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Nomograms of solid linear viscoelastic materials in time and frequency domains

Nomogrammes des matériaux viscoélastiques linéaires solides dans les domaines temporel et fréquentiel

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Abstract. The mechanical behavior of isotropic solid viscoelastic material (VEM) can be described both in time or frequency domain considering temperature effects. Thus, one can make use of viscoelastic functions such as Young's and/or shear moduli and the Poisson's ratio. The viscoelastic dynamic behaviors in different temperature and frequency or time ranges can be grouped into a single graph, named nomogram. The present work proposes a method for constructing nomograms for viscoelastic functions, Young's and shear relaxation moduli, and Poisson's ratio, defined in the time domain. It also proposed a nomogram for the complex Poisson's ratio in the frequency domain.

Résumé. Le comportement mécanique des matériaux viscoélastiques solides isotropes (VEM) peut être décrit à la fois dans les domaines temporel et fréquentiel, en prenant en compte les effets de la température. Ainsi, il est possible d'utiliser des fonctions viscoélastiques telles que les modules de Young et/ou de cisaillement, ainsi que le coefficient de Poisson. Les comportements dynamiques viscoélastiques dans différentes plages de température, de fréquence ou de temps peuvent être regroupés en un seul graphique, appelé nomogramme. Ce travail propose une méthode pour construire des nomogrammes pour les fonctions viscoélastiques, les modules de relaxation de Young et de cisaillement, et le coefficient de Poisson, définis dans le domaine temporel. Il propose également un nomogramme pour le coefficient de Poisson complexe dans le domaine fréquentiel.

Keywords. Viscoelastic behavior, Viscoelastic functions, Complex Poisson's ratio, Complex Young's modulus, Complex shear modulus.

Mots-clés. Comportement viscoélastique, Fonctions viscoélastiques, Coefficient de Poisson complexe, Module de Young complexe, Module de cisaillement complexe.

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1. Introduction

In recent decades, the use of viscoelastic materials (VEMs) has been increasing in several engineering areas, such as civil construction, the automotive and aerospace industries, wind energy, transportation, and the biomedicine. Examples of VEMs include asphalt mixtures, pads, bumpers, water piping, packaging, household utensils, gastrointestinal probes, and syringes. To estimate the mechanical behavior of these materials, it is necessary to characterize the parameters of viscoelastic rheological models, which can be done in frequency or in the time domain [1–3].

Considering problems described solely in the frequency domain, several studies have been conducted in recent decades aimed at identifying the complex Young's and/or shear moduli functions. There are various methodologies proposed with this focus [2, 4–7]. Similarly, considering the time domain, some works [8–17] present methodologies for identifying the relaxation and/or creep moduli (in traction or shear). Another important viscoelastic function for predicting VEMs behavior is Poisson's ratio, which can also be defined in time or in frequency domain. Aiming at identifying this function, several works have been published [18–24]. According to authors [3–5, 25], the dynamic properties of complex viscoelastic functions can be expressed in terms of a loss factor and the dynamic modulus, over large ranges of temperature and frequency. On the other hand, those functions can be grouped into a single graph, named a nomogram, which results in a compact, simple, and fast way of obtaining the properties of the material under analysis.

In this context, the main contribution of the present work is to propose the construction of a nomogram for the complex Poisson's ratio in the frequency domain. Constructions of nomograms for the relaxation moduli (Young's and shear), defined in the time domain, are also proposed. It is important to highlight that in all the mentioned methodologies, the VEMs are considered to be isotropic, linear, and thermorheologically simple. Among numerous engineering applications, these nomograms can be used to obtain the materials' viscoelastic properties for use in both the frequency and time domains to control vibrations in structures involving rubbery materials, as presented in the works [26–28].

2. Theoretical concepts

VEMs are materials defined by their elastic and viscous behaviors simultaneously. Such mechanical behavior can be described through rheological models involving springs and dampers, where the spring represents the elastic part and the damper represents the viscous part. Thus, for any association, whether in series, in parallel, or in a combination of those arrangements involving springs and Scott-Blair's fractional dampers, fractional viscoelastic rheological models are obtained [29]. According to [30], for one-dimensional problems, the modeling of those systems results in fractional differential equations such as

$$\left[1+\sum_{r=1}^{n} a_r \frac{\mathrm{d}^{\beta_r}}{\mathrm{d}t^{\beta_r}}\right]\sigma(t)=\left[k+\sum_{r=1}^{n} b_r \frac{\mathrm{d}^{\beta_r}}{\mathrm{d}t^{\beta_r}}\right]\varepsilon(t). \tag{1}$$

In this equation, n, k, a_r, and b_r ($r = 1,\ldots,n$) are parameters associated with the chosen rheological model, $\sigma(t)$ is the stress function, $\varepsilon(t)$ is the strain function, t is time, and $\mathrm{d}^{\beta_r}(\cdot)/\mathrm{d}t^{\beta_r}$ represents a differential operator of non-integer order β_r, which is given by $\beta_r = r+\beta-1$, where $0<\beta<1$ [29]. In the present study, Riemann–Liouville's definition of the fractional derivative are the most appropriate, since it is assumed that the structural system is initially at rest, and there is no need to treat the information that occurs for a time $t<0$ [29]. Thus, considering a function

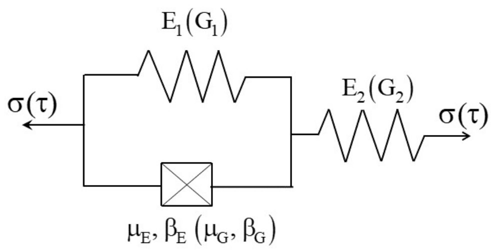

Figure 1. Fractional Zener model illustration. The parameters are related to uniaxial and shear tests.

$f(t)$, the Riemann–Liouville's definition of fractional derivative on the left, with an order β, is given by

$$\frac{\mathrm{d}^{\beta} f(t)}{\mathrm{d}t^{\beta}} = {}_0D_t^{\beta}[f(t)] = \frac{1}{\Gamma(m-\beta)}\frac{\mathrm{d}^m}{\mathrm{d}t^m}\int_0^t \frac{f(\tau)}{(t-\tau)^{1+\alpha-m}}\mathrm{d}\tau, \quad \text{where } m-1<\beta<m, \tag{2}$$

in which β is a positive real number, m is a positive integer number and $\Gamma(\cdot)$ is the Euler's Gamma function [29].

According to [29, 31, 32], there are several constitutive models involving fractional derivatives, such as Maxwell, Kelvin-Voigt, and Zener. Pritz [33] and Ciniello et al. [34] claim that the Fractional Zener Model (Figure 1) has shown to be efficient in predicting the behavior of linear VEMs in time and/or frequency domains.

Considering that the mechanical behavior of VEMs can be described by the rheological model presented in Figure 1, the differential equation that relates stress and strain can be shown as:

$$\left(1+\frac{\mu_E}{E_1+E_2}\frac{\mathrm{d}^{\beta}}{\mathrm{d}t^{\beta}}\right)\sigma(t) = \left(\frac{E_1E_2}{E_1+E_2}+\frac{E_2\mu_E}{E_1+E_2}\frac{\mathrm{d}^{\beta}}{\mathrm{d}t^{\beta}}\right)\varepsilon(t), \tag{3}$$

where E_1 and E_2 are stiffness moduli of the elastic elements, and μ_E is the intensity of the Scott-Blair element [29, 31, 32, 34].

Defining the auxiliary parameters $E_\infty = E_1E_2/(E_1+E_2)$, $E_0 = E_\infty r_E$, $r_E = (E_1+E_2)/E_1$, and $\tau_E^{\beta} = \mu_E/(E_1+E_2)$. it is possible to rewrite Equation (3) as

$$\left(1+\tau_E^{\beta}\frac{\mathrm{d}^{\beta}}{\mathrm{d}t^{\beta}}\right)\sigma(t) = \left(E_\infty + E_0\tau_E^{\beta}\frac{\mathrm{d}^{\beta}}{\mathrm{d}t^{\beta}}\right)\varepsilon(t). \tag{4}$$

Equation (4) is written for a uniaxial tension test. This equation also has a corresponding form when considering shear tests. In this case, the stress–strain relation in shear test can be placed as:

$$\left(1+\tau_G^{\beta}\frac{\mathrm{d}^{\beta}}{\mathrm{d}t^{\beta}}\right)\tau(t) = \left(G_\infty + G_0\tau_G^{\beta}\frac{\mathrm{d}^{\beta}}{\mathrm{d}t^{\beta}}\right)\gamma(t), \tag{5}$$

where $\tau_G^{\beta} = \mu_G/(G_1+G_2)$, $G_\infty = G_1G_2/(G_1+G_2)$, $G_0 = G_\infty r_G$ and $r_G = (G_1+G_2)/G_1$. The G_1 and G_2 parameters are the stiffness moduli of the elastic elements, and μ_G is the intensity of Scott-Blair element for shear constitutive model.

According to Tschoegl [30] and demonstrated by Sousa et al. [24], applying the Laplace transform to all terms of Equation (4), for $t > 0$, and considering a steady-state sinusoidal excitation of axial frequency, the steady-state response can be expressed as

$$\sigma(s) + \tau_E^{\beta}s^{\beta}\sigma(s) = E_\infty\varepsilon(s) + E_0\tau_E^{\beta}s^{\beta}\varepsilon(s). \tag{6}$$

Defining $\tilde{E}(s) = \sigma(s)/\varepsilon(s)$, where $\tilde{E}(s)$ is the function named Young's operational modulus [30], one obtains

$$\tilde{E}(s) = \frac{E_\infty + E_0\tau_E^\beta s^\beta}{1+\tau_E^\beta s^\beta}. \tag{7}$$

Analogously, applying Laplace transform to Equation (5) one can obtain

$$\left(1+\tau_G^\beta s^\beta\right)\tau(s) = \left(G_\infty + G_0\tau_G^\beta s^\beta\right)\gamma(s). \tag{8}$$

So, the shear operational modulus is obtained, defined as $\tilde{G}(s) = \tau(s)/\gamma(s)$, produces

$$\tilde{G}(s) = \frac{G_\infty + G_0\tau_G^\beta s^\beta}{1+\tau_G^\beta s^\beta}. \tag{9}$$

According to Tschoegl [30], from the operational moduli (Young's and shear), one can obtain the Poisson's operational ratio, denoted by $\tilde{\upsilon}(s)$, given by

$$\tilde{\upsilon}(s) = \frac{\tilde{E}(s)}{2\tilde{G}(s)} - 1. \tag{10}$$

Thus, from Equations (7), (9), and (10), one can obtain the Poisson's operational ratio, as

$$\tilde{\upsilon}(s) = \frac{E_\infty}{2G_\infty}\left[\frac{\left(r_E\tau_E^\beta\tau_G^\beta\right)s^{2\beta} + \left(r_E\tau_E^\beta + \tau_G^\beta\right)s^\beta + 1}{\left(\tau_E^\beta s^\beta + 1\right)\left(r_G\tau_G^\beta s^\beta + 1\right)}\right] - 1. \tag{11}$$

From the viscoelastic operational functions described in this section, the viscoelastic responses defined in the time domain can be obtained by applying the Laplace's inverse transform. These functions can also be described in the frequency domain for experiments in steady-state, where $s = \mathrm{i}\Omega$.

2.1. *The influence of temperature in the mechanical behavior of VEMs*

Temperature is a variable that directly influences the behavior of VEMs. According to Lakes [1] and Ferry [35], the viscoelastic response can be described by using the time-temperature superposition principle, which proposed a shift in the time (or frequency) scale of the viscoelastic response. This means that each time (t) or frequency (Ω) record at a temperature (T) can be shifted to a reference temperature (T_0) according to

$$t_\mathrm{R} = t/\alpha_T(T, T_0) \quad \text{and/or} \quad \Omega_\mathrm{R} = \alpha_T(T, T_0)\Omega, \tag{12}$$

where t_R and Ω_R are named 'reduced time' and 'reduced frequency', respectively [35–37]. In this case, $\alpha_T(T, T_0)$ is a function that represents the temperature shift factor.

The literature presents several models that describe the shift factor, including those by Goldstein, Bestul-Chang, Arrhenius, and Williams–Landel–Ferry. The most widely used in scientific works is the mathematical model proposed by Williams, Landel, and Ferry [38], named the WLF model, which is given by

$$\log\alpha_T(T, T_0) = \frac{-C_1^T(T-T_0)}{C_2^T + (T-T_0)}, \tag{13}$$

where C_1^T and C_2^T are intrinsic constants of the material.

2.2. *Viscoelastic functions in the frequency domain*

According to Tschoegl [30] and Park and Schapery [39], Young's modulus in the frequency domain, considering the influence of temperature, can be obtained by combining Equations (7) and (12), resulting in

$$E^*(\Omega_R) = \frac{\overline{E}_0 + \overline{E}_\infty \tau_E^\beta (i\Omega_R)^\beta}{1 + \tau_E^\beta (i\Omega_R)^\beta}, \tag{14}$$

where $\overline{E}_0 = E_\infty$, and $\overline{E}_\infty = E_0$. In addition, the complex shear modulus, $G^*(\Omega_R)$, can be obtained similarly, relating Equations (9) and (12), resulting in

$$G^*(\Omega_R) = \frac{\overline{G}_0 + \overline{G}_\infty \tau_G^\beta (i\Omega_R)^\beta}{1 + \tau_G^\beta (i\Omega_R)^\beta}, \tag{15}$$

where $\overline{G}_0 = G_\infty$, and $\overline{G}_\infty = G_0$. This model, Equation (14) or Equation (15), is known in the literature as a four-parameter fractional constitutive model [29, 31, 32].

Based on Equation (11), the complex Poisson's ratio can be posed in its final form as

$$\upsilon^*(\Omega_R) = \frac{\overline{E}_0}{2\overline{G}_0} \left[\frac{\left(r_E \tau_E^\beta \tau_G^\beta\right)(i\Omega_R)^{2\beta} + \left(r_E \tau_E^\beta + \tau_G^\beta\right)(i\Omega_R)^\beta + 1}{\left(\tau_E^\beta (i\Omega_R)^\beta + 1\right)\left(r_G \tau_G^\beta (i\Omega_R)^\beta + 1\right)} \right] - 1. \tag{16}$$

This complex function can be rewritten in its general form as given by $\upsilon^*(\Omega_R) = \upsilon'(\Omega_R) - i\upsilon''(\Omega_R)$, where $\upsilon'(\Omega_R)$ and $\upsilon''(\Omega_R)$ correspond to the real and imaginary parts, representing the dynamic modulus and the loss modulus of the complex Poisson's ratio, respectively. Thus, by relating these parts, one can obtain the loss factor of the complex Poisson's ratio, denoted as $\eta_\upsilon(\Omega_R)$, mathematically defined as

$$\eta_\upsilon(\Omega_R) = \upsilon''(\Omega_R)/\upsilon'(\Omega_R). \tag{17}$$

Therefore, based on the viscoelastic parameters, one can construct several curves associated with the complex viscoelastic functions presented in the current section. Hence, nomograms in the frequency domain can be obtained.

2.3. *Viscoelastic functions in time domain*

According to Ciniello et al. [34], the Young's relaxation modulus in the time domain, considering the influence of temperature for the fractional Zener model, can be obtained by applying the Laplace inverse transform to Equation (6), resulting in

$$E(t_R) = E_\infty \left[1 + r_E E_\beta \left(-(t/(\tau_E \alpha_T(T, T_0)))^\beta\right)\right], \tag{18}$$

where $E_\beta(\cdot)$ is the Mittag-Leffler function of order β [40].

Similarly, starting from Equation (9), the relaxation in shear modulus function is obtained considering temperature, which can be expressed as

$$G(t_R) = G_\infty \left[1 + r_G E_\beta \left(-(t/\tau_G \alpha_T(T, T_0))^\beta\right)\right]. \tag{19}$$

According to Sousa [41], the Poisson's ratio function, defined in the time domain, considering the effect of temperature, can be written as

$$\begin{aligned} \upsilon(t_R) = \frac{E_\infty}{2G_\infty} & \left[1 + \left(\frac{\tau_G^\beta \tau_E^\beta r_E - \tau_E^{2\beta} r_E - \tau_G^\beta \tau_E^\beta + \tau_E^{2\beta}}{\left(\tau_G^\beta r_G - \tau_E^\beta\right)\tau_E^\beta}\right)\left(E_\beta\left(-t_R^\beta/\tau_E^\beta\right)\right) + \cdots \right. \\ & \left. + \left(\frac{-\tau_G^\beta\left(\tau_G^\beta r_G^2 - \tau_E^\beta r_E r_G - \tau_G^\beta r_G + \tau_E^\beta r_E\right)}{\left(\tau_G^\beta r_G - \tau_E^\beta\right) r_G \tau_G^\beta}\right)\left(E_\beta\left(-t_R^\beta/r_G \tau_G^\beta\right)\right)\right] - 1. \end{aligned} \tag{20}$$

The viscoelastic functions presented in this section constitute a basis on which it is possible to predict the mechanical behavior of linear, isotropic, and thermorheologically simple VEMs

Table 1. Pseudo-algorithm implemented for constructing nomograms in the time and frequency domains

Step 1:	Obtain the mechanical properties of VEMs considering the influence of temperature.
Step 2:	Define the viscoelastic function to construct its nomogram.
Step 3:	Specify the reference temperature and the frequency/reduced time interval.
Step 4:	Plot the viscoelastic function(s) at the reference temperature within the defined interval for reduced frequency. The abscissa axis represents the reduced frequency (or reduced time), and the left ordinate axis shows the values of the viscoelastic functions.
Step 5:	Create a right ordinate axis while keeping the abscissa axis fixed.
Step 6:	Construct another graph for different temperatures on the same axis: reduced frequency versus frequency. The resulting curves are straight.

in the time domain, considering the effect of temperature. Thus, from the material parameters identified by Sousa et al. [24] or Sousa [41], and considering the viscoelastic functions presented here, nomograms are constructed in the time and frequency domains.

3. Methodology for construction and evaluating of nomograms in time and frequency domains

For the frequency domain, the dynamic properties of the complex viscoelastic functions, at several temperatures, can be presented in a single graph, called a nomogram. Similarly, in the time domain, the nomograms can be constructed for viscoelastic functions like the relaxation in traction/shear modulus and the Poisson's ratio.

Table 1 schematically presents a method for constructing nomograms in frequency or in time domain. The methodology consists of a pseudo-algorithm that describes a computational procedure used for construct nomograms of viscoelastic functions step-by-step.

3.1. *Obtaining dynamical properties from nomograms*

Figure 2 presents a nomogram constructed at a reference temperature of 5 °C and includes a time interval from 0.1 s to a value around 2 years (ordinate axis on the right) for graphical analysis. The method for obtaining the dynamic property corresponding to the relaxation modulus is described in the pseudo-algorithm presented in Table 2. Similarly, others viscoelastic functions can be constructed and evaluated—defined in time or frequency domain—to determine their dynamic properties.

3.2. *Viscoelastic parameters*

The viscoelastic functions can be constructed by simply replacing the viscoelastic parameters in the models discussed in Section 2. The adopted viscoelastic parameters (Table 3) were determined by Sousa [41] through an optimization procedure employing a hybrid method that integrates Genetic Algorithms and nonlinear programming. This approach utilized experimental data from the EAR®-C1002 material, as reported by Jones [42]. The minimized objective function was the mean squared error between the empirical data and the theoretical fractional Zener viscoelastic models, characterized by four parameters. These models are formulated in the frequency domain and incorporate temperature effects.

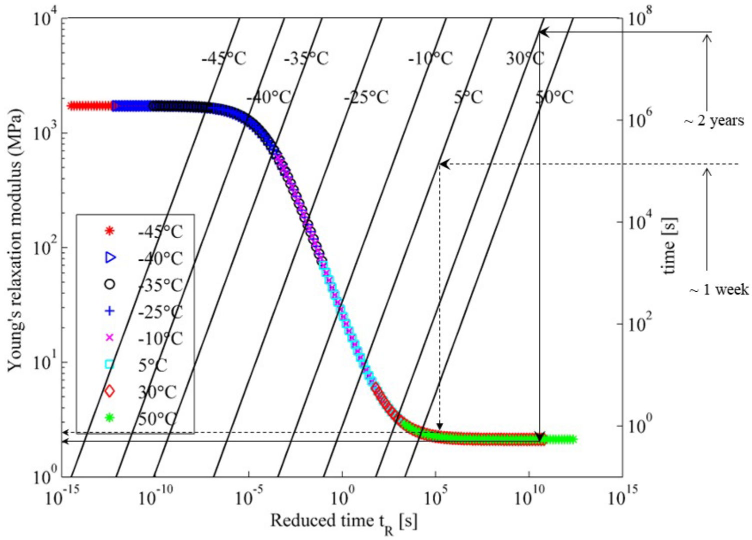

Figure 2. Nomogram in time domain for the Young's relaxation modulus.

Table 2. Pseudo-algorithm to obtain dynamic properties of VEMs, defined in time and frequency domains

Step 1:	Define the time and temperature to obtain the relaxation modulus (for example, 2 years and 30 °C).
Step 2:	From the specified time (e.g., 2 years), draw a horizontal line up to the desired temperature (e.g., 30 °C), as illustrated in Figure 2.
Step 3:	Draw a vertical line up to the relaxation curve, which corresponds the reference temperature.
Step 4:	Draw a horizontal line from the intersecting point of the relaxation curve up to the left ordinate axis.
Step 5:	Obtain the value of the relaxation modulus for the specified time and temperature.

Table 3. Viscoelastic parameters obtained from Sousa [41]

Viscoelastic parameters	Numerical values
E_0 (MPa)	$2.128667556644 \times 10^6$
E_∞ (MPa)	$1.723757249223 \times 10^9$
τ_E^β	$1.279369260811 \times 10^{-2}$
G_0 (MPa)	$7.119033966574 \times 10^5$
G_∞ (MPa)	$6.202192164498 \times 10^8$
τ_G^β	$1.189166029556 \times 10^{-2}$
β	$4.524176145415 \times 10^{-1}$
C_1^T	$1.433136320936 \times 10^1$
C_2^T (°C)	$1.031149458767 \times 10^2$

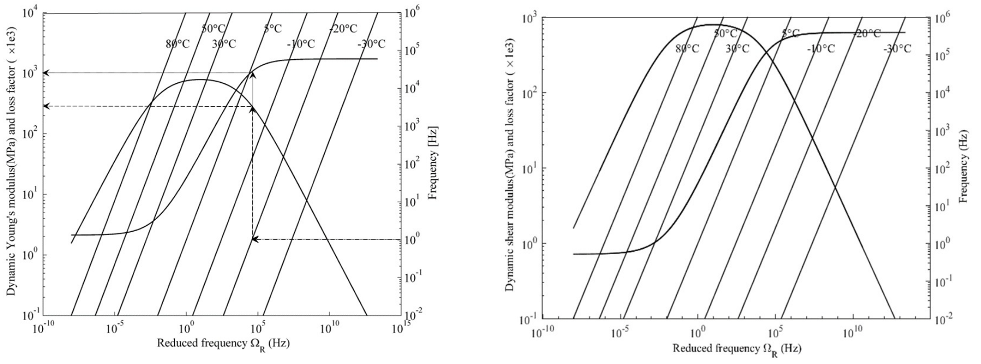

Figure 3. Nomogram of complex Young's (left) and shear (right) moduli.

4. Results and discussion

The current section presents the nomograms for the complex moduli (Young's and shear) and the proposed nomogram for the complex Poisson's ratio. Subsequently, the nomograms proposed for the relaxation moduli in tension and shear are shown, as well as the Poisson's ratio defined in the time domain.

4.1. *Nomograms of Young's and shear complex moduli, and complex Poisson's ratio: frequency domain*

Using the methodology presented for constructing nomograms and the mechanical properties (Table 3), nomograms are created for the complex viscoelastic functions (Young's and shear complex moduli, and complex Poisson's ratio). In all cases, the mechanical properties can be obtained over a frequency interval from 10^{-2} Hz to 10^{6} Hz for the following temperatures: −30 °C, −20 °C, −10 °C, 5 °C, 30 °C, 50 °C, and 80 °C. However, other temperatures can also be selected to be included in the nomogram.

Figure 3 presents the nomograms constructed for the Young's and shear complex moduli. In these graphs, the storage moduli and the loss factor are depicted together. It should be noted that the loss factor values shown in the graph need to be divided by 10^3. For instance, at a frequency of approximately 1 Hz and a temperature of −20 °C on the nomogram corresponding to the Young's complex modulus (Figure 3, left), the dashed line indicates a loss factor close to 0.1. Additionally, the value of Young's dynamic modulus is approximately 2000 MPa.

The complex Poisson's ratio can be obtained by inserting the parameters from Table 3 into Equation (16). Hence, Figure 4 proposes a nomogram for the complex Poisson's ratio, which presents its dynamic modulus and loss factor. These properties can be obtained in a similar manner. For example, at a frequency near 1 Hz and a temperature of 5 °C, the value of the dynamic modulus and that of the Poisson's loss factor are approximately 0.49 and 0.002, respectively.

The nomograms presented in this section establish a basis that can be used to quickly and approximately obtain the mechanical properties of VEMs. Furthermore, the methodology discussed in this paper enables the determination of the dynamic properties of the viscoelastic functions under analysis.

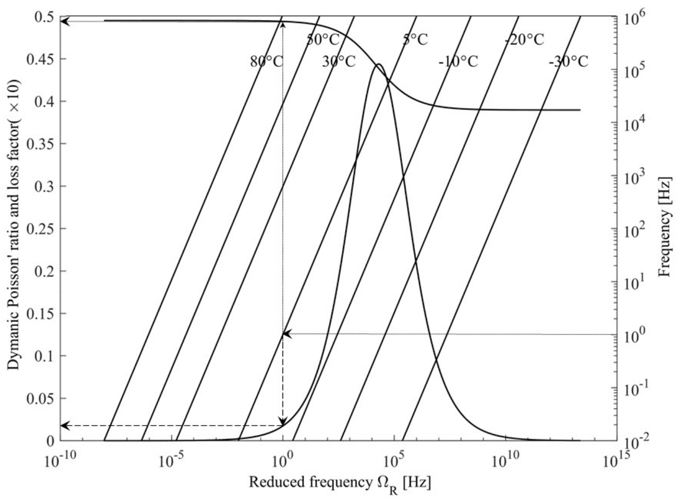

Figure 4. Nomograms of complex Poisson's ratio: dynamic modulus and its loss factor.

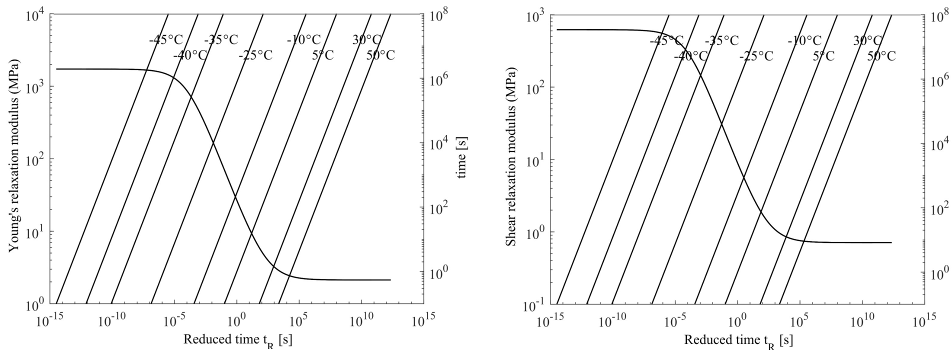

Figure 5. Nomograms in time domain: Young's (left) and shear (right) relaxation moduli.

4.2. *Nomogram of the shear traction relaxation modulus and the Poisson's ratio: Time domain*

Using the parameters presented in Table 3 to consolidate the viscoelastic functions for the time domain, nomograms are proposed for the traction and shear relaxation moduli, as shown in Figure 5. Subsequently, a nomogram for the Poisson's ratio in the time domain is presented in Figure 6. In all cases, these properties are observed over a time interval ranging from 1 s and 10^8 s (close to two years) at temperatures including: −45 °C, −40 °C, −35 °C, −25 °C, −10 °C, 5 °C, 30 °C, and 50 °C. However, other temperatures can also be selected for inclusion in the nomogram.

One observes that the graphs of the nomograms in the time domain are inverse forms of the corresponding nomograms in the frequency domain. For example, the relaxation modulus is a monotonically decreasing curve over time, whereas its counterpart in the frequency domain, the dynamic Young's modulus, exhibits an increasing trend with higher frequencies. Conversely, in the time domain, the Poisson's ratio shows a monotonic increase. However, the dynamic Poisson's modulus displays a decreasing trend as frequency increases. These results ensure a coherent physical interpretation of the material behavior [21, 41, 43], demonstrating the methodology's effectiveness and its potential for application across various engineering fields.

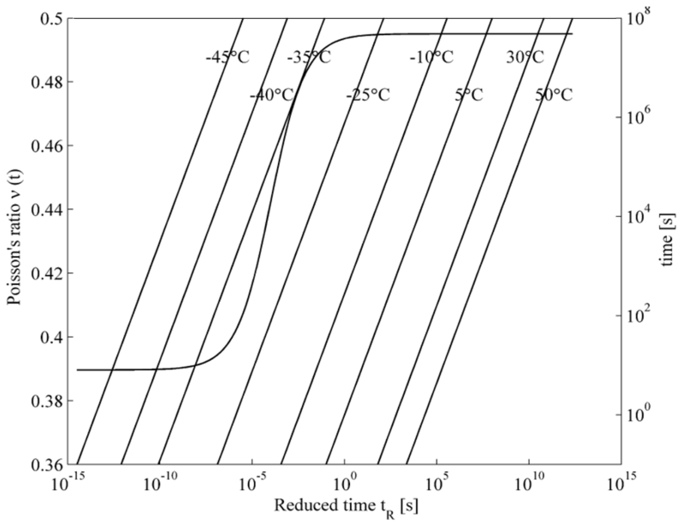

Figure 6. Nomogram of Poisson's ratio, defined in time domain.

5. Final remarks

The present work describes and implements a methodology for constructing viscoelastic functions in both the time and frequency domains. Initially, in the frequency domain, nomograms were presented for the complex Young's and shear moduli. Subsequently, a nomogram was proposed for the complex Poisson's ratio, incorporating two viscoelastic functions: the dynamic modulus and the Poisson's loss factor. These properties' values can be easily obtained for any desired frequency and temperature.

Nomograms of viscoelastic functions were proposed, defined in the time domain. Using the presented methodology, nomograms were created for the Young's and shear relaxation moduli, and subsequently, a nomogram for the Poisson's ratio was proposed in the time domain. The graphs obtained for these nomograms describe the inverse behavior of the corresponding nomograms in the frequency domain. These results ensure a coherent physical understanding of the material's behavior.

The use of a viscoelastic model based on a parametric constitutive model (here, the fractional Zener model) enables the derivation of two algebraic expressions for Poisson's ratio, one in the time-domain and the other in the frequency-domain. Accordingly, the methodology applied in this work is suitable whenever the constitutive model used allows the conversion between the time-frequency domains and the obtaining of the expression for the Poisson's ratio. This, in turn, enables the construction of a nomogram for this crucial material property.

An important point to highlight is that the proposed model can be applied in a finite element analysis involving VEMs and requires only the input of material properties. For finite element analyses in the time-domain, the mechanical properties of the medium are evaluated incrementally at each time interval. Thus, Poisson's ratio is updated at each time increment. For finite element analyses in the frequency-domain, such as harmonic analysis, the structure's response depends on the properties at each frequency. Therefore, the physical parameters (including Poisson's ratio) must be evaluated at each frequency value.

Note that in a time-domain FEM analysis, for very short times, Poisson's ratio has a value close to 0.39 (Figure 6). This corresponds to an analysis in the frequency-domain for very high frequencies (Figure 4). This behavior is close to that of a solid with low viscosity. On the other hand, for very long times, the value of Poisson's ratio is close to 0.49 (Figure 6). This behavior corresponds to an analysis in the frequency-domain for very low values of this variable (Figure 4), characterizing a behavior close to that of a nearly incompressible fluid.

As its main contribution to established knowledge, the present work proposes the construction of new nomograms of viscoelastic functions in the time and frequency domains, which allow for prompt and precise visualization of the dynamic behaviors of the VEM under analysis. Additionally, it is important to note that the methodology outlined for constructing nomograms can be extended to other viscoelastic functions (e.g., bulk modulus), various VEMs, and diverse combinations of constitutive models and shift factors.

Declaration of interests

The authors do not work for, advise, own shares in, or receive funds from any organization that could benefit from this article, and have declared no affiliations other than their research organizations.

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References

[1] R. S. Lakes, "Viscoelastic measurement techniques", *Rev. Sci. Instrum.* **75** (2004), pp. 797–810.

[2] J. Li, J. Zhang and S. Chen, "Study on dynamic viscoelastic properties and constitutive model of non-water reacted polyurethane grouting materials", *Measurement* **176** (2021), article no. 109115.

[3] J. J. Espíndola, C. A. Bavastri and E. M. O. Lopes, "Design of optimum systems of viscoelastic vibration absorbers for a given material based on the fractional calculus model", *J. Vib. Control* **14** (2006), pp. 1607–1630.

[4] T. L. Sousa, F. Kanke, J. T. Pereira and C. A. Bavastri, "Property identification of viscoelastic solid materials in nomograms using optimization techniques", *J. Theoret. Appl. Mech.* **55** (2017), pp. 1285–1297.

[5] F. O. Balbino, C. T. Preve, M. Munaro, P. J. Ribeiro Jr. and E. M. O. Lopes, "Wide estimation of dynamic properties of viscoelastic materials using bayesian inference", *J. Theoret. Appl. Mech.* **59** (2021), pp. 369–384.

[6] Z. Jingwei, L. Jia, H. Chuhao and C. Shuo, "Study on dynamic viscoelastic constitutive model of nonwater reacted polyurethane grouting materials based on DMA", *Rev. Adv. Mater. Sci.* **61** (2022), pp. 238–249.

[7] T. L. Sousa, J. Silva and J. T. Pereira, "Statistical analysis of identification of linear viscoelastic models", *Rheol. Acta* **63** (2024), pp. 301–318.

[8] S. W. Park and Y. R. Kim, "Fitting Prony-Series viscoelastic models with power-law presmoothing", *J. Mater. Civil Eng.* **13** (2001), pp. 26–32.

[9] I. Emri, R. Bernstorff, R. Cvelbar and A. Nikonov, "Re-examination of the approximate methods for interconversion between frequency- and time-dependent material functions", *J. Non-Newton. Fluid Mech.* **129** (2005), pp. 75–84.

[10] S. H. Chae, J. H. Zhao, D. R. Edwards and P. S. Ho, "Characterization of the viscoelasticity of molding compounds in the time domain", *J. Electron. Mater.* **39** (2010), pp. 419–425.

[11] M. Gergesova, B. Zupancic, I. Saprunov and I. Emri, "The closed form t-T-P shifting (CFS) algorithm", *Soc. Rheol.* **55** (2010), pp. 1–16.

[12] M. Gergesova, I. Saprunov and I. Emri, "Closed-form solution for horizontal and vertical shiftings of viscoelastic material functions in frequency domain", *J. Rheol. Acta* **55** (2016), pp. 351–364.

[13] J. G. Barruetabeña, F. Cortés, J. M. Abete, P. Fernandez, M. L. Lamena and A. F. Canteli, "Experimental characterization and modelization of the relaxation and complex moduli of a flexible adhesive", *Mater. Design* **32** (2011), pp. 2783–2796.

[14] Y. J. Lin, S. J. Hwang, H. H. Lee and D. Y. Huang, "Modeling of viscoelastic behavior of epoxy molding compound during and after curing", *Trans. Compon. Packag. Manuf. Technol.* **1** (2011), pp. 1755–1760.

[15] I. Saprunov, M. Gergesova and I. Emri, “Prediction of viscoelastic material functions from constant stress- or strain-rate experiments”, *Mech. Time-Dependent Mater.* **18** (2014), pp. 349–372.

[16] L. Rouleau, R. Pirk, B. Pluymers and W. Desmet, “Characterization and modeling of the viscoelastic behavior of a self-adhesive rubber using dynamic mechanical analysis tests”, *J. Aerosp. Technol. Manag.* **7** (2015), pp. 200–208.

[17] S. Liang, R. Luo and W. Luo, “Fractional differential constitutive model for linear viscoelasticity of asphalt and asphalt mastic”, *Constr. Build. Mater.* **306** (2021), article no. 124886.

[18] A. H. Tsou, J. Greener and G. D. Smith, “Stress relaxation of polymer films in bending”, *Polymer* **36** (1995), pp. 949–954.

[19] T. Pritz, “Frequency dependences of complex moduli and complex Poisson’s ratio of real solid materials”, *J. Sound Vib.* **214** (1998), pp. 83–104.

[20] T. Pritz, “Measurement methods of complex Poisson’s ratio of viscoelastic materials”, *Appl. Acoust.* **60** (2000), pp. 279–292.

[21] T. Pritz, “The Poisson’s loss factor of solid viscoelastic materials”, *J. Sound Vib.* **306** (2007), pp. 790–802.

[22] D. L. Chen, P. F. Yang and Y. S. Lai, “Interconversions between linear viscoelastic functions by using relaxation-creep duality representation”, *Math. Mech. Solids* **18** (2012), pp. 701–421.

[23] D. L. Chen, T. C. C. Chiu, P. F. Yang and S. R. Jiam, “Interconversions between linear viscoelastic functions with a time-dependent bulk modulus”, *Math. Mech. Solids* **23** (2018), no. 6, pp. 879–895.

[24] T. L. Sousa, J. Silva and J. T. Pereira, “Indirect Identification of the Complex Poisson’s Ratio in Fractional Viscoelasticity”, *Lat. Am. J. Solids Struct.* **15** (2018), no. 9, pp. 1–21.

[25] D. I. G. Jones, “Temperature-frequency dependence of dynamic properties of damping materials”, *J. Sound Vib.* **33** (1974), pp. 451–470.

[26] F. E. C. Silva and C. A. Bavastri, “A methodology for an optimal design of physical parameters, positions, and viscoelastic materials of simple dynamic absorbers for passive vibration control”, *J. Vib. Control* **25** (2019), pp. 1133–1147.

[27] S. O. Park, W. H. Choi and G. L. Park, “Dynamic response optimization of structures with viscoelastic material using the equivalent static loads method”, *J. Automob. Eng.* **235** (2021), pp. 589–603.

[28] C. T. Preve, F. O. Balbino, P. J. Ribeiro Jr. and E. M. O. Lopes, “On the use of viscoelastic materials characterized by Bayesian inference in vibration control”, *J. Theoret. Appl. Mech.* **59** (2021), pp. 385–399.

[29] F. Mainardi, *Fractional Calculus and Waves in Linear Viscoelasticity — An Introduction to Mathematical Models*, Imperial College Press, London, 2010.

[30] N. W. Tschoegl, *The Phenomenological Theory of Linear Viscoelastic Behavior: An Introduction*, Springer, New York, 1989.

[31] R. L. Bagley and J. Torvik, “On the fractional calculus model of viscoelastic behavior”, *J. Rheol.* **30** (1986), pp. 133–155.

[32] A. C. Galucio, J. F. Deu and R. Ohayon, “Finite element formulation of viscoelastic sandwich beams using fractional derivative operators”, *Comput. Mech.* **33** (2004), pp. 282–291.

[33] T. Pritz, “Five-parameter fractional derivative model for polymeric damping materials”, *J. Sound Vib.* **265** (2003), pp. 935–952.

[34] A. P. D. Ciniello, C. A. Bavastri and J. T. Pereira, “Identifying mechanical properties of viscoelastic materials in time domain using the fractional Zener model”, *Lat. Am. J. Solids Struct.* **14** (2016), pp. 131–152.

[35] J. D. Ferry, *Viscoelastic Properties of Polymers*, John Wiley & Sons, New York, 1980.

[36] A. D. Nashif, D. I. G. Jones and J. P. Henderson, *Vibration Damping*, John Wiley & Sons, New York, 1985.

[37] J. Capodagli and R. Lakes, “Isothermal viscoelastic properties of PMMA and LDPE over 11 decades of frequency and time: a test of time–temperature superposition”, *Rheol. Acta* **47** (2008), pp. 777–786.

[38] M. L. Williams, R. F. Landel and J. D. Ferry, “The temperature dependence of relaxation mechanisms in amorphous polymers and other glass-forming liquids”, *J. Am. Chem. Soc.* **14** (1955), pp. 3701–3707.

[39] S. W. Park and R. A. Schapery, “Methods of interconversion between linear viscoelastic material functions. Part I - A numerical method based on Prony series”, *Int. J. Solids Struct.* **36** (1999), pp. 1653–1675.

[40] F. Mainardi and G. Spada, “Creep, relaxation and viscosity properties for basic fractional models in rheology”, *Eur. Phys. J. Spec. Top.* **193** (2011), pp. 133–160.

[41] T. L. Sousa, *Indirect Identification of Viscoelastic Materials Properties in Time and Frequency Domains, Considering Temperature Effects*, Doctoral Thesis, Federal University of Paraná, Paraná, Brazil, 2018.

[42] D. Jones, *Results of a round-robin test program: complex modulus properties of a polymeric damping material*, 1992. USAF Report WL-TR-92-3104.

[43] N. W. Tschoegl, W. G. Knauss and I. Emri, “The effect of temperature and pressure on the mechanical properties of thermo–and/or piezorheologically simple polymeric materials in thermodynamic equilibrium - A critical review”, *Mech. Time-Dependent Mater.* **6** (2002), pp. 53–99.