walk (Figure 3).

2.2. *Background*

The distribution $p(x,t)$ was first derived by Rayleigh [10] in an interpretation of Maxwell's theory of gases, and arose similarly in an analysis by Kohlrausch & Shrödinger [11] concerning the approach and fluctuations about equilibrium in the Ehrenfest's problem [12]. In order to interpret Eq. (3), these authors had even coined the image of a brownian particle fitted with an "elastic force" pulling it back to the origin.

Eq. (5) is familiar in the context of scalar mixing [13] as it describes the concentration of a passive scalar diffusing in a diluting environment stretched at a rate γ, and the change of variables $\{x,t\} \to \{\xi,\tau\}$ in Eq. (6) is usually attributed to Ranz [14] in this context, although it was already mentioned in-between the lines by Kraichnan [15]. The tolerance ℓ is the finest spatial modulation in a passive scalar field and is called the Batchelor scale [16] in that context.

The ratio of the target width w to the length ℓ is a measure of the "fussiness" of the trajectory, and of the precision of the aiming operation. This ratio may, in analogy with the field of mixing recalled above, be called a Péclet number Pe such as

$$\sqrt{Pe} = \frac{w}{\ell}, \quad \text{or} \quad Pe = \frac{\gamma w^2}{D}. \tag{12}$$

Trajectories with $Pe \gg 1$ are almost certainly ultimately aligned with the target, whereas for $Pe \ll 1$, there is a nonzero probability that the pointer sits outside from it, even at large times, as we quantify in Section 3.2 below.

3. Applications to aiming and pointing

The structure of Fitts law is readily obtained from the mean position of the pointer. As Eq. (9) shows, the maximum of the pointer probability of presence $p(x,t)$ is in $s(t) = ae^{-\gamma t}$. The target is reached, in the mean, at a time such that the mean separation of the pointer with the source $s(t_p)$ is of the order of the target half width w or, said differently, when

$$t_p = \gamma^{-1} \ln\left(\frac{a}{w}\right), \tag{13}$$

which is essentially Eq. (1), with the characteristic time T now interpreted as the inverse of the rate involved in the definition of the mean drift velocity $u(x)$.

Because the pointer trajectory is noisy, there is obviously a distribution $j(w,t)$ of arrival times t in $x = w$ such that

$$\begin{gathered} \int_0^t j(w,t')\mathrm{d}t' = \int_{-\infty}^{w} p(x,t)\mathrm{d}x \\ \text{or} \quad j(w,t) = \partial_t\left(\int_{-\infty}^{w} p(x,t)\mathrm{d}x\right), \end{gathered} \tag{14}$$

expressing that all trajectories ending in $x < w$ at time t arrived earlier in $x = w$. This distribution which identifies with the flux $j(w,t)$ in Eq. (2) is sometimes called *first-passage time* to the target distribution [17]. Consistently, $j(w,t)$ is broader for $Pe \approx 1$ than for $Pe > 1$ an when $Pe \gg 1$, it peaks at $t = t_p$, as seen in Figure 2.

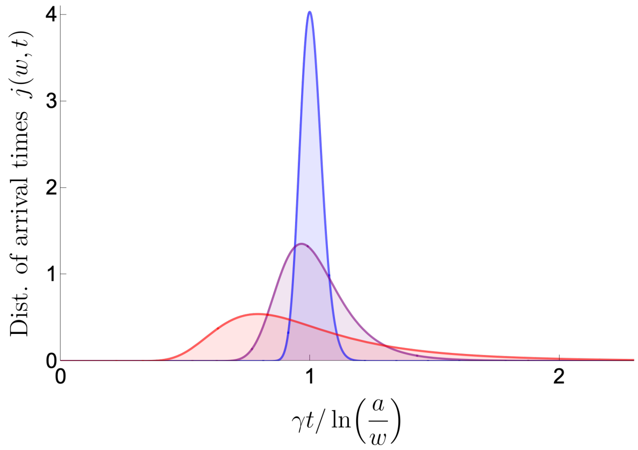

Figure 2. The distribution $j(w,t)$ of arrival times in $x = w$ given in Eq. (14) as a function of time t scaled by the kinematic pointing time t_p given in Eq. (13), for three values of the Péclet number $Pe = (w/\ell)^2$ defined in Eq. (12): $Pe = 100$ (Blue), $Pe = 10$ (Purple) and $Pe = 1$ (Red).

The structure of Fitts law suggests that the pointing characteristic time results from an a-priori observation and measurement by the aiming individual, who is pre-informed of the target location, and manipulates the pointer accordingly. The trajectory fuzziness is the *consequence* of the pointer's lack of control, not the *cause* of the successful target search. In other words, the time t_p in Eq. (13) scales like γ^{-1} reflecting an informed motion, and is not of order a^2/D, which would reflect a random exploration of space ending when, by chance, the pointer has hit the target (see e.g. [18] and the coincidental $\ln(a/w)$ correction to a^2/D in two-dimensions (2D), not in 3D though where the correction is different, while Fitts law is known to apply in 3D as well [7, 19]).

Fitts law describes relatively fast (and possibly efficient, see Section 3.2) aiming operations in the sense that the noise, while certainly present, is sub-dominant ($Pe > 1$), meaning that $w^2/D > \gamma^{-1}$ so that, a-fortiori, $a^2/D \gg \gamma^{-1}$ (since in practice $a \gg w$).

Fitts law does not reflect blind random search, nor does it express a process where the pointer is intermittently but continuously informed about the possible location of the target, and where his knowledge refines as it approches it (also called "infotaxis" [20]): a directed random walk towards a target initially located with certainty is not a pure random walk, nor an infotactic walk.

3.1. *Net probability of presence*

However, since Eq. (9) incorporates the full information about the pointer probability of presence, not only its mean, this allows to make additional predictions. We have already exploited the distribution of arrival times to the target $j(w,t)$, but a successful shot not only requires moving the pointer to the right place (which takes t_p), but also staying there with certitude.

We now compute the net probability P to find the pointer overlapping with the target, namely to lie in the interval $-w < x < w$, as a function of time

$$P = \int_{-w}^{w} p\,(x,t)\mathrm{d}x = \frac{1}{2}\left(\mathrm{erf}\left(\frac{ae^{-\gamma t}+w}{\ell\sqrt{2\left(1-e^{-2\gamma t}\right)}}\right) - \mathrm{erf}\left(\frac{ae^{-\gamma t}-w}{\ell\sqrt{2\left(1-e^{-2\gamma t}\right)}}\right)\right). \tag{15}$$

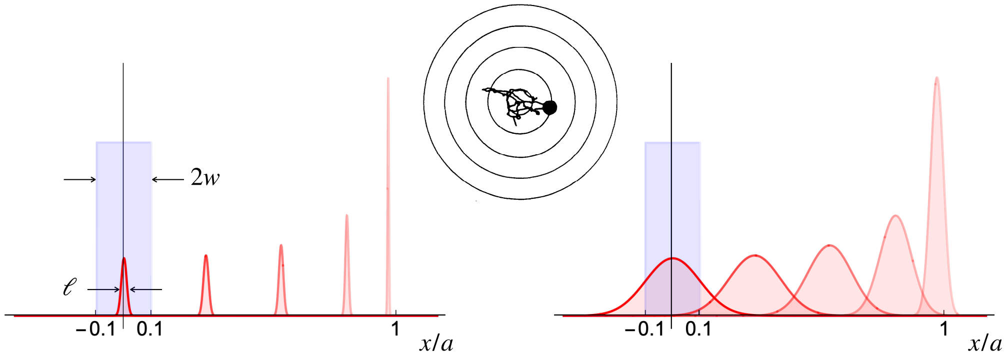

Figure 3. The deformation of $p(x,t)$ given in Eq. (9) as the pointer (initially in $x = a$) approaches the target (in $x = 0$, width $w = 0.1$ in units of a), shown at five consecutive times $\gamma t = 0.03, 0.2, 0.55, 1.2$. Left: $\ell = 0.01$ (meaning $Pe = 100$) the pointer remains localized within the target at large time ($E = 1$), Right: $\ell = 0.1$ (meaning $Pe = 1$) there is a nonzero probability that the pointer is outside the target ($E < 1$). The center image, taken from Ref. [9], shows the trace of a (real) rifle trajectory aiming at a two-dimensional target (the concentric circles). The value of ℓ is in that case clearly larger than the center circle radius, and the shot has been missed.

When $a \gg w$ (a typically small target reached from a distant location), P is obviously initially zero (see Figure 1), and increases sharply (when $Pe > 1$) as $s(t)$ become of the order of w, consistently with Eq. (13), thus providing another way of understanding Fitts law in Eq. (1).

3.2. *Pointing Efficiency*

The final value of P however depends on the value of the tolerance ℓ intrinsic to the motion in Eq. (10) relative to the target size w, namely on the Péclet number:

- If $\ell \ll w$ (or $Pe \gg 1$), then the pointer will most certainly remain within the target boundaries at large time ($t > t_p$); in other words, the target has been hit almost certainly.
- But if $\ell \gg w$ (or $Pe \ll 1$) on the other hand, the localization of the pointer is so loose around the target that, no matter how long the shaky aiming operator waits, there is a nonzero probability that the pointer lies outside from the target.

The efficiency E of the of the pointing operation giving, for instance, the fraction of shots which have successfully hit the target in rifle shooting is thus

$$E = \lim_{t\to\infty} P = \mathrm{erf}\left(\sqrt{\frac{Pe}{2}}\right) \tag{16}$$

which incorporates quantitatively the limit trends mentioned above, namely $E \to 1$ for $Pe \gg 1$, and $E \sim w/\ell < 1$ for $Pe < 1$ (see Figures 1 and 3).

Fitts law describes the pointing time t_p, but the present model provides in addition the efficiency of the pointer localization, which may not be perfect, after t_p.

4. Conclusion

We have given a new interpretation of Fitts law, not invoking a discretization of space and time as in the original information theory representation of Fitts [1], but involving a simple restrained random walk on a continuum, in space and time.

This model, consistent with the relation observed by Fitts (Eqs. (1) and (13)) and subsequently by many others (see [2, 3] for reviews), has two parameters: a rate γ which specifies the speed at which the pointer moves to the target, and a diffusion coefficient D which reflects the "shaky" character of the otherwise deterministic motion of the pointer. These parameters must have a physiological reality, or at least should be relatable to some aspects of the aiming operator's psycho/physiology: invoking a velocity $u(x)$ function of the distance from the target x explicitly assumes that the operator is fitted with a memory, being able to adjust the intensity of his movement to a pre-located destination. As for the origin of the noise, we may recall that any nonlinear dynamical system, as the human sensory-motor system amplifies minute initial differences, leading to divergent trajectories [2, 21, 22]. The value of D should be very different comparing an operator suffering from neurological deficiencies, being on drugs or simply drunk (large D), with another operator being well fit and trained at controlling its actions (small D). For instance, and this can be considered as a consistent prediction of the present model regarding the speed/accuracy tradeoff issue [3], among valid rifle shooters, those who point faster are also those who are more precise [9]: in the language of Eq. (10), a larger γ for a given D implies a smaller ℓ, therefore an enhanced ability to localize the pointer close to the target center, improving the efficiency E of the gesture (larger Pe in Eq. (16)).

We have also underlined why the present directed random walk model differs from a pure Brownian exploration protocol [18], and why it is not either a process where the pointer is intermittently but continuously informed about the possible location of the target, although some similarities with infotaxis [20] may arise in situations where the target moves itself hieratically in space, not considered here.

In psychology, or should we say in psychophysics since it was the ambition of Fechner [23] to make psychology an exact science, a logarithm as the one in Fitts law (Eq. (1)) naturally evokes the celebrated Weber-Fechner relationship relating the intensity of the perception I with the one of its stimulus S

$$I \sim \ln S \tag{17}$$

or, more precisely $\Delta I \sim \Delta S/S$: above some detection threshold, the increment of perception is proportional to the *relative* stimulus increment. For example, sound is measured in decibels, stars brightnesses and earthquakes magnitudes are counted on a logarithmic scale, and we have the same feeling of extra weight between 1 and 1.1 kg than between 10 and 11 kg (and not 10.1 kg, which we are likely to confuse with 10 kg). Bergson [24] has criticized Eq. (17) for not considering the time of application (the *duration*) of the stimulus in the definition of the perception (see also Delboeuf [25]). Although this is remotely related to this debate, it is interesting to see that Fitts law connects space and time by a logarithm, as a simple property of the exponential function, itself a consequence of a velocity diminishing with distance to destination.

Finally, let us mention that some observations have suggested that the amplitude of the shaky movements is itself a decreasing function of x, as is the position of the pointer, a property which has been qualitatively and somewhat improperly termed "Weber–Fechner compatible" [8]. In the language of the present model, this would mean that the diffusion coefficient $D(x)$ goes to zero as the pointer approaches the target, an effect which would reinforce localization, and hence the precision of the operation, as we have explained.

Declaration of interests

The authors do not work for, advise, own shares in, or receive funds from any organization that could benefit from this article, and have declared no affiliations other than their research organizations.

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