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Physical digital twins for ancient stone masonry informed of original construction techniques: the case of Sardinian nuraghi

Jumeaux numériques physiques de maçonnerie antique basés sur les techniques de construction originelles : le cas des nuraghi *sardes*

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Abstract. A short review is provided regarding modern technical tools allowing to build digital twins for modelling ancient stone masonry structures and their physical behaviour. The objective of such tools is to assess the structural safety of cultural heritage masonry structures. The present work focuses on the particular case of Sardinian nuraghi, which are ancient corbelled stone masonry structures whose typical form is a truncated cone. As a starting point we consider a careful historical analysis of the construction techniques of those nuraghi. From this analysis, we address the choice of theoretical and numerical tools apt to construct a digital twin of complex nuraghi, in addition to delineating future challenges in building digital twins capable of simulating any physical process which may be relevant to ancient buildings.

Résumé. Le présent article passe en revue les outils techniques modernes aptes à modéliser les structures antiques en maçonnerie et leur comportement physique. L'utilisation de ces outils vise à estimer la sûreté structurelle d'ouvrages en maçonnerie faisant partie du patrimoine culturel. Le présent article considère le cas particulier des *nuraghi* sardes : il s'agit de structures antiques en maçonnerie construites en encorbellement et présentant une forme typique de cône tronqué. Le point de départ consiste en une étude historique attentive des techniques de construction de ces structures. De cette étude sont ensuite déduits quels types d'outils théoriques et numériques semblent adaptés à la construction de jumeaux numériques pour des *nuraghi* complexes, ainsi que les défis à relever pour que de tels jumeaux puissent modéliser n'importe quel phénomène physique pouvant affecter ces structures antiques.

Keywords. Digital twins, ancient masonry, cultural heritage, nuraghi, multiphysical modeling, weathering.

Mots-clés. Jumeaux numériques, maçonneries antiques, patrimoine culturel, nuraghi, modélisation multiphysique, météorisation.

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1. Introduction

A general definition and methodology is presented as a framework for modelling historic structures and estimating their preservation state. More details about this issue can be found in specialised works (see for example Croci’s book [1] for a general review, or Amer et al. [2] for a complete case study).

Safety assessment of historic constructions consists in studying their structural stability in order to determine their preservation state and collapsing risks. More specifically, one may define three objectives: (i) to determine the present damage state of the studied structure; (ii) to understand the possible causes which have lead to the observed damage state; and finally (iii) to estimate its potential evolution.

Various means are available to reach these objectives; they are summarised below in two categories: data collection methods to investigate the past and present states of a structure, and physical-mathematical modelling methods to bridge the present state with its possible causes, as well as to predict potential evolutions.

Data collection methods allow to retrieve both qualitative and quantitative informations regarding the present state of a structure, as well as its history. A first source of information lies in available documentation regarding the studied structure, such as architectural drawings or historical records mentioning events involving the structure. More generally, studying such documentation allows to retrieve already-processed data regarding the structure and its environment. Field surveys are another way to collect data on a given structure. Contrarily to documentary studies, field surveys lead to generating new data on the present state of the structure or its environment. Laboratory tests are also a way to generating new data on the present state of the studied structure; however, they are more specific than field surveys. Indeed, the results of laboratory tests are quantitative values characterising the studied system, and those values are defined only within the framework of a specific physical-mathematical model.

The physical-mathematical modelling of a structure is a representation of its behaviour within a certain validity range, itself depending on a set of hypotheses regarding how to describe both the studied system and the phenomena affecting it. The objective of such a model is to bridge the documented and/or observed state of the system to its possible causes and potential evolutions. The modelling process may be summarised into several steps. The first one is choosing a set of hypotheses on which to build the model, as well as a modelling approach, based on the collected data. The hypotheses may regard which variables are to be used in order to describe the studied system, and which phenomena should be accounted for and how. The modelling approach includes which physical principle should drive the derivation of a mathematical problem representing the studied structure, as well as which types of study should be carried out. Then, available data should be integrated into this mathematical framework in order to bridge the physical system and the theoretical model. This can be made by estimating parameter values based on the surveys and laboratory tests from the data collection step. Finally, once the model is set up, theoretical results can be derived by solving the defined mathematical problem, analytically when possible, or numerically. However, the relevance of the model with respect to the study case must be assessed first by checking its validity for known problems. Note that, in the case of a numerical resolution, another modelling step is needed to evaluate the correspondance between the original analytical problem and the numerical problem which is actually solved. Afterwards, studies can be carried out to conclude on the present state of the structure and its potential evolutions.

Several aspects may be considered in order to evaluate the current damage state of a historical structure and its ability to resist present and future loads. Those aspects are complementary and each of them contributes to the final safety assessment. A brief presentation of those aspects is given in the following; more details may be found in specialised works, such as [1,3,4], for example. The historical approach to safety assessment relies on learning as much as possible about the past events involving the studied structure and its environment, in order to estimate their structural effects on the current damage state and potential future loads. This aspect relies heavily on the available historical documentation regarding the structure. The qualitative-empirical approach uses the current state of the structure as a guide to find the possible causes of the current damage state and their potential consequences. This approach relies on the observation of the current state of the structure, and the use of experience as an intuitive guide. The quantitative-analytical approach consists in building a model based on all acquired data and the phenomena which have been selected to be investigated. The results of this model are to be validated first by comparison with the current state of the structure, then used to predict potential evolutions of the system. This aspect is determined by the chosen physical hypotheses and principle, which in turn must take into account the survey and characterisation of the studied structure.

Keeping all the previous general considerations as a framework, the aim of the present work is to introduce a modelling approach for studying a specific type of historic masonry, as well as to identify available tools and methods which may be used. More specifically, Section 2 provides a short review of the physical modelling of masonry structures: what types of phenomena may be relevant to be accounted for, and which tools may be used to formulate them. Then, Section 3 focuses on the specific case of Sardinian nuraghi[1]: how they were built, what is their current state, and how those data may be taken into account if their structural modelling. Finally, the concluding remarks gathered in Section 4 summarise the main points of this study, along with directions towards future works on this matter.

2. Physical modelling of masonry structures

The specific characteristics of masonry structures lead to their specific behaviour and failure modes; this must be taken into account in the structural modelling of a historical masonry.

Since masonry is an assembly of units and mortar, an alteration of the physical properties (including non mechanical ones) of either of those constituents may have an impact on the mechanical behaviour of the structure. Moreover, the interaction between those constituents may also lead to other types of damage.

Historic structures have been interacting with their environment for typically extended periods of time. As a first consequence of this, weathering cannot be neglected since it can have structural effects through potentially complex and multi-physical damaging mechanisms. A second consequence is that, even regardless of weathering, the mechanical behaviour of a historic structure may change because of the long timescales involved: the same loads (constant or cyclic) may lead to local damage over time. A brief overview of physical phenomena potentially leading to damage within masonry units and/or mortar is given in the following; more details can be found in specialised texts (see for example the books [1,5,6]. One of the main physical damaging factors comes from water and humidity, not only because of direct mechanical erosion, but

[1]We use the Italian words: singular nuraghe, plural nuraghi.

also through the activation of salt diffusion and crystallisation within the masonry, leading to crystallisation damage. Indeed, when a salty solution flows into the masonry and dries up, the salts may crystallise into *florescences*, whether on visible external surfaces (*efflorescences*) or hidden within the masonry (*cryptoflorescences*). The latter may lead to internal damage by applying pressure to the surrounding material. This phenomenon may become cyclic if the environment is such that a wetting-drying cycle takes place, thus damaging the masonry progressively. It is also noteworthy to specify that direct wetting is not always necessary: so-called hygroscopic salts may take in enough water from relative humidity to activate their dissolution-crystallisation cycles. Less directly, water and humidity may also lead to modifications in the physical properties of the material itself, especially for mortar or masonry units made from raw clay or sedimentary rocks.

In a similar way, the ambient atmosphere content may also lead to erosion, as well as activating chemical corrosion through acid-base reactions or reduction-oxydation. Such phenomena may be worsened by air pollution, due to the consequent higher concentrations of acidic gases. The specific chemical species and reactions involved depend on the types of materials considered. One example which may be relevant to the study of historic masonries is the degradation of limestone by sulfur dioxide (SO_2). This gas can combine with water into sulfurous (H_2SO_3) or sulfuric (H_2SO_4) acids, which then corrodes limestone ($CaCO_3$) to form gypsum ($CaSO_4$,$2H_2O$). More examples may be found in specialised texts, such as [5].

Temperature, and especially its variations, may have a high impact on the physical properties of the materials, such as density or stiffness. An increase in temperature may also activate or accelerate chemical processes which may lead to damage, and periodic temperature changes may influence the already-mentioned wetting-drying cycles and the induced crystallisation damage. Those effects may be even greater if they are unevenly distributed throughout the structure, leading to high heterogeneities in the material properties. In a similar way, direct radiation from sunlight may also lead to uneven temperature changes, regarding both the surfaces of masonry structures and the ambient air. This may worsen the cyclic phenomena mentioned above.

Biological factors may also be taken into account: growing vegetation can lead to high direct mechanical effects, depending on the environment of the studied structure (see for example Croci's study of Angkor temples [7], where growing roots contributed to the unstability of the architectural remnants). Other biological factors may also activate or worsen physical (e.g. density modification, ion diffusion and crystallisation…) and chemical (acid-base or redox reactions) processes damaging the structure.

The difference in properties between mortar and units may lead to internal effects, possibly leading to or worsening internal damage. For example, under compressive loads, if the mortar's stiffness is too low, local tangential stresses may appear at the contact surface between unit and mortar, thus leading to internal tensile stress within the unit, even if the external load is compressive. Other damaging mechanisms due to internal interaction may also appear when one of the constituents has already been damaged. For example, an eroded surface can lead to increased ion diffusion and salt crystallisation within the structure; in reinforced masonry, the steel bars can also become corroded and weaken the masonry around the surface contact.

Research efforts have been made in the existing literature to review and classify masonry models (see for example [8,9]), depending on different criteria characterising the involved modelling method.

Masonry structures may be studied by separating them into distinct well-defined geometrical entities, or as a whole continuum. In the case of distinct geometrical elements, those elements may also be modelled as continua themselves. Thus, one may define three general types of kinematic descriptions: discrete, semi-discrete, and continuous ones. In discrete descriptions, a finite set of kinematic descriptors is considered sufficient to model the studied masonry system.

Such a description may be relevant for models where the bricks are considered rigid with respect to the mortar [10,11], and allows to model precisely the geometry of the studied masonry. In semi-discrete descriptions, the system is described by assembling a finite number of continua. This approach allows a detailed geometrical description of the position of relevant geometrical elements, while allowing a continuous description of their deformation. The involved elements may be bricks and mortar [12,13], or macro-elements [14] identified as relevant to the studied system and behaviours. In fully continuous descriptions, the whole studied system is represented by a continuum model. This approach may be relevant for studying the global behaviour of an entire masonry structure [15,16] while reducing the computational cost which would be needed for a more detailed description.

Depending on the aims of a study, time and inertia may or may not be taken into account when studying masonry structures. In static analyses, time is not considered, and only the equilibrium configuration of the studied system is investigated. This kind of analysis may be used to verify the minimal conditions for a structure to hold, independently from its history [17,18]. When considering quasi-static evolution, the history of the behaviour of the studied system is investigated, while assuming that this behaviour remains slow enough not to involve inertial effects. In dynamic analyses, the time evolution of the system is studied, including inertial effects. This type of study is relevant when studying the vibrational behaviour of masonry structures, for example for seismic applications [19].

Variational principles have been used to drive the definition of physical models and their mathematical formulation for complex systems. Note that, here, the expression "variational principle" is intended not only from a mathematical point of view (i.e. minimisation problems and/or weak forms depending on test functions), but also from a physical one (i.e. physical principles where the modelled phenomena are characterised by their reaction to virtual perturbations). A general review of this double aspect may be found for example in [20,21]; see also [22–24] for more specific details on the principle of virtual power. Those variational principles have been used to model complex multiscale systems, such as bones [25–28]. Variational principles have also lead to systematic modelling methods allowing to build and compute generalised models, such as higher-gradient continua [29,30] and micromorphic continua [31,32]. In turn, those generalised theories have been used to model various types of phenomena [33,34], as well as complex systems whose behaviour cannot be modelled with classical approaches (see for example [35, Figure 3] for an explicit comparison of validity ranges). In order to build and identify such generalised models, one may use homogenisation methods, which allows to build effective continuum models by bridging simple models at local scale (e.g. Lagragian discrete description or Cauchy continuum) and generalised models at global scale (higher-gradient and/or micromorphic continua) [36–40]. In particular, there have been many applications to metamaterials, i.e. synthetic materials which have been designed to have specific microstructures determining their behaviour (see for example [41] for a review about mechanical metamaterials, [42–46] for an example modelled by second-gradient continua, and [47] for third gradient), ranging from analytical results ([48–52]) to additive-manufacturing aspects [53,54]. Generalised models have also been shown to allow a finer modelling of damaging phenomena [55–58]. Another notable application is granular micromechanics: recent research efforts have been made to build generalised continuum models based on local interactions between grain pairs [59–62], with applications to cementitious materials [63,64], chiral materials [65], and masonry structures [66–68]. Such generalised models also require generalised computing methods; for example, McAvoy and Barchiesi present an adaptation of the finite element method to a novel tetraskelion metamaterial in [69], while Turco et al. provide a comparative study of computational methods based on discrete and continuum models in [70,71]. Other innovative computational methods have also been studied by using swarm robot dynamics [72], applied to both continuum [73,74] and discrete [75] systems.

The concept of digital twins has been used progressively more in the last few decades, both in industry and academia. Zhou et al. [76] reviewed the history of this and related concepts, and provided a definition of its main characteristics. According to them, the first use of the expression "digital twin" was by Hernández in 1997 [77] regarding a three-dimensional digital model of several examples of civil engineering structures. One of the characteristic ideas of digital twins is the emphasis on the real-time aspect: it is a modelling and simulation environment, built not only on a certain knowledge of the studied system, but also on information communication technologies allowing to measure its state in real-time and visualise it in a virtual environment. This characterisation of digital twins sets them apart from other related concepts, such as building information modelling (BIM) which focuses on design efficiency an maintenance planning instead of real-time states.

The following paragraphs illustrate the modelling approaches previously presented and general methodologies from Section 1 with examples from the recent literature. Two types of works are considered: reviews regarding methods and means for the structural stability assessment of historical masonry, and case studies focusing on specific monuments.

Works such as those from Lourenço [3] and Roca [4] provide general guidelines regarding the assessment and restoration of historic masonry structures. This includes step-by-step methodologies, as well as tools which may be used for data collection or structural modelling. More details on the same subjects may be found in books, such as those from Croci [1] and Feilden [6]. Other works also give a more detailed review of specific subjects. Latifi et al. [78] present diagnostics methods for detecting cracks in historic masonry structures, including automatic procedures based on machine learning and deep learning. They also review common crack patterns in historic masonry and strengthening techniques, both traditional and modern. The static and kinematic principles of limit analysis are also presented for modelling masonry in a rigid, no-tension framework. In a similar way, Valluzzi [79] reviews common vulnerabilities of historic masonry structures and methodologies for their survey, modelling and mitigation. Castellazzi et al. [80] provides a review which is more focused on the data collection aspect: they present how building information modelling (BIM) may improve the data collection step, in a way which can also be used for numerical analysis.

Amer et al. [2] give an example of a complete structural assessment methodology for historic masonry structures. All steps are presented: data collection through documentation, field surveys and laboratory experiments; numerical modelling using three-dimensional finite element method in a macro-modelling approach; static and seismic analyses based on all gathered data, providing a better understanding of the current damage state and an estimation of structural stability. Mirtaheri et al. [81] provides another example of complete study: they present a methodology for the seismic protection of a historic minaret. The data collection step is based on geometrical survey, laboratory experiments to characterise the materials' mechanical behaviour, and on-site experiments to estimate the natural frequency of the structure. Those data are then taken as parameters in a finite element model (Abaqus software) returning a numerical estimation of the natural frequency. This value is then compared to the experimental one, and this procedure is used to study several reinforcement options. As another example, D'Altri et al. [82] use 3D surveys and numerical modelling for the safety assessment of leaning masonry towers. More specifically, they present an automatic procedure allowing to manage both geometrial survey and numerical analysis in an economical way, through an algorithm which identifies which parts of the studied structure require heavier calculations. Croci, in a short paper from 2001 [7], presents a methodology for the structural stabilisation of Angkor temples. He gives an overview of the structural state and damages before the intervention, and describes the monitoring of soil movements prior to stabilisation measures. In particular, this work gives an example of how much a historic structure may be damaged by vegetation and weathering.

3. The specific case of nuraghi

The aim of the present work is to introduce an adaptation of the previous general guidelines to build a physical digital twin of Nuragic structures. A flowchart of the intended general process is given in Figure 1: starting from a real structure, two ingredients are necessary to build a physical digital twin, namely a precise representation of its geometry, and a mechanical model of its behaviour. Once the digital twin is assembled from those two elements, it must first be validated with experimental data from the real system. After the validity range of the digital twin has been ascertained, it may be used to study the present state of the system and predict its evolution.

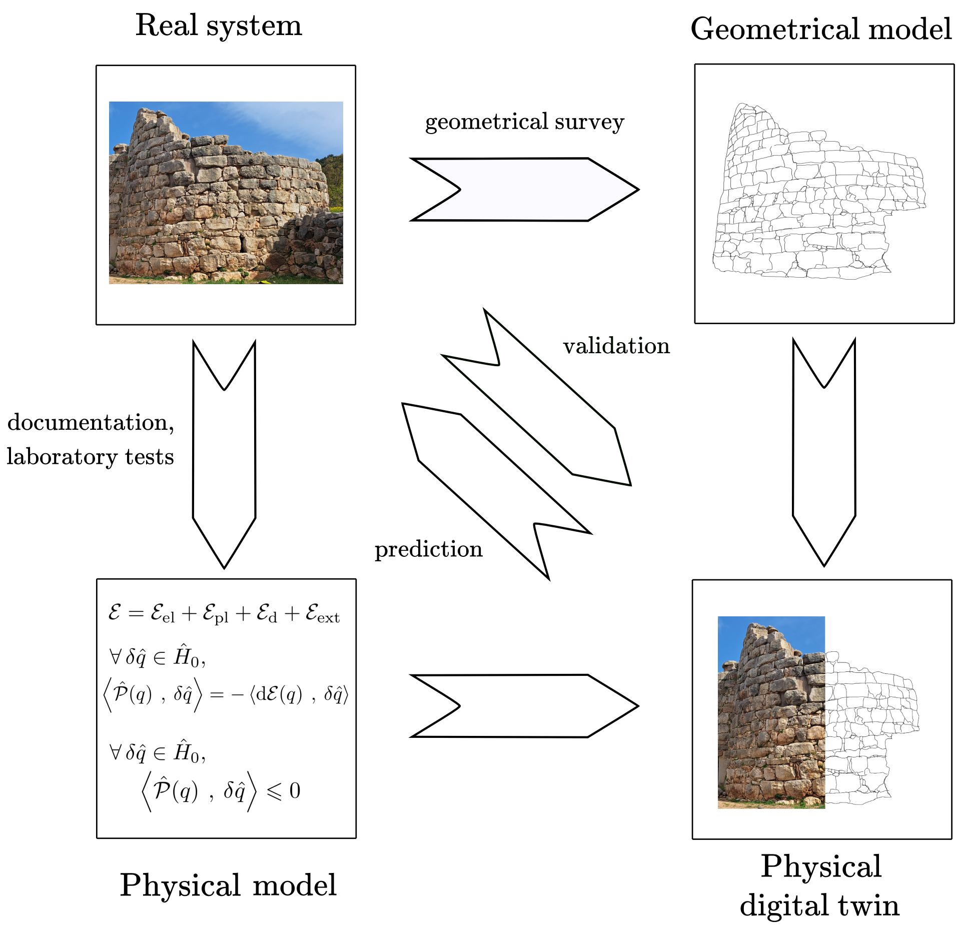


Figure 1. Building process of the physical digital twin of a real system based on a survey of its geometry and a modelling of its behaviour.

As a preliminary step, the rest of this section aims at identifying the main characteristics of Nuragic structures (Section 3.1), especially regarding how they were built (Section 3.2), in order to propose modelling approaches (Section 3.3) to be used in upcoming structural models for Nuragic structures.

3.1. *Main characteristics*

The spread and number of nuraghi in Sardinia does not allow an easy synthesis of building typologies. The presence of more than 8,000 monuments and their diffusion in a geographical context of more than 24,000 square kilometres makes the amount of data extremely heterogeneous, with several variables to be taken into account and different influences that may have played a role in the dynamics of planning and construction of each nuraghe. Therefore, presenting architectural types and building techniques from the Nuragic period in a synthetic presentation is not simple, although a long tradition of study and the extensive bibliography on the subject allows us to outline some main features (Lilliu 1962 [83]; Lilliu 1963 [84, pp. 136–232]; Lilliu 1982 [85]; Contu 1985 [86]; Blake 1998 [87]; Ugas 2005 [88, pp. 35–44]; Depalmas 2009 [89–91], 2015 [92]; Tanda 2015 [93]; Melis 2017 [94]; Depalmas 2018 [95]).

First, regarding the architectural model, two main types can be distinguished on the basis of morphological and chronological criteria. On the one hand, the so-called *protonuraghe*, an older example than the classic and better known architectural model (2nd millennium BC). It is characterized by a rather simple construction system, which shows a certain variety of planimetric forms that often feature an inner corridor and side rooms (Webster 1996 [96, pp. 62–84]; Melis 2003 [97, pp. 9–10]; Ugas 2005 [88, pp. 70–81]; Spanedda and Camara Serrano 2009 [98, p. 671]; Moravetti 2015 [99]; Depalmas 2015 [92, p. 76]; Depalmas 2018 [95]). They are less numerous and their territorial distribution is mainly focused in the central part of the island and in low-lying areas (Moravetti 2017 [100, pp. 11, 24]) (Figure 2).

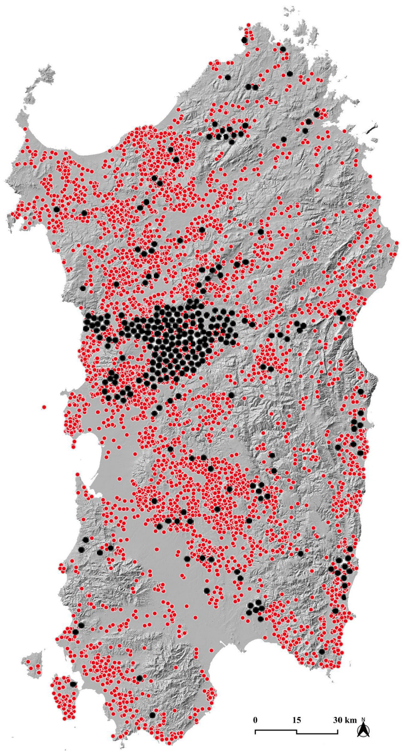


Figure 2. Distribution map (base DTM, Regione Sardegna) of the so-called protonuraghi (in black) and nuraghi (in red).

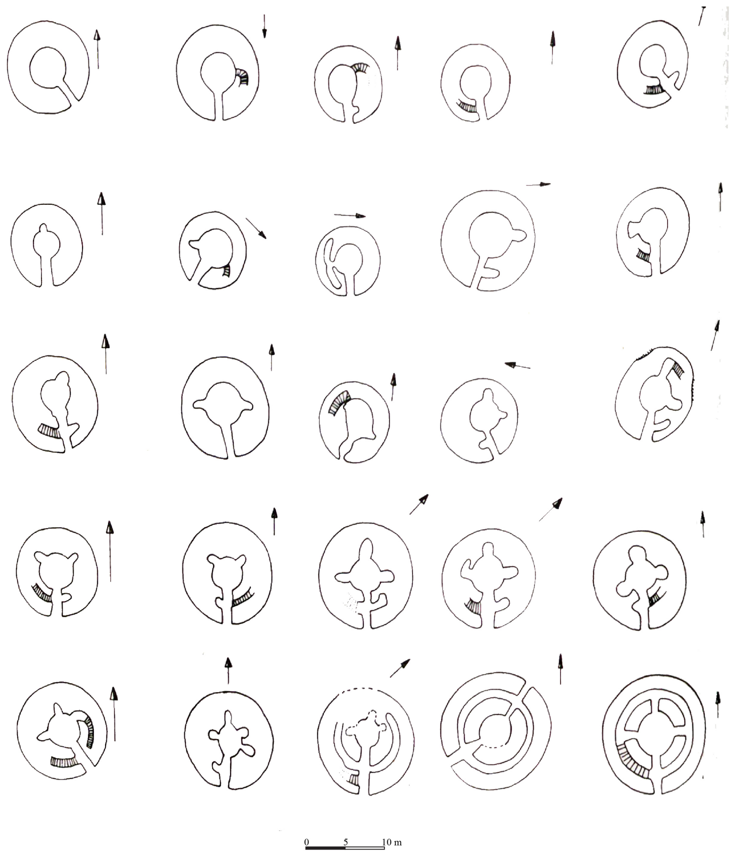


Figure 3. Different plans of *tholos* nuraghi (adapted from Lilliu [83]).

More common are the *tholos*[2] nuraghi (17th-14th BC), which can be recognized by the classic tower shape and the short entrance corridor leading to a circular chamber that organizes the interior space (Figure 3). In this case, the variety is mainly related to the alteration of the basic module (Melis 2017 [94, p. 29]). In the case of complex nuraghi, systems of rectilinear and curvilinear walls may be added as a connection to the circular towers, in order to define polygonal patterns around the central structure (Contu et al. 2004 [101, pp. 383–388]; Depalmas 2018 [95, pp. 57–58]). Regarding the organization of the interior space, the most common modular layout includes an entrance (whose closing system remains unclear), a vestibule, a central room, a stairwell to access the upper part of the building and the upper rooms (Melis 2017 [94, p. 32]).

The vestibule leads to the central room. Usually, it is built beside the tower with greater wall thickness. Its roof may feature a sequence of transverse slabs or an overhang with an ogival cut, which creates a progression in height of the corridor up to the entrance of the central room (Figure 4). In most Nuragic structures, the staircase and a niche were located on either side of the vestibule. If the staircase was not open on the side of the entrance corridor, it was usually located inside the room, spiralling into the thickness of the masonry, and featuring a varying inclination and path length. Sometimes, at the end of the short corridor, there were small, narrow openings to the outside which might open directly into the room.

[2]We use the Greek word: singular *tholos*, plural *tholoi*.

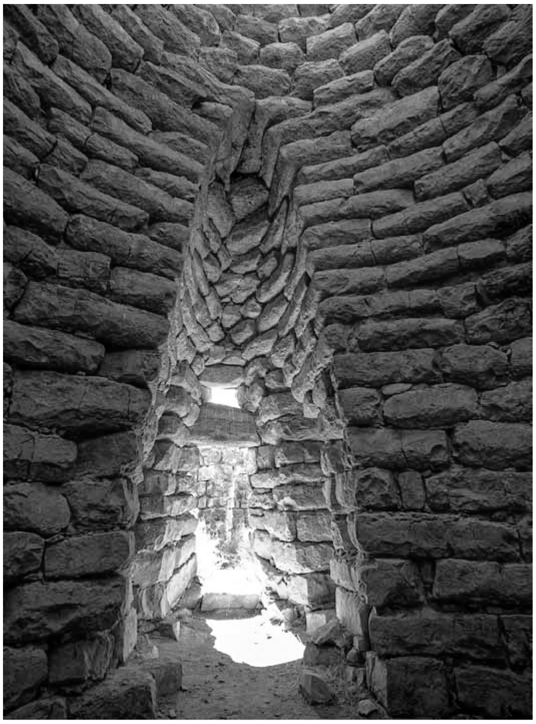


Figure 4. Example of a nuraghe entrance and vestibule (view from inside the *tholos*) (adapted from Melis 2017 [94]).

The main central chamber usually has a circular floor plan with a diameter varying between 4 and 5 metres (sometimes up to 7 metres) for an overall height of about 12 metres. In most cases, on its sides there are smaller rooms whose number may vary from 1 to 4. Considering the overall height, it is possible that within the *tholos* there were overlapping and independent rooms, separated by wooden floors: it has been hypothesized that these floors could be supported by trunks inserted between the gaps in the masonry, by pillars or by additional wooden supports (Melis 2017 [94, p. 38]). The access to the upper chambers was provided by a stairwell that could be opened at the entrance or directly in the central room (see Figure 3). Finally, some smaller and multifunctional rooms could have been located within the *tholos*.

3.2. *Construction and structural aspects of nuraghi*

A first reflection arises on the constructive characteristics of the nuraghi and concerns their structural longevity. Beyond the legislative solutions and the resources allocated to their protection, the high number of Nuragic towers still in good condition leads to think about the guarantees offered by the architectural structure itself. The structural principle is that of *thrust-free* systems, which are capable of transferring weights and stresses to the ground along resulting forces close to the vertical axes. The formal concept recalls that of pseudo-domes. The structural system of nuraghi is achieved through polygonal blocks which overlap in concentric circles, according to a decreasing diameter: each block overhangs the one below, eventually forming a closure of the structure with a single block (Figure 5). Concerning the masonry, the main characteristic is the absence of binding material, with the exception of a few cases in which the presence of mud could be noted (Melis 2017 [94, p. 29]). A specific attention has been paid to the placement of

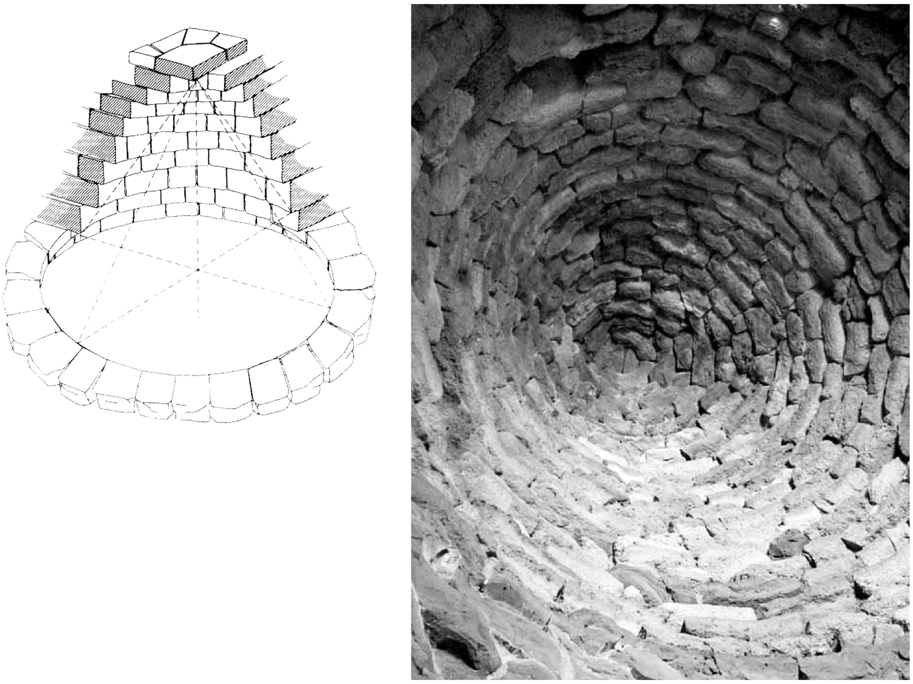


Figure 5. Roofing system of the central chamber of the *tholos* (adapted from Melis 2017 [94]). On the right, example of a structure with a pseudo-dome roof (adapted from Giuliani [102]).

the elements within the structure: larger ones (with a higher specific weight) were placed at the lower parts. Gradually, the lighter blocks were placed up to the top of the towers. These elements show a certain level of workmanship, which usually distinguishes nuraghi from protonuraghi: the "tail-like" or "T" shape successfully ensures a greater solidity of the masonry fabric and increased stability to the structure.

Regarding the foundation, it rests directly on the rock bed, on a leveled earth surface, or on artificial platforms created to level the foundation plane. A real foundation structure has been discovered just in few contexts, as Santu Antine and Terralba (Depalmas 2018 [95, p. 56]). Once the main elements are in place, variously sized stones were added as further leveling of the laying surfaces, as well as to facilitate the structural connection between the individual elements and prevent potential sliding phenomena.

All these choices have a static reason and are easily shared. On the one hand, the larger elements contribute to forming a solid foundation, while on the other hand, the Nuragic builders must have also considered the difficulties associated with the processing, transport, and lifting of the blocks. Unfortunately, the frequent collapse of the upper part prevents a clear understanding of its articulation, although the discovery of stone brackets suggests the presence of a wood balcony extending over the edge of the upper terrace. The lack of intact structures also prevents determining the height of the towers, which can only be hypothesized. Considering the base diameter of the most imposing buildings, which could reach 15 meters (in the *tholoi* or in the central towers of complex nuraghi it is around 10 meters), it is reasonable to assume that the Nuragic towers could have risen to almost 20 meters in height. Today, the tallest nuraghe is that of Santu Antine (Torralba), which reaches 18 meters in height, but even in this case, it is hypothesized that it could have reached a higher level (Melis 2017 [94, p. 31]).

Overall, the statics and mechanical resistance of the masonry rely exclusively on the mass and weight of the elements. The shape of the nuraghe and the masonry structure ensure the solidity of the entire building. The shape of the tower provides a masonry with inclination of about 10°, which reduces the risk of overturning and ensures a more balanced distribution of the weight (Melis 2017 [94, p. 29]).

It is evident how much the weight of the Nuragic structure influenced the design and construction phases. It is no coincidence that the only discontinuity in the masonry façade is related to the structure's entrance. In a way, the need to discharge the weight of the masonry along vertical axes imposed the use of a lintel system, which is composed of two vertical elements supporting an overlying beam. Although the mass supported by the horizontal element can be limited to that carried by an imaginary equilateral triangle with the length of the lintel as its base, the high bending stress must have required particular care in assessing the size of the openings, suggesting the creation of entrances with limited width. Sometimes, the trilithic system of the nuraghi is further assisted by a small opening made at the level of the upper row, serving as a relieving window.

The difficulties of extracting building materials, as well as those directly related to the elements transportation, should not be overlooked. Moreover, due to the stresses that the structure had to bear, even the choice of building material became crucial for the success of the project. For this reason, the extent and methods of distribution of the nuraghi across the island can be seen as an indication of a deep understanding of the soil characteristics, which was probably critical to the success of the architectural project. Despite a generally widespread distribution, the highest density on the island is recorded in the Marghine region, in the central-western part of the island. Here, the geological map identifies rock beds mainly composed of volcanic rocks, specifically basaltic rocks, which have a high degree of resistance and ideal physical-mechanical properties for heavy structures subjected to compression (Figure 6).

The already-mentioned work from Cavanagh and Laxton [103] provides an overview of the structural features of nuraghi, before focusing on a stability analysis of the corbelling vault of several specific nuraghi. Their model considers the studied systems as dry masonry bearing only compressive interactions; as a consequence, the stability of the corbelled vaults depends highly on its geometry.

Another example of stability assessment is given by Roberti and Spina [104]. Focusing on one specific nuraghe, they estimate the stability of two sub-systems modelled through discrete element method (DEM): a portion of a wall with an opening, and a section of a corbelled vault. Their model is dynamic: for a given load, they verify whether the studied system reaches static equilibrium or starts collapsing.

A lot of research effort has also been made regarding 3D recording techniques for the geometrical survey of nuraghi (see for example [105,106]). Such surveys provide a precise model of the geometry, which can help in detecting structural damage and be used as a basis to stability analysis.

3.3. *Possible roads towards physical digital twins*

This section provides a possible choice of modelling approach which is under investigation by the authors for future structural studies of nuraghi, based on all the previous considerations.

Based on what has been said in the previous sections, the structural study of Nuragic remains is a complex issue, if only due to the diversity of phenomena involved. Thus, a model for the mechanical behaviour of nuraghi should be built on a sound basis, allowing to model interactions between mechanical and other physical effects. For example, variational principles may be used, since they allow to model multiscale and multiphysics systems in a well-posed manner

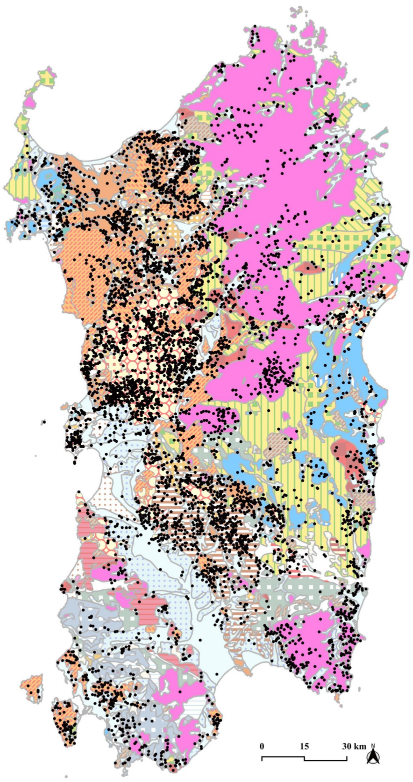


Figure 6. Distribution of nuraghi on a geological map, with a noticeable concentration in the N-NW part of the island. Most of the nuraghi are located on volcanic terrain (basalts, rhyolites, trachytes) related to the Miocene-Paleogene cycle (rhyolites) and the Quaternary cycle (basalts and trachytes). There is also a particular concentration in the SW, near the Campidano plain, where marl soils from the middle-lower Miocene epoch are exploited.

(see for example [20] for an overview of the history and technical advantages of variational principles). More specifically, recent research efforts have been made regarding the use of variational inequality principles to model damageable elastoplastic systems based on a granular micromechanics [59,107,108], with notable applications to cementitious materials [63,64].

A part of these efforts has also been adapted to masonry systems [66–68] and may serve as a basis to model nuraghi structures in future works.

Indeed, they are formulated by defining[3] a linear functional $\widehat{\mathscr{P}}(q)$ depending on the configuration q of a masonry system, and taking as argument a virtual variation of velocity $\delta\widehat{q}$ of the configuration. The evaluation of $\widehat{\mathscr{P}}(q)$ for any $\delta\widehat{q}$ is denoted by:

$$\left\langle \widehat{\mathscr{P}}(q), \delta\widehat{q} \right\rangle \tag{1}$$

which represents the (total) virtual power which would have been released for a variation $\delta\widehat{q}$ of the velocity. Then, for a given set $\widehat{H}_0$ of admissible velocity variations, the mathematical problem to be solved is formulated as the search for a configuration q such that the following inequality holds:

$$\forall\, \delta\widehat{q} \in \widehat{H}_0, \quad \left\langle \widehat{\mathscr{P}}(q), \delta\widehat{q} \right\rangle \leqslant 0 \tag{2}$$

[3] The notations presented here are different from the ones used in the cited papers but follow the same reasoning; in those papers, a discrete time is considered, so virtual works $\widehat{\mathscr{W}}$ are considered instead of virtual powers.

which is to say, we are looking for a configuration q such that its velocity maximises the released power. When modelling a specific situation, the total released power $\widehat{\mathscr{P}}$ may be additively separated into several terms, each representing a phenomenon to be taken into account. For example, in a similar way to the previously cited papers, we may consider damageable elastoplastic interactions between the masonry unit, as well as an external load, thus separating the total virtual power into four terms:

$$\widehat{\mathscr{P}} = \widehat{\mathscr{P}}_{\mathrm{el}} + \widehat{\mathscr{P}}_{\mathrm{pl}} + \widehat{\mathscr{P}}_{\mathrm{d}} + \widehat{\mathscr{P}}_{\mathrm{ext}} \tag{3}$$

Then, one way to identify those powers is by assuming that they derive from energy terms (see the papers cited above for specific examples of energies).

The next crucial point regards the type of kinematic description. As a first approach, and similarly to previous works on Nuragic structures, a discrete description may be considered, where the geometry of all the blocks are taken into account. If those blocks are considered rigid and interacting only in compression, such a model may lead to a first estimation of the necessary equilibrium conditions for te studied system.

A more refined description may also be considered, still with discrete blocks, by adding a continuous granular medium between the blocks, which may model modern concrete reinforcements or small packing stones. A similar approach was adopted in the previously-cited masonry models [67,68]: rigid discrete blocks were considered, while the damaged elastic deformation of the binding material was concentrated into so-called *vertex springs*. More specifically, *augmented bricks* were defined as made of an inner brick and a surrounding layer of mortar (Figure 7), and each pair of interacting augmented bricks was modelled as linked by vertex springs (Figure 8).

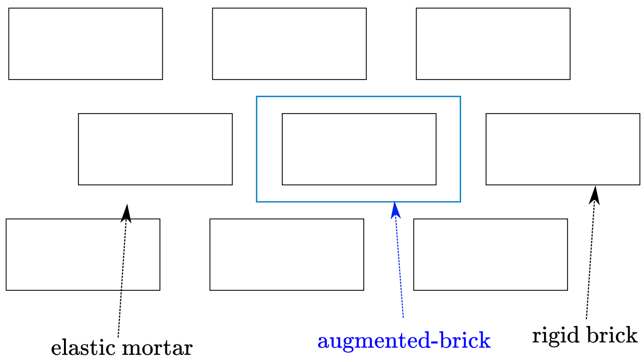


Figure 7. Discrete masonry description (adapted from [66]).

The specific choice of kinematic description depends on which (sub)system is studied: the local analysis of a Nuragic wall would need a precise description of the masonry texture, while the modelling of wall interactions or corbel vaults (similar to Cavanagh [103]) might need a different length scale.

The previous sections have hinted at the complex interactions which may be relevant for studying the present state of nuraghe structures. The following paragraphs summarise the phenomena which may be relevantly accounted for in upcoming models for nuraghi.

Regarding the behaviour of the studied structures, at least two cases may be identified. First is dry masonry: as stated previously, nuraghi are made by corbelling rocks without using mortar. As such, a modelling approach considering rigid blocks with only compression and friction may be relevant for a first estimation of the conditions for static equilibrium. More phenomena may also be added to study more complex cases: deformable blocks for a finer study of internal stresses, or inertial effects for dynamic analyses. Overall, this first modelling case remains similar to discrete-element models from the literature. However, another modelling case may be relevant when

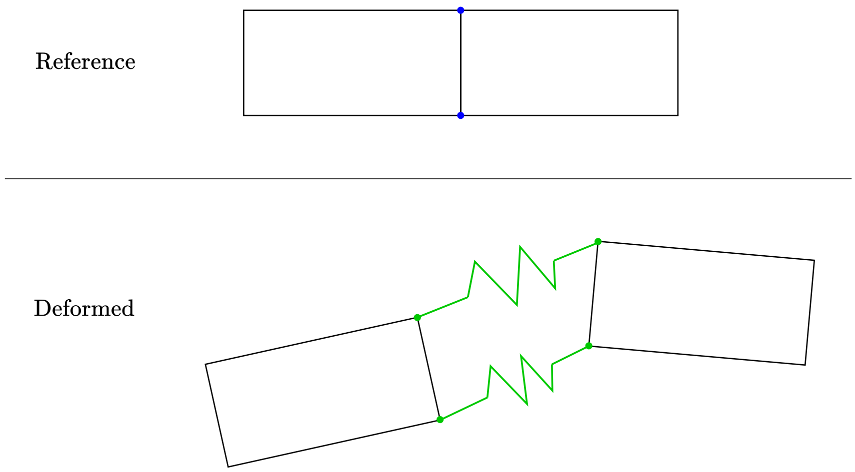


Figure 8. Vertex springs concentrating the deformation and damage effects (adapted from [66]).

(a) Concrete reinforcement

(b) Packing stones in-between larger blocks

Figure 9. Examples of situations which may be modelled using a binding material (Palmavera nuraghe, Alghero).

considering nuraghi which have either a modern concrete reinforcement (see Figure 9a), or many packing stones (Figure 9b) which are much smaller than the main blocks. In this case, a binding element may be added to the model, along with its deformation (elastic or elastoplastic) and damaging effects. Therefore, such a modelling approach would be similar to the variational-inequality-based models mentioned above.

After defining which kind of behaviour the studied system should have, the types of external effects should also be chosen. In the case of historic structures such as nuraghi, weathering (see Figure 10) is the predominant cause of deterioration which should be considered when studying the evolution of the preservation state. Starting with water diffusion and salt crystallisation, the induced damage can be modelled as depending on the density of diffusing species. An example of such a modelling approach in a variational framework may be found for example in [109,110]: in [109], the placement function is defined as χ, and its gradient $F_0 = \nabla\chi$ is decomposed into a so-called intercalation deformation gradient F_i depending on the concentration c of diffusing species, and an elastic deformation gradient F_e:

$$F_0 = F_e F_i \tag{4}$$

The intercalation deformation is defined more specifically as:

$$F_i = \beta^{1/3} I, \quad \text{where } \beta = 1 + \alpha(c - c_0), \tag{5}$$

thus modelling the effect of the concentration c on the deformation of the system. In [110], contrarily to the previous case, the concentration is taken into account in the energy dissipated through damage, instead of the deformation. Indeed, after defining the damage ω and the concentration c, the damage energy is defined as:

$$\mathscr{E}_\mathrm{d} = \int_{(0,L)} \left[K_{\omega 0}\,\omega + K_{c\omega}\,c\,\omega + \frac{1}{2} K_\omega\,\omega^2 \right] \mathrm{d}X \tag{6}$$

where the K are constants. These two examples may be used as a basis to model the influence of diffusing water and crystallising salts in future nuraghe models.

Other works such as [111,112] also give examples of hygrothermal models for masonry.

Regarding thermal effects, a recent variational modelling approach has been presented in [113] through a so-called *entropy displacement* vector $\mathbf{s}$ such that its time variation yields the entropy variation associated to a thermal flux $\mathbf{q}$ at temperature T:

$$\frac{\mathrm{d}\mathbf{s}}{\mathrm{d}t} = \frac{1}{T}\mathbf{q} \tag{7}$$

Thus, the volume density of entropy increase η may be expressed as the convergence of entropy displacement:

$$\eta = -\operatorname{div}\mathbf{s} \tag{8}$$

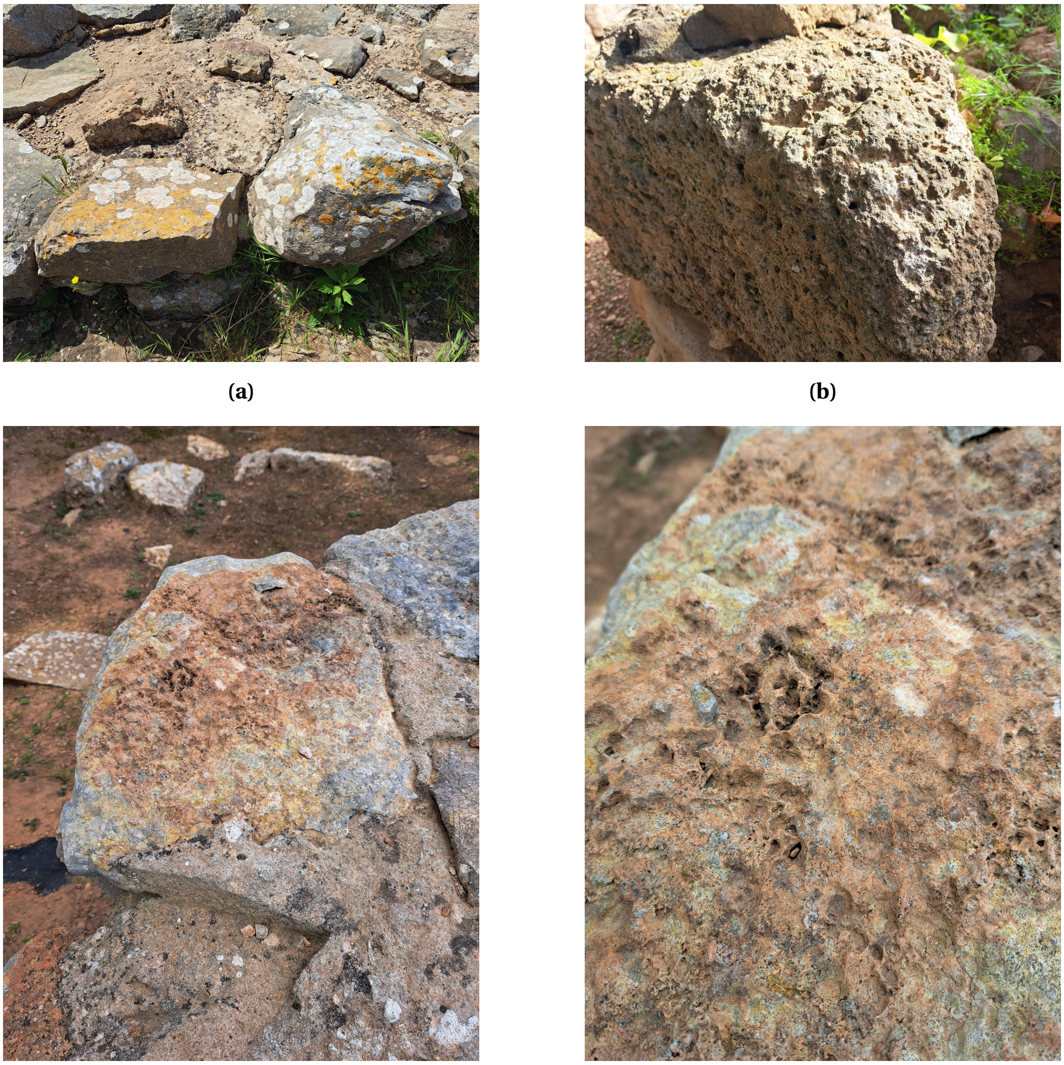
Those definitions allowed the authors of the cited paper to derive thermoelastic couplings through a Hamilton–Rayleigh principle in which the elastic and entropy displacements have similar roles.

Finally, chemical effects may be taken into account through a variable representing the extent of reaction, in a similar way to Biot [114, Section VI] or Morozov [115]: an extent of reaction ξ can be added as an independent variable to represent each chemical reaction to be taken into account, along with a chemical energy depending on the stoichiometric coefficients. In a variational approach, the chemical energy can then be added to the other energetic contributions from which virtual works/powers can be derived.

4. Final remarks and future challenges

The present work has provided a short review of previous studies in order to outline a methodology for assessing the preservation and damage states of Sardinian nuraghi. Regarding their mechanical behaviour more specifically, historical considerations about the construction and geographical distribution of Nuragic structures have lead us to propose several key points to be followed in upcoming research efforts towards the mechanical modelling of nuraghi.

The general objective is to build physical digital twins of Nuragic structures to study their current state and predicting their potential evolutions (see Figure 1 for a graphical summary). Such digital twins are intended as the sum of all available data regarding the studied structures, thus allowing a quantitative-analytical study of their behaviour. More specifically, 3D geometrical surveys would provide precise data for their present configuration, while historical documentation



(a) **(b)** **(c)** **(d)**

Figure 10. Examples of degradation by weathering (Palmavera nuraghe, Alghero).

and laboratory tests would guide the types of behaviours and external loads to be taken into account in the mechanical model of the studied structures. The sought digital twins would then be the combination of those geometrical and mechanical models.

The present work has raised several common characteristics regarding the structure and construction of nuraghi. First and foremost, they have been built by corbelling; this holds for the lower (and older) protonuraghi as well as for the higher *tholos* nuraghi, which are conic towers having possibly several floors. This corbelled structure highly influences the statics of nuraghi, and thus the types of mechanical models to be developed. Second, most if not all Nuragic structures were built without binding materials, so they behave as dry masonries highly relying on frictional effects. Third, the spatial distribution of nuraghi shows higher concentrations on specific types of soils; this can give information regarding the building materials, as well as how to model the interactions between the studied structures and the ground. Last but not least, considering the age of Nuragic remains, weathering phenomena cannot be neglected and must be taken into account as damaging factors; in particular, the proximity to the sea hints at the importance of water diffusion and salt crystallisation.

The previous considerations have lead us to propose several modelling key points. Firstly, regarding the kinematic description, it seems necessary to account for the precise geometry of the studied structures, since they are dry corbelled masonries. Thus, a discrete description of each individual block would seem relevant; however, for some cases it may also be relevant to add continuous parts to model small packing stones in-between much larger blocks, or modern concrete reinforcement. Secondly, regarding the phenomena to be taken into account, it will be necessary to account for friction, as well as defining a damage variable. As to what influences this damage, multiphysical loads will have to be taken into account to model weathering. The present work has raised modelling ideas from the literature concerning water diffusion and salt crystallisation, as well as thermal and chemical effects. Instead of adding all those phenomena at once, it might be relevant to add them progressively, focusing each time on the interactions between the phenomenon to be added and the ones which are already taken into account. Finally, as a consequence of the previous points, variational principles can be useful here, in order to build kinematically-consistent models which are able to account for complex interactions, namely the coupling between purely mechanical phenomena on the one hand, and damages from multiphysical phenomena on the other hand. Although non-variational models do exist for such interactions, a variational approach would allow to derive governing equations in a way which is less likely to induce loss of well-posedness.

Dedication

The manuscript was written through contributions of all authors. All authors have given approval to the final version of the manuscript. More specifically:

- general conceptualisation and supervision: E. Turco, E. Barchiesi;
- writing (first draft):
  - Sections 1, 2, 3.3, 4: C. A. Tran;
  - Sections 3.1, 3.2: R. Busonera, I. Trivelloni;
- writing (revision): all;
- French version of the title, abstract and keywords: C. A. Tran.

The figures were elaborated by:

- C. A. Tran: Figures 1, 7, 8, 9, 10;
- I. Trivelloni: Figures 2, 6;
- R. Busonera and I. Trivelloni: Figures 3, 4, 5.

Declaration of interests

The authors do not work for, advise, own shares in, or receive funds from any organization that could benefit from this article, and have declared no affiliations other than their research organizations.

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