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Supplementary Information for “Eddy-driven Zonal Jet Flows in the Laboratory”

Matériel supplémentaire pour “Jets Zonaux en Expériences de Laboratoire”

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1. Zonostrophy index estimates for gas giants and Earth's oceans and atmosphere

We calculate the zonostrophy index for planetary fluid layers in Table 2 of the main text from

$$R^s_\beta = \left(\frac{C_Z}{C_K}\right)^{3/10}\left(\frac{2^5U^5\beta}{\epsilon^2}\right)^{1/10}, \tag{1}$$

where we used $(C_Z/C_K)^{3/10} \approx 0.5$ (equation (16) in the main text). Parameters used for gas giants and Earth's oceans and atmosphere are provided in Table 1.

	Ω (rad/s)	R (km)	D (km)	β or β_T ($\mathrm{m^{-1}s^{-1}}$)	ϵ ($\mathrm{m^2s^{-3}}$)	U (m/s)	R^s_β	E
Earth Oceans	7.29×10^{-5}	6371	1	1.6×10^{-11}	10^{-9}	0.1	1.2	1×10^{-8}
Earth Atmosphere	7.29×10^{-5}	6371	10	1.6×10^{-11}	10^{-4}	15	1.4	2×10^{-9}
Jupiter	1.76×10^{-4}	69,911	2,000	7.6×10^{-12}	10^{-5}	50	3.9	4×10^{-16}
Saturn	1.66×10^{-4}	58,232	7,000	1.1×10^{-11}	2×10^{-6}	40	4.9	4×10^{-17}

Table 1. Parameters used for estimating the zonostrophy index of gas giants and Earth's oceans and atmosphere. ϵ values are taken from [1] (Table 13.1) for the Earth, from [2] for Jupiter and [3] for Saturn. Velocities are taken from [1]. The β-effect is estimated at a reference latitude of 45° using the shallow β-effect for Earth oceans and atmosphere (equation (4) in the main text), and the topographic β-effect (equation (7) in the main text) for gas giants. R is the outer radius, and D the thickness of the spherical shell.

2. Zonostrophy index estimates for icy moon subsurface oceans

We calculate the zonostrophy index for icy moon oceans from equation (1), with parameters reported in Table 2. We again use the topographic β-effect. Because no direct measurements of the flow in subsurface oceans is available, the typical speed U has to be estimated from ϵ.

	Europa	**Enceladus**
Ω ($\mathrm{s^{-1}}$)	2.1×10^{-5}	5.3×10^{-5}
r_o (km)	$1531 - 1556$	$202 - 242$
D (km)	100	$10 - 50$
β_T ($\mathrm{m^{-1}s^{-1}}$)	4.4×10^{-11}	$6.8 - 12 \times 10^{-10}$
Estimate 1		
ϵ ($\times 10^{-13}\ \mathrm{m^2s^{-3}}$)	$5 - 80$	$\lesssim 2$
$R^{s,\mathrm{I}}_\beta$	$2.0 - 3.2$	$\gtrsim 2.0 - 4.1$
Estimate 2		
ϵ ($\times 10^{-13}\ \mathrm{m^2s^{-3}}$)	$14 - 97$	$0.04 - 2.63$
$R^{s,\mathrm{II}}_\beta$	$11 - 12$	$5.9 - 8$
E	5×10^{-12}	$8 \times 10^{-12} - 2 \times 10^{-10}$

Table 2. Parameters used for estimating the zonostrophy index of icy moon oceans. Parameters for *Estimate 1* (equation (3)) are taken from Jansen et al. [4]. Parameters for *Estimate 2* (equation (5)) are taken from Cabanes et al [5] and Soderlund et al [6].

2.1. *Estimate 1: using a drag coefficient*

Jansen et al. [4] estimate typical flow velocities assuming that most dissipation occurs in turbulent boundary layers near the seafloor and ice–ocean interface. The kinetic energy of the geostrophic flow, E_g, is then estimated as

$$E_g \approx \frac{D^{2/3}\epsilon^{2/3}}{2^{5/3}C_D^{2/3}} \tag{2}$$

(their equation 25), where D is the ocean depth, ϵ the kinetic energy dissipation rate, and C_D a drag coefficient. From this, one can get a flow speed estimate, $U \approx \sqrt{2E_g}$. Jansen et al. [4] obtain flow velocities of up to a few cm/s for Europa and up to one cm/s for Enceladus. Substituting the expression for U in equation (1) leads to a first estimate for the zonostrophy index:

$$R_\beta^{\mathrm{s,I}} = 0.5\left(\frac{2^{10}\beta^3 D^5}{\epsilon C_D^5}\right)^{1/30}. \tag{3}$$

The values used for ϵ and D are reported in Table 2and taken from Jansen et al. [4]. We use their range of drag coefficients $C_D = 0.001 - 0.01$. We obtain zonostrophy index ranging from the transitional to the zonostrophic regime for both Europa and Enceladus.

2.2. *Estimate 2: using Ekman friction*

Alternatively to using somewhat arbitrary drag coefficients, Cabanes et al. [5] estimate flow speeds by assuming that dissipation occurs through the Ekman boundary layer at the outer solid boundary. Using their equation (26) at a latitude of 45°, we obtain a different flow speed estimate,

$$U^2 \approx \frac{r_o\epsilon}{2^{3/4}\sqrt{\nu\Omega}}, \tag{4}$$

where r_o is the outer radius of the ocean, ν the kinematic viscosity of the fluid and Ω the rotation rate. This estimate leads to significantly larger flow speeds compared to Jansen et al. [4], from 20 to 60 cm/s for Europa, and 0.3 to 5 cm/s for Enceladus [5]. Substituting U in equation 1 gives a second estimate for the zonostrophy index:

$$R_\beta^{\mathrm{s,II}} = 0.5\left(\frac{2^{25/4}\beta^2 r_o^5\epsilon}{(\nu\Omega)^{5/2}}\right)^{1/20}. \tag{5}$$

We assumed molecular value for the viscosity, i.e. $\nu \sim 1\times10^{-6}\ \mathrm{m^2s^{-1}}$, the values used for r_o, ϵ, Ω are reported in Table 2. This second zonostrophy index estimate is largely zonostrophic for both Europa and Enceladus. This suggests that the first estimate (represented on Figure 1 in the main text) is a lower bound for the zonostrophic index of icy moon oceans.

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