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L'EFFET HALL QUANTIQUE FRACTIONNAIRE

THE FRACTIONAL QUANTUM HALL EFFECT

Fractional statistics, Hanbury-Brown and Twiss correlations and the quantum Hall effect

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Abstract The direct detection of the statistics of the quasiparticles in the quantum Hall effect has so far eluded experimental discovery. Here a quantum transport geometry is analyzed, which could provide a link to the fractional statistics via the measurement of low frequency noise correlations. The proposal constitutes an analog of the Hanbury-Brown and Twiss experiment, this time for three chiral edges – one injector edge and two collectors. Luttinger liquid theory reveals that the real time correlator decays much slower than in the case of fermions, and exhibits oscillations with a frequency scale corresponding to the applied bias multiplied by the quasiparticle charge. The zero frequency noise correlations are negative at filling factor $1/3$ as for bare electrons (anti-bunching). However they are strongly reduced in amplitude, which constitutes a first evidence of unusual correlations. The noise correlations become positive (suggesting bunching) for $\nu \leqslant 1/5$, however with a much reduced amplitude, when one computes the noise assuming that only the most relevant operators contribute. ***To cite this article: R. Guyon et al., C. R. Physique 3 (2002) 697–707.***

noise correlations / fractional statistics / edge states / quasiparticles / quantum Hall effect

Statistiques fractionnaires, corrélations de Hanbury-Brown et Twiss et effet Hall quantique

Résumé Jusqu'à présent, la statistique des quasiparticules de l'effet Hall quantique n'a pu être directement mise en évidence expérimentalement. Ici, une géometrie qui procure une connexion à la statistique fractionnaire par la mesure des corrélations de bruit est proposée. Celle ci constitue un analogue de l'expérience de Hanbury-Brown et Twiss, adaptée à trois états de bord chiraux – un bord injecteur et deux détecteurs. La théorie des liquides de Luttinger révèle que le corrélateur en temps réel décroît plus lentement que pour des fermions, et oscille à la fréquence spécifiée par le voltage source–drain, multipliée par la charge effective des quasiparticules. Pour un facteur de remplissage $1/3$, les corrélations

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de bruit à fréquence nulle sont négatives comme pour les électrons, mais leur amplitude est fortement réduite : une manifestation d'une statistique inhabituelle. Les corrélations de bruit deviennent positives pour $\nu \leqslant 1/5$, suggérant un comportement bosonique, toutefois avec une amplitude très réduite. Cependant le calcul présenté ne tient compte que des opérateurs les plus pertinents. ***Pour citer cet article : R. Guyon et al., C. R. Physique 3 (2002) 697–707.***

corrélations de bruit / statistique fractionnaire / effet Hall quantique / états de bord / quasiparticules

1. Introduction

In condensed matter physics, the interactions between the constituents of the system are typically known, and have been since the nineteenth century. This contrasts strongly with the field of high energy physics where the search for elementary interaction processes constitutes a dominant theme. Although the building blocks of condensed matter systems are mere electrons, protons and neutrons, their collective behavior has been shown to lead to a variety of astonishing phenomena. Classic examples of such quantum correlated systems are superconductivity [1,2], superfluidity [3] and the fractional quantum Hall effect [4,5]. In these instances, the departure from usual behavior is often symptomatic of the presence of a non trivial ground state: a ground state which cannot be described by a systematic application of perturbation theory on the non interacting system.

Investigations of such ground states naturally lead to that of the elementary excitations of the system. One typically probes the system with an external interaction which triggers the population of excited states. The subsequent measurement of the thermodynamical properties then provides some crucial information. From a different angle, transport measurements deal with open systems connected to reservoirs. In the last two decades, mesoscopic physics has been concerned with the analysis of open electron systems using current and noise measurements.

In the fractional quantum Hall effect (FQHE), quasiparticle excitations and quasiparticle exchange properties were first discussed theoretically in bulk systems using the Laughlin wave function [6]. The elementary excitations – such as quasi-holes – bear a fractional charge and under exchange a fractional statistical phase $\pm \mathrm{i}\nu\pi$ is generated (ν is the electron filling factor, here chosen to be the inverse of an odd integer).

The above results were generated for an infinite disc-shaped geometry. Transport measurements typically involve two dimensional electron gas samples with boundaries, connected to current and voltage contacts. In the integer quantum Hall effect (IQHE), the chiral waves which propagate along the sample are the quantized version of the classical skipping electron orbits. In fractional quantum Hall fluids with boundaries, the excitations are not electron-like as in the IQHE, yet they also propagate as chiral edge waves which carry the current. Long wave length edge excitations can be described by a Luttinger liquid, as described by the hydrodynamic model of [7,8]. In a Hall bar, backscattering can be induced by bringing together two counter-propagating edges using a point contact. The prediction of such a system is that in the absence of impurities or backscattering, the maximal edge current is $I_{\mathrm{M}} = \nu e^2/h$, while for weak backscattering, the current voltage characteristic is highly non linear for Laughlin fractions, i.e. $\langle I_{\mathrm{B}}\rangle \sim V^{2\nu-1}$ ($\langle I_{\mathrm{B}}\rangle$ is the average backscattering current and V is the voltage bias between the two edges). However, the current alone does not provide direct information about the charge and the statistics of the elementary excitations.

It was suggested [9,10] that a two terminal noise measurement performed on a gated mesoscopic device in the weak backscattering regime provides a direct link to the quasiparticle charge. If quasiparticles are scattered from one edge to the other one by one, the usual Shottky formula $S_{\mathrm{B}} = 2e^*\langle I_{\mathrm{B}}\rangle$ which relates the zero frequency backscattering noise (see Eq. (1) below) to the average current flowing between the two edges applies [11], except that the effective carrier charge $e^* = \nu e$ contains the electron filling factor. This fractional charge was measured recently by several groups [12,13]. These results constitute a successful test of the Luttinger liquid models [9,10] based on chiral edge Lagrangians [7,8]. On the theoretical side, these perturbative calculations of the noise have now been supplemented [14] by exact results using the Bethe ansatz solution of the boundary sine-Gordon model. This allows us to describe the whole parameter range from weak to strong backscattering at the point contact.

As noted above, the discussion of the physics of fractional Hall transport has been centered on the charge of the quasiparticles, rather than the statistics. Signatures of the statistics should be explicit in the noise characteristics of a two terminal (or two edge) sample, as long as one goes beyond the Poissonian noise. In Fermi systems, the Pauli principle leads to a quantum shot noise formula $S = 2e\langle I\rangle(1 - T)$ (T is the transmission probability) where the Shottky result is reduced at high transmission [15–19]. This reduction was observed in two experiments [20,21]. So far in the FQHE, the analog of the measurement of the noise reduction factor for fermions has not been conclusive [12]. Here [22], it is suggested that the statistics can be monitored via a Hanbury-Brown and Twiss experiment [23] – similar to the case of photons and electrons. Quasiparticles are emitted from one edge and tunnel through the correlated Hall fluid in order to be collected into two receiving edges (see Fig. 2). This constitutes a mesoscopic analogue of a collision process which involves many (2 or more) quasi-particles. A central result is that the noise correlations are negative, but strongly reduced in amplitude at $\nu = 1/3$ compared to the fermionic result. Below this filling factor, the noise correlations become positive but have a small amplitude.

2. Hanbury-Brown and Twiss correlations

Particularly interesting is the role of electronic correlations in quantum transport. Correlations can have several causes. First, they may originate from the interactions between the particles themselves. Second, correlations are generated by a measurement which involves two or more particles. In the latter case, non-classical correlations may occur solely because of the bosonic or fermionic statistics of particles, with or without interactions. The measurement of noise – the Fourier transform of the current–current correlation function – constitutes a two particle measurement, as implied in the average of the two current operators:

$$S_{\alpha\beta}(\omega = 0) = \int \mathrm{d}t \left(\left\langle I_\alpha(t) I_\beta(0) + I_\beta(0) I_\alpha(t)\right\rangle - 2\langle I_\alpha\rangle\langle I_\beta\rangle\right). \tag{1}$$

Here I_α is the current operator in reservoir α, and the time arguments on the average currents have been dropped, assuming a stationary regime.

Consider the case of photons propagating in vacuum: the archetype of a weakly interacting boson system. It was shown [23] that when a photon beam is extracted from a thermal source such as a mercury arc lamp, the intensity correlations measured in two separated photo-multipliers are always positive. On average, each photon scattering state emanating from the source can be populated by several photons at a time – due to the bunching property of bosons. As a result when a photon is detected in one of the photo-multipliers, it is likely to be correlated with another detection in the other photo-tube. The positive correlations can be considered as a diagnosis of the statistics of the carriers performed with a quantum transport experiment.

What should be the equivalent test for electrons? A beam of electrons can be viewed as a train of wave packets, each of which is populated at most by two electrons with opposite spins. If the beam is fully occupied, negative correlations are expected because the measurement of an electron in one detector is accompanied by the absence of a detection in the other one, as depicted in Fig. 1(a). The discovery of Hanbury-Brown and Twiss prompted a proposal to repeat the measurement using electron beams propagating in vacuum [24]. However, it was never possible to achieve the occupation (close to full occupancy of the scattering states) necessary to obtain a measurable anticorrelation signal. It was understood later [17–19] that if the electrons propagate in a quantum wire with few lateral modes, near

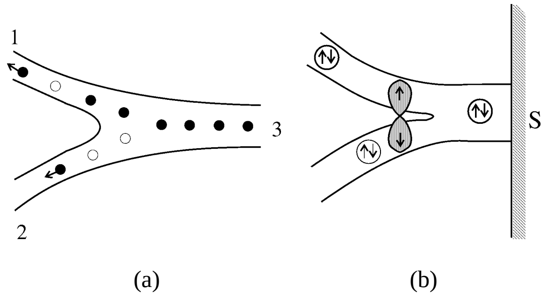

Figure 1. (a) Hanbury-Brown and Twiss geometry in a normal metal fork with electrons injected from 3 and collected in reservoirs 1 and 2. Occupied and empty electron wave packet states are identified as black and white dots, respectively. (b) Hanbury-Brown and Twiss geometry in a superconductor–normal metal fork. Cooper pairs are emitted from the superconductor, and the two constituent electrons can either propagate in the same lead, or propagate in an entangled state in both leads.

maximal occupancy could be reached, and the anticorrelation signal would then be substantial, and possibly measurable. Consider the device drawn in Fig. 1(a): electrons emanating from reservoir 3 have a probability $T_{1(2)}$ to end up in reservoir $1(2)$. The scattering theory of electron transport then specifies the noise correlations between the two branches in the presence of a symmetric voltage bias between 3 and 1, 2:

$$S_{12}(\omega = 0) = -4\big(e^3|V|/h\big)T_1T_2. \tag{2}$$

These electronic noise correlations were measured recently [25,26] by two groups working either in the IQHE regime or in the ballistic regime, with beam splitters designed with metallic gates. These challenging experiments confirmed that the Pauli principle is at work in forked sample geometries. Here, the negative correlations are used as a reference for comparison with the result for fractional quantum Hall edge transport.

Negative correlations for fermionic systems are most natural, yet there exist situations where they can be positive. If the reservoir which injects electrons in the fork is a superconductor as in Fig. 1(b), both positive and negative correlations are possible [27]. Charge transfer between the injector and the two collectors 1 and 2 is then specified by the Andreev scattering process, where an electron is reflected as a hole. Positive correlations are linked to the proximity effect, as superconducting correlations (Cooper pairs) leak in the two normal leads. Depending on the nature of the junction in Fig. 1(b), it may be more favorable for a pair to be distributed between the two arms than for a pair to enter a lead as a whole. The detection of an electron in 1 is then accompanied by the detection of an electron in 2, giving a positive correlation signal. Applying spin or energy filters to the normal arms 1 and 2, it is possible to generate positive correlations only [28]. As an illustration, energy filters with resonant energies symmetric above and below the superconductor chemical potential can select electrons (holes) in leads $1(2)$. The positive correlation signal then can be written:

$$S_{12}(\omega = 0) = 2\big(e^2/h\big)\sum_{\sigma=\uparrow,\downarrow}\int_0^{e|V|}\mathrm{d}\varepsilon\, R_{e1\sigma,h2-\sigma}(\varepsilon)\big[1 - R_{e1\sigma,h2-\sigma}(\varepsilon)\big], \tag{3}$$

where $R_{e1\sigma,h2-\sigma}$ is the Andreev reflection probability for a hole incoming from 2 to be scattered as an electron in 1. Negative bias voltage $eV < 0$ insures that the constituent electrons of a Cooper pair from the superconductor are emitted into the leads. The propagation of a Cooper pair in a given lead is blocked by the filters because of energy requirements. Note the similarity with the quantum noise suppression mentioned above. This is no accident: by adding constraints to our system, it has become a two terminal device, such that the noise correlations between the two arms are identical to the noise in one arm. In fact, the device of Fig. 1(b) with additional filters constitutes a source of entangled electrons, allowing us to probe the non local nature of quantum mechanics [29]. Bell inequalities [30] have been shown to be maximally violated in this stationary transport situation [31].

Negative and positive correlations are in principle possible in mesoscopic devices (with the latter effect tied to a pairing mechanism). Here, our goal is to extend these considerations to the FQHE. One could in principle start from an approach where the three reservoirs are filled with 'particles' which obey fractional statistics. A decade ago, exclusion statistics – a form of statistics which is intermediate between fermions and bosons – was proposed by Haldane [32]. A recent work where a one body scattering matrix specified the transmission of such particles showed that the shot/thermal noise crossover is indeed affected by the exclusion statistics [33]. However the zero frequency noise cross correlations were found to be similar to that of fermions [27].

Quite generally, in a physical system where many body interactions are present, the scattering properties of quasiparticles should be addressed from first principles. The quest for the signatures of unconventional statistics in noise correlations experiments has to be approached from the point of view of a microscopic model, using a non-equilibrium thermodynamics approach to describe transport.

3. Model Hamiltonian

The suggested geometry of our proposed experiment [22] requires *three* edges with two tunneling paths between them. Tunneling occurs through the quantum Hall fluid. It is drawn schematically with two point contacts in Fig. 2(a). Note that previous noise correlation measurements in the quantum Hall effect [12, 25,26] dealt with a single constriction and with tunneling between two edge states. It can be shown using current conservation that this latter geometry does not allow a direct probe of fractional correlations. Here, edge 3 injects quasiparticles in the system, which can subsequently tunnel through the fluid at the two possible locations.

When a quasiparticle tunnels to edge 1, it leaves a quasiparticle hole on edge 3. Removing another quasiparticle later on will be affected by the first tunneling event. In the case of fermions, the Pauli principle will prevent this second removal, leading to negative noise correlations. Clearly the present geometry is well adapted to address the same issue for fractional excitations. Alternatively, this setup can be considered as a detector of partition noise between edge 1 and 3, but in the presence of a 'noisy' injecting current (due to backscattering between 2 and 3).

The edge modes running along each gate, characterized by chiral bosonic fields ϕ_l $(l = 1, 2, 3)$ are described by a Hamiltonian:

$$H_0 = (v_F\hbar/4\pi) \sum_{l=1,2,3} \int \mathrm{d}s\, (\partial_s\phi_l)^2, \tag{4}$$

with s the curvilinear abscissa and v_F is the Fermi velocity. The ϕ_l satisfy the commutation relation $[\phi_l(s), \phi_{l'}(s')] = \mathrm{i}\pi\delta_{ll'}\,\mathrm{sgn}(s - s')$ [7,8]. The quasiparticle annihilation operators are written in a bosonized form $\psi_l(s) = (2\pi\alpha)^{-1/2}\mathrm{e}^{-\mathrm{i}\sqrt{\nu}\phi_l(s)}$ with α a short distance cutoff.

Because we are dealing with a non equilibrium thermodynamics problem, the noise will be computed below using the Keldysh technique. The elementary building blocks of perturbation theory are the Keldysh ordered Green's functions for the bosonic fields.

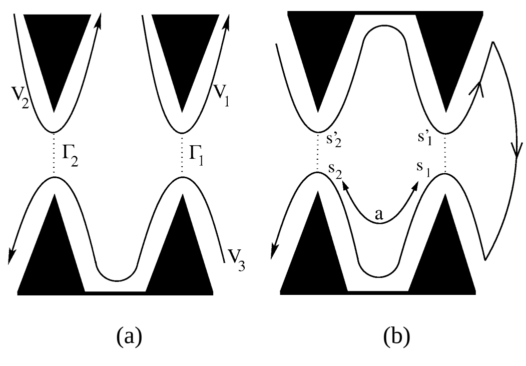

Figure 2. (a) Hanbury-Brown and Twiss geometry in the FQHE: 3 metallic gates (black) define 3 edge states, with 2 tunneling paths and with tunneling amplitudes Γ_1 and Γ_2. Voltage sources $V_{1,2,3}$ are attached to each edges. The outgoing currents are measured on edges 1 and 2. (b) Same geometry, except that the edges are linked together in order to specify the commutation relations between the quasiparticle fields.

Tunneling between edge 3 and the other edges is described by the time dependent hopping Hamiltonian [10]:

$$H_t(t) = \sum_{l=1}^{2} \sum_{\epsilon=\pm} \Gamma_l \, e^{-i\epsilon e^* \chi_l(t)/\hbar c} T_l^{(\epsilon)} \quad \text{where } T_l = T_l^{(+)} = \psi_3^\dagger(s_l)\psi_l(s_l') = T_l^{(-)\dagger}. \tag{5}$$

$\psi_l^\dagger$ creates a quasiparticle excitation with charge $e^* = \nu e$ on edge l. Γ_l is a quasiparticle hopping amplitude. A voltage bias can be imposed using the Peierls substitution, i.e. $e^*\chi_l(t) = \hbar\omega_0 ct$ for a constant DC bias $V_3 - V_l = \hbar\omega_0/e^*$ imposed between edges 3 and l. The two receiving edges are assumed to be decoupled.

4. Fractional statistics

Fractional statistics is a consequence of the Laughlin correlated state describing the FQHE. The commutation relations for the ψ_l's should originate from the derivation of the Chern–Simons boundary action. For the present problem, the procedure for taking fractional statistics into account has been described in detail in [34]. With the above definition of the quasiparticle fields, fractional statistics on a given edge is enforced with the help of the commutation relation [35]:

$$\psi_l(s)\psi_l(s') = e^{\pm i\sqrt{\nu}\pi\, \mathrm{sgn}(s-s')}\psi_l(s')\psi_l(s). \tag{6}$$

On the other hand, no similar commutation relations are available for fields with $l \neq l'$. A reasonable guess would be to include a phase factor $e^{\pm i\pi\nu}$ when two such fields are commuted. Yet the choice of the sign of this phase has to be justified by the topology of our experiment. Here, we conjecture that the commutation relations are similar to those for a system of connected edges.

Klein factors [34,36,37] are introduced, which means that each quasiparticle operator ψ_l is changed to $F_l\psi_l$. The requirement of unitarity, $F_l^\dagger F_l = F_l F_l^\dagger = 1$, is consistent with the fermionic case ($\nu = 1$).

Consider the closed system (Fig. 2(b)) where edges are connected with an open contour which brings all the edges into one. Klein factors are not necessary in this system because the commutation relations of tunneling operators are enforced by a single chiral bosonic field ϕ. Yet, by looking at the commutation relations of different tunneling operators we show below that for a system with the same topology but where edges are disconnected, the Klein factors can be computed. The operator describing tunneling from edge '3' (location s_l) to edge 'l' (location s_l') can be written $T_l = (2\pi\alpha)^{-1} e^{i\sqrt{\nu}\phi(s_l)} e^{-i\sqrt{\nu}\phi(s_l')}$. If the two tunneling paths do not cross, one can show that $[T_1, T_2] = 0$ using the Baker–Campbell–Hausdorff formula together with $\mathrm{sgn}(s_2 - s_1) = -\mathrm{sgn}(s_1' - s_2')$ which is imposed by chirality.

We turn to the same system depicted in Fig. 2 but where all edges are disconnected. In order to enforce fractional statistics, Klein factors are introduced in such a way that the tunneling operators become $T_l = \psi_3^\dagger(s_l)\psi_l(s_l')F_3F_l^\dagger$. We require that the commutator of the tunneling operators of this disconnected system give the same results as the connected one. The Klein factors thus have to satisfy the relation:

$$(F_3F_2^\dagger)(F_3F_1^\dagger) = e^{i\nu\pi\, \mathrm{sgn}(s_2-s_1)}(F_3F_1^\dagger)(F_3F_2^\dagger). \tag{7}$$

Equation (7) is compatible with fractional statistics: $F_lF_{l'} = e^{-i\pi\nu p_{ll'}}F_{l'}F_l$, where $p_{ll'}$ are the elements of an antisymmetric 3×3 matrix with elements which are tied to the topology. The connection between the elements $p_{ll'}$ and the sign of the algebraic distance reads: $\mathrm{sgn}(s_{l'} - s_l) = p_{l'3} + p_{3l} + p_{ll'}$. If, for instance, $l' = 2$ and $l = 1$ with $\mathrm{sgn}(s_2 - s_1) = 1$, we can choose: $p_{31} = -1 = -p_{32} = -p_{12}$. In particular, this insures that the fields ψ_j anti-commute for $\nu = 1$ and commute for $\nu \to 0$. It is then most convenient to compute the time ordered Klein factor product using a bosonization formulation similar to that used for the quasiparticle fields:

$$F_3 F_l^\dagger \equiv \mathrm{e}^{-\mathrm{i}\sqrt{\nu}\theta_l}. \tag{8}$$

Introducing the fact that $\mathrm{sgn}(s_2 - s_1) = 1$ in Eq. (7), we find (using the Hausdorff formula) that $[\theta_1, \theta_2] = \mathrm{i}\pi$. From the fields θ_l, annihilation and creation operators b and $b^\dagger$ can then be defined as $b \equiv (\theta_1 + \mathrm{i}\theta_2)/\sqrt{2\pi}$, with the usual property that $\langle b^\dagger b\rangle_0 = 0$ ('ground' state) and $\langle bb^\dagger\rangle_0 = 1$.

The ground state expectation value of products of the bosonic fields reads: $\langle\theta_1\theta_2\rangle_0 = -\langle\theta_2\theta_1\rangle_0 = \mathrm{i}\pi/2$ and $\langle\theta_1\theta_1\rangle_0 = \langle\theta_2\theta_2\rangle_0 = \pi/2$.

Because we are dealing with a non-equilibrium transport situation, a Keldysh matrix is introduced for the bosonic fields of Eq. (8):

$$g_{12}^{\eta_1\eta_2}(t) \equiv \left\langle T_K\left\{\theta_1(t^{\eta_1})\theta_2(0^{\eta_2})\right\}\right\rangle = \mathrm{i}\frac{\pi}{2}\left\{\frac{\eta_1+\eta_2}{2}\,\mathrm{sgn}(t) - \frac{\eta_1-\eta_2}{2}\right\}. \tag{9}$$

For the same tunneling events one finds $g_{ll}^{\eta_1\eta_2}(t) = \pi/2$.

5. Non equilibrium current cross correlations

The dynamics of the free fields brought together with the above statistical constraints are now applied to compute the transport properties of the three terminal device of Fig. 2. The perturbative computation of the correlator of Eq. (1) with currents in the Heisenberg representation is achieved by going to the interaction representation and expressing the noise in terms of time ordered products along the Keldysh contour. The lowest non-vanishing contribution to the noise in a given lead are the autocorrelations S_{11} and S_{22}: the noise in a given terminal, which to order $\mathrm{O}(\Gamma_l^2)$ $(l = 1, 2)$ gives the Poisson result. The lowest non-vanishing contribution to the noise cross correlator between terminals 1 and 2 is of order 4 $(\mathrm{O}(\Gamma_1^2\Gamma_2^2))$ in the tunneling amplitudes [22]:

$$\begin{aligned} S_{12}(t,t') = \frac{-2e^{*2}}{\hbar^2}\left(\frac{-\mathrm{i}}{\hbar}\right)^2 \sum_{\eta\eta_1\eta_2}\sum_{\epsilon\epsilon'\epsilon_1\epsilon_2} \epsilon\epsilon'|\Gamma_1\Gamma_2|^2\eta_1\eta_2 \\ \times \int \mathrm{d}t_1 \int \mathrm{d}t_2 \left\langle T_K T_1^{(\epsilon)}(t^\eta) T_2^{(\epsilon')}\left(t'^{-\eta}\right) T_1^{(\epsilon_1)}\left(t_1^{\eta_1}\right) T_2^{(\epsilon_2)}\left(t_2^{\eta_2}\right)\right\rangle \\ \times \exp\left[\mathrm{i}\epsilon\chi_1(t) + \mathrm{i}\epsilon'\chi_2(t') + \mathrm{i}\sum_{l=1,2}\epsilon_l\chi_l(t_l)\right], \end{aligned} \tag{10}$$

where η, η_1 and η_2 label the contours $(\pm)$ to which the times are assigned. Note that in this real time noise correlator, the product of the current averages – as they appear in Eq. (1) – have not yet been subtracted. To this order, the Keldysh ordered product in Eq. (10) contains the product of two contributions. The first one originates from the computation of the average of the bosonic fields associated with each edge: it contains the dynamical aspect of these fields, and gives rise to exponentiated chiral Green functions. The contraction of these chiral fields leads to quasiparticle conservation laws $\epsilon = -\epsilon_1$ and $\epsilon' = -\epsilon_2$. The other factor has no dynamics other than that specified by time ordering; it contains the Klein factors.

Exploiting the symmetry property of the Green's function of the chiral bosonic fields $G_{-\eta,-\eta'}(s,t) = [G_{\eta,\eta'}(s,t)]^*$, one obtains:

$$\begin{aligned} S_{12}(t) = 4\frac{|e^*\tau_0\Gamma_1\Gamma_2|^2}{(h\alpha)^4}\,\mathrm{Re}\int_{-\infty}^{\infty}\mathrm{d}t_1\int_{-\infty}^{\infty}\mathrm{d}t_2 \sum_{\epsilon,\eta_1,\eta_2=\pm}\epsilon\eta_1\eta_2\cos\left(\omega_0(t_1+\epsilon t_2)\right)\mathrm{e}^{2\nu[G_{+,\eta_1}(0,t_1)+G_{-,\eta_2}(0,t_2)]} \\ \times\,\mathrm{e}^{\nu\epsilon[\widetilde{G}_{+\eta_2}(-a,t+t_2)+\widetilde{G}_{\eta_1,-}(-a,t-t_1)]}\,\mathrm{e}^{-\nu\epsilon[\widetilde{G}_{+-}(-a,t)+\widetilde{G}_{\eta_1\eta_2}(-a,t+t_2-t_1)]}, \end{aligned} \tag{11}$$

where ϵ represents the product of the two charge transfer processes: $\epsilon = -/+$ when the quasiparticles tunnel in the same/opposite direction. In Eq. (11), the Green's function for edge 3 has been translated due to the Klein factors, which are given at zero temperature:

$$\widetilde{G}_{\eta\eta'}(-a,t) = G_{\eta\eta'}(-a,t) + g_{12}^{\eta\eta'}(t) = -\ln\left(\frac{t}{\tau_0} + \frac{a}{\tau_0 v_{\mathrm{F}}} + \mathrm{i}\eta\tau_0\right),$$
$$G_{\eta\eta'}(-a,t) = -\ln\left[1 + \frac{\mathrm{i}}{2}\left(\frac{t}{\tau_0} + \frac{a}{\tau_0 v_{\mathrm{F}}}\right)\left((\eta+\eta')\,\mathrm{sgn}(t) + (\eta-\eta')\right)\right], \tag{12}$$

where the times have been rescaled by the short time cutoff $\tau_0 \simeq \alpha/v_{\mathrm{F}}$ and $g_{12}^{\eta\eta'}(t)$ is defined in Eq. (9). The integrand in the double integral in Eq. (11) for $\nu < 1$ decays slowly with both time arguments. Absolute convergence is obtained for $\nu > 1/2$ from the power law decay in time. For $\nu < 1/2$ convergence is due to the oscillatory terms. The real time correlator is computed with the assumptions $a = 0$ and the biases between 1 and 3 (2 and 3) are chosen to be symmetric and equal to ω_0. For $a \neq 0$, the real time correlator is obtained using the property:

$$S_{12}(t) \equiv S_{12}(t,-a) = S_{12}(t + a/v_{\mathrm{F}}, a = 0). \tag{13}$$

The configuration $\eta_1 = -$; $\eta_2 = +$ is retained because first it provides the large time behavior and second, it corresponds to the contribution of the zero frequency noise correlations (to be computed later on). A leading contribution to $S_{12}^{-+}(t)$ is plotted in Fig. 3, as well as the excess noise at $\nu = 1$ for comparison. The latter oscillates with a frequency ω_0, and decays as t^{-2}. $S_{12}^{-+}(t)$ scales as $|\omega_0|^{4\nu-2} f(\omega_0 t)$, with $f(x)$ an oscillatory function which decays at least as $x^{-2\nu}$, thus a slower decay than that of electrons. At large times, the frequency of the oscillations stabilizes as $\omega_0 = e^* V/\hbar$.

The result in Eq. (11) is now integrated over t after subtracting the average current products. The sign and magnitude of the $\omega = 0$ correlations tell us the tendency for the quasiparticle to exhibit bunching or antibunching. At zero temperature only $\epsilon = -1$ in $\widetilde{S}_{12}(0)$ contributes, which gives the information that an 'exclusion principle' prohibits the excitations to be transferred from the collectors to the emitter. The zero

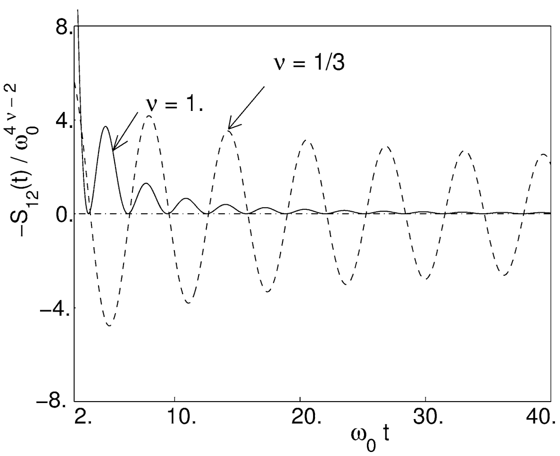

Figure 3. Contribution to the real time correlator $S_{12}^{-+}(t)$, for bias $\omega_0 = e^* V/\hbar$, normalized to $|\tau_0\Gamma_1\Gamma_2|^2|\omega_0|^{4\nu-2}$ for a filling factor $\nu = 1/3$ (dashed line). $2\langle I_1\rangle\langle I_2\rangle$ has been subtracted. The exact *excess* noise at $\nu = 1$: $S_{12}^{\mathrm{ex}}(t) \propto \sin^2(\omega_0 t/2)/t^2$ (full line), keeping all η configurations, is plotted for comparison.

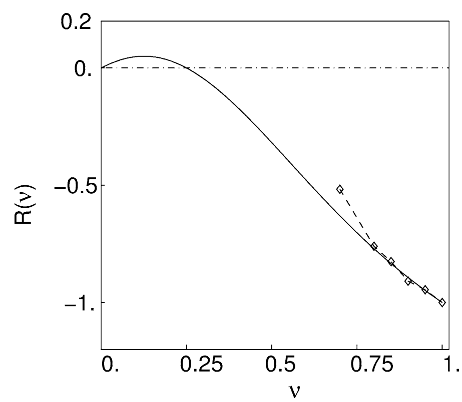

Figure 4. Normalized $\omega = 0$ correlations $R(\nu)$, plotted with the analytic expression of Eq. (16) for arbitrary ν, and compared to the direct numerical calculation for $0.7 \leqslant \nu \leqslant 1$ (lozenges).

frequency noise correlations have the general form:

$$\widetilde{S}_{12}(\omega=0) = \left(e^{*2}|\omega_0|/\pi\right) T_1^r T_2^r R(\nu), \tag{14}$$

where the renormalized transmission probabilities are $T_l^r = (\tau_0|\omega_0|)^{2\nu-2}[\tau_0\Gamma_l/\hbar\alpha]^2/\Gamma(2\nu)$, and the dimensionless function $R(\nu)$ characterizes the statistical correlations. Note the similarity with the non-interacting noise formula of Eq. (2), which has an extra factor 2 because of spin degeneracy. At $\nu = 1$, the cutoff dependence drops out of Eq. (14) and it is shown explicitly that $R(1) = -1$ using contour integration, so that $\widetilde{S}_{12}$ coincides exactly with the scattering theory result [17–19]. This issue represents a crucial test of the implementation of the Klein factors.

Moreover, for arbitrary ν, $R(\nu)$ could in principle be directly measured in an experiment. Indeed one can rescale the noise correlation $\widetilde{S}_{12}$ by the individual shot noises $\widetilde{S}_l \simeq 2e^*\langle I_l\rangle$ or equivalently (at this order) by the individual currents:

$$R(\nu) = |\omega_0|\widetilde{S}_{12}/\left[4\pi\langle I_1\rangle\langle I_2\rangle\right]. \tag{15}$$

An analytical expression for the function $R(\nu)$ in Eq. (14) is obtained for the filling factor range $1/2 < \nu \leqslant 1$:

$$R(\nu) = \frac{-\sin(\pi\nu)\Gamma^2(2\nu)}{2\sqrt{\pi}\,\Gamma(2\nu-1)\Gamma(2\nu-1/2)\Gamma(-\nu)} \sum_{n=0}^{\infty} \frac{\Gamma(n-\nu)\Gamma(n+1-\nu)\Gamma(\nu+n-1/2)}{n!\,\Gamma(n+\nu)\Gamma(n+3/2-\nu)}, \tag{16}$$

which converges as $n^{-\nu-2}$. Here, ν has a physical meaning only when it is a Laughlin fraction $1/m$ (m odd). At first glance the only physical filling factor which one can reach with this series is $\nu = 1$. Yet, it has been shown [22] that $R(\nu)$ can be analytically continued to the interval $[0, 1/2]$. Indeed, the terms of the series of Eq. (16) are still well defined for $\nu < 1/2$.

For $\nu \simeq 1$, a check is obtained (Fig. 4) by direct numerical integration of $S_{12}(t)$. The comparison between the series solution of Eq. (16) and the numerical data shows a fair agreement for $0.7 \leqslant \nu \leqslant 1$.

Starting from the IQHE and decreasing ν (Fig. 4), the noise correlations between the two collector edges are reduced in amplitude at any $\nu = 1/m$. When a quasi-particle is detected, in edge 1, one is less likely to observe a depletion of quasi-particles in edge 2 than in the case of noninteracting fermions. The reduction of the (normalized) noise correlations constitutes a direct prediction of the statistical features associated with fractional quasiparticles in transport experiments, and should be detectable at $\nu = 1/3$.

$\widetilde{S}_{12}$ vanishes at $\nu = 1/4$ and becomes positive for lower physical filling factors $(1/5, 1/7, \ldots)$, which is reminiscent of bosons bunching up together [23], or alternatively, of the positive noise correlations of a normal metal–superconductor Y-shaped junction [27].

The continuation procedure could be jeopardized if other tunneling operators generated by the renormalization group (RG) procedure happened to be more relevant at $\nu = 1/2$. Below the general expression for the Hamiltonian which describes multiple tunneling is specified together with the RG flow associated with each term:

$$\widetilde{H}_t \equiv \sum_{\vec{n}} V_{\vec{n}}\, \mathrm{e}^{\mathrm{i}\sqrt{\nu}\vec{n}\cdot\vec{\phi}}, \qquad \frac{\mathrm{d}V_{\vec{n}}}{\mathrm{d}l} = \left(1 - \frac{\nu}{2}\sum_{l=1}^{3} n_l^2\right) V_{\vec{n}}, \tag{17}$$

where $\vec{n} = (n_1, n_2, n_3)$ (n_l integer) satisfies quasiparticle conservation $n_1 + n_2 + n_3 = 0$ and $\vec{\phi} = (\phi_1, \phi_2, \phi_3)$ contains the fields of the three edges. The bare tunneling terms of Eq. (5) are relevant at $\nu < 1$, and always dominates all other $V_{\vec{n}}$, which become relevant below $\nu = 1/3$ at most. Note that here the noise correlations were computed while taking into account the most relevant tunneling operators. Situations where less relevant or marginally relevant operators contribute have been encountered [38], and here such additional tunneling operators could modify the predictions for $\nu < 1/3$.

6. Conclusions

To summarize, Hanbury-Brown and Twiss geometries provide a physical test of mesoscopic transport. They can be used to check the bosonic/fermionic statistics of the carriers, or alternatively to generate entangled streams of particles. Here we have argued that they provide direct information about fractional statistics in the FQHE. Edge states in the fractional quantum Hall effect constitute an ideal ground to probe the nature of quasiparticles excitations in transport measurements. The role of electronic correlations in transport has been emphasized by experimentalists and theorists alike in the measurement/prediction of the effective charge of quasiparticles, when the latter are scattered one by one – the Poissonian limit. The effect of the tunneling of one quasiparticle on another tunneling event has not been considered so far. Such information should be implicit in the Bethe ansatz formulations of the problem [14]. So far these approaches have not been adapted to deal with Hanbury-Brown and Twiss geometries. The present work has addressed the role of statistics in two quasiparticle scattering for a correlated one dimensional system using an intuitive, tunneling approach. Quite generally, information about statistics is necessarily contained in quantum measurements which involve two particles or more: here the zero frequency noise correlations play the role of the intensity correlator in the early quantum optics experiment of [23].

The noise correlations are found to be reduced in amplitude when compared to the fermionic case, a prediction which could be directly tested in experiments. At filling factor $\nu = 1/3$, they are negative, suggesting antibunching behavior. In our previous work [22], the assumption of an arbitrary small separation between the two tunneling locations was made. Clearly, this imposes drastic constraints on possible experimental implementations of this proposal: the two point contact shown in Fig. 2 have to lie closer than the relevant lengthscales describing the edge excitations. We have shown elsewhere [39] that this assumption is unnecessary when computing the zero frequency noise correlations, thus allowing more freedom in designing an experimental setup similar to Fig. 2, with two separated point contacts, within a phase coherence length.

A starting result is the positive correlations found for filling factors $\nu \leqslant 1/5$. Here, such correlations can be either attributed to the fact that the fractional statistics are bosonic near $\nu \to 0$, as suggested by the commutation relations, or can be linked to the eventual presence of composite bosons resulting from attachment of an odd number of flux tubes [40,41]. On the one hand, one is dealing with a fermionic system where *large* negative correlations are the norm. On the other hand, the presence of an external magnetic field and the (resulting) collective modes of the edge excitations favor correlations which can be positive for small enough ν, but which are always reduced in amplitude. In fact, the analysis of the charge probability distribution in a simple, two edge system [42] has confirmed that this distribution tends to a classical one when the limit $\nu \to 0$ is taken, which is in qualitative agreement with the present findings on the HBT geometry. The competition between these two tendencies yields a statistical signature which is close to zero – analogous to the noise correlations of 'classical' particles.

The tendency for the noise correlation ratio to be reduced compared to its non interacting value is consistent with the existing data for two-terminal devices [12], as a connection exists between the two types of measurements [17–19,27]. There, shot noise suppression was observed to be weaker than that of bare electrons, which then multiplies the shot noise by $1 - T$ [17–21], the reflection amplitude. However, a qualitative analysis of noise reduction in this situation is rendered difficult because of the nonlinear current–voltage characteristics. In contrast, Hanbury-Brown and Twiss experiments constitute a direct and crucial test of the Luttinger liquid models used to describe the edge excitations in the FQHE, as it addresses the role of fractional statistics in transport experiments. Similar analysis could be employed to deal with hierarchical fractions of the Hall effect, or alternatively to nonchiral Luttinger liquids, such as carbon nanotubes.

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