C. R. Physique 3 (2002) 677–684

Solides, fluides : propriétés électroniques et optiques/*Solids, fluids: electronic and optical properties*

L'EFFET HALL QUANTIQUE FRACTIONNAIRE
THE FRACTIONAL QUANTUM HALL EFFECT

DOSSIER

Is the chiral Luttinger liquid exponent universal?

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Received 9 March 2002; accepted 30 March 2002

Note presented by Guy Laval.

Abstract We present experimental evidence from electron tunneling measurements that the chiral Luttinger liquid power-law exponent, α, for tunneling into the fractional quantum Hall edge deviates substantially from the universal behavior predicted by theory. Our results suggest that the existing standard analyses based on effective Chern–Simon field theories deserve careful reexamination when applied to the dynamics at the Hall fluid edge. ***To cite this article: A.M. Chang, C. R. Physique 3 (2002) 677–684.***

quantum Hall effect / two-dimensional electron systems / Luttinger liquids / tunneling

L'exposant du liquide chiral de Luttinger est-il universel ?

Résumé A partir de mesures d'effet tunnel, nous présentons l'évidence expérimentale que l'exposant α du liquide de Luttinger chiral décrivant la loi de puissance pour l'effet tunnel d'électrons dans le bord de l'état Hall fractionnaire s'écarte de manière notable du comportement universel prédit par la théorie. Nos résultats suggèrent que les analyses éxistantes standard basées sur les théories de champ type Chern–Simon méritent d'être revues quand elles s'appliquent au bord du fluide Hall. ***Pour citer cet article : A.M. Chang, C. R. Physique 3 (2002) 677–684.***

effet Hall quantique / systèmes d'électrons à deux dimensions / liquides de Luttinger / effet tunnel

1. Introduction

The strongly interacting, 1-dimensional (1d) chiral Luttinger liquid (CLL) at the fractional quantum Hall (FQH) edge is emerging as a prototypical non-Fermi liquid system [1–13]. It offers advantages over other 1d systems, such as carbon nanotubes [14,15], quantum wires [16–18], quasi-1d organic conductors, or blue-bronze 1d conductors, for the investigation of the unusual power-law energy dependence in the single-particle tunneling-density-of-states as evidenced by the unsurpassed large dynamic range and high quality of power-law tunneling current–voltage (I–V) characteristics. I–V characteristics containing a power-law

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S1631-0705(02)01353-1/FLA

region with dynamic range in excess of 4 decades in I and 1.5 decades in V have been obtained which, when plotted on a log–log scale yields a nearly perfect straight line [12,13].

New and exotic conductors such as high T_c (normal state) conductors, dimensional conductors, as well as composite Fermion systems are pushing beyond the familiar Fermi-liquid scenario. The need to better understand non-Fermi liquid behavior has stimulated numerous investigations of the Luttinger liquid (LL) [19–22], which is believed to describe an interacting electron system in one dimension. Unlike the familiar Fermi liquid in which quasi-particles constitute the low energy excitations, this unusual 1d many body quantum system supports elementary excitations best described as phonon-like gapless modes, while at the same time possessing an unusual power law tunneling density of states for the tunneling of electrons. This power-law density gives rise to the power-law dependence of the tunneling I–V observable in experiment which represents a hallmark signature of LL behavior.

Historically, experiments to observe LL behavior at $B = 0$ have proven to be challenging due to the fact that residual backscattering of electrons from either disorder or from non-ideality of 1-dimensionality produce localization effects which readily obscure the LL characteristics [16,17], although significant progress is now being made in nanowires [18] and carbon nanotubes [14,15]. However in the fractional quantum Hall (FQH) effect, leading theorist such as Wen [1–4], Kane and Fisher [5–7], Moon et al. [9], and Fendley et al. [8] have shown in the past decade that the edge should behave as a chiral LL, where the chirality arises from the presence of the magnetic field. The quasi-1-dimensionality occurs naturally at the boundary of the 2DEG and imperfections only cause the boundary to meander while back-scattering is minimal. As a result, the chiral LL is much more robust. Moreover, tunneling exponent in the edge of the Laughlin Hall fluids is determined simply by the quantized reduce Hall conductance $g = \nu$.

2. Universal α?

The basis for our current understanding starts with the analysis of Wen [1–4] using a hydrodynamic approach as well as other equivalent but more powerful approaches such as the effective chiral boson theory approach. The 1-d effective theory can be derived from the bulk 2d Chern–Simon effective theories [23–27], which in principle capture all the essential 2d physics at low energies and becomes more exact as the electron–electron interaction approaches the idealized, $\delta''(\tilde{r})$ potential. Resulting from such analysis, the CLL power-law tunneling exponent, α, appears as a topological quantum number [1–4] which can be used to label and differentiate different, strongly-correlated states. Due to the topological nature, it is often argued that the edge properties should be directly tied to the bulk properties and should be insensitive to the details of the form of the interaction. A major consequence is the prediction of *universality* in the exponent value. For instance, between $\nu = 1/3$ to $1/2$, equivalently $1/\nu = 2$ to 3 (but more precisely the reduced Hall resistance, $\rho_{xy}/(h/e^2)$, between 2 to 3), the exponent for electron tunneling is predicted to take the value of exactly 3 [28,29,5–7,1–4]. For other filling fractions, e.g. $\nu = 1/2$ to 1, residual disorder is expected to also drive the exponent to universal values yielding a linear dependence on $1/\nu$ ($\rho_{xy}/(h/e^2)$) [5–7].

Cumulative experimental evidence to date, however, point to substantial deviations from universality. For instance, the exponent in the vicinity of $\nu = 1/3$ appears to exhibit a plateau near $\alpha \approx 2.7$ below 3. Furthermore, the exponent value does not appear to be tied to the Hall conductance (resistance) in the 2d bulk, but rather is sensitive to surface reconstruction at the edge arising from the long-range Coulomb potential, which results from a chemical potential imbalance across the tunnel barrier. This imbalance leads to charge distribution across the barrier and an edge electron density as well as a corresponding filling factor which can differ substantially from the 2d bulk values. Most notably, the predicted [28,29,5–7,1–4] step-like behavior in the α versus $1/\nu$ dependence – more precisely versus $\rho_{xy}/(h/e^2)$ dependence – with linear regions between steps, is not observed. (Note that the distinction between $1/\nu$ and $\rho_{xy}/(h/e^2)$ becomes significant near $\nu = 1/3$ where ρ_{xy} remains quantized over a finite range of ν.) This conclusion is reached based on the sensible premise that once α exceeds the value 3, the reduced Hall resistance must also have exceeded the value 3. Instead, experiment clearly indicates the tendency toward a behavior of $\alpha \propto 1/\nu$, where the proportionality constant is in the range of 1–1.4, rather than the theoretical behavior where α reaches the value 3 at $1/\nu = 2$ and in between steps the slope takes the value 2. These experimental results raise questions regarding our fundamental understanding of the connection between the edge dynamics and the topological characterization of the bulk fluids, even though the basic Hallmark feature of the Luttinger liquid, i.e. power-law tunneling, is unequivocally established. Indeed, recent finite size calculations are already beginning to indicate a possibility for the strong renormalization of the edge tunneling exponent due to the long-range Coulomb interaction and deviations from universality [30,31]. In spite of the fact that finite-size results cannot be taken as a definitive proof in the thermodynamic limit of an infinite number of particles, the combined experimental and computational evidence should stimulate a reexamination of the detailed properties of the rich and novel physics at the edge of the fractional quantum Hall fluids.

3. Experimental investigations

Our experiments make use of a novel tunneling geometry made possible by the CEO growth technique [32,33]. In this unique geometry, tunneling takes place from a bulk $n+$ doped GaAs metal overgrown on the (0 1 1) plane into the edge of a fractional quantum Hall fluid confined within a quantum well in the (1 0 0) plane; see inset to Fig. 1. Using this technique we are able to produce devices in which a tall, thin $Al_{0.1}Ga_{0.9}As$ barrier separates the structurally atomically sharp edge of the 2DEG, confined within a quantum well, from a heavily doped $n+$ GaAs bulk metal layer. The sharp edge is created by in-situ cleaving along the (0 1 1) direction followed by a regrowth of the thin barrier, a 150 Å region of undoped GaAs, and the heavily doped $n+$ GaAs metal on this (0 1 1) plane perpendicular to the conventional (1 0 0) growth plane. The barrier thickness is of order 50–125 Å while its height rises 70 meV above the 2DEG chemical potential far exceeding the 2DEG Fermi energy of < 3.6 meV. This tall, thin barrier has proven essential by enabling access to a significant range of dynamic range in the tunneling bias voltage without concomitant distortions of the barrier shape, and is opening up new possibilities for studying the chiral Luttinger liquid. The $n+$ GaAs is doped to $1.4–2 \cdot 10^{18}$ cm^{-3} carrier density yielding a chemical potential of 65–83 meV from the GaAs band bottom. This leads to an imbalance with the 2d electron gas. Charge redistribution can take place across the barrier due to this chemical potential imbalance. The actual density profile will in

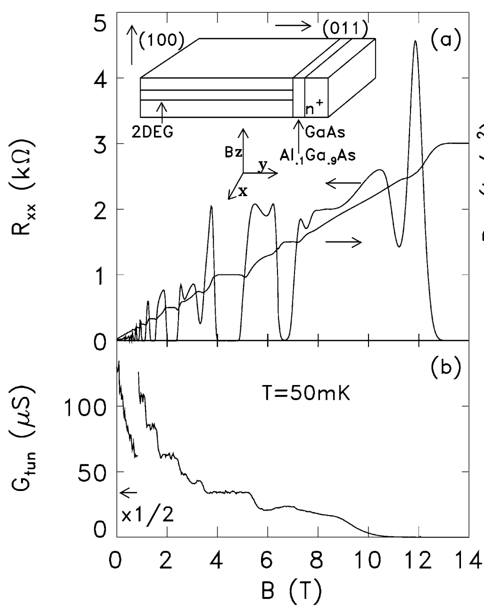

Figure 1. Magnetic field traces of (a) longitudinal resistance (R_{xx}) and Hall resistance (R_{xy}), (b) tunneling conductance (G_{tun}), at low bias for sample 1. The temperature is 50 mK.

addition depend on to what extent residual silicon dopants penetrate into the 150 Å undoped GaAs buffer layer separating the barrier from the heavily doped GaAs $n+$ metal during the regrowth process.

In Fig. 1(a) we show the sample geometry for a CEO grown device in the inset and the longitudinal and Hall resistances, and in Fig. 1(b) the tunneling conductance at low voltage bias at 50 mK temperature. The $1/3$ fractional quantum Hall effect is clearly visible centered at 13.4 T in magnetic field. Figure 2 contains a log–log plot of the first results on electron tunneling current between the $n+$ GaAs bulk metal and the edge of the $1/3$ fractional quantum Hall fluid as a function of the applied voltage bias. Data for two samples are included. At low voltage bias, the accessible energy scale in the tunneling process is determined by the thermal energy $\sim kT$ ($T = 25$ mK) and the I–V characteristics are linear. At a higher voltage bias corresponding to $eV = 2\pi kT$, the I–V crosses over to a power law dependence. These first data already showed the extraordinary quality of the power law behavior. The power law region exceeds 3 decades in current and 1.4 decades in bias. The large dynamic range enables us to rule out other competing functional forms. This power law represents the clearest evidence for Luttinger liquid behavior to date. In this Fig. 2 the solid curves represent fits to the Kane–Fisher universal scaling functional form $I \propto T^{\alpha}[x + x^{\alpha}]$, where $x = eV/(2\pi kT)$ [5–7]. Whereas in the theory, the exponent $\alpha = 3$ exactly [1–9], our curve fitting was achieved by allowing α to vary as a free parameter with the proportionality constant between I and V as a second parameter. We find $\alpha = 2.7 \pm 0.06$ and 2.65 ± 0.06, respectively, for the two samples (crosses, and dots), close to the theoretically predicted value of 3 [10].

In the bulk region of the 2-dimensional $\nu = 1/3$ fractional quantum Hall fluid, the excitation spectrum is well known to contain a gap above the ground state and there are no zero energy excitations. The edge of the fluid and its low energy excitations decoupled from the bulk at low temperatures and the edge can rigorously be treated as a one-dimensional system. The formation of a Luttinger liquid is then a natural consequence of the strong electron–electron interaction. On the other hand, the edge of a compressible fractional quantum Hall fluid such as the composite Fermion fluid at filling factor, $\nu = 1/2$, which does not

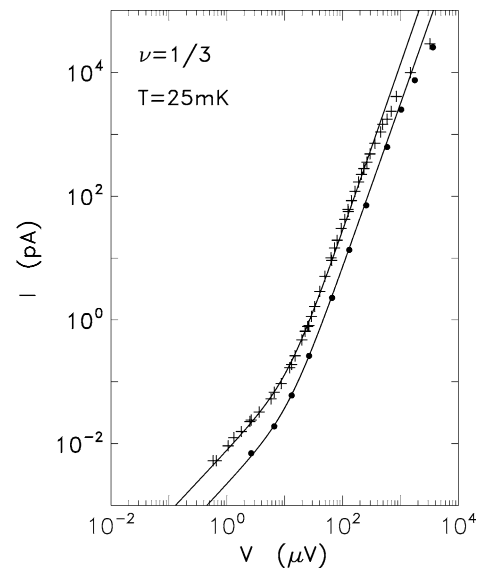

Figure 2. Current–voltage (I–V) characteristics for tunneling from the bulk-doped $n+$ GaAs into the edge of a $\nu = 1/3$ fractional quantum Hall effect for two samples 1 and 2 (crosses and solid circles, respectively) in a log–log plot. The solid curves represent fits to the theoretical universal form [5] for $\alpha = 2.7$ and 2.65, respectively.

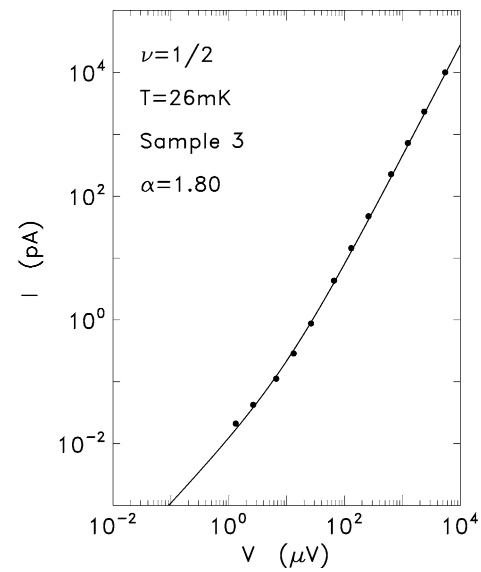

Figure 3. Current–voltage (I–V) characteristics for tunneling from the bulk-doped $n+$ GaAs into the edge of a $\nu = 1/2$ composite Fermion liquid for sample 3 in a log–log plot at $B = 9.28$ T. The solid curve represent a fit to theory [5,34] for $\alpha = 1.80$.

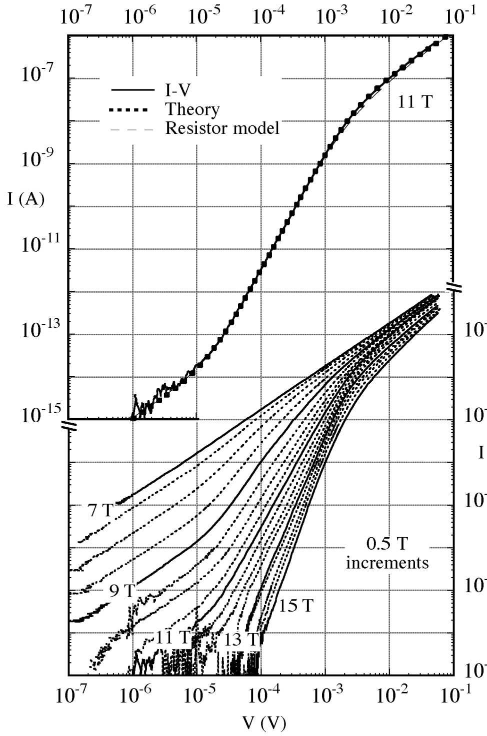

Figure 4. (Top) Log–log I–V for sample 2 at 11.0 T, $\nu = 1/3$. Theory of Chamon and Fradkin (dotted line) and simple series resistance model (dashed line) are overlaid for comparison. (Bottom) Log–log I–V for sample 2 at different values of B from 7.0 to 15.0 T in 0.5 T steps.

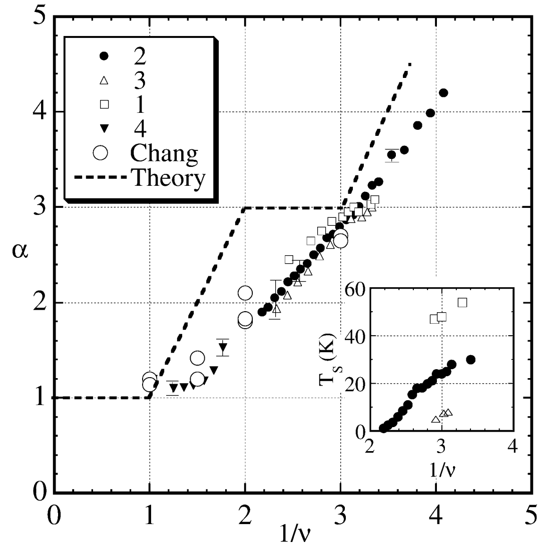

Figure 5. Power law exponent α versus $1/\nu$, the reciprocal of the filling factor, for four samples. (Inset) T_S versus $1/\nu$ for three samples whose traces spanned high excitations.

contain a bulk excitation gap is more complicated. The absence of a bulk excitation gap in a compressible fluid enables the edge dynamics to couple to the bulk excitations and a Luttinger liquid description may not be appropriate. It was far from clear that the tunneling of electrons into a composite Fermi system would necessarily entail a suppression of the tunneling density of states at low energies. In Fig. 3 we present power law tunneling behavior in the the edge of the $\nu = 1/2$ fluid. The exponent, $\alpha \approx 1.8$, was an early indication from this particular sample that deviations from universality may be occurring. Subsequent detail experiment at continuous values of $1/\nu$ revealed power law behavior at all accessible ν's as is shown in Fig. 4.

A systematic extraction of the exponents were performed by fitting the entire I–V curve for several devices to the Chamon–Fradkin scaling expression for the tunnel current, I, at voltage bias, V [34], with the notation $\beta = \alpha - 1$, $r = 2\pi T/T_S$:

$$I = \int_0^V \nu \frac{e^2}{h}\left(1 - \frac{\mathrm{e}^{-1/2r^\beta}}{[(V'/rT_S)^\beta(1-\mathrm{e}^{-\beta r^\beta/2})/\Gamma^2(\alpha+1)/2+1]^{\alpha/\beta}}\right)\mathrm{d}V' \tag{1}$$

with the resultant exponent values plotted in Fig. 5. What is strikingly clear is the absence of step-like features. Instead for several devices, the exponent, α, behaves roughly as $1/\nu$, with a proportionality constant in the range of 1.1–1.25. The observed behavior stands in direct contrast to theoretical expectations of step-like plateau features in α versus $1/\nu$ [5–7,28,29] (more precisely Hall resistivity, ρ_{xy}) and has presented a significant puzzle. Because the $\nu = 1/3$ FQH fluid possesses the largest gap and is robust, evidence for plateauing in the exponent is of critical importance.

Further detail experiment on the highest quality devices revealed the presence of a plateau feature for the α versus $1/\nu$ dependence with an α value close to 3. Characterizing the α versus $1/\nu$ plot by the slope, $S \equiv \mathrm{d}\alpha/\mathrm{d}(1/\nu)$, in the first set of samples S exhibits an abrupt change from 1.15 ± 0.3 reflecting a roughly $\alpha \sim 1/\nu$ dependence to 0.15 ± 0.15 as $1/\nu$ increases beyond 2.76, constituting a reduction of more than a factor of 7 in S. This plateau region of reduced slope extends from $1/\nu = 2.76$ to 3.33 before reverting to a value of ~ 1.05 above $1/\nu = 3.33$. Similarly in a second set S abruptly reduces from 0.33 ± 0.3 for $1/\nu < 4.12$ to -0.14 ± 0.18 for $4.12 < 1/\nu < 4.76$ before increasing rapidly, e.g. $S > 1.5$ for $1/\nu > 4.76$. In both sets of data the plateau region is centered about a value for α of 2.7, slightly below 3.

In Fig. 6 we presents log–log plots of the tunneling I–V characteristics. Successive curves are shifted in the positive direction on the horizontal axis by 0.3 units (a factor of 2) for clarity and curves for which sufficient dynamic range is available to yield a meaningful exponent are included. The dashed curves represent best fits to data described below. To establish the presence of a plateau feature in the exponent, α,

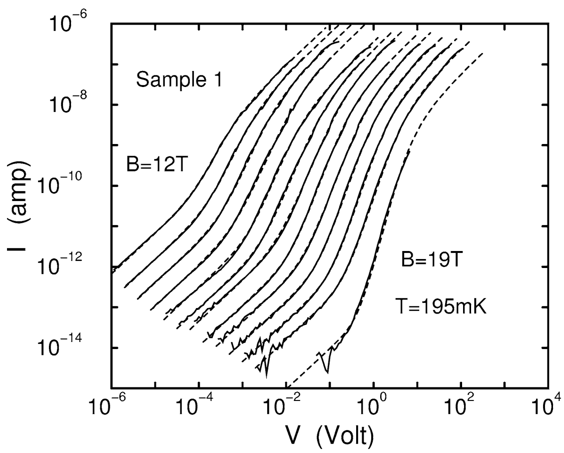

Figure 6. Log–log plot of the I–V characteristics (solid lines) for electron tunneling from the FQH edge into the bulk doped $n+$ GaAs in sample 1 at various magnetic fields from 12 to 19 T in steps of 0.5 T, 18 and 18.5 T excluded. Corresponding filling factors vary from 2.69 to 4.26. Dashed lines represent best fits to Eq. (1) with an additional series resistance, R_S. (Please see text.) Successive curves are shifted by 0.3 units (a factor of 2) in the x-direction for clarity.

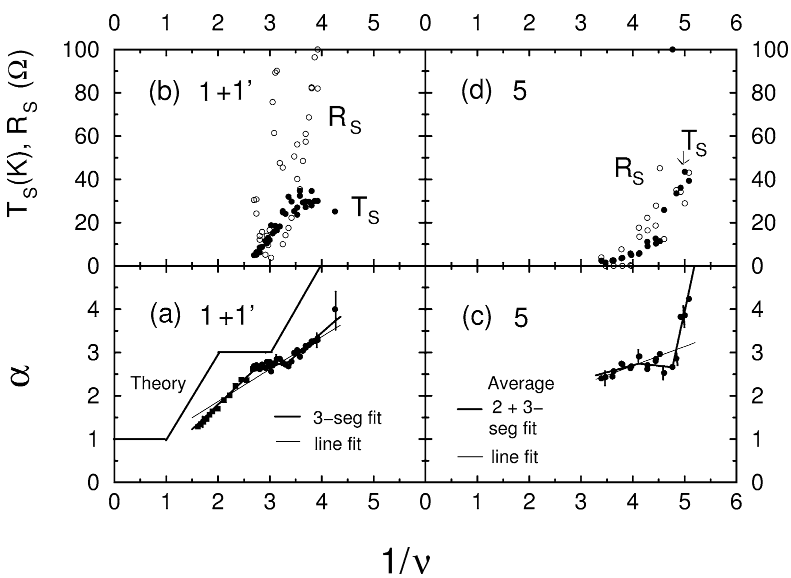

Figure 7. The power law exponent, α, for sample 1 and 1′, versus $1/\nu$, in (a), for sample 5 in (c). Representative error bars are as shown and solid curves are as labeled. The parameters T_S and R_s are summarized in panels (b) and (d).

we again extract α by fitting the entire I–V range containing the three bias regimes to the Chamon–Fradkin expression in conjunction with the constraint that $V_a = V + IR_s$ where V_a is the voltage applied on the device across contacts and R_s a 2DEG series resistance. Since the temperature is known, 3 parameters are needed: α, T_S, and a 2DEG series resistance R_s. The inclusion of R_s significantly improves the fits for traces taken at lower B fields.

Figure 7 summarizes the fitting parameters α, T_S, and R_s deduced for two sets of samples versus $1/\nu$. Results for samples 1 and $1'$ containing identical 2DEG are presented together, since they contain the identical 2DEG. We focus our attention on α in panels (a) and (c). To establish unequivocally the presence of a plateau feature we first fit our data to curves containing: (i) three line segments where the middle exhibits a reduced slope; (ii) two line segments; and (iii) a single straight lines, indexed by 3, 2, and 1, respectively. We next apply the statistical F-test on the resulting χ^2's. We find the 3-segment fit to be superior and are able to reject the competing 1- and 2-segment fits with a confidence exceeding 99%. The plateau occurs at a value of α near 2.7 close to the theoretical value of exactly 3. At the same time the $1/\nu$ positions are shifted to higher values than the theoretical prediction based on the bulk 2d electron density and filling factor. Let us remark that due to the chemical potential imbalance between the 2DEG and the 3d $n+$ doped GaAs, charge transfer must take place across the tunnel barrier. For the devices studied, this is expected to lead to an enhanced edge density which exhibits an inhomogeneous density profile near the tunneling edge. In the absence of a direct method to independently determine the edge density, we propose the following method to estimate the edge density which we argue should be accurate to 5–10%. Since by all reasonable analysis and sensible argument the exponent must remain nearly constant when the Hall resistance ρ_{xy} is approximately quantized at $3(h/e^2)$ (note $\rho_{xx} \ll \rho_{xy}$ always), and at the same time α can exceed 3 only when ρ_{xy} exceeds 3, we can determine the $\nu_{\text{edge}} = 1/3$ position by the $1/\nu_{\text{bulk}}$ value where α first exceeds 3, less the half-width of a typical 1/3 Hall plateau (roughly 5% of $\nu = 1/3$). For samples 1 and 2, this yields an edge density roughly equal to 1.05 the bulk density. Accordingly the plateau feature in α is more likely ascribable to the finite width of the Hall plateau rather than to a step of the type predicted by existing theories. In any case, the exponent at $\nu_{\text{edge}} = 1/2$ is highly unlikely to reach the value 3.

4. Discussion

What possibilities or non-ideality could lead to the discrepancy with theory? Two issues come to mind: (i) long range nature of the Coulomb interaction; and (ii) a non-constant density profile near the tunneling edge. Based on effective field theories, long range Coulomb interaction leads to a $\log(V)$ correction in the power law relation, with an increase in the exponent at low energies [35,1–4,28,29]. However, no evidence of this type is observable in the tunneling data despite the large dynamic range in the I–V. Furthermore, based on the dynamic range, a $\log(V)$ increase in the propagation velocity of the charged mode above the neutral modes is also not likely to sufficiently separate out the respective energy scales and lead to the apparent absence of a contribution from the neutral modes [36]. Regarding the density profile, even if the tunneling reflects an averaging over a strip of electron gas of non-constant density, an α of 3 should still be observable due to the width of the predicted $\alpha = 3$ step which spans $\Delta\rho_{xy}/(h/e^2) = 1 \approx \Delta(1/\nu)$.

In view of the evidence, one possibility which goes beyond the standard analysis deserving in depth investigation is a renormalization of the exponent from its universal value due to the long range nature of the Coulomb interaction. As mentioned above recent exact numerical diagonalization [30] as well as calculations based on composite Fermion edge-state wavefunctions [31] are indicating that for a 3d Coulomb interactions the exponent is no longer universal and takes on a value in the 2.5–2.75 range. These new developments suggest that the edge dynamics in the fractional quantum Hall regime may be more complex than previously thought and potentially will lead to further discoveries of more novel and interesting physics.

Acknowledgements. A.M.C. would like to thank Virginia Pang-Ying Yeh Chang for her many contributions. He gratefully acknowledges the contributions of his collaborators L.N. Pfeiffer, K.W. West, M. Grayson, D.C. Tsui, M.K. Wu and C.C. Chi, and also wishes to acknowledge stimulating conversations with M. Fisher, E. Fradkin, B. Halperin, C. Kane, L. Levitov and X.-G. Wen. Work at Purdue supported by NSF Grant #DMR-9801760. Work at Tsing-Hua University supported by the Taiwan National Science Council.

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