Supporting online material for:

Cloaking by Plasmonic Resonance among Systems of Particles: Cooperation or Combat?

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I. FIELD IDENTITY

We consider the problem of a finite cluster of identical coated cylinders with the core radius r_c and the shell radius r_s, having the core permittivity ε_c and the shell permittivity ε_s, embedded in a matrix (background) of permittivity ε_b. The cylinders are subjected to a uniform electric field sourced at infinity and the field of a polarizable line dipole added to the system so that it becomes a source of electric field. Fig. 1 shows a pair of nearest neighbours in the cluster.

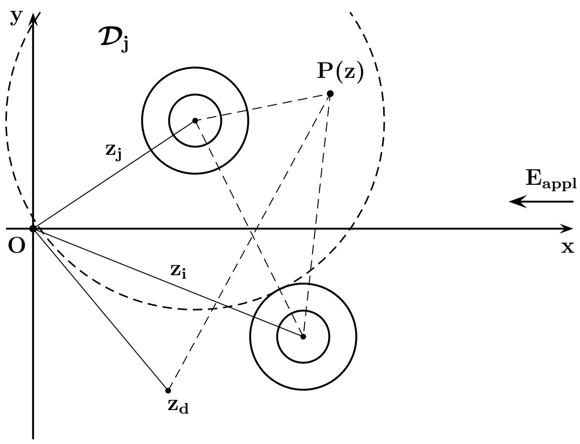

FIG. 1: Two coated cylinders, placed at the points z_1 and z_2, and a polarizable line dipole at z_d. $\mathbf{E}_{\rm appl}$ is an electric field with sources at infinity. The annular domain $\mathcal{D}_j$, between the shell of the cylinder j and the dashed circle, is chosen so that it contains no other cylinder or field sources. P(z) is an arbitrary point in the plane where the total electric potential is evaluated.

For a physical system comprising a cluster of N coated cylinders, a dipole of magnitude $\mathbf{d} = (k^{(e)}, k^{(o)})$ located at $z_d \neq 0$, and a uniform field $\mathbf{E}$, the analytic potential in the matrix is given by the Wijngaard type series [1]

$$f_b^{(p)}(z) = C^{(p)} + E^{(p)}\, z + q\, \frac{k^{(p)}}{z - z_d} + q \sum_{i=1}^{N} \sum_{\ell=1}^{\infty} \frac{B_\ell^{(p,i)}}{(z - z_i)^\ell}\,. \tag{1}$$

Here, the cylinders are represented by multipole series, $p = e, o$, and $q = 1$ if $p = e$, and $q = -1$ if $p = o$ [2, 3]. The complex potential is obtained from the formula [2, 3]

$$V_b(z) = [f_b^{(e)}(z) + f_b^{(e)}(\overline{z})]/2 + [f_b^{(o)}(z) - f_b^{(o)}(\overline{z})]/(2\mathrm{i})\,, \tag{2}$$

where $\mathrm{i} = \sqrt{-1}$ and the superposed bar denotes complex conjugation. Note that $f_b^{(p)}(\overline{z})$ means to take the complex conjugates of all complex coordinates z, z_d, and z_i.

Note the q in front of $k^{(p)}$ in (1). With this, applying the usual formula of the type (2) to the complex function

$$f_d^{(p)}(z, z_d) = q\,\frac{k^{(p)}}{z - z_d}\,,$$

we obtain

$$\begin{aligned} V_d(x, y, x_d, y_d) &= \frac{1}{2}\left[f_d^{(e)}(x + \mathrm{i}\,y, x_d + \mathrm{i}\,y_d) + f_d^{(e)}(x - \mathrm{i}\,y, x_d - \mathrm{i}\,y_d)\right] \\ &\qquad + \frac{1}{2\mathrm{i}}\left[f_d^{(o)}(x + \mathrm{i}\,y, x_d + \mathrm{i}\,y_d) - f_d^{(o)}(x - \mathrm{i}\,y, x_d - \mathrm{i}\,y_d)\right] \\ &= \frac{k^{(e)}(x - x_d) + k^{(o)}(y - y_d)}{(x - x_d)^2 + (y - y_d)^2}\,, \end{aligned} \tag{3}$$

which is the classical formula for the potential of a line dipole of magnitude $\mathbf{d} = (k^{(e)}, k^{(o)})$, located at $z_d = x_d + \mathrm{i}\,y_d$ [4]. Also, using $q\,k^{(p)}$ makes $k^{(p)}$ similar to the multipole coefficients $B_\ell^{(p,i)}$, so we are using systematically q for terms involving negative powers of z, $z - z_i$ etc.

Let j be a cylinder in the cluster and $\mathcal{D}_j$ an annular region free of any field sources or other cylinders/inclusions. For $z \in \mathcal{D}_j$ and inside the cylinder j, we may write the complex potential in terms of the analytic potentials

$$\begin{aligned} f_{\mathrm{out}}^{(p,j)} &= \sum_{\ell=1}^{\infty} A_\ell^{(p,j)}\,(z - z_j)^\ell\,, \quad f_{\mathrm{in}}^{(p,j)} = q\sum_{\ell=1}^{\infty}\frac{B_\ell^{(p,j)}}{(z - z_j)^\ell}\,, \\ \underline{f}_{\mathrm{out}}^{(p,j)} &= \sum_{\ell=1}^{\infty} C_\ell^{(p,j)}\,(z - z_j)^\ell\,, \quad \underline{f}_{\mathrm{in}}^{(p,j)} = q\sum_{\ell=1}^{\infty}\frac{D_\ell^{(p,j)}}{(z - z_j)^\ell}\,, \\ \underline{f}_{\mathrm{c}}^{(p,j)} &= \sum_{\ell=1}^{\infty} E_\ell^{(p,j)}\,(z - z_j)^\ell\,. \end{aligned} \tag{4}$$

Consequently, for $z \in \mathcal{D}_j$ we construct the complex potentials

$$V_{\mathrm{out}}^{(j)} = [f_{\mathrm{out}}^{(e)}(z) + f_{\mathrm{out}}^{(e)}(\overline{z})]/2 + [f_{\mathrm{out}}^{(o)}(z) - f_{\mathrm{out}}^{(o)}(\overline{z})]/(2\mathrm{i})\,, \tag{5}$$

$$V_{\mathrm{in}}^{(j)} = [f_{\mathrm{in}}^{(e)}(z) + f_{\mathrm{in}}^{(e)}(\overline{z})]/2 + [f_{\mathrm{in}}^{(o)}(z) - f_{\mathrm{in}}^{(o)}(\overline{z})]/(2\mathrm{i})\,, \tag{6}$$

the total complex potential in $\mathcal{D}_j$ being

$$V_{\mathcal{D}_j}(z) = \widetilde{A}_0^{(j)} + V_{\mathrm{out}}^{(j)}(z) + V_{\mathrm{in}}^{(j)}(z)\,, \tag{7}$$

where $\widetilde{A}_0^{(j)}$ is a constant to be determined. Also, in terms of analytic potentials we have

$$f_{\mathcal{D}_j}^{(p)}(z) = A_0^{(p,j)} + f_{\mathrm{out}}^{(p,j)}(z) + f_{\mathrm{in}}^{(p,j)}(z)\,, \tag{8}$$

which has to be identical to $f_b^{(p)}(z)$ for $z \in \mathcal{D}_j$. Hence, $f_b^{(p)}(z)$ is the analytical continuation of $f_{\mathcal{D}_j}^{(p)}(z)$ in the matrix. Consequently, for $z \in \mathcal{D}_j$ we have

$$A_0^{(p,j)} + \sum_{\ell=1}^{\infty}\left[A_\ell^{(p,j)}\,(z - z_j)^\ell + q\,\frac{B_\ell^{(p,j)}}{(z - z_j)^\ell}\right] = C^{(p)} + E^{(p)}\,z + q\,\frac{k^{(p)}}{z - z_d} + q\sum_{i=1}^{N}\sum_{\ell=1}^{\infty}\frac{B_\ell^{(p,i)}}{(z - z_i)^\ell}\,, \tag{9}$$

or

$$A_0^{(p,j)} + \sum_{\ell=1}^{\infty} A_\ell^{(p,j)}\,(z - z_j)^\ell = C^{(p)} + E^{(p)}\,z + q\,\frac{k^{(p)}}{z - z_d} + q\sum_{i\neq j}^{N}\sum_{\ell=1}^{\infty}\frac{B_\ell^{(p,i)}}{(z - z_i)^\ell}\,. \tag{10}$$

The constant $C^{(p)}$ can be set so that the potential at the origin of coordinates is zero ($f_b^{(p)}(0) = 0$):

$$C^{(p)} = q\,\frac{k^{(p)}}{z_d} - q\sum_{i=1}^{N}\sum_{\ell=1}^{\infty}\frac{B_\ell^{(p,i)}}{(-z_i)^\ell}\,. \tag{11}$$

Now, introduce the notations $\zeta_j = z - z_j$, $z_{jd} = z_d - z_j$, and $z_{ji} = z_i - z_j$, and note that for $z \in \mathcal{D}_j$ we have $|\zeta_j| < |z_{jd}|$ and $|\zeta_j| < |z_{ji}|$, $\forall i \neq j$ (see Fig. 1), so that we may use the series expansions

$$\frac{1}{\zeta_j - z_{jd}} = -\frac{1}{z_{jd}} - \frac{1}{z_{jd}} \sum_{s=1}^{\infty} \left(\frac{\zeta_j}{z_{jd}}\right)^s , \tag{12}$$

$$\frac{1}{(\zeta_j - z_{ji})^\ell} = \frac{(-1)^\ell}{z_{ji}^\ell} \sum_{s=\ell-1}^{\infty} \begin{pmatrix} s \\ \ell-1 \end{pmatrix} \left(\frac{\zeta_j}{z_{ji}}\right)^{s-\ell+1} = \frac{(-1)^\ell}{z_{ji}^\ell} + \frac{(-1)^\ell}{z_{ji}^\ell} \sum_{s=\ell}^{\infty} \begin{pmatrix} s \\ \ell-1 \end{pmatrix} \left(\frac{\zeta_j}{z_{ji}}\right)^{s-\ell+1} , \tag{13}$$

to rewrite (10) in the form

$$A_0^{(p,j)} + \sum_{\ell=1}^{\infty} A_\ell^{(p,j)} \zeta_j^\ell = C^{(p)} + E^{(p)}(\zeta_j + z_j) - q \frac{k^{(p)}}{z_{jd}} \sum_{\ell=0}^{\infty} \left(\frac{\zeta_j}{z_{jd}}\right)^\ell + q \sum_{i\neq j}^{N} \sum_{m=1}^{\infty} \sum_{s=m-1}^{\infty} B_m^{(p,i)} \frac{(-1)^m}{z_{ji}^m} \begin{pmatrix} s \\ m-1 \end{pmatrix} \left(\frac{\zeta_j}{z_{ji}}\right)^{s-m+1} . \tag{14}$$

The zeroth order term is

$$\begin{aligned} A_0^{(p,j)}(z_j, z_{jd}, z_{ji}) &= C^{(p)} + E^{(p)} z_j - q \frac{k^{(p)}}{z_{jd}} + q \sum_{i\neq j}^{N} \sum_{\ell=1}^{\infty} B_\ell^{(p,i)} \frac{(-1)^\ell}{{z_{ji}}^\ell} \\ &= E^{(p)} z_j + q k^{(p)} \left[\frac{1}{z_d} - \frac{1}{z_{jd}}\right] + q \sum_{i\neq j}^{N} \sum_{\ell=1}^{\infty} B_\ell^{(p,i)} \frac{(-1)^\ell}{{z_{ji}}^\ell} - q \sum_{i=1}^{N} \sum_{\ell=1}^{\infty} \frac{B_\ell^{(p,i)}}{(-z_i)^\ell} . \end{aligned} \tag{15}$$

Taking into account that

$$V_{\mathcal{D}_j}(z) = \frac{1}{2}\left[f^{(e)}_{\mathcal{D}_j}(z) + f^{(e)}_{\mathcal{D}_j}(\overline{z})\right] + \frac{1}{2\mathrm{i}}\left[f^{(o)}_{\mathcal{D}_j}(z) - f^{(o)}_{\mathcal{D}_j}(\overline{z})\right] ,$$

we may express the constant $\widetilde{A}_0^{(j)}$ from (7) in the form

$$\widetilde{A}_0^{(j)} = \frac{1}{2}\left[A_0^{(e,j)}(z_j, z_{jd}, z_{ji}) + A_0^{(e,j)}(\overline{z_j}, \overline{z_{jd}}, \overline{z_{ji}})\right] + \frac{1}{2\mathrm{i}}\left[A_0^{(o,j)}(z_j, z_{jd}, z_{ji}) - A_0^{(o,j)}(\overline{z_j}, \overline{z_{jd}}, \overline{z_{ji}})\right] . \tag{16}$$

For $m \geq 1$ we have

$$A_m^{(p,j)} = E^{(p)} \delta_{m,1} - \frac{q\, k^{(p)}}{{z_{jd}}^{m+1}} + q \sum_{i\neq j}^{N} \sum_{\ell=1}^{\infty} B_\ell^{(p,i)} \frac{(-1)^\ell}{{z_{ji}}^\ell} \begin{pmatrix} m+\ell-1 \\ \ell-1 \end{pmatrix} \frac{1}{{z_{ji}}^m} , \tag{17}$$

or

$$A_m^{(p,j)} - q \sum_{i\neq j}^{N} \sum_{\ell=1}^{\infty} B_\ell^{(p,i)} \frac{(-1)^\ell}{{z_{ji}}^{\ell+m}} \begin{pmatrix} m+\ell-1 \\ \ell-1 \end{pmatrix} = E^{(p)} \delta_{m,1} - q \frac{k^{(p)}}{{z_{jd}}^{m+1}} . \tag{18}$$

From the boundary conditions at the shell-matrix interface we have

$$B_m^{(p,i)} = \beta_m^{(i)} A_m^{(p,i)} , \tag{19}$$

where

$$\beta_m^{(i)} = \frac{{r_c^{(i)}}^{2m} ({\varepsilon_s}^{(i)} - {\varepsilon_c}^{(i)})(\varepsilon_b + {\varepsilon_s}^{(i)}) + {r_s^{(i)}}^{2m} ({\varepsilon_s}^{(i)} + {\varepsilon_c}^{(i)})(\varepsilon_b - {\varepsilon_s}^{(i)})}{{r_c^{(i)}}^{2m} ({\varepsilon_s}^{(i)} - {\varepsilon_c}^{(i)})(\varepsilon_b - {\varepsilon_s}^{(i)}) + {r_s^{(i)}}^{2m} ({\varepsilon_s}^{(i)} + {\varepsilon_c}^{(i)})(\varepsilon_b + {\varepsilon_s}^{(i)})} {r_s^{(i)}}^{2m} . \tag{20}$$

Here, $r_c^{(i)}$ and $r_s^{(i)}$ denote the core and shell radii of the cylinder i. Also, ${\varepsilon_c}^{(i)}$ and ${\varepsilon_s}^{(i)}$ are respectively the core and shell permittivities, while ε_b represents the permittivity of the matrix (background). Finally, by substituting Eq.(19) into Eq.(18) we obtain the linear system

$$A_m^{(p,j)} - q \sum_{i\neq j}^{N} \sum_{\ell=1}^{\infty} \frac{(-1)^\ell}{{z_{ji}}^{\ell+m}} \begin{pmatrix} m+\ell-1 \\ \ell-1 \end{pmatrix} \beta_\ell^{(i)} A_\ell^{(p,i)} = E^{(p)} \delta_{m,1} - q \frac{k^{(p)}}{{z_{jd}}^{m+1}} . \tag{21}$$

II. POLARIZABLE LINE DIPOLE

The equation (21) can be used when we know the magnitude of the dipole. In the case of a polarizable dipole, the magnitude of the dipole is determined by the total electric field at the point where the dipole is located. Thus, the total electric field at an arbitrary point z is $\mathcal{E} = -\nabla V_b(z)$ [4] , which is singular at $z = z_d$. Hence, we have to remove the dipole term from $V_b(z)$ or, equally well, from $f_b^{(p)}(z)$ defined in Eq.(1), and use

$$\widetilde{f}_b^{(p)}(z) = E^{(p)}\, z + q \sum_{i=1}^{N} \sum_{\ell=1}^{\infty} \frac{B_\ell^{(p,i)}}{(z - z_i)^\ell}\,. \tag{22}$$

Note that, we have also removed the constant which vanishes after differentiation. From (22) we derive the expression of the complex potential

$$\widetilde{V}_b(z) = \frac{1}{2}\left[\widetilde{f}_b^{(e)}(z) + \widetilde{f}_b^{(e)}(\overline{z})\right] + \frac{1}{2\mathrm{i}}\left[\widetilde{f}_b^{(o)}(z) - \widetilde{f}_b^{(o)}(\overline{z})\right], \tag{23}$$

and the total electric field at z_d

$$\mathcal{E} = -\,\nabla\, \widetilde{V}_b(z)\Big|_{z=z_d}\,. \tag{24}$$

Now,

$$\partial_x\, \widetilde{V}_b(z) \;=\; \frac{1}{2}\left[\widetilde{f}_b^{(e)\prime}(z) + \widetilde{f}_b^{(e)\prime}(\overline{z})\right] + \frac{1}{2\mathrm{i}}\left[\widetilde{f}_b^{(o)\prime}(z) - \widetilde{f}_b^{(o)\prime}(\overline{z})\right], \tag{25}$$

$$\partial_y\, \widetilde{V}_b(z) \;=\; \frac{\mathrm{i}}{2}\left[\widetilde{f}_b^{(e)\prime}(z) - \widetilde{f}_b^{(e)\prime}(\overline{z})\right] + \frac{1}{2}\left[\widetilde{f}_b^{(o)\prime}(z) + \widetilde{f}_b^{(o)\prime}(\overline{z})\right], \tag{26}$$

where

$$\widetilde{f}_b^{(p)\prime}(z) \;=\; E^{(p)} - q \sum_{i=1}^{N} \sum_{\ell=1}^{\infty} \frac{\ell\, B_\ell^{(p,i)}}{(z - z_i)^{\ell+1}}\,, \tag{27}$$

$$\widetilde{f}_b^{(p)\prime}(\overline{z}) \;=\; E^{(p)} - q \sum_{i=1}^{N} \sum_{\ell=1}^{\infty} \frac{\ell\, B_\ell^{(p,i)}}{(\overline{z} - \overline{z_i})^{\ell+1}}\,. \tag{28}$$

Therefore,

$$\partial_x\, \widetilde{V}_b(z_d) \;=\; E^{(e)} - \sum_{i=1}^{N} \sum_{\ell=1}^{\infty} \ell \left[B_\ell^{(e,i)}\, \mathrm{Re}\left(\frac{1}{{z_{id}}^{\ell+1}}\right) - B_\ell^{(o,i)}\, \mathrm{Im}\left(\frac{1}{{z_{id}}^{\ell+1}}\right)\right], \tag{29}$$

$$\partial_y\, \widetilde{V}_b(z_d) \;=\; E^{(o)} + \sum_{i=1}^{N} \sum_{\ell=1}^{\infty} \ell \left[B_\ell^{(e,i)}\, \mathrm{Im}\left(\frac{1}{{z_{id}}^{\ell+1}}\right) + B_\ell^{(o,i)}\, \mathrm{Re}\left(\frac{1}{{z_{id}}^{\ell+1}}\right)\right], \tag{30}$$

and

$$k^{(e)} = -\alpha\, \partial_x\, \widetilde{V}_b(z_d) \;=\; -\alpha\, E^{(e)} + \alpha \sum_{i=1}^{N} \sum_{\ell=1}^{\infty} \ell \left[B_\ell^{(e,i)}\, \mathrm{Re}\left(\frac{1}{{z_{id}}^{\ell+1}}\right) - B_\ell^{(o,i)}\, \mathrm{Im}\left(\frac{1}{{z_{id}}^{\ell+1}}\right)\right], \tag{31}$$

$$k^{(o)} = -\alpha\, \partial_y\, \widetilde{V}_b(z_d) \;=\; -\alpha\, E^{(o)} - \alpha \sum_{i=1}^{N} \sum_{\ell=1}^{\infty} \ell \left[B_\ell^{(e,i)}\, \mathrm{Im}\left(\frac{1}{{z_{id}}^{\ell+1}}\right) + B_\ell^{(o,i)}\, \mathrm{Re}\left(\frac{1}{{z_{id}}^{\ell+1}}\right)\right]. \tag{32}$$

Finally, by substituting $k^{(e)}$ and $k^{(o)}$ into (21) we obtain the linear system

$$\sum_{\ell=1}^{\infty}\left[\delta_{\ell m}+\frac{\alpha\,\ell}{{z_{jd}}^{m+1}}\mathrm{Re}\left(\frac{1}{{z_{jd}}^{\ell+1}}\right)\beta_\ell^{(j)}\right]A_\ell^{(e,j)}-\sum_{i\neq j}^{N}\sum_{\ell=1}^{\infty}\left[\frac{(-1)^\ell}{{z_{ji}}^{\ell+m}}\binom{m+\ell-1}{\ell-1}-\frac{\alpha\,\ell}{{z_{jd}}^{m+1}}\mathrm{Re}\left(\frac{1}{{z_{id}}^{\ell+1}}\right)\right]\beta_\ell^{(i)}\,A_\ell^{(e,i)}$$
$$-\sum_{i=1}^{N}\sum_{\ell=1}^{\infty}\frac{\alpha\,\ell}{{z_{jd}}^{m+1}}\mathrm{Im}\left(\frac{1}{{z_{id}}^{\ell+1}}\right)\beta_\ell^{(i)}\,A_\ell^{(o,i)}=\left(\delta_{m,1}+\frac{\alpha}{{z_{jd}}^{m+1}}\right)E^{(e)}\,, \tag{33}$$

$$\sum_{\ell=1}^{\infty}\left[\delta_{\ell m}+\frac{\alpha\,\ell}{{z_{jd}}^{m+1}}\mathrm{Re}\left(\frac{1}{{z_{jd}}^{\ell+1}}\right)\beta_\ell^{(j)}\right]A_\ell^{(o,j)}+\sum_{i\neq j}^{N}\sum_{\ell=1}^{\infty}\left[\frac{(-1)^\ell}{{z_{ji}}^{\ell+m}}\binom{m+\ell-1}{\ell-1}+\frac{\alpha\,\ell}{{z_{jd}}^{m+1}}\mathrm{Re}\left(\frac{1}{{z_{id}}^{\ell+1}}\right)\right]\beta_\ell^{(i)}\,A_\ell^{(o,i)}$$
$$+\sum_{i=1}^{N}\sum_{\ell=1}^{\infty}\frac{\alpha\,\ell}{{z_{jd}}^{m+1}}\mathrm{Im}\left(\frac{1}{{z_{id}}^{\ell+1}}\right)\beta_\ell^{(i)}\,A_\ell^{(e,i)}=\left(\delta_{m,1}-\frac{\alpha}{{z_{jd}}^{m+1}}\right)E^{(o)}\,. \tag{34}$$

This system can be written in matrix form as

$$\begin{bmatrix}\mathbf{M}^{(e)} & \mathbf{N}^{(o)}\\ \mathbf{N}^{(e)} & \mathbf{M}^{(o)}\end{bmatrix}\begin{bmatrix}\mathbf{A}^{(e)}\\ \mathbf{A}^{(o)}\end{bmatrix}=\begin{bmatrix}\mathbf{S}^{(e)}\\ \mathbf{S}^{(o)}\end{bmatrix}, \tag{35}$$

where $\mathbf{N}^{(e)}=-\mathbf{N}^{(o)}$, and

$$\mathbf{A}^{(p)}=\begin{bmatrix}\mathbf{A}^{(p,1)}\\ \mathbf{A}^{(p,2)}\\ \vdots\\ \mathbf{A}^{(p,N)}\end{bmatrix},\quad \mathbf{A}^{(p,j)}=\begin{bmatrix}\mathbf{A}_1^{(p,j)}\\ \mathbf{A}_2^{(p,j)}\\ \vdots\\ \mathbf{A}_{\mathrm{N_{trunc}}}^{(p,j)}\end{bmatrix},\quad \mathbf{S}^{(p)}=\begin{bmatrix}\mathbf{S}^{(p,1)}\\ \mathbf{S}^{(p,2)}\\ \vdots\\ \mathbf{S}^{(p,N)}\end{bmatrix},\quad \mathbf{S}^{(p,j)}=E^{(p)}\begin{bmatrix}1+q\,\alpha/z_{jd}^2\\ q\,\alpha/z_{jd}^3\\ \vdots\\ q\,\alpha/z_{jd}^{(\mathrm{N_{trunc}}+1)}\end{bmatrix},\quad p=e,o, \tag{36}$$

$\mathrm{N_{trunc}}$ being the truncation order for the series in ℓ and m.

Note that, in the case $E^{(e)}=E^{(o)}=0$ the linear system (33 – 34) becomes homogeneous and the solution is the trivial one, except the resonant states. Therefore, all the multipole coefficients vanish and there is no effect. Actually, with no external source the whole system polarizable dipole + cylinders is "in darkness" all the components being thus invisible.

The resonant states, if they exist, are interesting because the only parameters which can be varied to achieve the resonance effect are the polarizability of the dipole and the distances between the dipole and cylinders. The arrangement of cylinders could be important too.

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