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Tribute to an exemplary man: Yves Couder

Instabilities, patterns

The Kelvin–Helmholtz instability, a useful model for wind-wave generation?

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Abstract. The Kelvin–Helmholtz instability, one of the most classical instabilities in fluid mechanics, was initially introduced to describe the generation of waves by wind. This instability is relevant to several natural and engineering flow configurations, and has led to many experimental variants, in tilted channel with stratified fluids, in circular cells or quasi-2D Hele-Shaw cells. As many physicists, Yves Couder, who passed away in 2019, was fascinated by the problem of wind-wave generation, but also frustrated by the fact that, despite considerable studies over the years, it was still poorly understood. Yves was always eager for simple explanations about physics phenomena, with qualitative arguments rather than equations, and this problem was for him and his collaborators a recurrent source of inspiration. Sharing views about physics with him, making experiments, or even contemplating natural phenomena, was always highly stimulating. In the spirit of these discussions with Yves, we recall here the basics of the Kelvin–Helmholtz instability, and discuss to what extent it may be relevant to the problem of wind-wave generation — not in the air–water case but, rather surprisingly, for very viscous liquids.

Résumé. L'instabilité de Kelvin–Helmholtz, l'une des instabilités les plus classiques de la mécanique des fluides, a été introduite initialement afin de décrire la génération des vagues par le vent. Cette instabilité est présente dans un certain nombre d'écoulements naturels ou industriels, et a donné lieu à de nombreuses variantes expérimentales, en canal incliné avec des fluides stratifiés, en cellules circulaires ou en cellules de Hele-Shaw quasi 2D. Comme beaucoup de physiciens, Yves Couder, disparu en 2019, était fasciné par ce problème de la génération des vagues par le vent, mais également frustré par le fait que, malgré un nombre considérable d'études, ce problème restait très mal compris. Yves avait toujours une préférence pour les explications simples sur les phénomènes physiques, avec des arguments qualitatifs plutôt que des équations, et cette instabilité a été pour lui et ses collaborateurs une source d'inspiration récurrente. Partager avec lui des points de vue sur la physique, faire des expériences, ou même contempler des phénomènes naturels, a toujours été très stimulant. Dans l'esprit de ces discussions avec Yves, nous rappelons ici les fondements de l'instabilité de Kelvin–Helmholtz, et discutons dans quelle mesure elle peut être pertinente pour le problème de la génération de vagues par le vent — non pas dans le cas de l'air et de l'eau mais, de façon plus surprenante, pour les liquides très visqueux.

Keywords. Kelvin–Helmholtz instability, Wind-waves.

2020 Mathematics Subject Classification. 76E05.

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1. Introduction

The Kelvin–Helmholtz instability is one of the most classical instabilities in fluid mechanics. In the simplest case, it describes the destabilization of the parallel flow of two perfect fluids moving at different velocities (Figure 1). In this configuration, if we ignore gravity and surface tension, the flow is always unstable: it is only when the stabilizing effects of gravity and surface tension are added that a critical velocity difference is found above which the flow becomes unstable. Although originally introduced to describe the generation of waves by wind [1], this instability is relevant to a number of natural and engineering applications involving liquid and gas transport, such as the atomization of fast moving films [2, 3] or the slug formation in two-phase pipe flows [4].

Lord Kelvin, beside being a talented mathematician and physicist, was also a sailor and enjoyed sailing with other scientists on his yacht "Lalla Rookh" [5]. This is certainly why he became interested in problems such as the wave pattern observed behind sailing boats [6] or the onset of wind wave formation [1], a problem that also inspired his friend Helmholtz a few years before [7]. For this problem Kelvin assumed a discontinuous velocity profile between two perfect fluids and derived a critical velocity difference for the instability. However, in the air–water case, this onset largely overestimates the observations by a factor of 6 or more. He concluded his letter by these sentences "*Observation shows the sea to be ruffled by wind of a much smaller velocity than this. Such ruffling, therefore, is due to viscosity of the air*" [1]. Indeed, by preventing discontinuity in the velocity profile, viscosity dramatically changes the nature of the problem. Another complication arises from the homogeneity and stationarity of the base flow assumed in the analysis: In open conditions, the development of boundary layers on each side of the interface breaks this assumption, and it is only by adding confinement or periodic boundary conditions that a homogeneous and stationary base flow can be achieved.

As many physicists, Yves Couder was fascinated by the problem of wind-wave generation. This problem was among the experimental projects he liked to propose to his students in his famous lecture "Physique expérimentale" in the 90's [8]. The Kelvin–Helmholtz instability was also the first problem he used to address in his lecture on hydrodynamics instabilities in the DEA (Master's degree) "Physique des Liquides". He used this example to introduce the fundamental concepts of linear stability analysis, perturbations in terms of Fourier modes, sub/super-criticality, on which he further developed more advanced situations. In these lectures he was particularly keen to explain phenomena using simple and intuitive mechanisms, limiting formalism and equations to what was strictly necessary. We follow here this approach, describe some experiments he directly introduced or inspired to colleagues, and discuss to what extent this instability may be a useful model to describe the formation of waves by wind.

2. Qualitative description

We first recall here the basics of the Kelvin–Helmholtz instability. We assume that the base flow is a discontinuous velocity profile and consider the frame of reference of the initially deformed interface. In this frame the upper fluid moves in one direction and the lower fluid in the other. Such a discontinuous velocity profile is a solution of flow equations as long as the fluid viscosities are not taken into account. We assume first that the two fluids have identical physical properties (no surface tension and no density difference).

Any initial deformation of the interface, that can be generated by any type of fluctuations, can be decomposed in Fourier modes. By linearity, the stability of the flow can be considered for each mode separately. If we consider a simple sinusoidal deformation (Figure 2), continuity implies that the streamlines follow on both side the shape of the interface, with an amplitude that

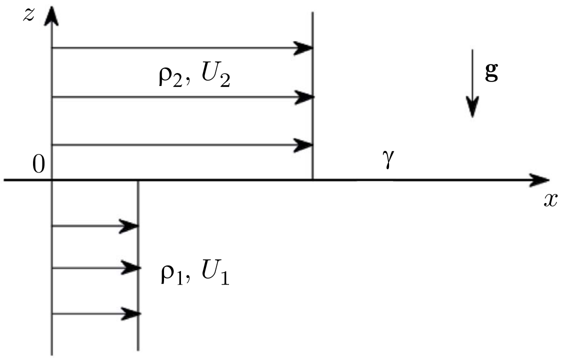

Figure 1. Parallel flow of two immiscible fluids of velocity U_1 and U_2 and density ρ_1 et ρ_2 ($\rho_1 > \rho_2$) separated by an interface of surface tension γ.

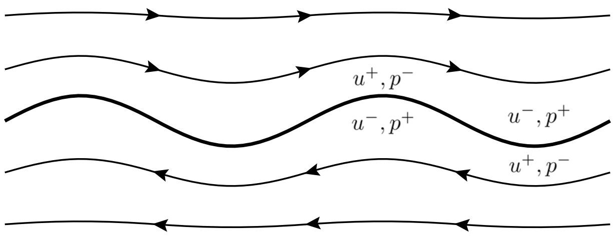

Figure 2. Sinusoidally perturbed interface (bold), and corresponding streamlines on each side of the interface, sketched in the reference frame of the wave. Because of energy conservation along streamlines, the pressure variations are opposite to those of the velocity, so the pressure difference across the interface amplifies the initial deformation.

decreases away from the interface on the scale of the wavelength. By mass conservation between streamlines, the fluid is accelerated above the crests and decelerated above the troughs, and this on both side of the interface.

In the absence of viscosity, which is a necessary condition to maintain a discontinuous velocity profile, and assuming that the temporal growth rate of the instability remains small, we can use the stationary form of Bernoulli equation: along each streamline the quantity $p + \frac{1}{2}\rho u^2$ remains constant, so velocity changes along a streamline correspond to pressure changes of opposite sign. Pressure difference at the interface therefore acts in a way that amplifies the initial disturbance, so the flow is unstable for all velocity differences and all wavelengths. We will see later that short wavelengths are more unstable than long wavelengths.

Introducing restoring forces such as surface tension and gravity yields a finite velocity threshold for instability. This velocity threshold can be seen as the minimum velocity such that the aerodynamic pressure exceeds the restoring forces [9]. We naturally expect long wavelengths to be stabilized by gravity and small wavelengths by surface tension, suggesting that, for a discontinuous profile, the capillary length is the most unstable wavelength. Considering both restoring and viscous forces is more complex, because a dissipation length is also present in addition to the capillary length; this case is discussed later.

Such explanation of the Kelvin–Helmholtz instability is elegant and simple but, as stressed by Yves, a quick-thinking student might make the following embarrassing remark. Since the Bernoulli relation involves the squared velocity it is not sensitive to the flow direction. What if we mechanically generate waves in water and air at rest and consider them in the frame of their phase velocity c_0? In this moving frame, both air and water move at velocity $-c_0$, so from the Bernoulli argument discussed above a destabilizing pressure difference should arise on the crests and troughs: waves between two fluids at rest should be self-amplified by their own motion! What's wrong? How the direction of the flow could be involved in the problem? The good answer is that if waves propagate at c_0 it is because gravity and/or surface tension act as restoring forces, and the minimum velocity to overcome these restoring forces is always larger than the phase velocity of the waves, making such self-amplification impossible.

3. With some equations now

Stability analysis for a discontinuous velocity profile. We now turn to a quantitative description of the instability, following the classical approach of linear stability analysis [10–12]. We still assume a basic discontinuous flow as in Figure 1, with an infinitesimal surface disturbance in the form $\propto \exp \mathrm{i}k(x-ct)$, with real k and complex c. This corresponds to a temporal description of the instability: writing $c = c_r + \mathrm{i}c_i$, we see that the flow is unstable if c_i is positive, with c_r the phase velocity.

By solving Euler equation in each fluid and assuming (i) vanishing velocity far from the surface, (ii) continuity of normal velocities at the interface, and (iii) Laplace pressure jump at the interface given by surface tension, we obtain a second-degree equation for c, of solution

$$c(k) = \frac{\rho_1 U_1 + \rho_2 U_2}{\rho_1 + \rho_2} \pm \left[c_0^2(k) - \frac{\rho_1 \rho_2}{(\rho_1+\rho_2)^2} \Delta U^2 \right]^{1/2}, \tag{1}$$

with $\Delta U = U_2 - U_1$ and $c_0(k)$ the phase velocity of linear gravity-capillary waves propagating at the interface between two fluids at rest,

$$c_0(k) = \frac{c_{0\,\mathrm{min}}}{\sqrt{2}} (k_c/k + k/k_c)^{1/2}. \tag{2}$$

Here k_c is the capillary wave-number and $c_{0\,\mathrm{min}}$ the minimum phase velocity

$$k_c = \sqrt{\frac{\Delta\rho\, g}{\gamma}}, \quad \text{and} \quad c_{0\,\mathrm{min}} = \left(2 \frac{\Delta\rho}{\rho_1+\rho_2} \frac{g}{k_c} \right)^{1/2}, \tag{3}$$

with $\Delta\rho = \rho_1 - \rho_2$ the density difference. In the air–water case $2\pi/k_c \simeq 1.7$ cm and $c_{0\,\mathrm{min}} = 23\ \mathrm{cm\cdot s^{-1}}$.

If c is real ($c = c_r$) the flow is stable: waves propagate at a phase velocity $c(k)$ modified by the two fluid velocities, without amplification (since dissipation is not considered, stability here does not mean damping of disturbances). This criterion is satisfied if

$$\Delta U \leqslant \frac{\rho_1+\rho_2}{\sqrt{\rho_1\rho_2}} c_0(k). \tag{4}$$

When the velocity difference becomes larger than this value, c becomes complex and the flow is unstable: waves propagate at $c_r = (\rho_1 U_1 + \rho_2 U_2)/(\rho_1+\rho_2)$ and are amplified with a temporal growth rate $\sigma = kc_i$,

$$\sigma(k) = k \left[\frac{\rho_1\rho_2}{(\rho_1+\rho_2)^2} \Delta U^2 - c_0^2(k) \right]^{1/2}. \tag{5}$$

The onset of the instability is the minimum value of ΔU, which is proportional to the minimum of the phase velocity $c_{0\,\mathrm{min}}$: the critical wave number is therefore the capillary wave number k_c, and

$$|\Delta U|_{\mathrm{min}} = \frac{\rho_1+\rho_2}{\sqrt{\rho_1\rho_2}} c_{0\,\mathrm{min}}. \tag{6}$$

The marginal stability curve $\Delta U(k)$, obtained by solving $\sigma(k) = 0$, and the squared growth rate are shown in Figure 3. For $|\Delta U| > |\Delta U|_{\mathrm{min}}$, the flow is unstable for the range of wave numbers

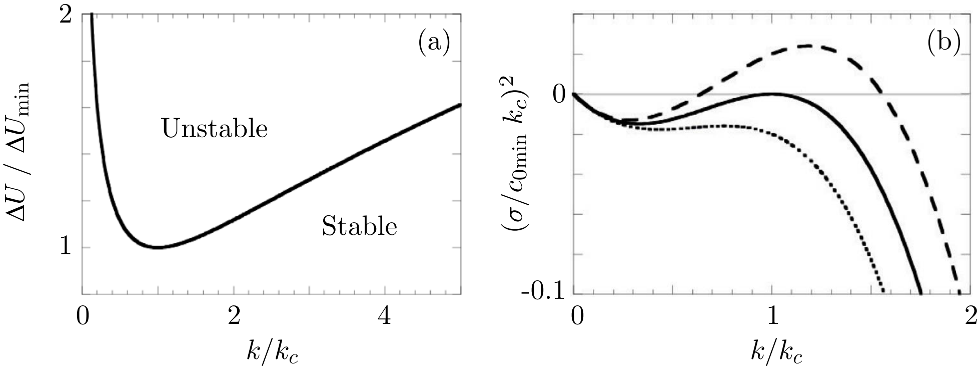

Figure 3. (a) Marginal stability curve of the Kelvin–Helmholtz instability as a function of the dimensionless wavenumber k/k_c (4). (b) Non-dimensional squared growth rate $\sigma^2(k)$ (5) for three velocity differences, $\Delta U/\Delta U_{\min} = 0.9$, 1, and 1.1, showing the range of unstable wavenumbers centered around k_c.

centered around k_c such that $\sigma^2(k)$ is positive. As anticipated, small values of k are stabilized by gravity and large values of k are stabilized by surface tension.

Application to wind blowing over the sea. Although this analysis was originally introduced to explain the generation of sea waves by the wind, it does not successfully predict the wind onset and critical wavenumber. In the air–water configuration we can assume $\Delta U \simeq U_2 \gg U_1$ and $\rho_2 \ll \rho_1$, and the criterion for instability becomes

$$\sigma = k\sqrt{\frac{\rho_2}{\rho_1}}\left[\Delta U^2 - \frac{\rho_1}{\rho_2}c_0^2(k)\right]^{1/2} > 0 \tag{7}$$

with a critical wind velocity

$$|\Delta U|_{\min} = \sqrt{\frac{\rho_1}{\rho_2}}\,c_{0\min}. \tag{8}$$

This simple result reflects the balance between the aerodynamic suction over the wave crests, of the order of $\rho_2\Delta U^2$, and restoring forces (weight and surface tension), of the order of $\rho_1 c_{0\min}^2$.

For the air–water configuration the critical wavelength is $2\pi/k_c \simeq 1.7$ cm and the critical velocity is $|\Delta U|_{\min} \simeq 6.7$ m·s^{-1}. As noticed by Kelvin himself, this wind value is much higher than in reality: capillary ripples are already present for wind velocity of the order of 1 m/s, whereas at 6 m/s well developed white horses (whitecaps) are already present.

Since the work of Kelvin many attempts have been made to better understand the onset of wave formation. In 1957 two landmark papers where published, still neglecting the fluid viscosity: Phillips considers the resonant amplification of pressure fluctuations traveling in the turbulent boundary layer in the air [13], while Miles performs a stability analysis of the mean velocity profile in air [14]; Miles model can be considered as quasi-laminar as the turbulence of the air flow is taken into account only through the average of the velocity profile. However quantitative agreement with these models is still lacking, at least in the air–water case, and identifying the true mechanism governing the generation of the first wind waves is still considered as a major challenge for oceanographers.

Viscous effects. The classical derivation based on discontinuous velocity profiles with zero viscosity clearly oversimplifies reality. Even a small viscosity has significant effects, by smoothing the velocity profile of the base flow, modifying the dispersion relation and damping the energy of the disturbance.

A natural extension, first proposed by Lord Rayleigh [15], consists in applying the linear stability analysis for continuous velocity profiles, while ignoring dissipation in the fluid motion. In the simplest case of a single fluid (shear layer configuration: no surface tension and $\rho_1 = \rho_2$), the problem is governed by an ordinary differential equation, the Rayleigh equation, for the stream function of the disturbance [16]. This equation predicts a necessary condition for the instability: the existence of an inflection point in the velocity profile, i.e. a vorticity extremum. However, this criterion is of little value in practice, because again in the absence of stabilizing effects there is no threshold in velocity: the flow is always unstable for all wavelengths, as in the original Kelvin–Helmholtz problem with a discontinuous velocity profile in a single fluid.

Incorporating viscous dissipation in the problem makes it considerably more difficult. A simplified approach where viscous effects are considered in the lower fluid only was proposed by Lamb [17]: neglecting the viscosity and the density of the upper fluid (a reasonable assumption for air), a discontinuous profile as in Figure 1 can still be assumed, and the problem can be solved exactly for arbitrary viscosity in the lower fluid. Lamb derived the full dispersion relation for this problem for a free surface (§. 349), and provided the appropriate boundary conditions for a purely tangential or purely normal forcing at the surface (§. 350). Taylor extended this work and derived the stability criterion in the case of purely normal forces [18]. The key result is that the onset of the instability and the most unstable wavelength are not affected by viscosity and are still given by the inviscid two-fluid Kelvin–Helmholtz theory (8). It is only the phase velocity of the wave and its growth rate that are decreased by the viscosity.

A further improvement was proposed by Miles, taking into account an inviscid wind with a realistic velocity profile (logarithmic) [9]. He obtains a velocity threshold larger than the flat-profile prediction (8), which follows from the fact that a larger wind is required for the velocity to reach the Kelvin–Helmholtz threshold just above the interface. He also obtains a critical wavelength larger than the capillary length, which can be explained by the fact that larger wavelengths, by inducing a disturbance in the air flow further above the interface, experience a larger aerodynamic suction and are more amplified. Interestingly, these predictions are in fair agreement with experiments in very viscous fluids [19, 20], but, as for the original Kelvin–Helmholtz theory, they still do not explain the much lower velocity threshold in the air–water case. We shall return to this result in Section 5.

The full viscous problem, considering viscosity in both fluids and hence a continuous velocity profile, is naturally of an even higher level of difficulty. The linear stability of a prescribed velocity profile for a single fluid (no density difference, no surface tension) is governed by a higher-order differential equation, the Orr–Sommerfeld equation, which generalizes the Rayleigh equation for non-zero viscosity. A finite velocity threshold is obtained by numerically solving the eigenvalue problem for a given velocity profile. Note that the Rayleigh inflection-point criterion is not necessarily satisfied in the full viscous problem, as shown by Hinch [21] in the case of two fluids with a viscosity contrast.

4. Experiments

There have been many attempts to explore the Kelvin–Helmholtz instability in the laboratory using model experiments. The interest is not only on the instability threshold, but also on the resulting flow pattern (referred to as Kelvin cat's eye vortex pattern) and its subsequent nonlinear evolution. These experiments face a difficulty unavoidable in open systems: because of viscous diffusion the velocity profile evolves, either in time or in space, making difficult connection with the idealized situation of an imposed velocity profile. A now famous experimental setup to overcome this problem was introduced by Thorpe, using a gravity current between stably

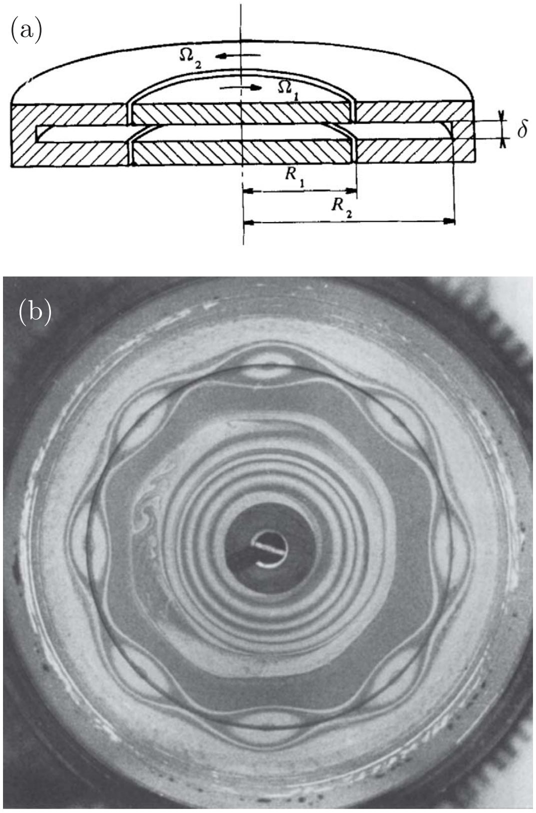

Figure 4. (a) Geometry of the circular shear layer where air is in solid rotation both in the center and in the outer part of the cylindrical cell of thickness δ. The counter-rotation of the disks generates an unstable annular shear layer of thickness of the order of δ. (b) Visualisation of the flow pattern using a soap film in the middle plane of the cell, showing a saturated mode 8, for an angular velocity difference slightly above the onset of the instability. Adapted from Ref. [25].

stratified fluids in a tilted channel [22, 23]. The resulting shear layer is almost homogeneous in space, but spreads over time.

Inspired by this work, Yves Couder introduced at the beginning of the 80's a geometry where the shear layer does not evolve either in time or in space: a confined circular shear layer (Figure 4) [24, 25]. This was his first contribution to the field of hydrodynamic instabilities, after early works in solid state physics in the 70's. In this set-up a thin layer of air is forced in differential rotation by the cell boundaries. The inner air part rotates rigidly with the angular rotation Ω_1 driven by the two disks of radius R_1 whereas the outer air part rotates at Ω_2 in opposite direction. A shear layer develops at the radius R_1, thin and intense close to the wall and of thickness of the order of δ in the middle plane. Because of the periodic boundary condition, a steady saturated regime emerges with a fixed number of vortices along the perimeter. To visualise the flow, a soap membrane was stretched in the middle plane of the cell, and its illumination with monochromatic light produced interference fringes mapping the streamlines of the flow. Using soap films to investigate the dynamics of two-dimensional flows was the first of a long history of experiments initiated by Yves, with further development in two-dimensional grid turbulence, vortex dipoles in wakes, etc. [26–28].

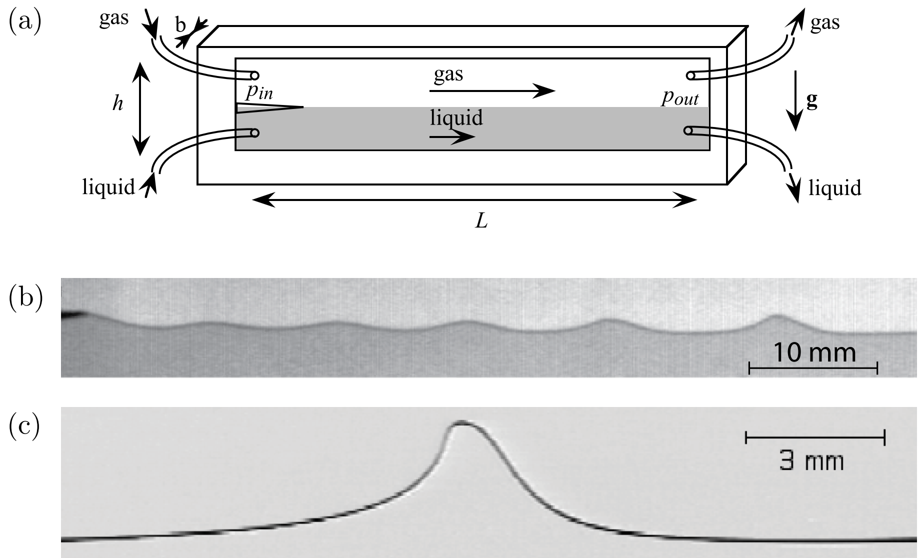

Figure 5. Waves at the interface between air and oil in a Hele-Shaw cell. (a) Experimental setup. (b) Surface deformation for a velocity 2% larger than the threshold velocity, showing a sinusoidal wave train gradually evolving into solitary waves. (c) Fully developed asymmetric solitary wave at a large downstream distance. Adapted from Refs [30, 31].

To predict the linear stability of such a shear layer of constant thickness, a simple phenomenology was proposed, which yields the correct scaling for the velocity threshold without the complication of the full Orr–Sommerfeld procedure. The idea is to account for the viscous effects by assuming that the inviscid growth rate $\sigma = k\Delta U/2$ (see (5)) is decreased by the viscous diffusion both in the plane of the shear layer $(-\nu k^2)$ and in the confining direction $(-\nu/\delta^2)$, where ν is the kinematic viscosity of the air. This simple argument naturally yields a critical Reynolds $Re = \Delta U\delta/\nu \simeq O(1)$ and a most unstable wavelength of the order of δ, in correct agreement with experiments [25]. A refined version of this argument, proposed by Villermaux [29], provides a low Reynolds-number correction to the most unstable wavelength.

Geometrical confinement was also used a few years later to study wave formation at the air/oil interface, using the Hele-Shaw geometry (Figure 5) [30]. This problem is closer to the initial problem of wind-wave generation, but because of the transverse confinement, the flow is laminar in both phases and the small gap size implies a thin uniform shear layer between the fluids. Averaging the Navier–Stokes equation across the gap replaces the dissipative Laplacian term by a damping Darcy term proportional to the fluid velocity. This Kelvin–Helmholtz–Darcy model was used to build a spatial stability analysis that successfully described the onset of wave formation. Here again, only the growth rate is affected by viscosity, while the velocity threshold and the most unstable wavelength remain given by the inviscid Kelvin–Helmholtz prediction.

5. Relevance of the Kelvin–Helmholtz mechanism in highly viscous liquids: viscous solitons

Although the liquid viscosity is a critical parameter to discriminate between the various models for wind-wave generation, this parameter is not varied in most experiments which are naturally performed in the air–water configuration only. Recently we addressed the problem with a new experimental setup, in which water is replaced by more viscous fluids such as glycerine mixtures,

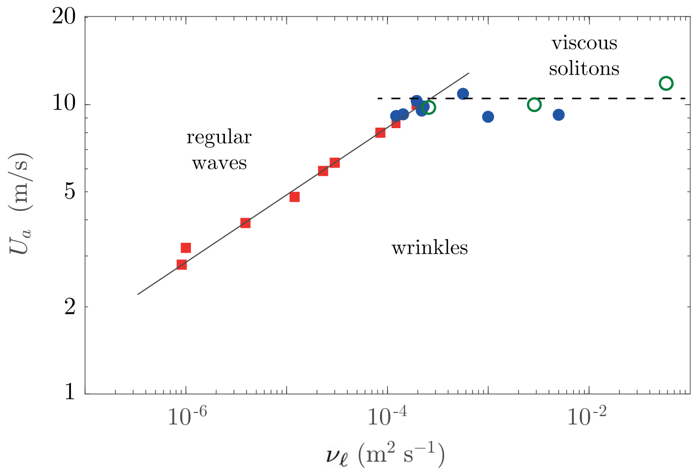

Figure 6. Regime diagram showing the various wave states generated by air wind blowing at velocity U_a over a liquid of kinematic viscosity ν_ℓ. At low viscosity ($\nu_\ell < 10^{-4}$ m^2·s^{-1}), the system shows disorganized small-amplitude wrinkles at low velocity and regular waves at larger velocity, delimited by a critical velocity $U_a \propto \nu_\ell^{1/5}$ (red squares). At large viscosity ($\nu_\ell > 2\times10^{-4}$ m^2·s^{-1}), subcritical viscous solitons are generated, beyond a velocity threshold $U_a \simeq 10$ m·s^{-1} nearly independent of viscosity. Filled symbols: data from Paquier *et al.* [33] and Aulnette *et al.* [35]; open circles: data from Francis [19, 20]. Adapted from Ref. [33].

sugar syrups or silicon oils [32, 33]. A fresh look at this old problem has also been made possible thanks to a new optical method, Free-Surface Synthetic Schlieren, developed a few years before [34]. This method, based on image correlation of random patterns refracted by the wave, allows us to detect tiny surface deformations with micrometer accuracy, much better than with conventional probes.

The main results of these experiments are summarized in Figure 6. For moderate liquid viscosity, up to about 200 times the water viscosity, the surface deformation shows a transition from small-amplitude disorganized fluctuations elongated in the streamwise direction at low wind velocity U_a, that we call *wrinkles*, to well-defined nearly chromatic regular waves above a critical velocity. The transition, which increases as $\nu_\ell^{0.2}$, is smooth at low viscosity, but becomes rather sharp at larger viscosity. We analyzed in Perrard *et al.* [36] the wrinkles as the superposition of incoherent wakes originating from the pressure fluctuations traveling in the turbulent boundary layer in the air: they can be seen as the viscous-saturated wave state excited by the inviscid resonant growth mechanism of Phillips [13]. Their characteristic amplitude is

$$\frac{\zeta_{\rm rms}}{\delta} \simeq C\frac{\rho_a}{\rho_\ell}\left(\frac{u^{*3}}{g\nu_\ell}\right)^{1/2}, \tag{9}$$

where δ is the boundary layer thickness, u^* the friction velocity (a parameter that measures the applied wind shear stress $\tau = \rho u^{*2}$ at the surface, with $u^* \simeq 0.05U_a$ for the Reynolds numbers considered here) and $C \simeq 0.02$. This dependence with liquid viscosity, $\zeta_{\rm rms} \propto \nu_\ell^{-1/2}$, clearly rules out the Kelvin–Helmholtz mechanism (and its generalization including viscous effects [9]), which predicts a threshold essentially independent of the liquid viscosity. A simple phenomenological argument reproduces the observed viscosity dependence of the velocity threshold: according

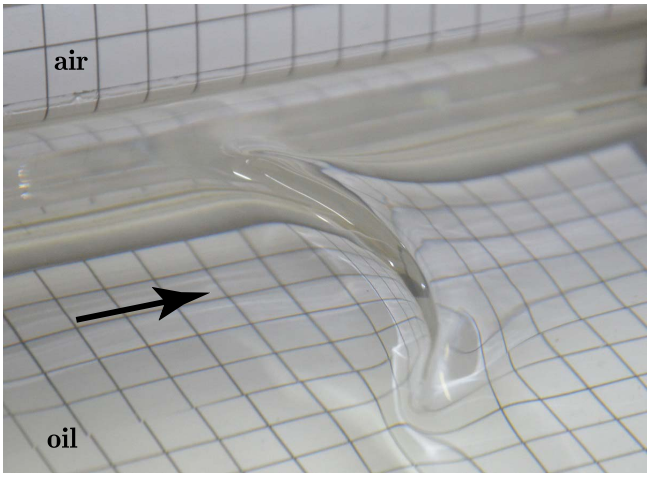

Figure 7. A *viscous soliton* propagating from left to right, generated by a wind of velocity $U_a = 9.63$ m/s on a silicon oil bath of viscosity $\nu = 1000$ mm^2·s^{-1} and depth $h = 35$ mm. The size of the grid pattern is 1.3 cm. Adapted from Ref. [35].

to (9), the wrinkle amplitude reaches a fraction of the viscous sublayer thickness ν_a/u^* (where ν_a is the kinematic viscosity of the air), for a critical friction velocity

$$u_c^* \propto \left(\frac{\rho_\ell}{\rho_a}\right)^{2/5} \left(\frac{g\nu_\ell \nu_a^2}{\delta}\right)^{1/5}, \tag{10}$$

in good agreement with the data of Figure 6 where $U_a \propto \nu_\ell^{1/5}$. In this scenario, the wrinkles appear as a natural "noise" state from which regular waves grow at sufficient wind velocity.

This scenario dramatically changes for larger liquid viscosity, for $\nu_\ell > 2\times10^{-4}$ m^2·s^{-1}: An initial wave train appears at a nearly constant wind velocity, $U_a \simeq 10$ m/s, which becomes strongly unstable even very close to the threshold. It rapidly evolves over a distance of the order of one wavelength into a large fluid bump pushed by the wind (see Figure 7) [35]. We call these strongly nonlinear objects, first observed by Francis [19, 20], *viscous solitons*. They are typically 2–4 mm high, 10–20 mm wide in the streamwise direction, with an almost vertical rear facing the wind and a weak slope at the front. Their propagation velocity results from a balance between the drag force in the air and the viscous friction in the liquid. Their finite amplitude even close to the onset suggests that they result from a subcritical instability. This is confirmed by additional experiments, in which solitons are excited for a wind slightly lower than the natural onset by using a small immersed wave-maker producing a finite-amplitude disturbance.

A remarkable result is that the critical wind velocity for the onset of viscous solitons, $U_a \simeq 10$ m/s, is almost independent of the viscosity, in good agreement with the generalized Kelvin–Helmholtz approach proposed by Miles [9]. This suggests, quite unexpectedly, that the Kelvin–Helmholtz mechanism, which is fundamentally an inertial instability mechanism, provides a better description of the wave generation in liquids of large viscosity.

Systematic measurements of the soliton amplitude as a function of the downstream distance and wind velocity show that their dynamics is governed by the local wind shear stress: In the absence of mechanical forcing, viscous solitons spontaneously emerge from the initial wave packet that forms at small fetch, from which they escape and propagate in the subcritical region over an appreciable distance. At large wind velocity, the emission frequency of solitons increases, resulting in a long-range sheltering of downstream mature solitons by newly formed upstream solitons.

The general picture in Figure 6 therefore suggests that at least two different instability mechanisms are at play in the wind-wave generation problem, with a crossover for a liquid viscosity $\nu_\ell \simeq 2\times 10^{-4}$ m$^2\cdot$s^{-1}. It confirms that the Kelvin–Helmholtz instability, though not relevant for the air–water case, provides a good description of the wave generation for highly viscous liquids. The story certainly does not end here: a number of questions remain open, in particular the origin of the critical liquid viscosity delimiting the two regimes, and the physical parameters governing the transition from supercritical to subcritical at large viscosity.

6. Conclusion

The generation of waves by wind has inspired generations of talented scientists and, despite decades of continuous efforts, it remains a difficult problem in fluid mechanics. This problem has motivated major developments in the theory of hydrodynamic instabilities. Considerable progress has been made recently thanks to increasing computational power, which now make possible the full numerical simulation of turbulent two-phase flows [37, 38]. But above all, this problem motivated over the years elegant experiments, ingenious visualization methods, and fruitful connections between various fields of physics and mechanics. Some of these experiments inherited from Yves Couder's approach, and we hope that they will help to shed light on this complex but fascinating problem in the future.

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