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PHYSIQUE DE LA MATIÈRE EN GRAINS

PHYSICS OF GRANULAR MEDIA

Dense flows of dry granular material

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Abstract The behavior of dense assemblies of dry grains submitted to continuous shear deformation is still not well understood. Recently it has been the subject of several experiments and discrete particle simulations. For both confined and free surface geometries, we present the general features of such flows as well as grain-level information. We then describe the main rheological models and their predictions. ***To cite this article: O. Pouliquen, F. Chevoir, C. R. Physique 3 (2002) 163–175.*** © 2002 Académie des sciences/Éditions scientifiques et médicales Elsevier SAS

granular media / hydrodynamics / rheology

Écoulements de milieux granulaires secs

Résumé Le comportement d'assemblées denses de grains secs soumises à un cisaillement rapide est encore mal compris, et a fait récemment l'objet de nombreuses études expérimentales et de simulations numériques discrètes. Nous présentons les caractéristiques générales de tels écoulements tant à surface libre que confinés, ainsi que les information obtenues à l'échelle du réseau de contact. Nous décrivons ensuite les principaux modèles rhéologiques, ainsi que leurs prédictions. ***Pour citer cet article : O. Pouliquen, F. Chevoir, C. R. Physique 3 (2002) 163–175.*** © 2002 Académie des sciences/Éditions scientifiques et médicales Elsevier SAS

milieux granulaires / hydrodynamique / rhologie

1. Introduction

Granular matter shows both solid and fluid behavior. Of interest in many industrial processes (handling devices) and in geophysics (rockfalls, sand dunes), granular flows are the focus of very active researches, at the frontier between mechanics and physics [1–9]. They are very sensitive to various parameters: geometry of the flow, wall roughness, flow rate, shape of the grains, granulometry, and coupling with the interstitial

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fluid. If the grains are large enough (size $d > 100$ µm), and if the viscosity of the interstitial fluid is small enough, it becomes possible to neglect electrostatic, aerodynamic and capillary forces, and one speaks of dry granular matter, without cohesion. Then the rheology is solely dictated by transfer of momentum and energy dissipation (friction and inelastic collisions) occurring in direct contacts between grains and with the walls. Despite the apparent simplicity of the system, the behavior of granular matter is very rich and extends from solid properties to gaseous properties depending on the flow regime.

When slow deformations are imposed to the material, it behaves more or less like a solid. This quasi-static limit is described in term of plasticity and has been thoroughly studied in soil mechanics [10,11]. Macroscopic observations (triaxial test) have motivated phenomenological models, whose microscopic origins start to be precisely studied with numerical simulations [12]. Basically, the strength of the material in this regime is characterized by an angle of internal friction ϕ: the material yields along slip plane, also called shear bands, when the ratio of tangential to normal stress reaches $\tan\phi$. When sheared, an initially loose piling compacts and an initially dense piling dilates, because of disentangling of the grains (Reynolds dilatancy). More precise models taking into account such volume changes [13–15] have been developed.

At the other extreme when a granular material is submitted to strong agitation, the medium is dilute and the grains interact through binary collisions. An analogy with a gas is then possible where grains play the role of molecules. A granular temperature is also defined related to the velocity fluctuations. During the last twenty years, the statistical tools of kinetic theory have been used and have led to a quasi-hydrodynamic treatment of this collisional regime [16].

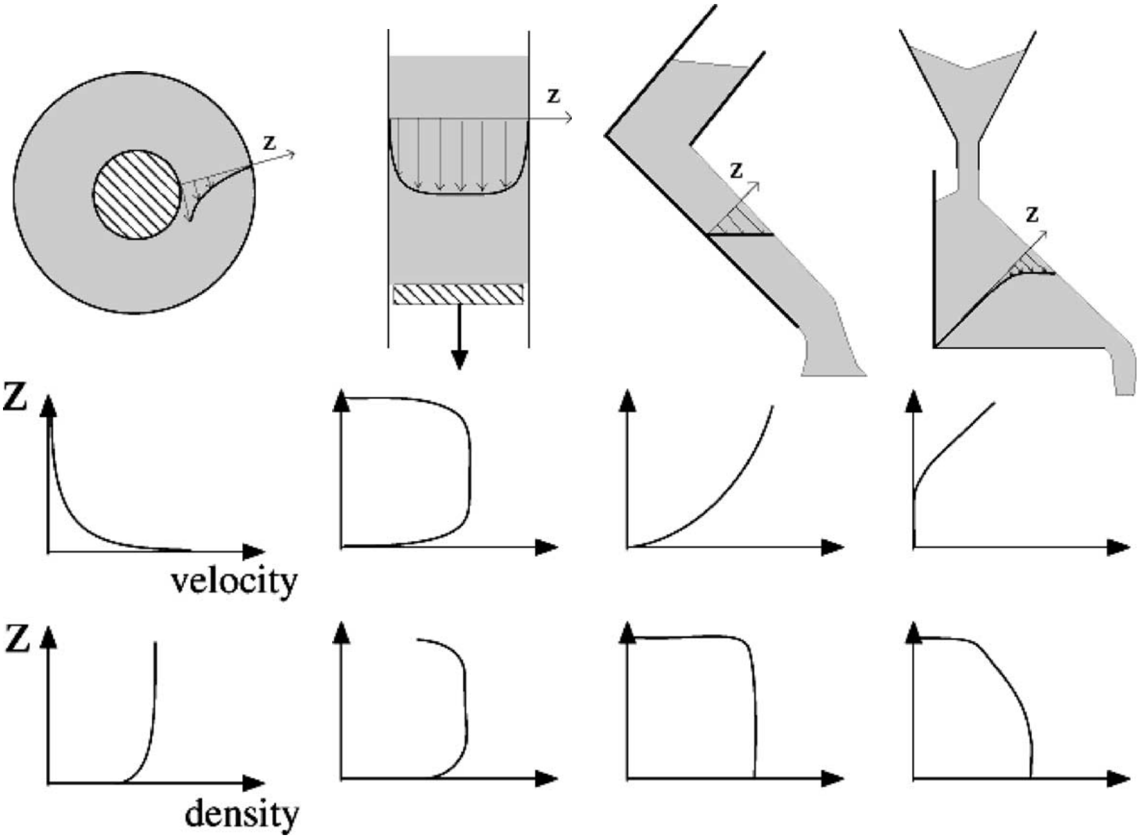
However, due to dissipation which favours clustering and/or due to confinement, the flows are usually dense, with a high solid fraction ν between random loose packing and random close packing. This is for example the case in sand avalanches at the surface of a pile. The motions of the grains are then strongly constrained, forces are transmitted through a percolating network of persistent contacts and the two basic assumptions of kinetic theory, i.e. binary collisions and molecular chaos, fail. The material in this dense regime is reminiscent of liquid behavior but our understanding of the dynamics and of the rheology is still incomplete. Recently, careful experiments (using optical imagery, photo-elasticity, X-ray tomography, magnetic resonance imagery or diffusing wave spectroscopy) and discrete particle simulations on model systems have provided a wealth of information which gives the basis for detailed description of the rheology. In this paper we shall first review the gross features of such flows and discuss the grain level mechanics, before presenting the different rheological models which are so far proposed.

2. The different configurations of dense flows

Dense granular flows are usually studied in one of the four configurations presented in Fig. 1. They are simple geometries where a uniform shear can be obtained which allow to test rheological models. In the following we shall try to describe and compare the properties of these different flows.

The two first configurations belong to confined geometries i.e. the material is confined between walls. The first configuration is the shear cell experiment where the material is sheared between two walls in relative motion. Coaxial cylinders [17–25] as well as parallel planes [26–29]. The control parameter can be either the velocity of the moving wall or the shear stress applied to the moving wall, but most of the results found in the literature are for fixed velocity. The second configuration is the silo where the material flow under gravity between two rough walls [30–35]. In this configuration one controls the flow rate by the aperture or by imposing a mean vertical velocity.

The two other configurations we shall discuss are free surface flows. The third configuration in Fig. 1 is a flow on a rough inclined plane [37–42,44–47] (as we are interesting in strongly sheared flow we will not discuss the case of a smooth wall [36]). The two experimental control parameters are the inclination of the plane and the flow discharge from the hopper. The last configuration of interest is the flow on a pile [43,48,49]. In this case there is only one control parameter which is the flow rate, the system adjusting by



(a) (b) (c) (d)

Figure 1. Sketches of dense granular flows and typical velocity and solid fraction profiles: (a) shear cell; (b) vertical chute flow; (c) flow on inclined plane; (d) flow on a heap.

itself the slope inclination. Flows in rotating drums [9,48,51,52] belong to this class (see also Gray: Notes added in proof 1), the flow rate being controlled by the rotation frequency.

In these four configurations, steady uniform flows can be observed in some range of the parameters giving information about the rheological properties. By studying the analogies and differences between the different flows, the aim is to try to understand the basic mechanisms controlling dense granular flows. However, we will show that the task is not easy and a lot of questions remain open. We will first briefly described the flow structure for each of the four configurations in terms of velocity profile, solid fraction profile and mean flow properties before discussing the differences and presenting the observations at the grain level.

3. Flow structure

3.1. Shear cell

The shear cell is a classical rheological apparatus. By imposing a shear rate we can measure the shear stress and observe the velocity profile. The first result is that steady flows are observed only above a critical shear rate [27,53], otherwise stick-slip instabilities are observed. In the steady regime the velocity profile is localized close to the moving boundary as sketched in Fig. 1. The width of the shear layer is typically of the order of ten grains. The decay far from the wall is an exponential for spherical grains and a gaussian for other shapes or polydispersity. The local solid fraction also varies across the layer, the material being looser in the high sheared region [19,23,25]. This means that the material dilates in the shear region. Both the density profile and the velocity profile seem independent of the velocity imposed at the moving wall.

Measurements of the stress components at the wall have been carried out for various shear rate, solid fraction and normal stress, both in experiments [54–56] and in discrete particles simulations [26–29]. The transition between a shear rate independent regime and shear rate squared regime has been shown [26,29] at constant density, as well as the validity of the Mohr–Coulomb criterion at the fluid–solid transition [28]. At constant pressure, the effective friction coefficient is found to slightly increase with the shear rate [27,54].

3.2. Vertical chute

Because of its practical application the case of the silo has been intensively studied. A steady and uniform regime is observed far above the exit which controls the flow rate, and for large enough exit to prevent jamming [1]. However, the flow may become intermittent in some range of shear rate [34,57]. The velocity profile across the silo consists of a plug flow in the center and of two shear zones close to the walls. The thickness of the shear zones (again typically ten grains) varies very little with the width of the channel but can be changed when the silo is inclined [32]. Like in the shear cell, a variation of the solid fraction of the medium is observed with dilatation close to the walls. We are not aware of systematic measurement of the shear stress that develops at the wall as a function of the mean discharge in a large range of chute velocities.

3.3. Flow on rough inclined planes

Flows on inclined planes which are reminiscent of geophysical events like land slides have been the subject of many studies but we will restrict ourself to results concerning steady uniform flows. They are observed only in a finite range of inclinations and thicknesses. If the layer is too thick and/or the plane too inclined, accelerated flows are observed [41,42,46,47]. For moderate inclinations θ and thickness h the region of steady uniform flows is bounded by a curve in the phase diagram (θ–h). For each inclination θ there exists a minimum thickness $h_{\text{stop}}(\theta)$ below which no steady flow is observed. The critical thickness h_{stop} is thinner for high inclination and diverges to infinity when inclination approaches the angle of repose θ_{r} of the material. This critical curve seems to result from the boundary condition imposed by the rough bottom.

Above this critical thickness, a steady uniform flow develops with a concave velocity profile with a maximum shear rate located at the fixed bed (Fig. 1(c)) [4,47]. Numerical simulations seem to show that the velocity profile tends to a Bagnold profile (i.e. a shear rate proportional to square root of the depth) in the limit of thick layers ($h > 50d$) [47]. An important information given by numerical simulations is that the velocity profile is insensitive to the value of the coefficient of restitution between particles, once it is smaller than about 0.7 [40].

Because a dense assembly of grains dilates when submitted to shear, one expects the solid fraction to vary in chute flows. A dilatation of the superficial layer is indeed observed before avalanching [27,28]. This is how Bagnold first explained the difference between the angle of repose θ_{r} and the avalanche angle θ_{m}. However, by contrast with what has been reported for the confined flow, the solid fraction in the steady regime is constant across the layer as shown in Fig. 1(c). The value of this uniform solid fraction varies when varying the control parameters: it increases when increasing the inclination θ but seems independent of the thickness h [46,47].

In order to get information about the rheology, systematic measurements of the mean velocity as a function of the inclination and thickness have been carried out [38,41,46]. The depth averaged velocity is found to vary as $h^{3/2}$ and an empirical correlation is found relating the velocity and the function $h_{\text{stop}}(\theta)$. From this scaling an empirical friction law has been proposed to describe the interaction between the flowing layer and the rough surface [41].

Notice the analogy with the motion of a single grain down a rough inclined plane. It is steady in a range of inclination and roughness, below which the grain stops and above which the grain accelerates [58]. The friction force has a finite value at rest (Coulomb friction), and following Bagnold's argument, increases as the velocity squared. Then an analysis of potential energy dissipation predicts a minimum of this friction force at small velocity [59]. This would explain the hysteresis between the angle of repose and the angle of maximum stability θ_{m}. Furthermore, the balance between gravity, dissipation by collisions and trapping in the roughness predicts how the average velocity depends on inclination [58].

3.4. Flow on a pile

The case of the flow on a pile is rather different from the flow down inclines as there is only one control parameter which is the flow rate. The slope of the pile is chosen by the system itself. In this configuration, steady uniform region are observed only for sufficiently high flow rate (or rotating frequency for the case of a rotating drum). Below this critical flow rate, the flow is intermittent and occurs through avalanches. In this regime, the slope of the pile oscillates between θ_{r} and θ_{m} [9,43,60]. In the continuous flow regime, the slope is constant and experiments reports a slight increase with the flow rate. However, this slope increase could be due to finite size effects as it seems to disappear for a wide enough channel.

The flow is localized close to the free surface with a velocity profile which is linear [9,48,51] with an exponential tail [50]. The velocity gradient in the linear part seems to be roughly independent of the flow rate, which means that the role of an increase of the flow rate is to increase the thickness of the flowing layer.

The solid fraction in this case is not constant as for flow on inclined plane but decreases close to the free surface in the region corresponding to the linear profile [51].

3.5. Discussion

The four configurations of Fig. 1 correspond to different boundary conditions and different stress conditions. It is thus not surprising that the structure of the flow varies from one configuration to another. However, the analogies and differences are not very easy to understand and raise several questions.

A first analogy concerns the two confined flow configurations. In both cases the shear is localized close to a wall but the stress distributions are not the same. The shear stress vary across the silo due to gravity and decreases from the wall in coaxial cylinders shear cells. It would be interesting to know if localization still persists when the stress components are uniform in the sample.

The major differences between the two free surface flows are also interesting. Naively one would think that the localized flow at a free surface of a pile is equivalent to a flow on a rough surface, the surface playing the role of the first layer of beads which are static in the pile. The observations show that it is more complex than this simple picture. The velocity and density profiles are different, although the stress distributions are similar. Two reasons could be responsible for the differences. First the bottom boundary condition plays a crucial role: a rigid boundary is not equivalent to an erodable boundary. Secondly, flows on a pile are observed for inclination slightly larger than the angle of repose, whereas flows on rough planes are studied above this critical angle. Flows on piles thus perhaps give information on the rheology for the peculiar value of the stress distribution corresponding to the onset of flow.

A last observation concerns the solid fraction profile. It is intriguing that the density profiles are not always correlated to the velocity profiles. In confined flows or flows on piles, regions with higher shear are more dilated than regions with low shear. For flows down inclined planes a variation of shear does not induce a variation of solid fraction. To our knowledge, no explanation of this difference has been so far proposed.

In order to better understand these properties, we investigate the dynamics at the grain level in order to evidence the relation between the flow structure described above and the microstructure. Another approach is to try to develop continuous models reproducing the results presented above and to seek for the minimum ingredient which are necessary in a continuum description. Both methods of investigations are discussed in the following.

4. Properties at the grain level

The study of granular material at the grain level has been possible with the development of efficient numerical simulations or with new experimental technics like high-speed cameras, X-ray tomography, MRI, ... First questions of interest concern the fluctuations of the grains around their mean motion. The problem of the amplitude of the fluctuation, of the existence or not of correlations in the fluctuating motion have been addressed. Other studies address the problem of the contact and force network in the material which plays an important role in the dense regime.

4.1. Fluctuations and correlations of the grain motion

In the collisional regime, incessant collisions are responsible for large velocity fluctuations δv, of the order of the average advection velocity. The fluctuation scales with the shear rate: $\delta v \sim \dot{\gamma}$ [40]. In dense flows, each grain is caged by its nearest neighbours, and has to escape from this cage to allow shear. These rearrangements are responsible for small velocity fluctuations, as compared to the average velocity. In the confined flow configurations of Fig. 1, the velocity fluctuations are found to be maximum near the rough walls and then decreases exponentially [19,21,25,35,46]. In these cases a scaling of the velocity fluctuations with the square root of the shear rate has been observed: $\delta v \sim \dot{\gamma}^{1/2}$ [20,22]. However, those fluctuations do not seem to be individual but concern clusters of particles. Due to steric hindrance, the motions of the grains are strongly correlated. Clusters or columns exist but are rapidly broken by the shear [37,40]. The role of these collective motions seems to be very important to understand the dynamics of dense granular flows. So far only few studies address the problem of space and time correlations of these collective motions [22, 35,61].

Other correlations are also induced by the presence of walls. Near the walls, particles are aligned in layers that move relative to each other [21,25,35,46,47].

4.2. Contact and force network

The correlation observed for the fluctuating motions in dense flows are due to the existence a percolating contact network as experimentally evidenced by photo-elasticity [18]. This network is not fixed but continuously changes as the material is sheared.

The contact network for dense flows is made of particles that are in long term contact and do not interact by quasi instantaneous collisions. The proportion of lasting and collisional contacts has been discussed in experiments [33] and in discrete particle simulations [26,35]. Quantitative analysis have been performed for high shear rates [23,25,26,35,46]. The number of lasting contact is found to vary in a non trivial way with the mean solid fraction, the confining pressure and the shear rate. Moreover, it is shown that the contact network in flow is strongly anisotropic with preferential orientations in the direction of shear and gravity.

This contact network carries forces whose distribution can be analyzed in the same spirit as what has been done in static piles [62–64]. As in the quasi-static limit, the force network shows a very large distribution of forces, with an exponential decay of the large force probability. It can be divided in a strong and a weak network, corresponding to forces respectively larger and smaller than the mean.

This contact network is extremely sensitive to shear (the so-called fragility [65]) provoking large space and time fluctuations of the forces which have been measured on the walls [17,19,24,29]. The dynamics of arching [66] (formation and breaking of force chains) is of interest to understand how the outlet regulates the flow rate in hoppers (‘free fall arch’ model [1]). There exists also a coupling between the contact network and the small variation of the solid fraction induced by the shear. Structural transitions can be observed (analogous to solid–glass–liquid transitions) [19,24,29], which could be responsible for abrupt changes of the average stress components [67]. For example, densification of the contact network together with an increase of the coordination number lead to jamming [26,29,46,68].

An important point is to understand how the contact and force network changes with the shear. A first interesting quantity is the proportion of contacts where the friction is fully mobilized i.e. where the two particles are sliding. The mobilization of friction (and its anisotropy) has been studied in quasi-static deformations [69], near the onset of flow [64], and in dense flows [46]. The proportion of rolling contacts, where friction is not fully mobilized and which do not contribute to energy dissipation, increases with the friction coefficient and with the intensity of the normal force. This question is connected to the organization of the grain rotation [25,62,66,70]. If the mean rotation velocity is equal to half the shear rate [19,23,46,67], the fluctuations of the rotation velocity are large, which might lead to a reduction of effective friction [46].

A last important point concerns the transfer of momentum which occurs during shear. As dense flows belong to a multi-contact regime, transfers of momentum are non-local: a shock between two particles diffuses in the entire contact network, so that the collision can become completely inelastic. This might be the reason why the velocity profile does not depend on the restitution coefficient in dense flows [9,40]. Coherent transfer of momentum associated to collective rearrangements of grains has also been discussed by [35].

4.3. Discussion

Information is now available at the grain scale. However the link between the dynamics at the grains level and the structure of flow (velocity, density profiles) is not yet clear. In our opinion an important idea which seems to emerge from the microscopic analysis is the presence of collective motions and correlation between the particles motion and forces. However, the role and dynamics of these correlations are still fuzzy.

5. Theoretical descriptions

Beside the numerical and experimental works presented in the previous sections, a great effort is devoted to the theoretical description of dense granular flows. Finding hydrodynamic equations which could describe velocity, density profiles and stress distribution in the different configurations of Fig. 1 represents a serious challenge not yet achieved. One of the difficulties is that dense flows often occur on length scales of the order of few grain diameters. We are then seeking for a continuous description which captures variation at the mesoscopic scale. Another serious difficulty lies in the boundary conditions. Most of the shear occurs close to a boundary or at a free surface.

So far no unified theory exists, but different approaches are explored which we succinctly review in the following. Some approaches are inspired either by plasticity models or by kinetic theory of rapid flows but recent approaches also try to take into account more explicitly the collective effects at the mesoscopic scale.

5.1. Plasticity model

It is known from triaxial test that for large but slow deformations, stress and solid fraction reach a stationary state independent of initial conditions, called the critical state. By including dependencies on solid fraction in rigid-plastic constitutive laws, flows in hoppers have been described [6,15]. In the same spirit a model taking into account friction and dilatancy has been proposed for avalanches [71]. Based on contact network analysis, a texture dependent rigid–plastic model has been analyzed [62].

Recently, Mohan et al. [72,73] have proposed a modified Cosserat plasticity theory to describe the fully developed flow in silo or shear cell. The idea of Cosserat approach consists in introducing a new degree of freedom in term of the local angular velocity associated to the local spin of material elements. An equation has then to be written for the balance of the angular momentum. Within this approach, Mohan et al. are able to predict velocity profiles observed in silo [72] and shear cell [73]. However, density profiles are not in agreement with the experiments.

In these approaches, the material behavior remains plastic which means that there is no rate dependence of the stress. Consequently, steady granular flows on inclined plane or on heap could not be described by this model.

5.2. Hydrodynamic models inspired by the kinetic theory

The kinetic theory of rapid granular flow has been originally derived for dilute and rapid flows in a regime where the particles interact only via instantaneous binary collisions [16]. For this regime a granular temperature is defined as the magnitude of the velocity fluctuations, and hydrodynamic equations are derived. One way to describe dense granular flows is to modify the kinetic theory in order to extend its domain of validity for high volume fraction. Different modifications have been proposed.

A first attempt consists in modifying the stress tensor by writing a rate independent term in addition to the collisional term [74–76]. The idea was to take into account the friction between grains which becomes important at high volume fraction. This approach has been applied to vertical chute flow [77] and to the flow on inclined plane [78]. The existence of a steady uniform flow in a finite range of inclination was predicted by this approach. However, the influence of the rough boundary was not captured. Moreover, the authors point out that the results were very sensitive to any small changes in the parameters.

Another modification of the kinetic theory has been recently proposed by Bocquet et al. [79,80] for describing the flow in a shear cell. By analogy with what is proposed for the glass transition, they have changed the dependence of the viscosity with the density. In their model the viscosity diverges more rapidly than in the classical kinetic theory when the density approaches the maximum density. The underlying idea is that at high density, particles are trapped in cages and the momentum transport coefficient is modified whereas the heat transport coefficient remains unchanged. The velocity profile and velocity fluctuation profile computed by the model are in good agreement with the experimental observation in a shear cell. The relation between velocity fluctuations and shear rate is also predicted.

Savage [81] has used a rather different approach to derive a set of hydrodynamic equations for dense granular flows. He considers a plasticity model with large shear rate fluctuations. When fluctuations are larger than the mean, a viscous behavior is recovered. Based on this idea, Savage derived equations for the density, the velocity and the shear rate fluctuation, which is assumed to be identical to the granular temperature. The result for dense flows is a set of hydrodynamic equations similar to the kinetic theory but with a viscosity which decreases with the temperature, and a different expression for the energy dissipation term. This model qualitatively predicts the flow in a silo but has not been applied to other configurations.

5.3. Two phase flow models

A granular medium undergoes a transition between a solid-like behavior and a liquid-like behavior when the shear stress exceeds a certain threshold. This property has motivated the description of dense granular flows as a multi phase system [5].

One model is proposed by Aranson and Tsimring [82,83]. The material is seen as a mixture of grains in a solid state and grains in a fluidized state. The relative concentration of the two populations is described by a local order parameter equal to 1 in the fully solid state and 0 in the fully fluidized state. The order parameter is governed by a Landau equation by analogy with phase transitions. The stress in the momentum conservation equation is then expressed as a sum of a newtonian viscous liquid part and of a solid part proportional to the order parameter. This approach is able to properly capture the hysteretic character of the flow threshold of granular material and the related complex dynamics of avalanches. It also gives good predictions for velocity profiles in the shear cell experiments, but fails to give the measured rheology on inclined plane, perhaps due to the simple assumption of a newtonian description for the fluid phase.

We mention a recent model proposed by Lemaitre [84], inspired by a plasticity model [85] which is also based on the transition between two populations of grains. The two states represent clusters of grains sheared on the right or on the left. The rate of transformation from one state to another is governed by the fluctuating motion (the granular temperature) which helps the transition depending on the local stresses. At the same time the mean flow continuously destroys and creates new clusters. So far it has been applied to the flow on an inclined plane and is able to predict correct velocity profile and threshold of the flow.

5.4. Nonlocal description

The models presented so far are more complex than the simple Navier–Stokes equation for a newtonian liquid, as they introduce an additional field: the granular temperature for the kinetic theory or an order parameter for two phase flow model. However, the rheology remains local, i.e. the shear stress at some point depends on the local value of the shear rate and of the fields at this point. Other approaches are proposed with a non local rheology.

The first one was proposed by Mills et al. [86,87] for describing the flow of granular material on inclined planes. The medium is described as a newtonian viscous fluid in which arches are present, propagating the stresses through the layer in a non local way. They end up with a non local description of the flow and were able to describe the scaling law observed experimentally. The shape of the velocity profile is correct, the amplitude satisfying in the limit where there are only arches.

Another nonlocal approach has been developed by Jenkins and Chevoir [46,88] for flow on inclined planes. The picture of the flow they proposed is the following: when one particle moves relatively to the adjacent layer, dissipation occurs when it falls in the interstice between two grains. However, in a dense regime this motion involves the displacement of the whole column of grains above the moving particle. The energy dissipation due to the column displacement is then non local. The model predicts correct scaling for the mean velocity as a function of the inclination and thickness of the flowing layer and correct velocity profiles for moderate height. We have to mention that the two models by Mills et al. and Jenkins and Chevoir are equivalent for some limit of the parameters [46].

As a nonlocal approach we mention the semi-discrete model by Andreotti and Douady [89]. They have carefully studied the dynamics of one bead rolling on a layer of fixed beads [58]. They then describe the flow of a granular layer as a stacks of layers by coupling several of single bead systems. A nonlocal coupling is introduced due to the propagation of the impulsion through the whole material during a shock between two adjacent layers. Some properties of the flow on heap are recovered within this approach.

5.5. Granular flow as an activated process

A last approach to describe dense granular flow is based on activated process. Thermally activated process is a common idea which has been successfully applied to explain many physical phenomena. The idea is that the thermal fluctuations help the material to yield by helping the particles trapped in some energy well to escape. However, for granular material thermal fluctuations are negligible and can not be the source of the activation.

A first attempt to apply this idea for granular flows has been made by Pouliquen and Gutfraind [32] for the flow in a silo. In this study the source of activation was proposed to be the stress fluctuations. Despite a crude description of the fluctuations, the velocity profiles in an inclined silo was predicted. More recently, Debregeas et al. [61] have developed similar ideas and succeeded in describing the velocity profile experimentally observed in a shear cell with wet foam (bubbles playing the role of grains). An interesting point is that this approach allows to predict some non trivial velocity correlation put in evidence in the experiments. In Debregeas's model, the source of the fluctuation is the shear localized between the rough wall and the material. By generalizing this idea, i.e. stipulating that shear everywhere in the material can be a source of a stress fluctuation which can activate a shear motion somewhere else, Pouliquen et al. [90] have proposed a nonlocal activated model.

6. Discussion and conclusions

A first conclusion one can raise from the multiplicity of models proposed is that dense granular flows are far from being well understood! Despite the number of numerical and experimental studies we do not really know what are the basic ingredients which control the flow in the dense regime, resulting in a large variety of approaches explored. The task is not easy as it seems that experimental observations like for example the existence of shear bands in shear cell can be captured by very different models ranging from plasticity model to kinetic theory. To our opinion a first step to discriminate between the different approaches would be to check their prediction in the four configurations of Fig. 1. The observation of shear bands, the important role of the boundaries observed in all configurations suggest that they are governed by the same physical mechanism and we believe that they should be described by a common approach. Experimental and numerical data are now available in the four configurations for velocity profiles, density profiles and rheological properties, so that precise comparisons are possible.

Despite the number of different approaches we can try to extract some basic common ideas. A first set of models are inspired by the kinetic theory based on the introduction of a granular temperature. Numerical simulation reveals that particles undergo multibody interactions and keep their neighbors during a finite time. The particle are not giggling around in cage interacting via collisions. Velocity fluctuations may well be relevant for dense flows, however, the derivation of transport coefficients or energy dissipation should not be solely based on collisions [81].

In a second set of models a central idea seems to be the presence at a mesoscopic scale of collective effects such as cluster motion or stress arching. Some models take them into account through non local formulation, some other introduce an order parameter to account for the presence of rigid cluster, but the idea of collective dynamics appears to be present in many models. This notion of collective effect needs to be precisely studied and defined but seems to our opinion an important ingredient. Ongoing experimental researches on this collective motion through for example the study of correlation [61] or the tracking of cluster should give crucial information for further development of models (see Bonamy et al.: Notes added in proof 2).

In this paper we have restricted ourselves to the properties of steady uniform flows which are the first step if one want to explore the rheological properties of granular flows. However most applications occur in unsteady non uniform configurations. In this regime, other approaches based on depth averaged equations have been successfully developed [91,92]. These approaches do not describe in detail the interior of the flowing layer but rather allow to predict the evolution of mean velocity and thickness. In this depth averaged model, the introduction of an empirical friction law is needed to describe the interaction between the flowing layer and a static layer. The understanding of this empirical rheological law requires the understanding of the rheology of dense granular flows. The link between depth averaged approach and fully 3D approaches is an important issue (this link has for example been carried out in the framework of Aranson and Tsimring model) [83].

Other important open questions concern the limit of the dense flow regime. Is the transition toward gaseous regime a continuous transition? How to connect results for dense flows to the solid regime and soil mechanics description? The transition between different regimes represents a serious challenge.

Finally we want to emphasize that this short review is certainly not exhaustive. Important areas of research on granular flows have been omitted and the recent explosive activity make the exercise difficult. We take refuge behind this excuse for all the contributions that have been omitted.

Notes added in proof

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