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Research article

Vibrational characteristics analysis of a thermoelastic nanoparticle submerged in an incompressible viscous fluid

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Abstract. In this article, we develop an analytical approach to characterize the breathing mode vibration of a thermoelastic nanosphere submerged in an incompressible fluid. The inclusion of temperature is under the concept of heat wave and the energy equation is combined with elastic theory in the fluid-structure interaction method. The bi-harmonic function is derived from the coupling of the velocity and temperature fields by the coupled thermoelasticity theory. Whereas for an incompressible fluid, these two fields are decoupled. This leads to the convenience of separating thermal conduction and dynamic viscosity parts in the frequency equation. The validation of frequency equation is confirmed by comparing other literatures. The thermal damping and viscosity are represented by Péclet number and Reynolds number respectively. The effects of two parameters on the vibration of the system are analyzed with multiple plots. The analysis could be a useful interpretation of experimental observation and an applicable measurement for vibrational and rheological properties of solids and fluids.

Keywords. Coupled thermoelastic vibration, breathing mode, analytical approach.

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1. Introduction

During recent decades, vibrational properties of nanoparticles embedded in fluid media have garnered substantial attention, due to both fundamental and technological motivations [1–5]. The vibration of an elastic spherical nanoparticle in vacuum was firstly investigated theoretically by Lamb in 1881 [6]. He developed the proportionality between the period of breathing mode and the sphere radius, which was proven by Hodak in 1999 [2]. Because the theoretical and experimental properties found in the vibration of nanoparticles, there have been a wide range of applications, for example, deigning mechanical and biological sensors [5,7,8] with high sensitivity, characterizing rheological properties of fluids, and utilizing resonant vibration mode excited by ultrasound to destroy viruses and tumors [9,10]. Consequently, the capability to the performance of such fluid-structure interaction (FSI) is necessary for their proper conception and application. Among these deployed nanoparticles, the study of vibrational properties of noble-metal spherical nanoparticles [11,12] such as gold is on the top of interests due to the effectiveness of gold thermal properties in oncology.

Despite the acknowledged development of previous researches, it is necessary to mention that most results obtained are based on the continuum elastic model [13,14]. It is proposed that small-scale effects cannot be ignored at nanoscale. Therefore, Ghavanloo and Fazelzadeh [15,16] studied the small size effect on the radial vibration of nanospheres using nonlocal elasticity. In 2023 Huang et al. [17] modelized the nanosphere submerged in viscoelastic fluid with the consideration of small-scale by Eringen nonlocal elastic theory. A further study of the comparison and combination of nonlocal elasticity and strain gradient theory for the modeling of vibrations of nanosphere was carried out by Ducottet and El Baroudi [18] and Huang et al. [19].

Furthermore, it is also important to point out that these analytical and experimental studies are based on the isothermal elasticity theory. In order to take into account the temperature field and energy conservation law, we could use the coupled thermoelastic theory [20]. The thermoelastic theory combines heat transfer with the theory of elasticity. The concept of heat wave where the temperature propagates through a continuous body as second sound rather than classical diffusion theory is very topical in the research literature [21]. Liang and Scarton [22] studied three-dimensional mode shapes of an annular elastic cylinder using the coupled thermoelastic theory. A generalized thermoelastic diffusion theory was derived by Sherief et al. [23]. Shibutan et al. [24] studied the thermoelastic-piezoelectric couplings for the application of nondestructive observation. In 2025, Huang et al. [25] developed an analytical approach to investigate the thermal effects on the vibration of a dry nanosphere.

In this article, the breathing mode (Figure 1) frequency equations of a nanosphere submerged in an incompressible fluid will be derived from the coupled thermoelasticity theory and thermodynamics of incompressible fluids. The effects of thermal damping and dynamic viscosity on vibrational properties will be studied and illustrated.

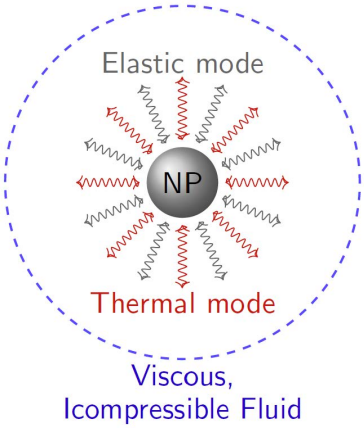

Figure 1. Schematic representation of the thermo-radial vibration of a nanosphere in an incompressible fluid.

2. Thermoelastic nanosphere

Based on the linear thermoelastic theory [20,23], the constitutive equations of a thermoelastic material can be expressed as:

$$\boldsymbol{\sigma} = \lambda(\mathrm{tr}\,\boldsymbol{\varepsilon})\mathbf{I} + 2\mu\boldsymbol{\varepsilon} - \beta T_s \mathbf{I}, \tag{1}$$

where $\boldsymbol{\sigma}$ is the stress tensor, $\boldsymbol{\varepsilon} = \frac{1}{2}\left[\nabla\mathbf{u} + (\nabla\mathbf{u})^T\right]$ is the infinitesimal strain tensor, $\mathbf{u}$ is the displacement vector, $\mathbf{I}$ is the identity tensor, λ and μ are the isothermal Lamé elastic constants. In addition, the increment of temperature T_s in solids and reference temperature T_0 are in the relation of $|T_s/T_0| \ll 1$. The parameter $\beta = \alpha(3\lambda + 2\mu)$ is thermal elastic constant for isotropic materials, and α is the linear coefficient of thermal expansion. In this section, the momentum and energy equations for an isotropic thermoelastic material are derived based on the linear thermoelastic theory [20,23]. In the absence of body forces, the dynamic behavior is governed by Newton's second law:

$$\rho_s \frac{\partial^2 \mathbf{u}}{\partial t^2} = \nabla \cdot \boldsymbol{\sigma} \tag{2}$$

where ρ_s represents the mass density of the nanosphere. Applying the divergence operator to both sides of Eq. (1) and substituting the results into Eq. (2), the following motion equation is obtained:

$$\rho_s \frac{\partial^2 \mathbf{u}}{\partial t^2} = \mu\nabla^2\mathbf{u} + (\lambda + \mu)\nabla\nabla\cdot\mathbf{u} - \beta\nabla T_s. \tag{3}$$

By introducing the isothermal sound velocities c_l (longitudinal wave) and c_t (transverse wave) defined as

$$c_l^2 = \frac{\lambda + 2\mu}{\rho_s}, \qquad c_t^2 = \frac{\mu}{\rho_s},$$

the motion equation (3) can be written in the form:

$$\frac{\partial^2 \mathbf{u}}{\partial t^2} = c_t^2\nabla^2\mathbf{u} + (c_l^2 - c_t^2)\nabla\nabla\cdot\mathbf{u} - \frac{\beta}{\rho_s}\nabla T_s. \tag{4}$$

The linearized form of energy equation for a thermoelastic material is:

$$\nabla\cdot\mathbf{q} = -\rho_s c_v \frac{\partial T}{\partial t} - \beta T_0 \frac{\partial(\nabla\cdot\mathbf{u})}{\partial t}, \tag{5}$$

c_v is the heat capacity at constant volume, $\mathbf{q}$ is heat flux, which can be expressed by Fourier's law $\mathbf{q} = -k_s\nabla T$, and k_s is the thermal conductivity of solids. Therefore we obtain:

$$k_s\nabla^2 T = \rho_s c_v \frac{\partial T}{\partial t} + \beta T_0 \frac{\partial(\nabla\cdot\mathbf{u})}{\partial t}. \tag{6}$$

Equations (4) and (6) are the momentum equation and energy equation for solids respectively.

2.1. *Solutions of breathing mode wave equation*

Generally, the vibrations of a spherical particle are classified into two categories: spheroidal (e.g., radial or breathing, dipolar, and quadrupolar) and toroidal modes [6]. This section focuses exclusively on the breathing mode. Notably, for a nanosphere of radius a and density ρ_s, the radial motion is considered. The origin is conveniently placed at the center of the nanosphere in a spherical coordinate system (r, θ, φ). Following the Helmholtz decomposition [26], the displacement potential function ϕ is introduced to solve Eq. (4). Since the motion is radial, the displacement vector $\mathbf{u}$ can be expressed as $\mathbf{u} = \nabla\phi$. Substituting this into Eq. (4), the momentum equation in the context of thermoelastic theory is written:

$$\nabla\left(\frac{\partial^2\phi}{\partial t^2} + \frac{\beta}{\rho_s}T - c_l^2\nabla^2\phi\right) = 0.$$

This leads to the expression of temperature in the forms of the corresponding scalar potential ϕ:

$$T_s = \frac{\rho_s c_l^2}{\beta}\left(\nabla^2\phi - \frac{1}{c_l^2}\frac{\partial^2\phi}{\partial t^2}\right). \tag{7}$$

Substituting the above expression of T_s into the energy equation (6) yields:

$$\left(\nabla^2 - \frac{1}{c_l^2}\frac{\partial^2}{\partial t^2}\right)\left(\nabla^2\phi - \frac{1}{D_s}\frac{\partial\phi}{\partial t}\right) - \frac{T_0\beta^2}{\rho k_s c_l^2}\nabla^2\left(\frac{\partial\phi}{\partial t}\right) = 0, \tag{8}$$

where $D_s = k_s/(\rho_s c_v)$ is thermal diffusivity of nanosphere. To solve Eq. (8), we assume the displacement potential function ϕ varies harmonically with time as $\phi(r,t) = \phi(r)e^{-j\omega t}$ (ω is the angular frequency and $j = \sqrt{-1}$). Substituting this into Eq. (8), the following differential equation is derived:

$$\nabla^4\phi + \left[\frac{\omega^2}{c_l^2} + \frac{j\omega}{D_s}(\epsilon+1)\right]\nabla^2\phi + \frac{j\omega}{D_s}\frac{\omega^2}{c_l^2}\phi = 0, \tag{9}$$

where we have introduced dimensionless thermoelastic coupling constant ϵ of nanosphere:

$$\epsilon = \frac{T_0\beta^2}{\rho_s c_v(\lambda+2\mu)}.$$

Eq. (9) is a bi-harmonic function which can be factorized and written in the equivalent form [22,27]:

$$(\nabla^2 + K_1^2)(\nabla^2 + K_2^2)\phi = 0.$$

The wavenumbers K_1 and K_2, derived from the parameters appearing in Eq. (9), are expressed as follows:

$$K_1^2 = \frac{A + \sqrt{A^2 - 4B}}{2}, \qquad K_2^2 = \frac{A - \sqrt{A^2 - 4B}}{2},$$

where

$$A = \frac{\omega^2}{c_l^2} + \frac{j\omega}{D_s}(\epsilon+1), \qquad B = \frac{j\omega}{D_s}\frac{\omega^2}{c_l^2}.$$

The general solution ϕ of Eq. (9) is the sum of ϕ_c and ϕ_t [22]. These two waves functions are respectively predominantly elastic ϕ_c and thermal ϕ_t. Therefore, the general solution of Eq. (8) is written as:

$$\phi = \phi_c + \phi_t,$$

which satisfy

$$\nabla^2\phi_c + K_1^2\phi_c = 0, \qquad \nabla^2\phi_t + K_2^2\phi_t = 0.$$

These are two dilatational waves, one (ϕ_c) predominantly elastic in character, named as a compressional propagating mode and the other (ϕ_t) predominantly thermal, called a thermal mode. Considering the interaction with fluid, we take the form

$$\phi(r) = C_1 j_0(K_1 r) + C_2 j_0(K_2 r), \tag{10}$$

where $j_0(Kr)$ is the first kind of zeroth-order of spherical Bessel function. The displacement $u(r)$ is derived as the gradient of ϕ, resulting in the following expression:

$$u(r) = \frac{C_1}{r}\frac{\sin(K_1 r)}{K_1 r}\left[\frac{K_1 r}{\tan(K_1 r)} - 1\right] + \frac{C_2}{r}\frac{\sin(K_2 r)}{K_2 r}\left[\frac{K_2 r}{\tan(K_2 r)} - 1\right]. \tag{11}$$

The temperature is then determined by inserting Eq. (10) in Eq. (7) as follow:

$$T_s(r) = \frac{\rho_s c_l^2}{\beta}\left[C_1\left(\frac{\omega^2}{c_l^2} - K_1^2\right)\frac{\sin(K_1 r)}{K_1 r} + C_2\left(\frac{\omega^2}{c_l^2} - K_2^2\right)\frac{\sin(K_2 r)}{K_2 r}\right]. \tag{12}$$

An alternative expression for the temperature can be derived by relating the thermoelastic constant to the longitudinal sound velocity, transverse sound velocity and solid density:

$$\beta = 3\alpha\rho_s\left(c_l^2 - \frac{4c_t^2}{3}\right).$$

Substituting the newly derived expression for the thermoelastic constant into Eq. (12) yields Eq. (13):

$$T_s(r) = \widetilde{\alpha}\left[C_1\left(\frac{\omega^2}{c_l^2} - K_1^2\right)\frac{\sin(K_1 r)}{K_1 r} + C_2\left(\frac{\omega^2}{c_l^2} - K_2^2\right)\frac{\sin(K_2 r)}{K_2 r}\right], \tag{13}$$

where

$$\widetilde{\alpha} = \frac{1}{3\alpha\left(1 - \frac{4c_t^2}{3c_l^2}\right)}.$$

The normal stress and heat flux expressions can also be derived by the same method:

$$\sigma_{rr}^s(r) = \frac{4\rho_s c_t^2}{r^2}\left\{C_1\frac{\sin(K_1 r)}{K_1 r}\left[1 - \frac{K_1 r}{\tan(K_1 r)} - \left(\frac{\omega r}{2c_t}\right)^2\right] + C_2\frac{\sin(K_2 r)}{K_2 r}\left[1 - \frac{K_2 r}{\tan(K_2 r)} - \left(\frac{\omega r}{2c_t}\right)^2\right]\right\}, \tag{14}$$

$$q_r^s(r) = \frac{k_s}{r}\widetilde{\alpha}\left\{C_1\left(\frac{\omega^2}{c_l^2} - K_1^2\right)\frac{\sin(K_1 r)}{K_1 r}\left[1 - \frac{K_1 r}{\tan(K_1 r)}\right] + C_2\left(\frac{\omega^2}{c_l^2} - K_2^2\right)\frac{\sin(K_2 r)}{K_2 r}\left[1 - \frac{K_2 r}{\tan(K_2 r)}\right]\right\}. \tag{15}$$

3. Thermodynamics of incompressible fluid

This study investigates the vibrational signature of a nanosphere embedded in a viscous, semi-infinite, and incompressible fluid. To begin, the constitutive equation governing the fluid behavior is expressed as follows [28]:

$$\boldsymbol{\sigma} = -p\mathbf{I} + \eta\left[\nabla\mathbf{v} + (\nabla\mathbf{v})^T\right], \tag{16}$$

where $\boldsymbol{\sigma}$ is the stress tensor in the fluid, $\mathbf{v}$ is the fluid velocity field, p is the fluid pressure field and η is the fluid dynamic viscosity. Moreover, when the fluid velocity is low compared to the characteristic dimensions of the system, the inertial term in the momentum conservation equation becomes negligible. As a result, the fluid's behavior is primarily dictated by the principles of mass and momentum conservation:

$$\nabla\cdot\mathbf{v} = 0, \tag{17}$$

$$\frac{\partial\mathbf{v}}{\partial t} = -\frac{\nabla p}{\rho_f} + \nu\nabla^2\mathbf{v}, \tag{18}$$

where ρ_f is the fluid density and ν is the kinematic viscosity. Using the definition of the specific heat at constant pressure $c_p = T(\partial s/\partial T)_p$, the energy equation can be written as follows:

$$\nabla^2 T_f - \frac{1}{D_f}\frac{\partial T_f}{\partial t} = 0, \tag{19}$$

where $D_f = k_f/(\rho_f c_p)$ represents the thermal diffusivity for the fluid and k_f is thermal conductivity.

3.1. *Radial vibration solution*

For an incompressible fluid, the velocity field and the temperature field are not coupled, therefore, we can solve them separately. For the velocity field of harmonic motion $\mathbf{v} = v(r)e^{-j\omega t}\mathbf{e}_r$, the solution of $v(r)$ is obtained from the continuity equation (17). The general solution is in the form:

$$v(r) = \frac{C_3}{r^2} \tag{20}$$

where C_3 is an unknown constant. Therefore, using the momentum equation (18), the expression of pressure is obtained as follows:

$$p(r) = -C_3 \frac{j\omega\rho_f}{r}. \tag{21}$$

And the same process applies for temperature field. The increment of temperature is solved by energy equation (19). Considering the semi-infinite boundary condition for fluids, the general solution for the temperature field is expressed in the following form:

$$T_f(r) = C_4 h_0(K_3 r), \tag{22}$$

where C_4 is an unknown constant, h_0 is the first kind of zeroth-order spherical Hankel function and $K_3 = \sqrt{\frac{j\omega}{D_f}}$ is the heat conduction wavenumber. The normal stress is derived by constitutive equation (16):

$$\sigma_{rr}^f(r) = C_3\left(\frac{j\omega\rho_f}{r} - \frac{4\eta}{r^3}\right). \tag{23}$$

In a similar way, we can express the heat flux of fluid by Fourier's law:

$$q_r^f(r) = -C_4 k_f \frac{j + K_3 r}{K_3 r^2} e^{iK_3 r}. \tag{24}$$

4. Fluid-structure interaction

This section derives the eigenvalue equation governing the natural frequencies of an elastic nanosphere oscillating within an incompressible fluid, based on the enforcement of suitable boundary conditions. At the interface between the nanosphere and the surrounding fluid ($r = a$), the continuity of velocity, normal stress, temperature, and heat flux with neglecting of surface tension [27], is imposed:

$$v(a) = -i\omega u(a), \qquad T_f(a) = T_s(a), \qquad \sigma_{rr}^f(a) = \sigma_{rr}^s(a), \qquad q_r^f(a) = q_r^s(a), \tag{25}$$

which can be written in the form of matrix and vector:

$$[\mathbf{M}]\{\mathbf{C}\} = 0$$

where $\mathbf{M}$ is a 4×4 matrix and $\{\mathbf{C}\} = [C_1\ C_2\ C_3\ C_4]^T$. In addition, before the calculation of the eigenvalue equation, it is convenient to introduce the non-dimensional wavenumber x_1, x_2, x_3,

$$x_1 = K_1 a, \qquad x_2 = K_2 a, \qquad x_3 = K_3 a. \tag{26}$$

Similarly, we can also introduce the non-dimensional density, thermal conductivity and wave speeds:

$$\bar{\rho} = \frac{\rho_f}{\rho_s}, \qquad \bar{k} = \frac{k_f}{k_s}, \qquad c_1 = \frac{\omega a}{c_l}, \qquad c_2 = \frac{\omega a}{2c_t}. \tag{27}$$

Meanwhile, the zeroth-order spherical Hankel function can be expressed by exponential constant $h_0(x) = -je^{jx}/x$. Then we use the non-dimensional parameters to represent the expressions of radial displacement, velocity, temperature and heat flux:

$$\begin{aligned}
u(a) &= \frac{C_1}{a}\frac{\sin x_1}{x_1}\left(\frac{x_1}{\tan x_1} - 1\right) + \frac{C_2}{a}\frac{\sin x_2}{x_2}\left(\frac{x_2}{\tan x_2} - 1\right),\\
v(a) &= \frac{C_3}{a^2},\\
\sigma_{rr}^s(a) &= -\frac{\rho_s\omega^2}{c_2^2}\left[C_1\frac{\sin x_1}{x_1}\left(\frac{x_1}{\tan x_1} + c_2^2 - 1\right) + C_2\frac{\sin x_2}{x_2}\left(\frac{x_2}{\tan x_2} + c_2^2 - 1\right)\right],\\
\sigma_{rr}^f(a) &= C_3\frac{\rho_f\omega}{a}\left(j - \frac{4}{\mathrm{Re}}\right),
\end{aligned}$$

$$
\begin{aligned}
T_s(a) &= \frac{\tilde{\alpha}}{a^2}\left[C_1\frac{\sin x_1}{x_1}(c_1^2-x_1^2)+C_2\frac{\sin x_2}{x_2}(c_1^2-x_2^2)\right],\\
T_f(a) &= -C_4 i\frac{e^{ix_3}}{x_3},\\
q_r^s(a) &= -\tilde{\alpha}\frac{k_s}{a^3}\left[C_1\frac{\sin x_1}{x_1}(c_1^2-x_1^2)\left(\frac{x_1}{\tan x_1}-1\right)+C_2\frac{\sin x_2}{x_2}(c_1^2-x_2^2)\left(\frac{x_2}{\tan x_2}-1\right)\right],\\
q_r^f(a) &= -C_4\frac{k_f(j+x_3)e^{jx_3}}{ax_3}.
\end{aligned}
$$

By substituting these equations in the boundary conditions, we obtain the matrix

$$
\mathbf{M}=\begin{bmatrix}
\frac{\sin x_1}{x_1}\left(\frac{x_1}{\tan x_1}-1\right) & \frac{\sin x_2}{x_2}\left(\frac{x_2}{\tan x_2}-1\right) & \frac{1}{j\omega a} & 0\\
\frac{\sin x_1}{x_1}\left(\frac{x_1}{\tan x_1}-1+c_2^2\right) & \frac{\sin x_2}{x_2}\left(\frac{x_2}{\tan x_2}-1+c_2^2\right) & \frac{\bar{\rho}c_2^2}{\omega a}\left(j-\frac{4}{\mathrm{Re}}\right) & 0\\
\frac{\sin x_1}{x_1}(c_1^2-x_1^2) & \frac{\sin x_2}{x_2}(c_1^2-x_2^2) & 0 & \frac{ja^2e^{jx_3}}{\tilde{\alpha}x_3}\\
\frac{\sin x_1}{x_1}(c_1^2-x_1^2)\left(\frac{x_1}{\tan x_1}-1\right) & \frac{\sin x_2}{x_2}(c_1^2-x_2^2)\left(\frac{x_2}{\tan x_2}-1\right) & 0 & -\frac{a^2\bar{k}(x_3+j)e^{jx_3}}{\tilde{\alpha}x_3}
\end{bmatrix}.
$$

The manipulation of determinant of matrix leads to the following eigenvalue equation:

$$P_0+P_1+P_2+P_3=0.$$

The equation is divided into four parts by non-dimensional parameters $\bar{\rho}$ and $\bar{k}$. P_0 is the term without the two parameters, P_1 is the term with only $\bar{\rho}$, P_2 is term with only $\bar{k}$ and P_3 with both parameters:

$$
\begin{aligned}
P_0 &= \frac{c_1^2-x_1^2}{g_2}-\frac{c_1^2-x_2^2}{g_1}+\frac{x_2^2-x_1^2}{c_2^2},\\
P_1 &= \bar{\rho}\left(1+\frac{4j}{\mathrm{Re}}\right)(x_2^2-x_1^2),\\
P_2 &= \bar{k}(jx_3-1)\left[\frac{x_1^2-x_2^2}{g_1g_2}+\frac{1}{c_2^2}\left(\frac{c_1^2-x_1^2}{g_2}-\frac{c_1^2-x_2^2}{g_1}\right)\right],\\
P_3 &= \bar{\rho}\left(1+\frac{4i}{\mathrm{Re}}\right)\bar{k}(jx_3-1)\left(\frac{c_1^2-x_2^2}{g_2}-\frac{c_1^2-x_1^2}{g_1}\right),
\end{aligned}
$$

where

$$g_1=\frac{x_1}{\tan x_1}-1,\qquad g_2=\frac{x_2}{\tan x_2}-1,\qquad \mathrm{Re}=\frac{\rho\omega a^2}{\eta}.$$

Analysis of these expressions indicates that P_0 is the only term that does not involve the two non-dimensional parameters $\bar{\rho}$ and $\bar{k}$ defined in Eq. (27). In contrast, P_1 and P_2 include exclusively $\bar{\rho}$ and $\bar{k}$, respectively. Lastly, P_3 incorporates both $\bar{\rho}$ and $\bar{k}$ simultaneously. Note that $P_0=0$ is the frequency equation in vacuum for insulated boundary condition [25].

5. Results and discussions

If we minimize the effect of temperature field, we obtain the same value of quality factor in the article of Galstyan [13], $Q=841$. With the effect of the temperature, thermal parameters of gold [25] and water, shear viscosity $\eta=0.894\times10^{-3}$ Pas, thermal conductivity $k_f=0.6095$ W/(mK), specific heat capacity at constant pressure $c_p=4.181\times10^3$ J/(kgK), the result is $Q=257$. From this point of view, the thermal effects are not negligible.

To investigate the effects of viscosity and thermal dissipation, four separate frequency equations — $\mathbf{F}_{\text{Total}}$, $\mathbf{F}_{\text{Viscous}}$, $\mathbf{F}_{\text{Thermal}}$, and $\mathbf{F}_{\text{Vacuum}}$ are established:

$$\begin{aligned}
\mathbf{F}_{\text{Total}} &= P_0 + P_1 + P_2 + P_3,\\
\mathbf{F}_{\text{Viscous}} &= P_0 + P_1,\\
\mathbf{F}_{\text{Thermal}} &= P_0 + P_2 + \bar{\rho}(x_2^2 - x_1^2) + \bar{\rho}\bar{k}(ix_3 - 1)\left(\frac{c_1^2 - x_2^2}{g_2} - \frac{c_1^2 - x_1^2}{g_1}\right),\\
\mathbf{F}_{\text{Vacuum}} &= P_0.
\end{aligned}$$

$\mathbf{F}_{\text{Total}}$ contains the two parameters $\bar{\rho}$ and $\bar{k}$. By contrast, the thermal parameter is not taken into account in $\mathbf{F}_{\text{Viscous}}$. Lastly, viscosity is assumed to be negligible in $\mathbf{F}_{\text{Thermal}}$ formulation.

5.1. *Effects of Péclet number*

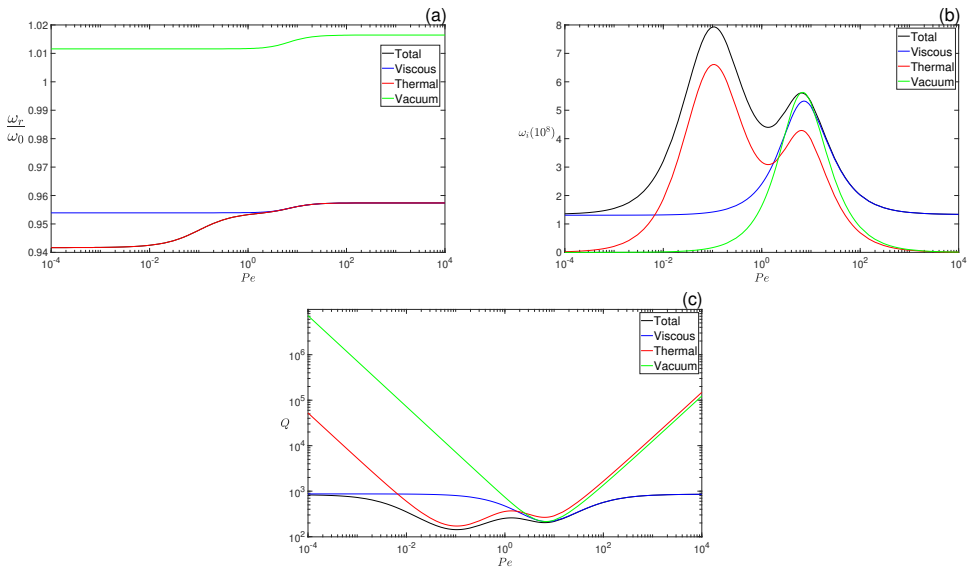

Figure 2. Effects of Pe on the frequency, attenuation and quality factor. ω_0 is the frequency of an isothermal nanosphere vibrating in vacuum with the same radius.

The Péclet number Pe = $(v_c a)/D_s$ is defined as the ratio of the rate of advection to the rate of diffusion, which can be considered as a thermal damping factor of nanosphere [25]. The Péclet number has very small effect on the frequency ($0.01 < \text{Pe} < 100$). The plateaus in Figure 2(a) demonstrate the regime dominated by elasticity ($\text{Pe} < 0.01$) or thermal isolation ($\text{Pe} > 100$) of the nanosphere material.

The attenuation behavior influenced by the Péclet number, as depicted in Figure 2(b), reveals intriguing patterns. Notably, the total and thermal frequency responses exhibit two distinct peaks, whereas the viscous and vacuum frequency curves display only one. The vacuum curve aligns well with the findings reported in [25]. Interestingly, the second peak arises from thermal dissipation within the fluid. Furthermore, in the low Péclet number regime (corresponding to the elastic behavior of the nanosphere), a clear distinction emerges between total attenuation (black curve) and thermal attenuation (blue curve). In contrast, the viscous (blue) and total attenuation (black) curves overlap. A similar overlap is observed at high Péclet numbers, representing thermal isolation. These two non-zero plateaus are therefore attributed to viscous dissipation mechanisms.

From the quality factor analysis shown in Figure 2(c), three distinct regimes emerge, each governed by different dominant mechanisms. The total attenuation (black curve) and thermal contribution (blue curve) both display two noticeable valleys. As the Péclet number increases, the first valley arises from thermal dissipation within the fluid, while the second is driven by thermal dissipation in the nanosphere. In the third regime — corresponding to very small or very large Péclet numbers — viscous effects become the prevailing factor.

5.2. *Effects of Reynolds number*

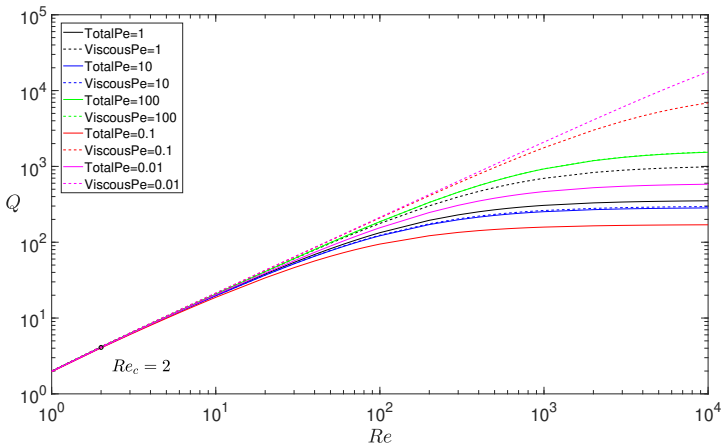

Figure 3. Effects of Re on the behavior of quality factor with different Péclet numbers (0.01, 0.1, 1, 10, 100). Dashed lines are plotted by the viscous frequency equation.

The behavior of the quality factor as a function of Reynolds number differs significantly from its variation with the Péclet number. In this analysis, both total and viscous frequency equations are employed to plot the quality factor against Reynolds number. A reduction in Reynolds number corresponds to an increase in fluid viscosity. As shown in Figure 3, when the Reynolds number is very low — indicating a viscosity-dominated regime) the curves for different cases converge, suggesting that thermal effects become negligible under such conditions. As the Reynolds number increases, the quality factor approaches a constant value. The specific Reynolds number at which this plateau occurs depends on the Péclet number. Interestingly, according to Figure 2(c), there is no monotonic relationship between the quality factor and Péclet number, meaning the curves exhibit complex, non-linear behavior with changes in Reynolds number. Nevertheless, thermal dissipation by the fluid plays a significant role. This is evident when comparing dashed curves (which exclude fluid thermal conduction) to solid ones. Some solid lines stabilize much earlier than their dashed counterparts as Reynolds number increases. This observation indicates that thermal dissipation in the system leads to a limiting value for the quality factor. The underlying cause can be attributed to acoustic radiation within the thermoelastic material and the thermodynamic response of the incompressible fluid.

5.3. *Effects of fluid Péclet number*

Since the fluid is incompressible, the sound speed propagating in this media is infinite. In order to investigate the influence of fluid thermal diffusivity, the Péclet number of fluid is defined as $\mathrm{Pe}_f = (v_c a)/D_f$.

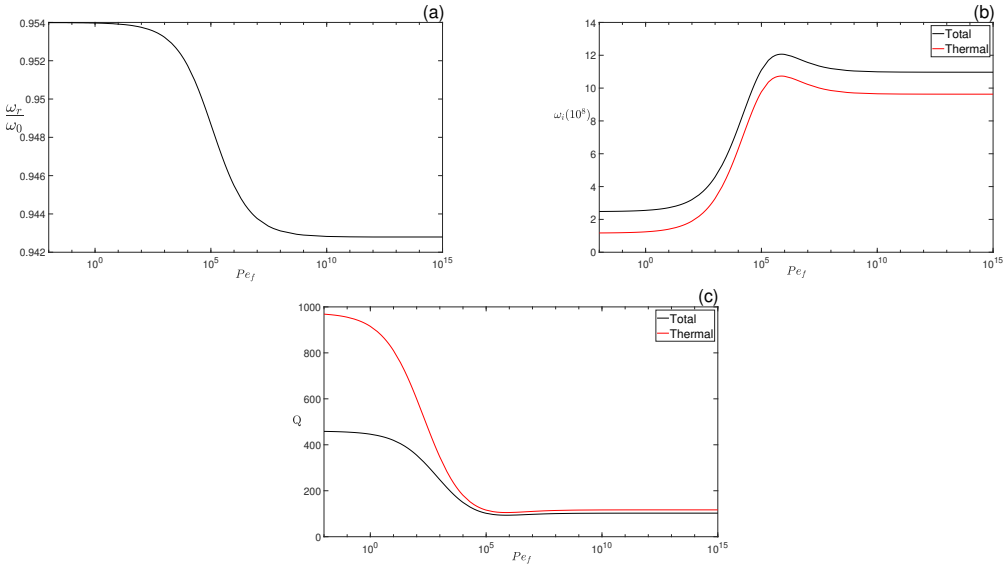

Figure 4. Effects of Pe_f on the frequency, attenuation and quality factor.

Comparing with Figure 2(a) and Figure 4(a), it is evident that the effect of fluid thermal diffusion are opposite to the nanosphere diffusion. When the fluid is considered isothermal, the frequency is higher than when it is thermal isolated. The frequency decreases with the increase of Pe_f between two constant values.

To study the effect of thermal diffusion on attenuation, we illustrate the attenuation on the function of Pe_f by two frequency equations ($\mathbf{F}_{\mathrm{Total}}$ and $\mathbf{F}_{\mathrm{Thermal}}$). It is important to note that eliminating the viscosity will not change the curve's shape. By the variance of fluid Péclet number, there are peaks like Figure 2(b). However, the attenuation for fluid with little thermal diffusion is much larger than isothermal one. This is different from the nanosphere Péclet number and is the contrary to a dry nanosphere with different boundary conditions [25]. It is necessary to distinguish the solid Péclet number, the boundary conditions for a dry nanosphere and the fluid Péclet number. For the investigation of the vibration in a FSI system, the nanosphere is considered as a heat source in our situation. For the convenience of the description, we name the three cases as: isothermal and insulated nanosphere, isothermal and insulated BCs and isothermal and insulated fluid. The physical meaning of isothermal can be interpreted by an infinite thermal diffusion and the physical meaning of insulated is zero thermal diffusion. In the condition of isothermal or insulated nanosphere ($\mathrm{Pe} = 0$ or $\mathrm{Pe} = \infty$), the solid is either elastic or thermally isolated, which both indicate that there is no heat conduction inside the nanosphere. Consequently, the thermal dissipation is zero for isothermal and insulated nanosphere, nevertheless it is a single dry nanosphere or it is embedded in incompressible fluid. The only dissipation under this circumstance is caused by fluid viscosity. Comparing the isothermal and insulated BCs of a dry nanosphere with the case when it is embedded in isothermal and insulated incompressible fluid, the larger thermal diffusion of BCs means more energy lost by the solid as a heat source, whereas the larger thermal diffusion of fluid means less energy absorbed by the surrounding fluid environment. This explains when Pe_f is larger, which means smaller thermal diffusion of fluid and more energy consumed by fluid, the thermal dissipation of FSI system is larger.

Last but not least, in Figure 4(c), it is observed that the difference of quality factors of viscous and inviscid fluids is much larger for isothermal fluid than that for insulated fluid. Compared to

thermal damping, the viscosity of an incompressible insulated fluid can be negligible.

6. Conclusion

In this article, we develop an analytical approach to characterize the breathing mode vibration of a thermoelastic nanosphere submerged in an incompressible fluid. For a coupled thermoelastic nanosphere, there are two modes of dilatational wave propagation, composed by an elastic and a thermal wave equations. While the velocity field and the temperature field are independent for an incompressible fluid. The FSI method is used to ensure the continuity of velocity, stress, temperature, and heat flux at the interface of the nanosphere and fluid. This analytical approach is validated by comparing the results of classical articles [13,14]. Below are the conclusions drawn by this method.

- The Péclet number is considered as a thermal damping factor. The thermal dissipation effect can be ignored whether nanosphere Péclet number is too small (isothermal condition) or too large (insulated condition). Under these circumstances, the energy dissipation is mainly caused by dynamic viscosity of the fluid.
- When the thermal dissipation is dominant, it is divided into two regimes. With the increase of the Péclet number of nanosphere, firstly the thermal dissipation of fluid is important, then the thermal dissipation of nanosphere predominates.
- Since the nanosphere is considered as a heat source and fluid as energy consumer in a FSI system, the increase of fluid Péclet number means the increase of thermal damping of the system. Furthermore, if the fluid can be seen as insulated, the effect of viscosity is so small to be ignored.
- If Reynolds number is small enough, which means that viscosity of fluid is large enough, the change of Péclet number is trivial. It is concluded that the energy dissipation of vibration caused by thermal conduction can be neglected under three conditions: isothermal nanosphere condition, insulated nanosphere condition and highly viscous condition.
- The radiation of sound waves by thermal damping in thermoelastic material and incompressible thermodynamic fluid restricts the upper value of the quality factor under inviscid condition of this system.

The modeling of fluid media is simplified by incompressibility for the convenience of calculation. However, compressibility in the fluid has a considerable effect on the attenuation [13]. For further studies, it is important to compare different fluid medium, such as incompressible viscous fluid, compressible inviscid fluid and thermoviscous fluid. The size effect cannot be ignored when the radii of nanoparticles reach a certain limit. It is also important to take into account small size effect by the use of models like nonlocal strain gradient theory [19]. In addition, the thickness of fluid is also important by considering the energy consumption of it.

Declaration of interests

The authors do not work for, advise, own shares in, or receive funds from any organization that could benefit from this article, and have declared no affiliations other than their research organizations.

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