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Tribute to an exemplary man: Yves Couder

Bouncing drops, memory

Wave-particle duality: tribute to the memory of Yves Couder

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Abstract. The story of wave-particle duality in quantum mechanics has been told a thousand times. This modest contribution is to get back to a few points, including memories of a discussion we had on a possible experimental evidence of quantum effects at macroscopic scale, but in a way totally unrelated to the experiments he and his group made quite later to exhibit a model of wave-particle duality. The more general question of interpreting/understanding quantum mechanics will be also scrutinized once more.

Keywords. Yves Couder, Quantum mechanics, Fluid mechanics, Capillarity.

1. Introduction

This contribution is dedicated to the memory of Yves Couder, whence its rather personal tone and slightly unusual style. Nevertheless it concerns a point of science, wave-particle duality, that was completely renewed by Yves Couder and that led many people, including this author, to think again to this fairly non trivial question of modern physics.

Duality is a word and even a concept quite foreign to exact sciences (by “exact” is meant here sciences having been mathematized over the years like physics has). On the contrary, duality is a word fairly common in philosophy and related fields: body/soul, evil/good etc. Somehow the very idea of duality goes against exactness: something has only one unique existence, if it is singularized as an object, a being or even an idea. The trick of quantum mechanics is that the singular existence of objects is lost: one has to manipulate conceptually objects existing or not existing at the same time. Let us take a photon as such an object and consider the spontaneous decay of an atom from an excited state to the ground state. As well known this is accompanied by the emission of a photon and this emission occurs at random times by a Poisson process whose rate was computed in 1927 by Dirac [1] at the age of 24. The conclusion of Dirac’s subtle calculation was that the electromagnetic field emitted by the atom is made of a photon having a probability of existence per unit time. Such a probability of existence clashed with the standard view of classical deterministic physics where things exist or not. There is an apparent paradox there, as often noticed, because the solution of deterministic dynamical equations with well defined initial conditions ends up by predicting a phenomenon with a probability of occurrence,

probability being a notion opposite to determinism. The full understanding of this paradox is due to a graduate of Princeton University, Hugh Everett, who realized [2] that every photon with a certain probability of presence lives so-to-say in its own Universe, separated of other Universes where the photon was emitted at a different time. This existence of many Universes eliminates conceptual difficulties which would come for instance from the possibility of harnessing the energy of photons in different Universes if so to say one could navigate from one Universe to the other, and so violate the principle of conservation of energy. Somehow this gave a clear understanding of a deep result in quantum mechanics. Besides the case considered by Dirac it remains hard to deal with situations slightly more complex than the one of an atom initially in an excited state decaying spontaneously to the ground state. The first part of a recent book [3] deals with this subject in a well defined case. More details can be found there by an interested reader, the main lines being exposed below.

One of the major difficulty with the statistical interpretation of quantum mechanics is that, formally, this theory is deterministic, that is it deals, like classical mechanics, with evolution equations (at least in the non relativistic case) predicting the future state of the system (I use here this vague word of “system”) supposing the state of this system (practically its wave function) known at an initial time. So the big difficulty is how to derive from such a deterministic theory a probabilistic one? “Practically” means there that from the base equation one has to derive a dynamical equation for a probability distribution. In the reference just given this derivation is worked out in a specific problem of atomic physics that has its own interest. The starting point is an atom existing in two different states, one excited and the ground state. This atom is illuminated by a classical light beam at the frequency difference between the two atomic states. In the realistic limit where the intensity of the beam is not too big, the state of the atom makes regular oscillations, called optical Rabi oscillations between the two possible quantum states. So its state is a linear superposition of the two possible states in the form $\Psi = \cos(\varphi(t))\Psi_0 + i\sin(\varphi(t))\Psi_1$ where Ψ_0 and Ψ_1 are the eigenfunctions of the ground state and of the excited state. Because those states are orthogonal, and because one assumes that they are normalized to one, the norm of Ψ as given by this formula is

$$|\Psi|^2 = \cos^2(\varphi(t))|\Psi_0|^2 + \sin^2(\varphi(t))|\Psi_1|^2 = 1.$$

Therefore the state of the atom, even though it depends on time, remains normalized to one. The phase $\varphi(t)$ grows linearly with time, according to the law:

$$\varphi(t) = \Omega t$$

with $\Omega = |d|\mathcal{E}/2\hbar$ where $|d|$ is the electric dipole computed between the states Ψ_0 and Ψ_1 , and \mathcal{E} is the amplitude of electric field of the incoming classical wave. This describes the Rabi oscillations between the two atomic levels. This picture of the Rabi oscillations must be changed because, when the atom is in the state Ψ_1 , it can decay spontaneously to the ground state Ψ_0 by emission of a photon at the frequency difference between the frequencies of the atomic oscillations, as it results from the Planck–Einstein law. This process of emission of photons has been described precisely by Dirac, as said before. Dirac theory shows that this emission takes place with a small rate, because of the relatively weak coupling between electrons and electromagnetic waves which depends on the small constant of fine structure. The mathematical image of this process cannot be caught by ordinary perturbation theory because the transition is from state Ψ_0 to state Ψ_1 that differ by a finite amount. The correct way to describe this statistically is to use Kolmogorov method for dealing with rare but finite amplitude jumps, a way to put in a coherent framework the result of Dirac calculation of the rate of transition. This yields a Kolmogorov-like equation for the probability distribution $p(\varphi, t)$ that describes the evolution by both the deterministic Rabi

oscillations and the random jumps from state the excited state to the ground state by emission of photons. This equation reads:

$$\partial_t p + \Omega \partial_\varphi p = \gamma \left(\delta(\sin(\varphi)) \left(\int_{-\pi/2}^{\pi/2} d\varphi' p(\varphi', t) \sin^2(\varphi') \right) - p(\varphi, t) \sin^2(\varphi) \right),$$

where ∂_t is for the partial derivative with respect to time and γ for the rate of decay of the excited state to the ground state by spontaneous emission of a photon. The fundamental object of this theory is a probability distribution in the classical sense (a real positive function) normalized as

$$\int_{-\pi/2}^{\pi/2} d\varphi p(\varphi, t) = 1.$$

The left-hand side of the dynamical equation is for the deterministic part of the evolution including the Rabi oscillations whereas the right-hand side is for the effect of the random quantum jumps to the ground state accompanied by the emission of a photon. A detailed analysis of solutions of this equation as well as the way to get from it the fluctuations of the fluorescence light can be found in the first part of the book [3]. The relationship between this statistical picture and Everett' idea is that the statistics is carried over all possible Universes, which are infinitely many in the realistic limit where the rate of transition is far smaller than the frequency of the emitted light. A last word on the origin of stochasticity in this problem: it is a consequence of the randomness of the time of emission of a photon by the excited atom. This randomness is a consequence of the existence of infinitely many degrees of freedom of the electromagnetic field in free space as it results from Dirac's calculation of the rate of emission. Without such an infinite number (namely a continuous spectrum) there is no stochasticity in quantum processes.

Although this may look a bit trivial, compared to other theories for this physical process, the present one has the advantage of conserving the total probability at all times and to keep the positiveness of this probability. Moreover it allows to compute fluctuations, something that is very important for describing the fluorescence observed with three levels, one having a very small rate of decay. As imagined and observed by Dehmelt this yields long periods without fluorescence when a single ion is submitted to a resonant radiation [4, 5].

Another non trivial question to answer is how to explain the derivation of a time irreversible dynamics from the formally time reversible quantum dynamics of electrons interacting with electromagnetic radiation. Here this answer is not as complicated as it is in classical many body systems, no need of a kind of Stosszahlansatz. The irreversibility is already in Dirac' calculation of the rate of transition: this assumes that the state of the electromagnetic field are initially unoccupied, so that the emission process is irreversible because it amounts simply to put a photon in the radiation state, something that is already in the classical calculation of Hertz of the radiation of an oscillating dipole: the field has to satisfy the radiation condition at infinity, which is where the difference between the two possible directions of time is introduced. This amounts to make an assumption about the initial condition that exists in our Universe.

The wave-particle duality brings us back to the understanding of what is a wave and what is a particle. Newton's Principia are universally seen as the book that revolutionized our understanding of the world. This claim is, rightly, associated to Newton's elucidation of the laws of dynamics and to his derivation of Kepler's laws for the orbits of the planets. It is less well known that Newton gave also the first account of what is a wave in fluid mechanics. Before that waves and moving objects were not seen as different. Newton showed that waves are a kind of ordered motion of a material medium. He applied this idea to water waves and showed that their velocity is proportional to the square root of their wavelength (in modern terms). He considered also sound waves and showed that their speed is the square root of the compressibility of the medium where they propagate. Newton made experiments to measure the speed of sound in air and found results

in rough agreement between the measured velocity and the square root of the isothermal compressibility of air, a result corrected by Laplace who showed that the adiabatic compressibility should replace the isothermal compressibility, the first (highly non trivial) result of the nascent science of thermodynamics, a science unknown to Newton. As an aside let us come back to the photon. Waves need (in Newton's approach) a physical substrate obeying his laws of dynamics. Maxwell realized there are waves described by his equations of electrodynamics without an obvious mechanical substrate. Because of that he imagined a clever mechanical model, perhaps not unrelated to the one of "wave-particle" duality of de Broglie and followers.

The other side of the wave-particle duality, the particles, is less easy to put in a historical framework. It is hard to trace back the idea of point particle. When Newton derived Kepler's laws from his dynamical theory and from the law of universal attraction, he took planets as points with a given mass. This is correct because their radius is far smaller than their distance to the Sun. At the end of his *Opticks* Newton outlines his view of the small scale structure of matter, made of particles interacting by a potential, presumably (although this is not said so clearly) being point wise. Although he had not invented it, Lucretius in classical antiquity brought forward the idea of atom in his poem "De natura rerum". He even suggested a way [6] to put atoms in evidence in air with something resembling Brownian motion of dust lighted by the Sun. He interpreted this motion as originating from collisions with atoms moving around randomly. Amazingly this idea (in modern terms use Brownian motion to get the Avogadro number) was realized by Jean Perrin in the early years of the twentieth century, following a suggestion by Einstein.

2. Quantum mechanics and fluid mechanics

The work of Yves Couder on the experimental modeling of quantum mechanics can be seen as belonging to the long and fruitful relationship between fluid mechanics and quantum mechanics.

One can see the starting point of this history in what is called the Kelvin model of the atom. Even though this model has only an historical interest nowadays and is quite forgotten it is worth pointing out some of its interesting features. It was suggested by Lord Kelvin as a possible explanation for the existence of atoms. Atoms and molecules exist in manifestly different forms (or states), but belong nevertheless to a finite number of types. Kelvin imagined those atoms as made by steady configurations of vortices in a "fundamental" fluid existing at small scales. Hicks (whithout referring explicitly to Kelvin) managed to write the equation satisfied by an axisymmetric vortex configuration with swirl, an equation extending one written down by Stokes in the case without swirl. This is for the stream function of the incompressible flow in the (r, z) coordinates (r radial distance to the symmetry axis and z coordinate along this axis). There is an azimuthal flow (the "swirl") but no dependence on the azimuthal angle. One assumes an inviscid and incompressible flow. The real valued stream function, denoted as $\Psi(r, z)$, obeys the Hicks equation [7]:

$$\left(\frac{\partial^2}{\partial z^2} + \frac{\partial^2}{\partial r^2} - \frac{1}{r} \frac{\partial}{\partial r} \right) \Psi - r^2 \frac{dH}{d\Psi} + B \frac{dB}{d\Psi} = 0,$$

with $H(\cdot)$ and $B(\cdot)$ are arbitrary functions of the stream function Ψ for the flow in the (r, z) half plane (without boundary conditions).

One can choose (as did Hicks) H and B to make Hicks equation linear and decompose in Fourier transform along the z -direction (not done by Hicks). This leaves a free weight function of the wavenumber along z . Hicks introduces now a parameter in the function $B(\Psi)$ and shows a kind of quantization of the solution as this parameter changes. This is not a quantization in the sense of Schroedinger equation. It is more a kind of periodic oscillation of the solution

as this parameter changes. The interesting part of Hicks paper is on the supposition he makes on the connections between the behavior of solution as this parameter changes and the periodicity of properties of the chemical elements (metal, non metal, etc.). Seen in the light of our present understanding of the atomic structure of matter this is weird, but not without interest. The time (1898) where this paper was written was before any serious attempt was made to mathematize the principles of quantum mechanics, at least for the structure of atoms. It reminds one of what Hilbert said about the quantization (in front of Schroedinger and Pauli): you should look for a differential equation, eigenvalues are always associated to a PDE! Even though Kelvin model of atom is completely out of favor in atomic physics, the (non trivial) Hicks equation keeps an interest in fluid mechanics, because its localized solutions make a seed for self-similar solutions of the Euler equations blowing-up in finite time. The existence of solutions of fluid equations blowing-up self-similarly in time was imagined first by Leray [8], and such solutions are associated [9] to the phenomenon of intermittency observed in fully developed turbulence.

Reviewing the connections between quantum mechanics and fluid mechanics, one cannot forget the experiment done by G. I. Taylor in the early years of the twentieth century [10], an experiment that played a role in the early development of quantum mechanics, but without making any scientific relationship between quantum mechanics and fluid mechanics. Such a connection was made much later by Couder and Fort [11, 12] in their model of bouncing droplets. At the time of Taylor's experiment the relation between the two types of mechanics was only through the man who did it and became later a famous fluid mechanistic. Taylor's paper, when read today, makes one wonder a little bit. Although it was written in 1908 no allusion to the idea of quantum of energy, already known (most likely to J. J. Thomson who advised G. I. Taylor) and published in 1900 by Planck to represent the black body radiation spectrum, and used by Einstein in 1905 to explain the photoelectric effect. The Taylor paper is however interesting: he illuminated a photographic plate uniformly with a light beam of constant intensity and changed the intensity of this beam by putting more and more identical absorbing screens with the same attenuating factor. With an attenuation by a factor of two (for instance) he checked that the photographic plate looked about the same when exposed twice longer. Somehow this was a continuation of the earlier work by Hertz who had shown that the photocurrent given by illuminating a plate is proportional to the intensity of the UV light responsible for the photoelectric effect, but that the voltage created is independent of this intensity. This is explained by the fact that this photocurrent is just a manifestation of singular events (the so-called quantum jumps) occurring randomly in time at an average frequency proportional to the intensity of the light beam. As said before the precise connection between this observation and the mathematical theory of quantum mechanics was the work of Dirac.

Perhaps a little arbitrarily, the next "relation/connection" between quantum mechanics and fluid mechanics is related to Heisenberg. The story has been told many times and I heard it from Aage Bohr who heard it from Heisenberg himself. Heisenberg, a very precocious student, had Sommerfeld as advisor. His PhD topic was the linear stability of the Poiseuille parallel flow [13]. Because the velocity field parallel to the plates has no inflection point it is linearly stable according to the Rayleigh stability criterion. Sommerfeld suggested Heisenberg to look at the same stability problem, but with viscosity included, that is to replace the Rayleigh equation by the Orr-Sommerfeld equation, linear but fourth order in order of derivation (instead of second order for the inviscid Rayleigh case), this taking into account the effect of viscosity. Heisenberg, with an amazing mathematical virtuosity, did look at the limit of a large Reynolds number of the Orr-Sommerfeld equation and showed the existence of two branches of unstable solutions in the plane α (wave number of perturbation along the flow), Re (Reynolds number). This was obtained by performing a kind of WKB expansion and a boundary layer analysis, because the relevant perturbations are localized near the boundaries. This amazing result (stability without friction

and instability with friction) took many years to be understood, partly because Heisenberg never went back to this work to explain what he had done. It showed however that mathematical virtuosity trained in fluid mechanics was helpful to do quantum mechanics!

The last example of interaction between quantum mechanics and fluid mechanics I have chosen is Landau's theory of superfluidity [14]. This requires a synthesis of both theories of quantum mechanics and of fluid mechanics. Contrary to what is often said, Landau assumes nothing on the statistics of the particles in the superflow and does not refer to Bose–Einstein condensation. The particles participating to the superflow can be either bosons or fermions. This remark is meaningful because one can explain the quantization of the circulation [15] and of the magnetic flux in supra conductors [16] without reference to the statistics of the fermions making them and without referring to pairing (in the latter example), quite hard to pinpoint in high T_c supra conductors by the way! Extending Onsager's explanation of the quantum of circulation in superfluids one can show that the quantum of magnetic flux is divided by two in superconductors (compared to a spinless case) because the ground state has the symmetry group $SU(2) \times U(1)$ where $SU(2)$ is for the rotation of the spin 1/2 part of the wave function and $U(1)$ for the symmetry under changes of the phase of the full wave function. Landau's theory of superfluidity shows that for spinless quantum particles near their ground state a superflow is represented by a slow variation in space and time of the global phase of the quantum state, this phase playing the same role as the velocity potential in the Bernoulli version of the inviscid Euler equation. The equations for a superfluid follow from that. The full Landau's equations includes also the dynamics of the gas of thermal excitations, namely the normal fluid. The final result is a set of equations for the normal fluids and superfluids, coupled both in the bulk and by the boundary conditions, since on boundaries normal fluid and superfluid may transform into each other.

3. A dinner at Chambord

Chambord castle is a beautiful Renaissance castle built 500 years ago in the heart of France by King François the first. There is a belief that it was at least partly designed by Leonardo, living nearby at this time. Since the time of its building the castle and the surrounding land have been the property of the French state. A meeting on Chaos and Complexity was held in the nearby city of Blois, by the Loire River and the conference dinner was in Chambord castle. We sat with Yves Couder near each other at this dinner and I had a rather unique opportunity to talk to him at length. Our talk was on quantum uncertainty in “macroscopic” world. How big is the spreading induced in the real world by Everett's multiworlds? A point in passing: this conference was held in June 1993 and the big scientific news of the day was the solution of Fermat Conjecture by Andrew Wiles, an announcement that went straight against the claim of one of the talks in Blois predicting (or claiming?) that Fermat conjecture belongs to the class of undecidable mathematical statements (as is well known Wiles's proof left some gaps and two more years of hard work by mathematicians were necessary to fill them).

A point of our discussion with Yves Couder was the idea of amplification of quantum fluctuations. Such an amplification takes place in the Schroedinger's cat example: a change of polarization of tiny emitted quantum particles yields at the end a big change in the state of the whole surrounding: the cat is alive or dead. Somehow this amplification relies on the nonlinearity (in the classical sense) of all the devices leading from the tiny quantum fluctuation to a big macroscopic change in the state of the cat. Indeed in Schroedinger's cat gedanken experiment, the outcome is somehow restricted to an indeterminacy of the immediate future. It is of interest to think to quantum fluctuations having even far bigger consequences than the killing of a cat. Such amplification in real world should concern, for instance, the sex of newborns in royal families of medieval Europe. Think if Empress Matilda (1102–1167), grand-daughter of William the Conqueror, had been

a man: this man could have united the Holy German Empire and the Kingdom of Normandy and England, likely changing the future of Europe and bringing very bad news for medieval France. However such an amplification of an initial quantum fluctuation is slightly ambiguous: Empress Matilda could have been a boy for other reasons than quantum fluctuations, any kind of thermal or chemical fluctuation could have changed the sex of the baby. There is however a deep conceptual difference between quantum and non-quantum fluctuations: the non-quantum fluctuations occur in a unique history and, in this unique history, Empress Matilda is never an Emperor whereas there is another Universe in the sense of Everett where an Emperor is born instead of an Empress and likely changes the course of history with respect to what happened over the last thousand years, all this because of the quantum fluctuations! But the two futures “exist” in two unrelated Universes.

This talk in Chambord was long before Yves Couder got his idea of modeling quantum mechanics with fluids. From what I recall, we tried to figure out a way of putting in evidence quantum fluctuations in small worms, all genetically identical but able to make binary choices. The idea was to relate the uncertainty in those binary choices to a kind of amplification of quantum fluctuations. I suspect that this was not a very realistic experiment. Can one measure freedom of choice? We thought of worms because they can be all identical and (presumably) without prejudice in their decision. Planck's constant h is very small and so to put in evidence quantum fluctuations one needs to amplify them hugely as in Schrodinger's cat experiment. So to speak, the multiverses of Everett are disconnected from each other and there is no way to know what happens in another Universe: quantum fluctuations do mix up with “ordinary” fluctuations described by classical Gibbs–Boltzmann thermodynamics.

However the disconnection of Everett's Universes is not absolute, just because the dynamics of those worlds are described by continuous equations. In the case of emission of photons for instance, there is a small overlap of different Universes during the short time of the emission, about the period of the light wave. Yves Couder has imagined [11, 12] a clever way to make this time slightly longer by sending back the photon to the atom. Somehow the reabsorption of the photon by the atom is a way for the system to return to its initial excited state by combining two different “Everett's universes”, the one where the emission took place and the other where it did not. The net effect of such an experiment is that the lifetime of the excited state will be longer if emitted photons can be reabsorbed. By itself it is not a very spectacular result and also an expected one. However if, during the very short journey of the photon after it is emitted some event takes place outside of the photon-atom system, the Universes branching at the first photon emission will be in different Everett's universes. This is slightly paradoxical, but not unexpected: different Universes are branching off all the time because quantum jumps happen all the time. All this points to a more restricted definition of an Universe in the sense of Everett. Such a Universe has to have, as any notion in physics, a definition that can be compared with possible measurements. If one defines as an Universe the quantum state of a macroscopic object, a living being or a piece of wood, this object has many degrees of freedom interacting all the time. Its dynamics is “classical” (meaning non quantum) so to speak if the time needed for the quantum phase to change by a finite amount is shorter than the time needed for a change of its macroscopic state. Let δt_c be the time of quantum coherence and δt_h for the time of macroscopic change by thermal fluctuations. The time δt_c is related to the energy fluctuation δE at the temperature T of the system. This is $\delta E \sim N^{1/2} k_B T$ where N is the number of degrees of freedom and k_B Boltzmann constant. Therefore $\delta t_c \sim \hbar / (N^{1/2} k_B T)$ because the energy fluctuation is the source of dephasing of the quantum state \hbar being 2π times Planck's constant. The time δt_h is estimated very simply by assuming that this macroscopic time is ruled by a diffusion process of diffusivity κ , which yields $\delta t_h \sim (1/\kappa)(N/n)^{2/3}$, where the size of the system under consideration is about $(N/n)^{1/3}$, n being its number density (one may think for

instance to a tiny droplet of number density n). Therefore the two time scales δt_c and δt_h are of the same order of magnitude if

$$N^{7/6} \sim \frac{\kappa \hbar n^{2/3} \kappa}{k_B T}.$$

This makes sense only if N is at least of order one or bigger. This may be hard to satisfy because of the extreme smallness of Planck's constant. However by looking at very small temperatures one could perhaps reach a domain where N as given by the above formula is macroscopic, or at least somewhat bigger than 1. In this domain it is, at least in principle, possible to do interferences between the "macroscopic" objects with N smaller than the value so derived.

4. Summary and conclusion

The exposition given above was like a journey in a subfield of the research by Yves Couder. It was not designed to be complete in even one of his subfields. However I hope that some of its inspiration is true to the message he left us, a message that non trivial ideas and new discoveries can show up in almost any field, irrespective of the way those fields are classified in organized science: classical physics, quantum physics, botany, etc. What matters most is to bring new relevant ideas in whatever field one does research, as Yves Couder did so well.

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