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A recurrence formula in reciprocal frame structures: the Da Vinci bridge

Une formule de récurrence pour les structures réciproque : le pont de Léonard De Vinci

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Abstract. This article studies a reciprocal system referred to as the “Da Vinci bridge” and first proposed by Leonardo Da Vinci in the Codex Atlanticus written in 1495. The concept of reciprocal systems consists in transferring forces using planar or elongated elements in such a way to form a closed circuit. Reciprocal systems are usually built from the repetition of a building block. In the case of the Da Vinci bridge, the building block is an assembly of four beams. This article uncovers a recurrence formula linking the number of building blocks to the reaction forces on the most loaded beams of the bridge. It is interesting to note that the recurrence formula differs if the system consists in an even or odd number of building blocks. The various hypotheses of the strength of material computations that led to the recurrence formula are validated by comparison with an experimental realization. This studies shows that, when designing reciprocal systems, an optimization has to be carried out in order to chose between using long beams or using a large number of building elements.

Résumé. Cet article étudie un système réciproque appelé « pont de Léonard De Vinci », proposé pour la première fois par Léonard de Vinci dans le Codex Atlanticus rédigé en 1495. Le concept des systèmes réciproques consiste à transférer les forces à l'aide d'éléments plans ou allongés de manière à former un circuit fermé. Les systèmes réciproques sont généralement construits à partir de la répétition d'un élément de construction. Dans le cas du pont de Léonard De Vinci, l'élément de construction est un assemblage de quatre poutres. Cet article démontre une formule de récurrence reliant le nombre d'éléments de construction aux forces de réaction sur les poutres les plus sollicitées du pont. Il est intéressant de noter que la formule de récurrence diffère si le système est constitué d'un nombre pair ou impair d'éléments de construction. Les différentes hypothèses de calculs de résistance des matériaux ayant conduit à la formule de récurrence sont validées par comparaison avec une réalisation expérimentale. Cette étude montre que, lors de la conception de systhèmes réciproques, une optimisation doit être effectuée afin de choisir entre l'utilisation de poutres longues ou l'utilisation d'un grand nombre d'éléments de construction.

Keywords. Reciprocal, Strength of materials, Da Vinci, Bridge, Recurrence.

Mots-clés. Réciproque, Résistance des matériaux, De Vinci, Pont, Récurrence.

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1. Introduction

In 1495, Leonardo Da Vinci proposed an innovative bridge design grounded in the concept of reciprocity, or self-support. This particular design, illustrated in the Codex Atlanticus, as referenced by a drawing [1] reproduced in Figure 1a, has been extensively analyzed in engineering education settings and has been constructed in multiple locations around the world. Instances can be found either as decorations such as a sculpture in Freiburg, Germany, or as real bridges in Morsø, Denmark or in Vancouvert, Canada. Some companies propose to sell elements to build reduced toy models of this bridge. Example instructions to build this bridge are presented in Figure 1b. The so-constructed bridge presents an out-of-plane curvature due to the thickness of the beams composing it; were these beams fictively without thickness, the bridge would lay flat.

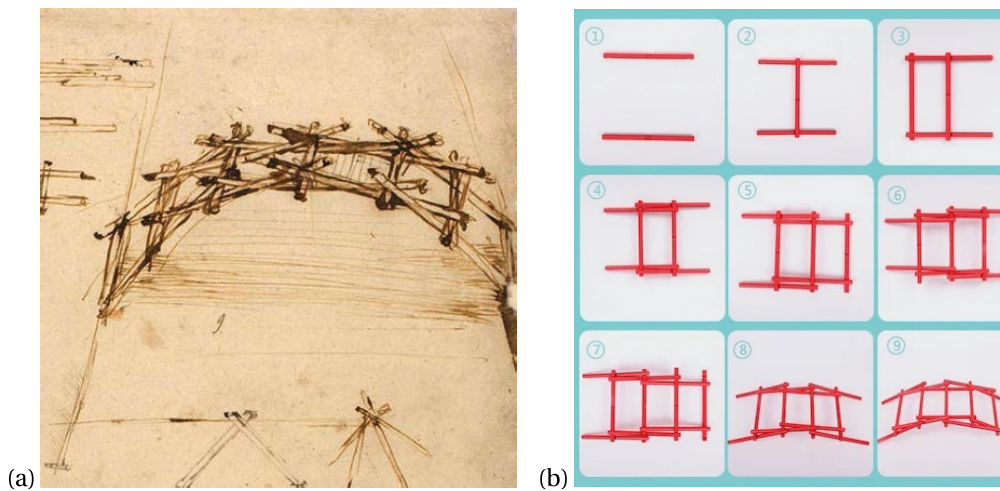


Figure 1. (a) Bridge proposed by Leonardo Da Vinci on p. 69 of the Codex Atlanticus. Image from [1] <https://codex-atlanticus.ambrosiana.it/>. (b) Da Vinci bridge construction instructions from <https://xtremeimports Kapit i.co.nz/products/da-vinci-bridge-3-pack>.

Reciprocal structures, also called self supporting structures, are defined by Olga Popovic Larsen, as “structures consisting of linear flat or inclined elements which support each other and are arranged in a way to form a closed circuit or unit” [2]. The closed circuit of the Da Vinci bridge consists of four beams that support each other as presented in Figure 2.

Reciprocal structures are usually built from the repetition of a building block. In the case of the Da Vinci bridge, the building block is an assembly of three beams composing a H shape (see subimage ② of Figure 1b). The reciprocal concept allows to span long distances with short elements [3] which is particularly convenient in bridge design, but most of all, it allows to build robust structures without using any assembling elements. An extensive review of reciprocal structures in the world is presented in the book [4] with a focus on their architectural use. The kinematic behavior of such structures has been studied by numerous researchers in the field of architecture or mechanics [5,6]. Software for reciprocal frame structures has been developed, drawing upon these articles, to ascertain the dimensions of the constituent elements necessary to achieve a specified three-dimensional geometry using reciprocal structures [6]. However, the mechanical behavior of reciprocal systems in terms of forces has been studied only in a very limited number of articles [7–9]. Among them, one could cite [8] which established an equivalent continuous medium able to capture the essential elastic properties of such structures.



Figure 2. Reciprocal unit circled in black on the simplest form of the Da Vinci bridge.

To the best of the authors' knowledge, no existing study has been conducted that allows the mechanical design of the structural elements constituting the bridge with respect to limit forces. Specifically, the authors raise a query concerning the closed reciprocal unit such as the one presented in Figure 2. Considering that the forces transferred from one element of the unit to another constitute a closed circuit, it appears intuitive that a recurrence formula could be derived, correlating the size of a reciprocal structure with the forces at the intersection of the elements. The objective of this article is to uncover this recurrence formula in the case of the Da Vinci bridge.

In Section 2, the closed form recurrence formula is established and the hypothesis for this computation are listed. In Section 3, based on this recurrence formula, design rules are set that predict the location of the maximum torque in the bridge thus identifying the critical beam. This section also presents an experimental validation of this prediction, thus validating the hypothesis listed in the previous section. Finally, a conclusion on the possibility to generalize these results is proposed.

2. Closed-form recurrence formula for the Da Vinci bridge

2.1. Model and hypothesis

The Da Vinci bridge is modeled as an assembly of beams and studied using strength of materials theory. Therefore, it is assumed that the beams are elongated and that the material stays in its linear elastic domain of behavior. The influence of the thickness of the beams on the out-of-plane orientation of the beams is neglected, along with its influence on the orientation of the reaction forces. As such, the reaction forces are supposed to all be aligned with the direction \vec{z} . The influence of these hypotheses will be assessed in Section 3.2 by comparison with experiments.

The Da Vinci bridge is studied for various bridge orders referred to by their number N of transversal beams. Transversal beams are the beam oriented along \vec{x} and colored in red. Bridges with different orders differ by the number of transversal beams that compose them while keeping the dimension of the composing beams constant. This means that different orders induce different bridge lengths in the longitudinal direction \vec{y} . Moreover, two important configurations are differentiated based on whether the bridge contains an even or odd number of transversal red beams. In odd cases, the load is applied as a point load to the center of the central transversal beam. In the even case, the load is evenly split in two and applied symmetrically to the center of

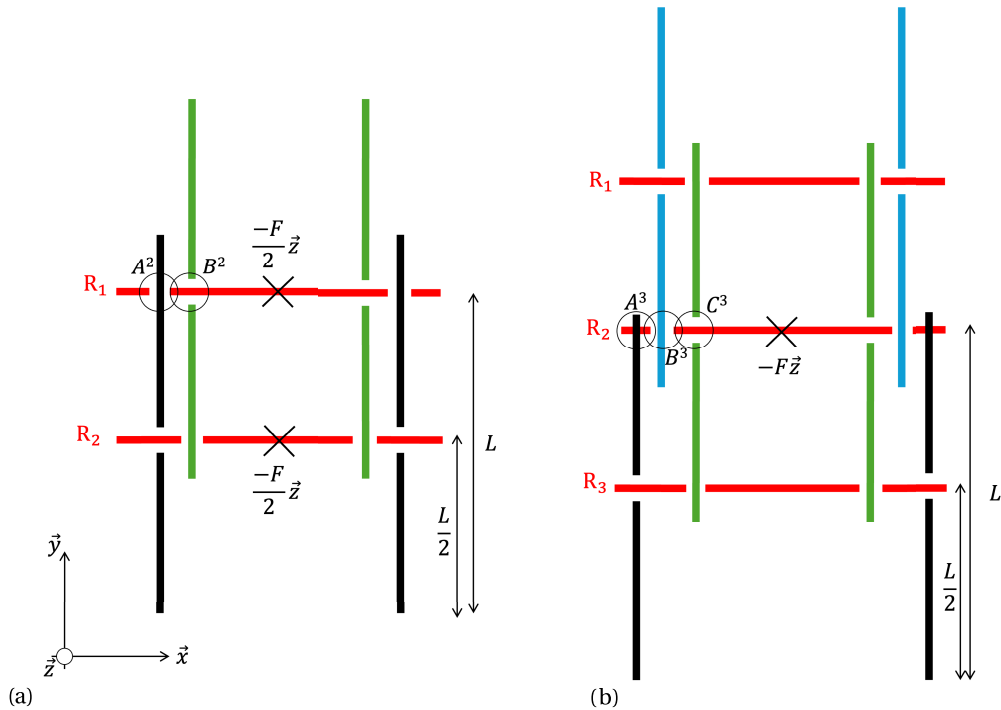


Figure 3. Top views of two studied bridges for the (a) even-order $N = 2$ and (b) odd-order $N = 3$ cases. Contact points on central transversal beams are indicated by circles and denoted A^N , B^N , C^N where N is the beam order.

the two transversal central beams. In both cases the load is considered to be applied downward in the out-of-plane direction, i.e. along $-\vec{z}$.

Top views of two bridges are presented in Figure 3 for the even-order $N = 2$ and odd-order $N = 3$ cases, from left to right. Thanks to the symmetry of both the structure and the loading, beams of similar colors are considered to be subjected to the same loading and thus have similar behavior. For the strength of material study, the spacing between the longitudinal green and black beams is neglected. Therefore, the structure can be considered to be symmetric with respect to both the $(O\vec{x}\vec{z})$ and the $(O\vec{y}\vec{z})$ planes passing through the center. Consequently, green and black beams in the even case and blue and black beams in the odd case are considered to be loaded similarly. The influence of this hypothesis will be assessed in Section 3.2 by comparison with experiments.

The reciprocal nature of the assembly is taken care of by considering unilateral contact forces, meaning that the direction of the forces are imposed based on the knowledge of the assembly: when a red beam is above a green beam, the load from red to green goes downward in the \vec{z} direction and vice-versa. In Figure 3, a beam passing under another one is represented as cut.

Finally, the transversal beams are supposed to be crossing the longitudinal beams at their centers.

2.2. Recurrence study

The present study is divided into 2 cases:

- (1) Structures with an even number of transversal beams, or even-order.
- (2) Structures with an odd number of transversal beams, or odd-order.

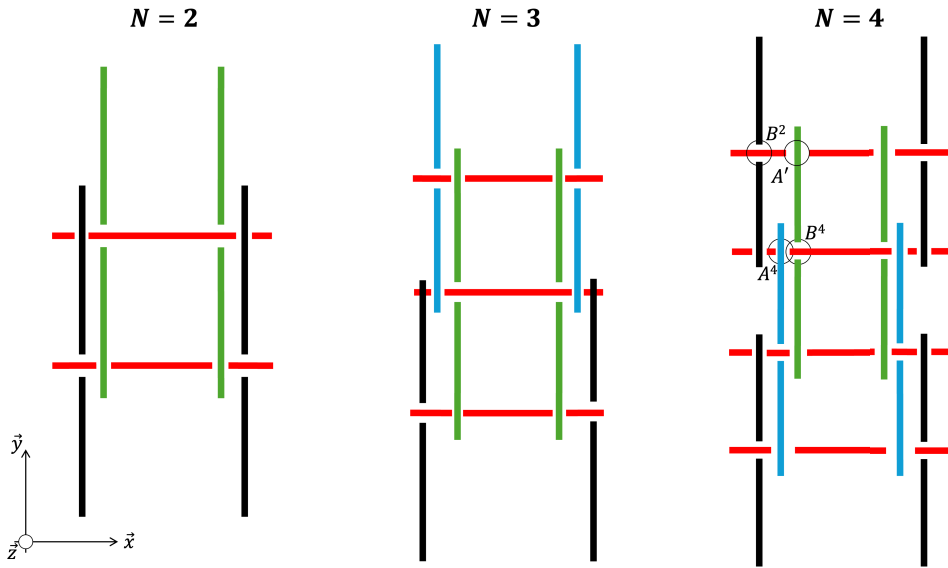


Figure 4. Example of three bridges configurations corresponding to various orders N .

As presented in the previous subsection, the study is carried out on bridges of increasing sizes indexed by their number N of transversal beams (or similarly their number of H-shaped building blocks) that will be called order from now on. Figure 4 presents three bridges structures with orders $N = 2$ to $N = 4$. It can be observed that the bridges are always constructed on the same manner, using the repetition of a H-shaped building block composed of one transversal and two longitudinal beams. The building blocks are then assembled by creating a top-bottom connection between the longitudinal and transversal beams.

The recurrence relation is established on the reaction forces A^N , B^N and eventually C^N applied to the central transversal beam, where the exponent N refers to the bridge order. The locations of these reaction forces are marked by circles in Figure 3 for the even-order $N = 2$ and odd-order $N = 3$ cases. For clarity and in anticipation of a recurrence proof, the reaction forces issued from the order $N = 2$ are indicated in the case $N = 4$ on Figure 4 with a subtlety on the reaction force denoted A' . This force is a modified A^2 reaction force which will be detailed in the coming subsection. Note that in the even-order cases the reaction forces applied to the other central transversal beam can be deduced by symmetry.

2.3. Results and recurrence formula

2.3.1. Even-order case

The full calculation is now detailed for the case $N = 2$. This case will serve as initialization of the recurrence proof.

As presented in the “model and hypothesis” Section 2.1, the structure has a symmetry plane ($O\vec{y}\vec{z}$) passing through the center. Moreover, the total load applied to the structure is a force of total magnitude F oriented along the negative vector \vec{z} . This force is separated into two forces evenly divided on the two central transversal beams. By neglecting the spacing between the longitudinal green and black beams, the fundamental principle of statics provides the four reaction forces applied by the ground to the bridge. These forces are all equal to $\vec{R}_g = (F/4)\vec{z}$.

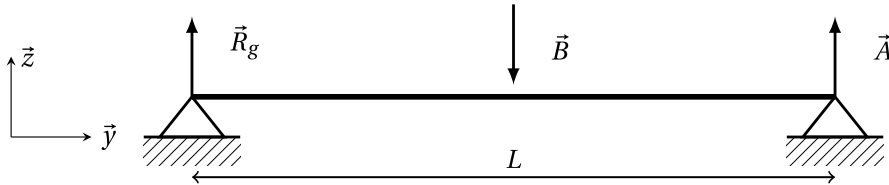


Figure 5. Free body diagram of one of the black beam in the case $N = 2$.

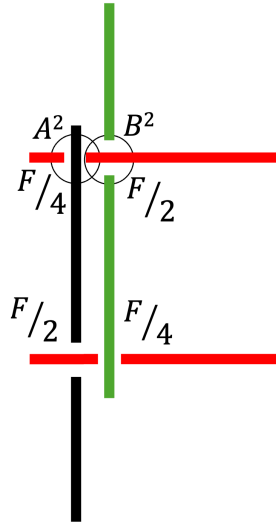


Figure 6. Reaction forces as function of applied force F on a bridge system constituted of $N = 2$ transversal beams.

One of the black beam is now isolated and its free body diagram is presented Figure 5. The black beam is on two supports, the ground and the R_1 red beam, while red beam R_2 applies a vertical force at the middle of the black beam which is then in a 3-point bending setting. Reaction forces from beams R_1 and R_2 are identified as forces A^2 and B^2 , respectively, corresponding to the forces at the contact points with similar names.

By application of the fundamental principle of statics in projection on \vec{z} and the static moment theorem at the point of contact with the ground, the reaction forces can be computed as follows:

$$\begin{cases} A^2 = \frac{F}{4} \\ B^2 = \frac{F}{2} \end{cases}$$

With this last computation, both reaction forces at contact points A^2 and B^2 are determined. These results are summed up on Figure 6.

Repeating this process on structures of $N = 4$ and $N = 6$ transversal red beams lead to the results gathered in Table 1. These results concern the reciprocal unit presented in Figure 6 that is located at the center of each bridge.

Table 1 presents a recurrence formula than is now proven taking the case $N = 2$ as initialization. When increasing the size of the beam going from order N to $N + 2$, the reaction force at contact point B^N is unchanged while the reaction force at contact point A^N is modified by the

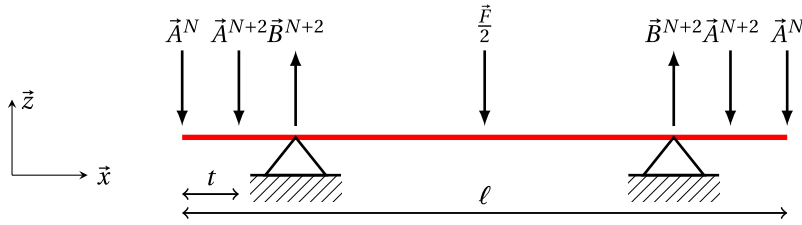


Figure 7. Free body diagram of one of the central transversal beams in any even-order case $N > 2$.

Table 1. Computation results of reaction forces on the central reciprocal unit along with established recurrence formula

	$N = 2$	$N = 4$	$N = 6$...	N
A^N	$\frac{F}{4}$	$\frac{F}{2}$	$\frac{3F}{4}$...	$\frac{FN}{8}$
B^N	$\frac{F}{2}$	F	$\frac{3F}{2}$...	$\frac{FN}{4}$

fact that the applied load $F/2$ is moved from this transversal beam to the new central transversal beam. Consequently, the new reaction force, noted A' is computed as $A' = A^N + F/4$ with symmetry arguments. The longitudinal beam where this reaction force A' acts is in a 3-point bending setting with central load B^{N+2} . Consequently, the reaction force B^{N+2} is computed as

$$B^{N+2} = 2A' = 2A^N + \frac{F}{2}.$$

Finally, the reaction force A^{N+2} can be computed from a fundamental principle of statics computation on the central transversal beam. The free body diagram of this beam is presented on Figure 7. This computation leads to

$$A^{N+2} = A^N + \frac{F}{4}.$$

Together with the initialization, these results yield the following recurrence formulae, also presented in Table 1:

$$A^N = \frac{FN}{8} \quad \text{and} \quad B^N = \frac{FN}{4}.$$

2.3.2. Odd-order case

The study of odd-order number of transversal beams is now presented. In this case, the load applied to the structure is a force of intensity F applied at the mid-span of the central transversal beam.

The calculation is detailed for the case $N = 3$ which serves as initialization for the recurrence proof. This case is presented in Figure 3b. In this case, longitudinal outmost black and blue beams are first isolated and the fundamental principle of statics gives the following results, where R_{Ri} is the reaction force at contact point between the longitudinal black beam and the transversal

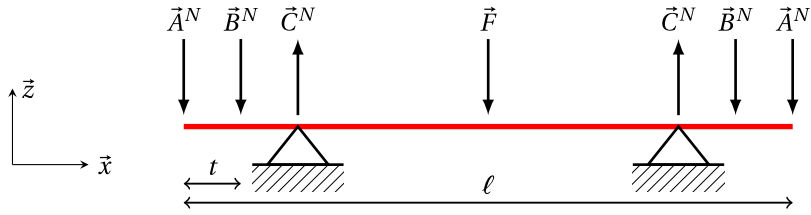


Figure 8. Free body diagram of one of the central transversal beams in any odd-order case $N \geq 3$.

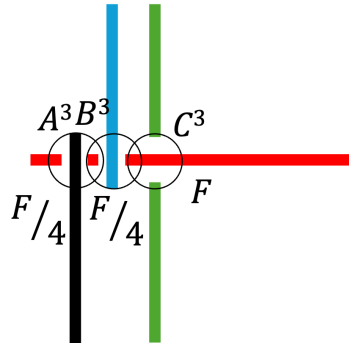


Figure 9. Reaction forces as function of applied force F on a bridge system constituted of $N = 3$ transversal beams.

red beam R_i . Note that, since the transversal beam R_2 is the central beam, R_{R_2} (resp. R_{R_2}) corresponds to the reaction force at contact point A^3 (resp. B^3).

$$\begin{cases} R_{R_2} = A^3 = \frac{F}{4} \\ R_{R_3} = \frac{F}{2} \\ R_{R_1} = \frac{F}{2} \\ R_{R_2} = B^3 = \frac{F}{4}. \end{cases}$$

The red central transversal beam R_2 can then be isolated. Its free body diagram is presented in Figure 8. Using reaction forces A^3 and B^3 , the fundamental principle of statics determines the last reaction force

$$C^3 = F.$$

With this last computation, all three reaction forces of the system are fully determined at the three contact points between the central transversal beam R_2 and the longitudinal beams. These results are summed up on Figure 9.

Repeating this process on structures of $N = 5$ and $N = 7$ transversal red beams lead to the results gathered in Table 2. These results concern the reaction forces presented in Figure 9 that are located on the central transversal beam of each bridge. The recurrence proof for the odd-order case stands on the same basis as for the even-order case. The reaction force B^N appearing on the $N + 2$ case is modified by the fact that the applied load F is moved to the new central transversal beam. The new reaction force B' is computed as $B' = B^N + F/2$. The longitudinal

Table 2. Computation results of reaction forces on the central reciprocal unit along with established recurrence formulae

	$N = 3$	$N = 5$	$N = 7$...	N
(1)	$\frac{F}{4}$	$\frac{F}{2}$	$\frac{3F}{4}$...	$F\frac{N-1}{8}$
(2)	$\frac{F}{4}$	$\frac{F}{2}$	$\frac{3F}{4}$...	$F\frac{N-1}{8}$
(3)	F	$\frac{3F}{2}$	$2F$...	$F\frac{N+1}{4}$

beam where this reaction force B' acts is again in a three-point bending setting with central load C^{N+2} . The, the reaction force C^{N+2} is computed as

$$C^{N+2} = 2B^{N+2} = 2B^N + F.$$

The two other longitudinal beams acting on the central transversal beam are in a three-point bending setting both with central load C^N . From this fact, the remaining reaction forces are computed as

$$A^{N+2} = B^{N+2} = \frac{C^N}{2}.$$

Altogether, these results yield the following recurrence formulae also referenced in Table 2:

$$A^N = B^N = F\frac{N-1}{8} \quad C^N = F\frac{N+1}{4}.$$

2.3.3. Verification of the recurrence formulae

All the recurrence formulae presented in Tables 1 and 2 have been verified by a full strength of material computation on two case studies of bridges of orders 15 and 16. The computations provided the following reaction forces of $A^{16} = 2F$ and $B^{16} = 4F$ in the case of the $N = 16$ bridge and $A^{15} = B^{15} = 7F/4$ and $C^{15} = 4F$ in the case of the $N = 15$ bridge. These results confirm the proposed recurrence formulae.

3. Design rules and experimental validations

3.1. Design rules

From the previously established recurrence formulae, it is possible to compute design rules for the Da Vinci bridge. Indeed, from the establishment of recurrence formulae, it appears that the most loaded beams in the Da Vinci bridge are the beams located around the center of the bridge. The longitudinal bars are always loaded in three-point bending while the transversal bars are in 7 point bending with varying forces orientations depending on the considered beam. Using this information, the recurrence formulae and basic strength of material, it is possible to draw diagrams to determine the most loaded bar of the whole bridge and thus design accordingly the Da Vinci Bridge.

Considering that the transversal and longitudinal beams can be of different lengths, denoted ℓ and L , respectively, and that the spacing of the longitudinal beams in the transverse \bar{x} direction is denoted t , one can define the following two non-dimensional quantities: the relative length ratio $\bar{\ell} = L/\ell$ and the spacing ratio $e = t/\ell$. From these quantities, phase diagrams can be computed for each bridge size N , stating whether the maximum torque is located in the longitudinal or the

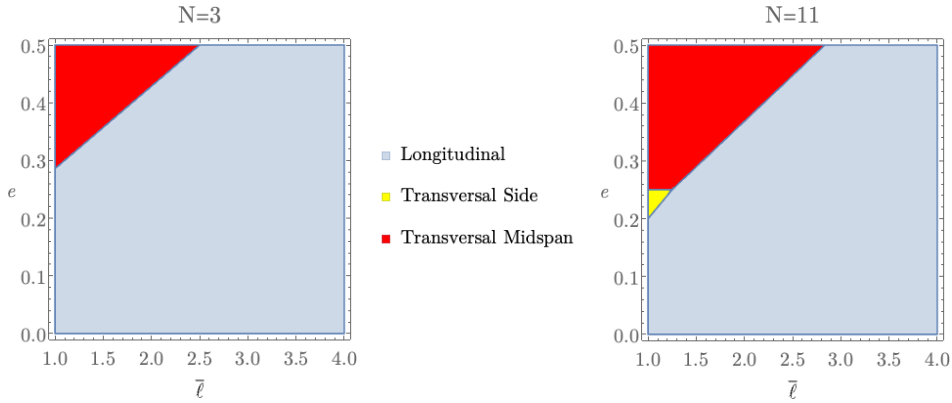


Figure 10. Phase diagrams indicating location of maximum torque for the odd cases $N = 3$ and $N = 11$.

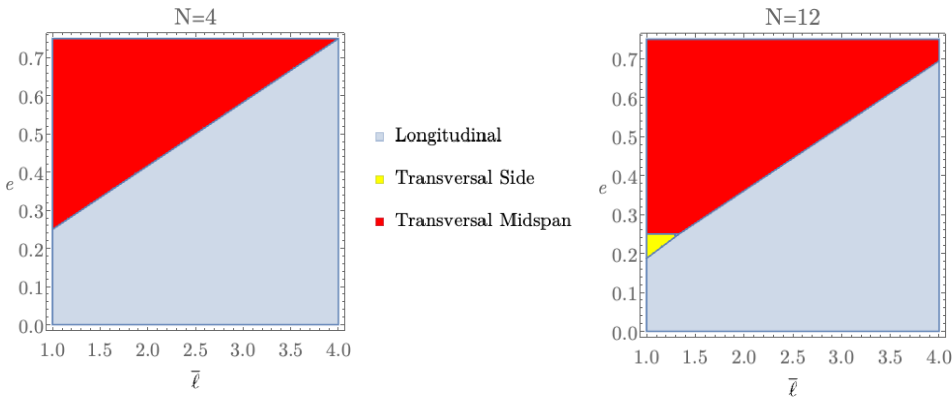


Figure 11. Phase diagrams indicating location of maximum torque for the even cases $N = 4$ and $N = 12$.

transversal beam, and where it is located in the transversal beam (namely at the midspan of the beam or on one of the side contact point). Phase diagrams differ for even-order and odd-order cases. Examples of such phase diagrams are displayed in Figures 10 and 11.

These diagrams are built from the following results:

- Maximum torque in the longitudinal beams for the odd case $M_{\max} = F(N + 1)L/16$ and the even case $M_{\max} = FNL/16$;
- Maximum torque on the side contact points of the transversal beams for the odd case $M_{\max} = 3F(N - 1)e/8$ and for the even case $M_{\max} = F(4 - 3N)e/8$ or $M_{\max} = F(2 - N)e/8$;
- Maximum torque at the midspan of the transversal beams for the odd case $M_{\max} = (F\ell/4) - (F(5 + 3N)e/8)$ and for the even case. $M_{\max} = F\ell/8$.

In the even case, the maximum torque differs on the side contact point depending on the considered central bar, hence the two possible expressions presented above.

An attentive reader would notice that the phase diagrams uncover the existence of a limit spacing ratio (ratio of spacing between longitudinal bars to the length of transversal bars) below which the maximum torque is always located in the longitudinal beam; and more precisely at the midspan of this beam. Moreover, it seems like the maximum torque can be located on the side contact point of the transversal bar only for bridges composed of enough reciprocal units.



Figure 12. Image of the test setup in the case of a bridge composed by square cross-section wooden beams.

3.2. *Experimental validation*

The presented computations have been validated by testing reduced size Da Vinci bridges built from circular cross-section wooden beams. These wooden beams have first been tested in three point-bending on an Instron test machine to determine their limit stress. The wooden beams have a cross-section diameter of 8 mm. The experiments have been carried out on five specimens leading to a mean limit stress $\sigma_l \approx 140$ MPa with standard deviation of 25 MPa. It is believed that the high standard deviation is due to the poor quality of the wood composing the beams.

The tested bridge was simply supported by two tables and a load was applied by means of a carabiner at the center of the bridge. The bridge structure was of order $N = 3$ (thus composed of 3 transversal beams). An image of the test setup is presented in Figure 12. Longitudinal beams had a length of 30 cm while transversal beams were 15 cm long, thus the relative length ratio was chosen to be $\bar{\ell} = L/\ell = 2$. As for the spacing between the longitudinal bars, it was minimal and thus corresponded to the diameter of the longitudinal beams cross section. Consequently, the spacing ratio was set to be $e = t/\ell \approx 0.053$. According to the phase diagram presented in Figure 10, these values predict a failure of the longitudinal beam at its midspan with a maximum force of 93 N corresponding to a total loading weight of 9.5 kg. In the experiments, a medium weight of 11 kg was used to reach the failure of one of the central longitudinal beam at its midspan, with a standard deviation of 1.5 kg. Both standard deviation of the structural and the material experiment are coincident which lead us to validate the predictions of the presented theoretical analysis.

4. Conclusion

In this paper, recurrence formulae are established that link the number of transversal beams of the classical Da Vinci bridge to the reaction forces on its central transversal beam. These recurrence formulae are established for two bridge configurations depending on the parity of the number of transversal beams composing the bridge. Using the recurrence formulae, phase diagrams are established that predict the location of the maximum torque in the beams

composing the bridge. These results are validated by experimental realization of a reduced wooden Da Vinci bridge composed of three transversal beams.

The main result of this article being the recurrence formulae, one could wonder if this recurring effect could be generalized to all reciprocal systems. Indeed, these recurrence formulae state that, in order to span a long distance and still hold a constant weight, an optimum must be found between using a large number of short longitudinal beams which would lead to an increase in the contact forces between the beams and using long longitudinal beams submitted to three point bending which could lead to large internal torque in these beams. Could this result be generalized to the design of a reciprocal floor? To which extent this result is limited by the use of strength of material hypothesis? These questions still remains unanswered to the authors and would merit further analysis probably including numerical simulations.

Declaration of interests

The authors do not work for, advise, own shares in, or receive funds from any organization that could benefit from this article, and have declared no affiliations other than their research organizations.

Dedication

The manuscript was written through contributions of all authors. All authors have given approval to the final version of the manuscript.

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