C. R. Physique 3 (2002) 717–732

Solides, fluides : propriétés électroniques et optiques/*Solids, fluids: electronic and optical properties*

L'EFFET HALL QUANTIQUE FRACTIONNAIRE

THE FRACTIONAL QUANTUM HALL EFFECT

DOSSIER

Quantum Hall effect, chiral Luttinger liquids and fractional charges

Patrice Roche [a]*, V. Rodriguez [a], D. Christian Glattli [a,b]

[a] **Service de physique de l'etat condensé, CEA Saclay, 91191, Gif-sur-Yvette, France**

[b] **Laboratoire de physique de la matière condensée, École normale supérieure, 24, rue Lhomond, 75231 Paris, France**

Received 30 June 2002; accepted 10 July 2002

Note presented by Guy Laval.

Abstract We review some basic properties of the Fractional Quantum Hall Effect and particularly address the physics of the edge states. The chiral Luttinger liquid properties of the edges are discussed and probed experimentally using transport measurements. Shot noise measurements, which allow determination of the quasiparticle charge are also discussed. ***To cite this article: P. Roche et al., C. R. Physique 3 (2002) 717–732.***

quantum Hall effect / fractional charge / Luttinger liquids / 2D electron gas

Effet Hall quantique, liquides de Luttinger et charges fractionnaires

Résumé Nous présentons une revue de quelques propriétés élémentaires de l'Effet Hall Quantique Fractionnaire et plus particulièrement des états de bord. Les aspects liquides chiraux de Luttinger des états de bords sont abordés et sondés expérimentalement par des mesures de transport. Les expériences de bruit de grenaille qui permettent de sonder la charge des quasiparticules sont aussi discutées. ***Pour citer cet article : P. Roche et al., C. R. Physique 3 (2002) 717–732.***

effet Hall quantique / charge fractionnaire / liquide de Luttinger / gaz d'électrons bidimensionnel

1. The quantum Hall effect

1.1. Introduction

The quantum Hall effect occurs in a two-dimensional electrons gas (2DEG) at low temperature in a high perpendicular magnetic field. The strong quantization of the cyclotron motion makes the 2DEG similar to a flat macroscopic atom made of 10^{11} electrons. Here, the magic quantum numbers are replaced with

* Correspondence and reprints.
E-mail address: roche@drecam.saclay.cea.fr (P. Roche).

S1631-0705(02)01375-0/REV

magic values of the filling factor $\nu = n_s/n_\Phi$ which measures the electron density n_s in units of the density n_Φ of the flux quantum $\phi_0 = h/e$. Sweeping this parameter, by varying the magnetic field or the density, reveals a series of new quantum fluids signaled by plateaus in the Hall resistance. The Hall plateaus take the remarkable values $\frac{q}{p}\frac{h}{e^2}$ each time ν is close to an integer value of p/q [1] ($q = 1$) or to a fraction with odd denominator [2,3] ($q = 2s + 1$). For integer values the physical origin of the phenomenon is the Fermi statistics and the cyclotron motion quantization in 2D, while for fractional values an additional ingredient is needed: the Coulomb interaction. These ingredients are the simplest one can imagine. No interaction with the host material is needed as in the case of superconductivity. It is remarkable that a so simple system has completely renewed our knowledge of quantum excitations. Topological fractionally charged excitations [3] with fractional anyonic or exclusonic quantum statistics [4–7], composite fermions [8] or composite bosons [9–11], skyrmions [12,13], etc., ... are the natural elementary excitations required to understand the quantum Hall effect. These concepts are not complicated nor difficult to understand. They all suggest how rich is the physics of the quantum Hall effect [14–17].

In this contribution we present the basic physics of the Integer and Fractional Quantum Hall Effect (FQHE) with more emphasis on the edge states properties. We show that the physics of the edges is connected to the physics of Luttinger liquids. Then, we discuss recent theoretical predictions and experimental investigations of the Luttinger liquid properties using transport measurements in the FQHE regime. Finally, we show how transport properties of the edges combined with shot noise measurements can be used to determine the charge of the fractional quasiparticle.

1.2. Realization of a clean 2D electron gas

A 2DEG is realized by confining electrons to a plane in a region free of positive compensating charges (for a review see [18,19]). The positive background can be either static ionized donors, randomly distributed in the host material far from the 2DEG, or the polarization charge of a positively biased metallic gate. This is different from the 3D case where the ionized impurities scatter the electrons with a dramatic decrease of the conductivity and electron–electron interactions effects. The electrostatics law requires another force to maintain electrons far from the donor plane. This is provided by a hard wall potential using a spatial conduction band discontinuity of the host material. The wall can be produced either using the interface between a semiconductor and an insulator or using two semiconductors with different conduction band energies. The first type, called a MOSFET, is found in silicon devices using Si/SiO_2 interface. It is used in the micro-electronic industry for micro-processors and random access memories. The second type is a semiconductor heterojunction usually $GaAs/Ga_xAl_{1-x}As$, but other III–V materials are possible. Heterojunctions find applications in high speed communication electronics. The purest systems need Molecular Beam Epitaxy (MBE) growth while standard heterojunctions can be realized using Chemical Vapor Deposition. Epitaxial growth provides atomically flat interface and a mean to modulate the doping.

In both systems, the conduction band is bent toward the potential wall in order to realize a triangular potential confining the charges close to the interface. In MOSFET, the electric field which bends the conduction band is provided by a positively polarized gate insulated by a dielectric oxyde layer. In GaAs/Ga(Al)As heterojunctions, electrons are trapped in the GaAs side, the Ga(Al)As region with larger gap provides the potential wall. The electric field is provided by donors situated in the Ga(Al)As region. Fig. 1 shows schematically the two systems and the band structure.

1.3. The integer quantum Hall effect

In two dimensions, the transport coefficients take very simple and remarkable values. For example, the conductivity σ (or resistivity) is expressed in conductance (resistance) units. In zero magnetic field, the Drude conductivity $\sigma_D = n_s e^2\tau/m^*$ can be rewritten as $\sigma_D = (e^2/h)k_F l$, where k_F is the Fermi wave vector, l is the mean free path and n_s the density of the two-dimensional electron gas (2DEG). The conductance quantum e^2/h appears as a natural unit. In this unit, the classical Drude conductivity is a

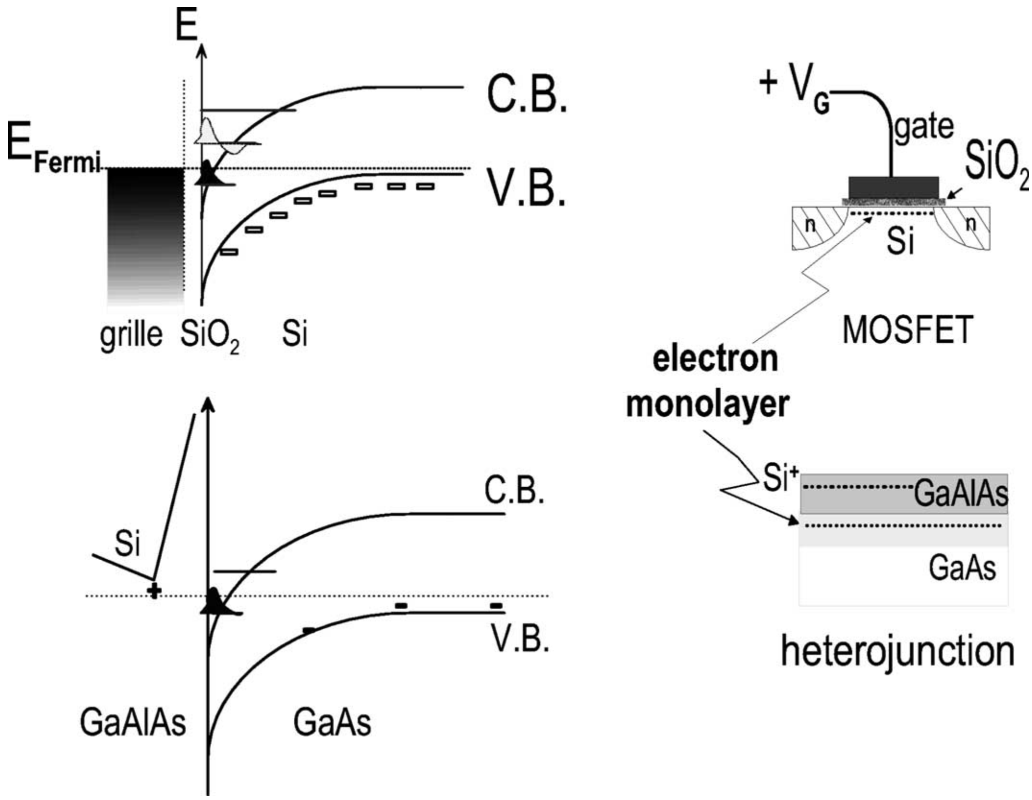

Figure 1. Band structure and confinement of a 2DEG using Si-MOSFET and GaAs/GaAlAs heterojunctions.

dimensionless number carrying information on the microscopic details (the ratio of the mean free path over the Fermi wavelength). Similar quantum units can be used for the *classical* Hall conductance in 2D. The Hall resistance is $R_H = B/en_s$. Expressing the magnetic field B in *quantum units*, i.e. by the density of flux quanta, $B = (h/e)n_\Phi$, the *classical* Hall conductance is:

$$\sigma_H = \frac{1}{R_H} = \frac{e^2}{h}\frac{n_s}{n_\Phi}. \tag{1}$$

In quantum units, σ_H is a measure of *the filling factor* $\nu = n_s/n_\Phi$.

Although, there was predictions that the longitudinal and Hall resistance in 2D was highly nontrivial [20, 21], the plateaus of the Hall resistance discovered by Klitzing, Dorda and Pepper [1] was not anticipated, nor the accurate (metrological) quantization. They appear at remarkable values:

$$R_H = \frac{h}{e^2}\frac{1}{p}, \tag{2}$$

where p is an integer. This is the Integer Quantum Hall Effect (IQHE). Eq. (1) suggests that for some range of magnetic field, the electrons participating to the Hall resistance have a density *pinned* to a multiple of the flux quantum density: $n_s = pn_\Phi$. The incompressibility of the 2DEG in the quantum Hall regime originates from the opening of an energy gap in the excitations which is directly manifested by a vanishing longitudinal resistance: a dissipationless state. Fig. 2 shows typical transport measurements in this regime.

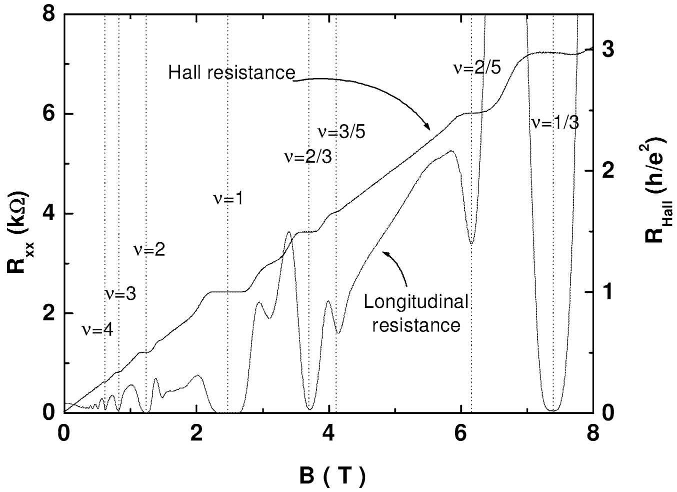

Figure 2. Typical longitudinal and Hall resistance observed at low temperature in high mobility samples. At low magnetic fields, Hall plateaus at h/pe^2 for $\nu \simeq p$ are well defined. Simultaneously the longitudinal resistance vanishes. For higher magnetic field, Hall plateaus at $\nu = 2/3$ and $\nu = 1/3$ with a simultaneous vanishing longitudinal resistance are also well defined.

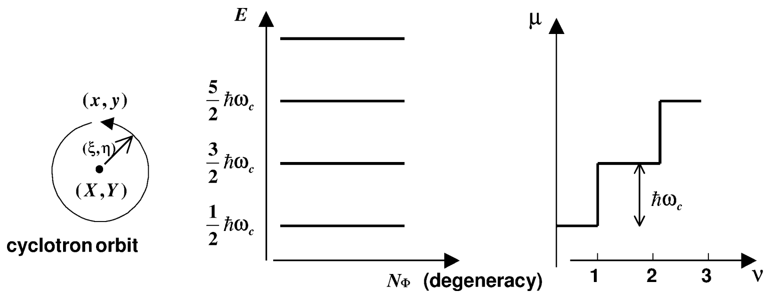

Figure 3. Decomposition of the electron motion (x, y) into orbit coordinates (ζ, η) and coordinates of the center of the cyclotron orbit (X, Y) (left); Landau levels (middle); chemical potential versus filling factor ν (right).

In the following we explain the basic physics of the IQHE. The kinetic energy $K = (\mathbf{p}+e\mathbf{A})^2/(2m^*)$ of an electron moving freely in a plane perpendicular to a magnetic field $\mathbf{B} = B\widehat{\mathbf{z}}$ is quantized into Landau levels:

$$E_n = \left(n + \frac{1}{2}\right)\hbar\omega_c, \tag{3}$$

where $\omega_c = eB/m^*$ is the cyclotron pulsation. As the energy depends on a single quantum number n while there are two degrees of freedom (the electron moves in 2D) there is a high degeneracy. Using the cylindrical gauge $\mathbf{A} = (-By/2, Bx/2, 0)$, the meaning of the degeneracy appears clearly if one replaces the conjugate pairs of electron coordinates $[x, p_x]$ and $[y, p_y]$ by a new set of conjugate pairs, see Fig. 3

$$(x, y) = (X + \xi, Y + \eta), \tag{4}$$

where

$$[\xi, \eta] = [v_y/\omega_c, -v_x/\omega_c] = -i\hbar/eB \quad \text{and} \quad [X, Y] = i\hbar/eB. \tag{5}$$

The Hamiltonian becomes $H = \frac{1}{2}m\omega_c^2(\xi^2 + \eta^2)$ and does not depend on (X, Y). The first pair of conjugate coordinates represents the fast cyclotron motion whose radius:

$$r_n = \langle n|\xi^2 + \eta^2|n\rangle^{1/2} = \left(n + \frac{1}{2}\right)^{1/2} l_c \tag{6}$$

increases with the orbital Landau level index n and scales as $l_c = (\hbar/eB)^{1/2}$, the magnetic length. The second pair is the center of the cyclotron orbit $\mathbf{R} = (X, Y)$. This is precisely the freedom to choose the center of cyclotron orbits which encodes the degeneracy of the Landau levels. The degeneracy (the number of way to place cyclotron orbit centers in the plane) equals the number $N_\Phi = n_\Phi S = eBS/h$ of flux quanta $\Phi_0 = h/e$ in the surface S. This is a direct consequence of the noncommutation of coordinates (X, Y). The 2D electron plane is analogous to the phase space $[P, Q]$ of a one-dimensional system, where the area of a flux quantum h/eB plays the role of the Planck constant h.

Assuming fully spin polarized electrons, it is easy to understand why there is a vanishing longitudinal resistivity ρ_l and conductivity ($\sigma_l = \rho_l/(\rho_l^2 + \rho_t^2)$) when the Hall resistance R_H is precisely h/pe^2, p integer. Because of the Pauli principle, a complete filling of the p lowest Landau levels occurs when the number of electrons $N = n_s S$ is exactly $pN_\Phi = pn_\Phi S$, i.e. filling factor $\nu = p$. There is an energy gap $\hbar\omega_c$ to create an electron–hole pair excitation. The 2DEG becomes a perfect insulator for longitudinal currents ($\sigma_l = 0$), while the current parallel to equipotential lines is exclusively made of dissipationless transverse currents ($\rho_l = 0$) and $\sigma_H = \frac{1}{R_H} = pe^2/h$. At finite temperature, longitudinal transport and dissipation are recovered via thermally activated inelastic processes with energy $\geqslant \hbar\omega_c$. Regarding thermodynamics, there is a jump $\hbar\omega_c$ of the chemical potential for the smallest change of the filling factor around the integer value p: removing a single flux quantum by lowering the field or adding a single electron. The system is *incompressible*.

The striking QHE features are expected only for an infinitely small width of the parameter ν around an integer. However experiments show a remarkable and accurately quantized Hall plateau *with finite width*. This is because in a large sample the chemical potential can be pinned between Landau levels by disorder which favors localized states. This can be understood by a quantum phase transition called localization. In clean and narrow samples the chemical potential is pinned by gapless excitations, called edges states, which will be discussed later.

1.4. The Fractional Quantum Hall Effect

We give here a very brief presentation of the FQHE. For simplicity electrons are assumed spin polarized and the first orbital Landau level partially filled: $\nu < 1$. Because of degeneracy, there is a large freedom to occupy the quantum states, i.e. to fill the plane, with electrons. However the Coulomb repulsion reduces our freedom to distribute electrons in the plane. This breaks the Landau level degeneracy and, for filling factors with odd denominator fractions: $\nu = 1/3, 1/5, 2/3, 2/5, 3/5, 2/7, \ldots,$ a *unique* collective wavefunction minimizes the energy.

The FQHE states form new types of quantum liquids with a ground state separated from a continuum of excitations by a gap Δ. For $\nu = 1/(2s+1)$, s integer, Laughlin proposed a trial wavefunction for the ground state which was found very accurate. The wavefunction is built from single particle states in the cylindrical vector potential gauge. Using a representation of electron coordinates as $z = x + \mathrm{i}y$ in unit of magnetic length l_c, the single particle states in the first Landau level are:

$$\varphi_m = \frac{1}{\sqrt{2\pi 2^m m!}} z^m \exp\bigl(-|z|^2\bigr). \tag{7}$$

It is instructive to look first at the Slater determinant of electrons at filling factor 1 which is a Van der Monde determinant. Its factorization gives the following wavefunction, up to a normalization constant:

$$\Psi_1 = \prod_{i<j\leqslant N} (z_i - z_j) \exp\left(-\sum_{i=1,N} |z_i|^2\right). \tag{8}$$

The polynomial part ensures a uniform distribution of electrons in the plane where on average each electron occupies the area of a flux quantum (one quantum state). The zeros at $z_i = z_j$ reflect the Pauli principle and their multiplicity 1 reflects the Fermi statistics. At filling factor $1/(2s+1)$ the polynomial for each z_i should be of degree of $(2s+1)(N-1)$ such that all electrons are also uniformly distributed on the $(2s+1)N$ states available. It should be symmetrical to ensures a uniform electron distribution in the plane. The Laughlin wavefunction has the simple polynomial form [3]:

$$\Psi_{1/2s+1} = \prod_{i<j\leqslant N} (z_i - z_j)^{2s+1} \exp\left(-\sum_{i=1,N} |z_i|^2\right). \tag{9}$$

The zeros of multiplicity $2s+1$ decreases the probability to find electrons close to each other and efficiently minimizes the correlation energy. By exchanging two electrons the wavefunction is multiplied by $(-1)^{(2s+1)}$ and satisfies the requirement that the bare electrons obey Fermi–Dirac statistics. But there is more: the extra phase factor $(-1)^{2s}$ obtained when moving two electrons around each other can be viewed as the Aharanov–Bohm flux of two fictive flux quanta bound to each electron. One can also say that electrons obey a super exclusion principle where each particle occupies $2s+1$ quantum states (i.e., the area of $2s+1$ flux quanta) so minimizing the interaction (there are deep connections with the concepts of exclusonic statistics and anyonic statistics [4–7]).

The excitations separated from the ground state by a gap carry fractional charge. The meaning of the excitations is particularly clear for the best known $\nu = 1/3$ state. The wavefunction cannot be continuously deformed and the only way to decrease the density is to empty a single particle quantum state, i.e. to create a hole having the area occupied by a single flux quantum (see Fig. 4). As the ground state corresponds to a uniform distribution of electrons, one electron per area occupied by three flux quanta, the hole carries a *fractional* charge $e^* = -e/3$. Theoretically, a hole excitation is built by multiplying the Laughlin wavefunction by $\prod_{i=1,N}(z_i - z_h)$ where z_h is the position of the hole. The energy cost Δ_h (the excitation gap) is approximately the energy required to create a disc of size Φ_0/B and charge $e/3 : (4\sqrt{2}/3\pi)(e/3)^2/4\pi\varepsilon\varepsilon_0 l_c$. Similarly, quasi-electron excitations carrying charge $e/3$ correspond to a local increase of the electronic density by removing a flux quantum. Quasi-electron or hole excitations with charge $\pm e/q$ can be generalized for other filling factor $\nu = p/q$ with q odd. An interesting theoretical issue is the statistics associated with the excitations. When moving adiabatically two quasi-holes around each other and exchanging their positions, the collective wavefunction picks up a Berry's phase factor $\exp(\mathrm{i}\pi/(2s+1))$. The excitations are not bosons nor fermions but obey anyonic statistics, a concept first introduced in the context of particle physics. While the observation of the fractionally charged excitations has been recently performed, as discussed later, evidence for fractional statistics remains an experimental challenge.

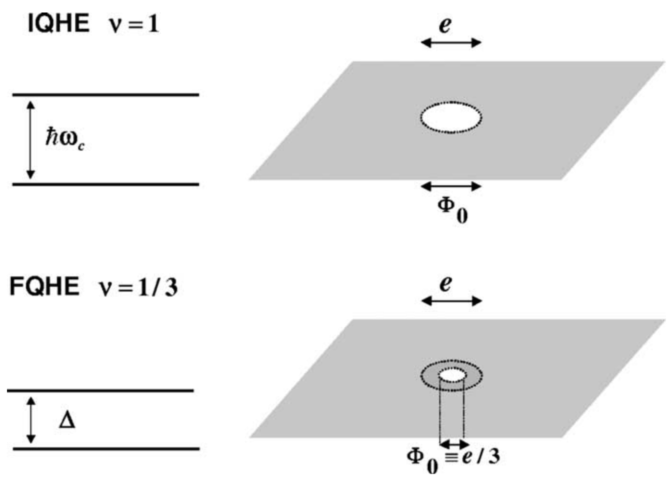

Figure 4. To create a hole having the area of a single flux quantum is equivalent to create a fractional charge $e/3$ for the $\nu = 1/3$ FQH state.

1.5. Edge channels

Here, we address the physics of the *gapless* excitations which appear at the periphery of a finite size quantum Hall conductor. The gapless excitations lead to the formation of chiral one-dimensional conduction modes called edge states. For better understanding, we first consider the noninteracting electrons of the IQHE regime. We will also neglect coupling between Landau levels, an assumption justified in most experimental situations.

Within the nth Landau level, the dynamics of electrons is described by the projected Hamiltonian:

$$H_n = \left(n + \frac{1}{2}\right)\hbar\omega_c + V^{(n)}(X,Y), \tag{10}$$

where $V^{(n)}(X,Y)$ is the confining potential $V(X,Y)$ at the edge averaged over the fast cyclotron motion of the nth level and $[X,Y] = \mathrm{i}\hbar/eB$. If the electric field due to confinement is along the $\widehat{y}$ direction, electrons drift along the boundary ($\widehat{x}$ direction) with a velocity $V_x = (1/eB)\partial V^{(n)}/\partial Y$. When populated by electrons the edge states give rise to a chiral persistent current concentrated at the edges, see Fig. 5. Because the nth Landau level is bent by the confining potential, there is a line along which the Landau level energies cross the Fermi energy E_F. The line of gapless excitations defines a one dimensional chiral conduction mode called an edge channel. At the opposite boundary, not shown in the figure, there are also similar gapless excitations running in opposite direction as the confinement electric field is of reversed sign. Thus for a filling factor $\nu = p$, there are p pairs of opposite edge channels associated with the p filled Landau levels in the bulk.

Edge channels are ideal one-dimensional (chiral) conductors: the physical separation between pairs of opposite edge channels prevents backscattering and electrons propagate elastically over huge distances (mm at low temperature) as phonon scattering is reduced. These properties have make them a convenient tool to test the generalized Landauer-Büttiker relations [22] in the context of the mesoscopic quantum transport [23]. In order to do that it is necessary to induce intentionally elastic backscattering in a controllable way. The tool used is a Quantum Point Contact (QPC) as shown in Fig. 5. A negative potential applied on a metallic gate evaporated on top of the sample depletes electrons to realize a narrow constriction in the 2DEG. This allows a controllable modification of the boundaries of the sample. The separation between opposite pairs of edges channels of a given Landau level can be made so small that the overlap between wavefunctions leads to backscattering from one edge to the other. Indeed, the QPC creates a saddle shape potential and when the potential at the saddle point is close but below the value $V_{F,n} = E_F - (n + \frac{1}{2})\hbar\omega_c$, electrons emitted from the upper left edge channel start to be reflected into the lower edge left channel with probability $R \ll 1$ but are still mostly transmitted with probability $T = 1 - R$ (see Fig. 5c). When the saddle point potential is above $V_{F,n}$ electrons are mostly reflected and rarely

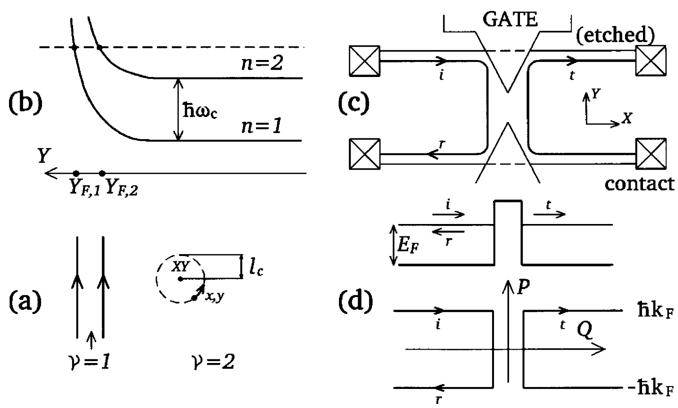

Figure 5. (a) Schematic representation of edge states and of (b) the Landau level bending; (c) reflection of an edge state by a controlled artificial impurity called Quantum Point Contact; (d) analogy with 1D semiclassical trajectories in the phase space.

transmitted $T \ll 1$. Fig. 5 shows the semi classical analogy between the real space coordinates (Y, X) of the 2D Hall conductor and the (P, Q) phase space coordinate of a real 1D conductor, as suggested by the noncommutation relations $[Y, X] = \mathrm{i}\hbar/|e|B$ and $[P, Q] = \mathrm{i}\hbar$. The physics of tunneling between opposite edge channels is clearly equivalent to that of the tunneling in a 1D system. However the chirality allows us to inject or detect electrons *at the four corners of the phase space*, something impossible with 1D systems.

Measuring the conductance is a good tool to know how many edge channels are transmitted. According to the Landauer formula, the conductance G defined as the ratio of the current I through the QPC when applying a voltage difference V between the upper left and lower right contacts is $G = Te^2/h$ if there is only one channel partially transmitted. $G = (e^2/h)(N - 1 + T)$ if there are $N - 1$ channels fully transmitted while the Nth channel is partially transmitted.

1.6. Chiral Luttinger liquids

While the Landauer formula describes the conduction of the edge channels in the IQHE regime, this is no longer the case in the fractional quantum Hall regime. A generalization of edge channels can be made in the fractional regime where the gap Δ plays the role of $\hbar\omega_c$. For fractions $1/(2s + 1)$ (Laughlin's states) there is a single fractional edge channel, while for fractions $\nu = p/(2ps \pm 1)$ (Jain's states) there are p edge channels similar to the case of p filled Landau levels in the IQHE [24–26]. However it has been predicted, and confirmed by experiments, that the conduction of the chiral modes is highly nontrivial. Because of interactions, 1D conductors are no longer Fermi liquids but Luttinger liquids [27]. In Fermi liquids the excitations which carry the current are charged quasiparticles with properties similar to that of free electrons. In Luttinger liquids, the elementary excitations are no longer quasiparticles but neutral collective modes (1D plasmons). A transport experiment, which involves the transfer of charges (i.e. quasiparticles), is now mediated by the collective modes. This leads to nonlinear current-voltage characteristics, a hallmark of Luttinger liquids.

Wen [28–31] has first shown the deep connection between fractional edge channels and the concept of Tomonaga–Luttinger liquids [32,33]. In a classical hydrodynamic approach and for a $\nu = 1/(2s + 1)$ Laughlin state, Wen has considered the periphery deformations of the 2D quantum Hall conductor which preserves the total area (like a 2D droplet of an ordinary incompressible liquid) as shown schematically in Fig. 6. If we denote $h(X, t)$ the deformation of the boundary which are respectively located at the radius $Y = Y_F$, the time varying electron density is given by:

$$n(X, Y, t) = n_s \Theta\big(Y - Y_F - h(X, t)\big), \tag{11}$$

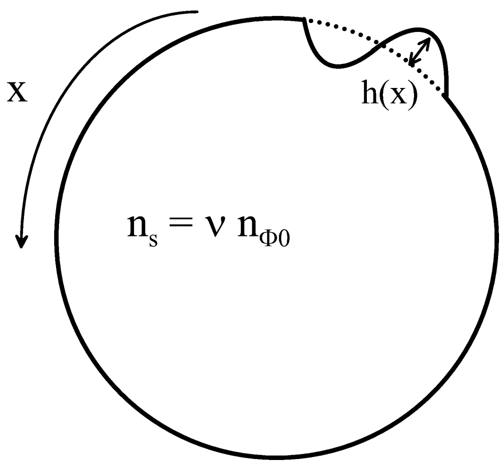

Figure 6. Edge deformation of the incompressible FQHE fluid following the hydrodynamic approach of Wen.

where $n_s = \nu eB/h$ and $\Theta(X)$ is the Heaviside function. To find the equations of motion for h we have to remember that, within the first Landau level, the single particle motion is given by the reduced Hamiltonian $H_1 = (1/2)\hbar\omega_c + V^{(1)}(X,Y)$ and that, X and Y being conjugate, the classical Poisson's bracket is $\{X,Y\} = h/eB$. Using the equation of motion for the 2D density $\partial n/\partial t + \{H_1, n\} = 0$ and assuming translation invariance along X, the propagation equation of the deformation h along the boundary $Y = Y_F$ is given by:

$$\frac{\partial h}{\partial t} + v_D \frac{\partial h}{\partial X} = 0, \tag{12}$$

where $v_D = \frac{1}{eB}|\partial V^{(1)}/\partial Y\rfloor_{Y=Y_F}|$ is the drift or local Fermi velocity. The potential energy associated with the deformation is

$$U = \frac{1}{2}\int \mathrm{d}X n_s h^2 \frac{\partial V}{\partial Y} = \frac{\hbar v_D}{\nu\pi}\int \mathrm{d}X (\pi n_s h)^2. \tag{13}$$

If we define $\widetilde{\phi}$ to be the charge variation integrated on the edge in units of π such that: $n_s h = \widetilde{\rho}(X) = \frac{1}{\pi}\frac{\partial\widetilde{\phi}}{\partial X}$, the action is $S = -\frac{\hbar}{\pi\nu}\int \mathrm{d}X\,\mathrm{d}t\,\frac{\partial\widetilde{\phi}}{\partial X}(\partial\widetilde{\phi}/\partial t + v_D\partial\widetilde{\phi}/\partial X)$ and the Hamiltonian $H = U$ is:

$$H = \frac{hv_D}{2\nu}\int \mathrm{d}X\,\widetilde{\rho}^2.$$

So far the model is purely classical. Defining the momentum conjugate to $\widetilde{\phi}$ as $\widetilde{\pi} = \frac{\hbar}{\pi\nu}\partial\widetilde{\phi}/\partial X$ and quantizing the bosonic field, we find:

$$\left[\widetilde{\pi}(X), \widetilde{\phi}(X')\right] = \mathrm{i}\hbar\delta(X - X'). \tag{14}$$

The equations above describe the dynamics of the neutral bosonic modes of the periphery deformations. The physics becomes nontrivial when considering the transfer of a bare electron or a Laughlin quasiparticle to an edge. The electron creation operator $\psi^\dagger$ must satisfy:

$$\left[\rho(X), \psi^\dagger(X')\right] = \delta(X - X')\psi^\dagger(X'). \tag{15}$$

On the other hand, the 1D excess density $\widetilde{\rho}$ is related to the conjugate of $\widetilde{\phi}$ by $\widetilde{\pi} = -\frac{\hbar}{\nu}\widetilde{\rho}$ and we have $[\widetilde{\rho}(X), \widetilde{\phi}(X')] = -\mathrm{i}\nu\hbar\delta(X - X')$, which immediately implies:

$$\psi^\dagger \propto \exp\left(\frac{\mathrm{i}\widetilde{\phi}}{\nu}\right). \tag{16}$$

$\psi^\dagger$ creates a unit charge at X and is an electron operator if it satisfies Fermi statistics. Exchanging two electrons at position X and X' gives $\psi^\dagger(X')\psi^\dagger(X) = \exp(-\mathrm{i}\frac{\pi}{\nu}\,\mathrm{sgn}(X - X'))\psi^\dagger(X)\psi^\dagger(X')$. The requirement that the bare particles are Fermions implies $\nu = 1/(2s+1)$. The Laughlin filling factors appear naturally as a consequence of *incompressibility and Fermi statistics*. Following a similar procedure, one can define the quasiparticle operator which creates a fractional charge $1/(2s+1)$ on the edge $\psi_{qp}^\dagger \propto \exp(\mathrm{i}\phi)$. It exhibits the *fractional* statistics $\psi_{qp}^\dagger(X')\psi_{qp}^\dagger(X) = \mathrm{e}^{(-\mathrm{i}\pi\nu\,\mathrm{sgn}(X-X'))}\psi_{qp}^\dagger(X)\psi_{qp}^\dagger(X')$ expected for fractionally charged Laughlin quasiparticles.

The coupling between $\psi^\dagger$ (or $\psi_{qp}^\dagger$) and the bosonic modes is strongly nonlinear. It characterizes a Luttinger liquid. The FQHE version is called a *Chiral Luttinger Liquid*. The conductance $\nu e^2/h$ corresponds to the conductance ge^2/h of a Luttinger Liquid and usually one identifies $g = \nu$. As for Luttinger liquids there is an algebraic decay of the correlation functions. One can show that the single particle Green's function decreases as $(X - v_D t)^{-1/\nu}$. The Tunneling Density of State for an electron at energy ε is $\sim |\varepsilon - E_F|^{(1/\nu - 1)}$. This means that, for electrons tunneling between a Fermi liquid and a chiral Luttinger fractional edge channel, the finite temperature conductance $G(T)$ and the zero temperature differential conductance $\mathrm{d}I/\mathrm{d}V$ show power laws variations: $G(T) \sim (T/T_B)^\gamma$ and $\mathrm{d}I/\mathrm{d}V \sim (V/V_B)^\gamma$ with $\gamma = 1/\nu - 1$ [34–38]. T_B and V_B are related to the coupling energy of the tunnel barrier.

2. Transport and shot noise properties in FQHE chiral Luttinger liquids

2.1. Tunneling electrons to a fractional edge

In order to probe this physics two kind of experiments are possible. The first one is an experiment where electrons tunnel from a metal to the fractional edge channel. The second type of experiment is an artificial scatter (a QPC) which allows the quantum transfer of fractional quasiparticles from one fractional edge to the corresponding counter-propagating fractional edge. Here, quasiparticle transfer is possible as it occurs via the fractional fluid. This configuration is also used in the shot noise experiments described at the end because it allows us to probe the quasiparticle charge. For the first kind of expriment, convincing results have been obtained by the group of Chang [39–41]. The tunnel contact is realized using the cleaved edge overgrowth technique. By epitaxial growth on the lateral side of a 2DEG, a large tunnel barrier is first defined followed by a metallic contact realized using heavy doped semiconductor. Power laws of the current with applied voltage $I \sim V^\alpha$, $\alpha = \gamma + 1$, are observed over several current decades for $\nu = 1/3$. The exponent α found is 2.7–2.65 close to the value $\alpha = 3$ predicted by the theory. For other filling factors similar algebraic variations are also observed but the exponents found differ from the predictions of simple chiral Luttinger liquid models. This is discussed in the contribution of Chang in this volume, [42].

2.2. Tunneling between fractional edges through a weak artificial impurity

In the following, we discuss the physics related to the second type of experiment, and show recent experimental results performed in this regime by our group. The transfer of electrons or of quasiparticles betweeen two opposite fractional edges is done using an *weak* artificial impurity, a Quantum Point Contact. We will not present any results for the case of a strong impurity leading to a high tunnel barrier between two nearly disconnected FQHE regions. In this case the behavior is expected to be similar to that of Chang's experiments. However, because the $\mathrm{d}I/\mathrm{d}V$ characteristics are proportional to the square of the TDOS, the exponents for the conductance are doubled. A difficulty is that sample inhomogeneities around the QPC leads to transmission resonances which are difficult to control. Exploiting Luttinger predictions for tunneling through such a resonant state has been used in [43].

Before showing experimental results, we will discuss the predictions for the case of a *weak impurity*. Without Luttinger liquid effects a weak impurity would lead to a weak coupling between upper and lower edges. Practically, the QPC gently pushes the upper edge close to the lower edge to induce a quantum transfer of particles from one edge to the other. Also the QPC potential is weak enough to not make appreciable the change of the local filling factor. Nevertheless, the Luttinger liquid theory predicts that in the limit of small energy ε ($k_B T$ or $eV \to 0$) the strength of the weak artificial impurity is *strongly renormalized* by interactions. The system flows to an insulating state and the conductance displays the same power law with T or V than the one expected for a strong impurity potential (tunnel barrier)

$$G \sim \frac{e^2}{3h}\left(\frac{\varepsilon}{T_B}\right)^{2(1/\nu - 1)} \to 0 \quad \text{for } \varepsilon \ll T_B, \tag{17}$$

where T_B is an energy scale related to the impurity potential. On the other hand, as the impurity is very weak one expects that at high energy a conductance close to, but smaller than the quantum of conductance

$e^2/(3h)$ is recovered. Indeed the Luttinger liquid theory predicts

$$G = \frac{e^2}{3h} - G_B \quad \text{with } G_B = \frac{e^2}{3h}\left(\frac{\varepsilon}{T_B}\right)^{2(\nu-1)} \to 0 \text{ for } \varepsilon \gg T_B. \tag{18}$$

G_B is called the backscattering conductance (if we call I the forward current, $I_0 = (e^2/(3h))V$ the current without impurity, the current associated with particles backscattered by the impurity is $I_B = I_0 - I$ from which one can define $G_B = I_B/V$). The above formula corresponds to *strong* and *weak* backscattering limits. In the first case there is a weak tunneling of particles between the left and right side, while in the second case there is a weak quantum transfer of particles between the upper edge and the lower edge. There is an interesting *duality* with $\nu \leftrightarrow 1/\nu$.

Recent work based on conformal field theory has shown that the problem of a Luttinger liquid with one impurity is fully integrable for all energies [44,45]. The approach is described in a contribution by Saleur in this volume, [46]. One can show that applying a voltage bias V between reservoirs emitting electrons in the upper and lower edges is, in the convenient basis for interacting electrons, equivalent to sending a regular flow of kinks which are randomly transformed into antikinks. Kink and antikink respectively contribute to the forward and backscattered current. The Landauer formula adapted to this approach gives the backscattering current which expresses simply as:

$$I_B(V, T_B) = ev_D \int_{-\infty}^{A(V)} \mathrm{d}\alpha \rho_+(\alpha) \left|S_{+-}(\alpha - \alpha_B)\right|^2 \quad \text{and} \quad I = \nu \frac{e^2}{h} V - I_B, \tag{19}$$

where $\rho_+(\alpha)$ is the density of incoming kink at energy parametrized by e^α, and

$$\left|S_{+-}(\alpha - \alpha_B)\right|^2 = \frac{1}{1 + \exp[2(1-\nu)(\alpha - \alpha_B)/\nu]} \tag{20}$$

is the probability for kink to anti-kink conversion (the scattering probability) with α_B related to the impurity strength T_B.

For finite temperature, one can show that the conductance is a function of the reduced variable T/T_B and $eV/2\pi k_B T$:

$$G(T, V) = \frac{e^2}{3h} f\left(\frac{T}{T_B}, \frac{eV}{2\pi k_B T}\right) \tag{21}$$

and measuring $G(T, 0)$ fixes the only parameter T_B. The V/T scaling law also can be tested very accurately. Fig. 7 shows theoretical calculations of the differential conductance for various values of the parameter T_B (the results of [44,45] have been used). We can see that, increasing the energy (the voltage or the temperature), leads to a progressive transition from the strong backscattering regime to the weak backscattering regime.

Fig. 8, left graph, shows experimental measurements of the differential conductance versus bias voltage for different values of the QPC gate voltage. An ncrease of the gate voltage corresponds to a decrease of T_B. As one can see, the observed nonlinearities compare well with the theoretical curves of Fig. 7. Fig. 8, right graph, shows recent data obtained in our group for the conductance in the strong backscattering regime for $\nu = 1/3$. Experiments are made in the intermediate regime (i.e., $G > 10^{-4} e^2/3h$). The effective exponent (power law fit for $0 \leqslant eV/(2\pi k_B T) \leqslant 2$) deduced from a series of $\mathrm{d}I/\mathrm{d}V$ curves for different impurity strength is compared with the effective one calculated using the finite temperature exact solution. The agreement is rather good. The theoretical graph in inset of the figure shows that the asymptotic scaling exponent $2(\nu^{-1} - 1) = 4$ is not expected except for conductance lower than 10^{-4}, which is experimentally difficult to obtain. It is noteworthy that there are no adjustable parameters.

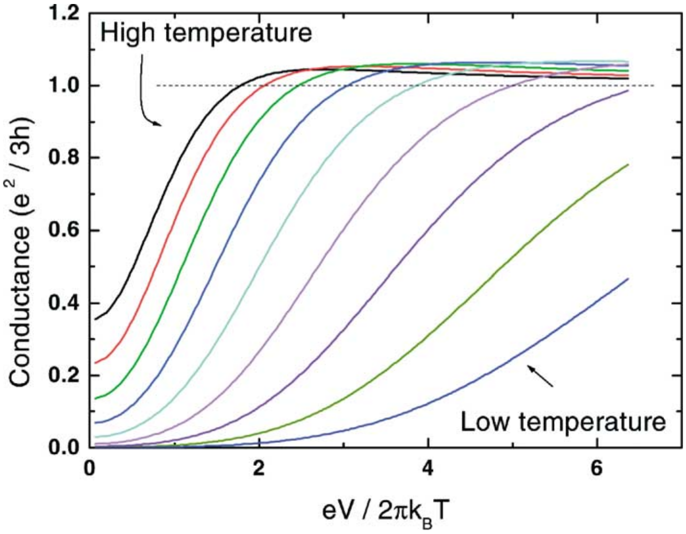

Figure 7. Theoretical curves for the differential conductance versus voltage calculated for values of $\ln(T/T_B)$ varying from -3 to -0.6 in steps of 0.3. The numerical exact solution of [44,45] is used. The conductance is a universal function of the variable T/T_B and $eV_{ds}/2\pi k_B T$.

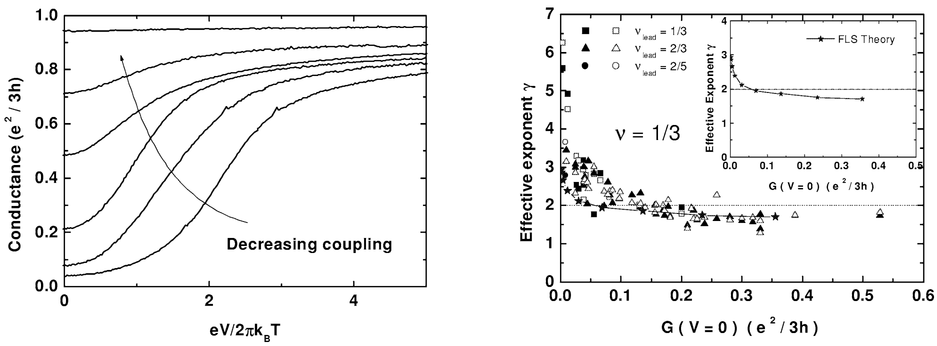

Figure 8. Differential resistance versus bias voltage for different values of the QPC gate voltage (left). Exponent of the algebraic variation of the differential conductance with voltage measured in the strong backscattering regime versus the zero bias conductance normalized to $e^2/3h$. The solide line is a comparison with the FLS predictions. The scaling exponent $\alpha = 2(\nu^{-1} - 1) = 4$ is only expected in a regime of extremely low conductance (10^{-4}) (right).

There are still many open problems for a quantitative description of conductance measurements using the Luttinger liquid model. Long range interactions are one of these. One can show that the dispersion relation of bosonic chiral edges modes which usually vary linearly with the wavenumber k get a contribution $k\ln(k)$. When the energy is low enough such that the wavelength is larger than the width of the sample, the Coulomb interaction couples the edges. The power law of the TDOS is lost. Instead the TDOS is expected to vary with energy like $\exp(const \times (\ln\varepsilon)^{3/2})$ [47–49].

2.3. Shot noise of fractional channels coupled by an artificial impurity: detection of fractional charges

The study of shot noise in quantum conductors has been of increasing importance in mesoscopic physics. This followed a series of theoretical papers [50] and experiments [51,52] showing that the quantum noise of electrons in conductors is naturally sub-Poissonian. The reduction of noise comes from the statistical interaction between electrons as the Pauli principle naturally regulates the electron flow. Another source of reduction can arise because of correlations when the repulsive interaction between electrons is important.

It was thus interesting to consider the noise properties of an interacting system such as a Luttinger liquid. When considering the fractional quantum Hall effect, another motivation is that shot noise provides an unambiguous evidence of the fractionally charged Laughlin excitations.

According to Schottky [53], the random transfer of charge q across a conductor generates finite temporal fluctuations of the current ΔI around the mean value $\overline{I}$ when observed during a finite time τ. The current is related to the average number of transferred electrons $\overline{N}$ via $\overline{I} = q\overline{N}/\tau$. If the statistics of transfer events is Poissonian $\overline{(\Delta N)^2} = \overline{N}$ this gives the Schottky formula:

$$\overline{(\Delta I)^2} = 2q\overline{I}\Delta f = S_I \Delta f, \tag{22}$$

where we have introduce the effective frequency bandwidth $\Delta f = 2/\tau$ (Nyquist theorem) and the current noise power S_I. The noise power is directly proportional to the carrier charge q.

Theoretical predictions for shot noise taking into account the specific Luttinger liquid dynamics have been given in [54]. An artificial impurity, as the one discussed in previous sections, couples the two opposite fractional edges. It induces tunneling in the strong backscattering regime or quantum transfer of quasiparticles in the weak backscattering regime. The results for both limits and $\nu = 1/3$ are respectively:

$$S_I \simeq 2eI \coth(eV/2k_B\theta), \qquad I \ll I_0 = \frac{1}{3}\frac{e^2}{h}V, \tag{23}$$

$$S_I \simeq 2(e/3)I_B \coth\big((e/3)V/2k_B\theta\big), \qquad I_B = I_0 - I \ll I_0. \tag{24}$$

Here, the Johnson–Nyquist thermal noise contribution $4Gk_BT$ has been subtracted for clarity. The zero temperature limit of expressions (23) and (24) have been also derived in [55] using Luttinger liquid models in the perturbative limit. For strong backscattering, low conductance ($I \ll I_0$), the rate of tunneling of charges is small compared to that of incoming charges. The statistics are Poissonian and one recovers the (finite temperature) Schottky formula. The charge found is that of electrons as intuitively expected. The system being insulating, correlations between the left and the right side of the conductor are lost and fractional excitations cannot survive. For weak backscattering, conductance close to $e^2/3h$, the backscattering current is now small compared to the incoming current and the statistics of quantum transfer of charge from the upper to the lower edge is Poissonian. The Schottky formula is again recovered but I_B replaces I and the effective charge is $e/3$. Here, fractional excitations are expected inside the fractional region virtually not perturbed by the weak impurity. In this limit, noise provides a *direct* way to measure the fractional Laughlin charge $e/3$.

A fractional charge is also found in the argument of the coth function although its meaning is different. The cross-over from thermal to shot noise occurs when the electro-chemical potential difference $\Delta\mu = eV/3$ is comparable to $k_B\theta$. This is not a measure of the fractional quasiparticle charge but a measure of the fractional filling of the quantum state at equilibrium, like the conductance $\nu e^2/h$ is. Only the shot noise $S_I \simeq 2(e/3)I_B$ really measures the quasiparticle charge. Nevertheless the observation of a three times larger voltage for the thermal cross-over in noise experiments has been an important confirmation of Eq. (24).

Beyond the perturbative limit, the impurity problem has an exact solution. The FLS model presented in the previous section allows not only to calculate the current in all regime but also to calculate the noise [44]. To obtain the noise, one can mimic the wavepacket approach used by Martin and Landauer for the noise of noninteracting Fermions [56,57]. The incoming kinks correspond to a regular flow of charged solitons. The regular flow is noiseless but the random scattering of kinks into anti-kinks produces noise in the outgoing current. When $|S_{+-}(\alpha - \alpha_B)|^2 \ll 1$ this a Poissonian processus while if $|S_{+-}(\alpha - \alpha_B)|^2$ is not negligible, the statistics is binomial and the fluctuations are proportional to $|S_{+-}(\alpha - \alpha_B)|^2(1 - |S_{+-}(\alpha - \alpha_B)|^2)$. The expression for the noise at zero temperature is thus:

$$S_I(V) = 2e^2 v \int_{-\infty}^{A(V_{ds})} \mathrm{d}\alpha\, \rho_+(\alpha) \big|S_{+-}(\alpha - \alpha_B)\big|^2 \big(1 - \big|S_{+-}(\alpha - \alpha_B)\big|^2\big). \tag{25}$$

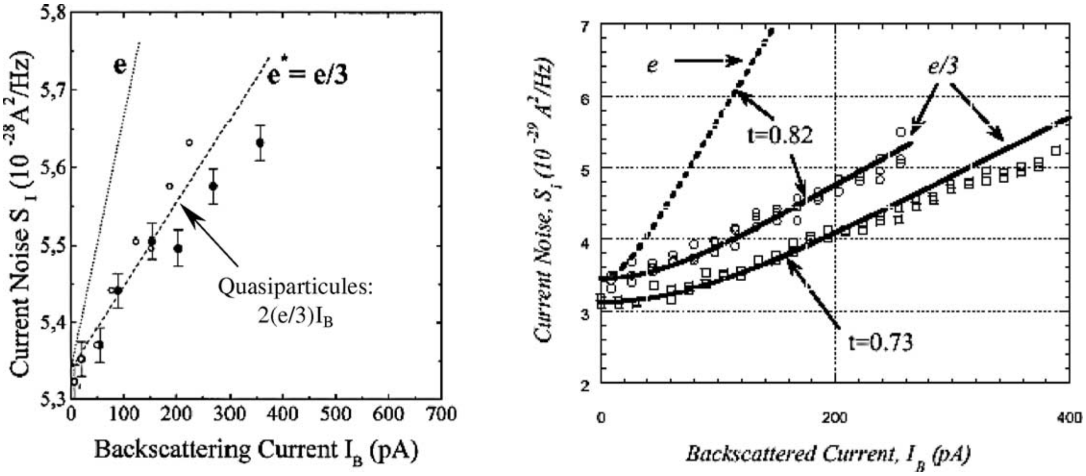

Figure 9. Experimental Poissonian noise of the fractionally charged excitations of the FQHE, from [58] (left) and [59] (right).

Exact expression and technical mathematical details can be found in [44]. The special simple form of $|S_{+-}(\alpha - \alpha_B)|$ leads to a relation between current and noise where

$$S_I = \frac{v}{1-v}\left(V\frac{\mathrm{d}I}{\mathrm{d}V} - I\right) = \frac{v}{1-v}\left(I_B - V\frac{\mathrm{d}I_B}{\mathrm{d}V}\right).$$

Whence, using the weak and strong backscattering limits of the Luttinger theory, we can easily check that $S_I \to 2(\nu e I_B)$ and $2eI$ respectively in agreement with the zero temperature limit of Eqs. (23) and (24). Finite temperature predictions can also be found in [45].

Shot noise measurements in this regime have been performed by two groups [58,59]. A difficulty of shot noise measurements in the FQH effect is that shot noise levels are extremely small both due to the smaller charge and the small available current. The latter is restricted by the fact that the FQH effect breaks down when the applied voltage is larger than the excitation gap. It is of the order of a few 100 μeV, leading to shot noise levels in the 10^{-29} A^2/Hz range.

A QPC is used in order to realize a local and controllable coupling between two $\nu = 1/3$ fractional edges to partially reflect the incoming current. The experiments are designed to have an optimal sensitivity for the weak coupling limit where Poissonian noise of the $e/3$ Laughlin quasiparticles is expected. In the experiment of Ref. [58], a cross correlation technique detects, at low frequency, the anticorrelated noise of the transmitted current I and the reflected current I_B, i.e. $S_{I,I_B} = \langle \Delta I \Delta I_B \rangle / \Delta f \simeq -2(e/3)I_B$. The magnetic field corresponds to a filling factor $2/3$ in the bulk of the sample and a small region of filling factor $1/3$ is created close to the QPC using the depletion effect of the gates. The size of the $1/3$ region is estimated to be about $150\phi_0$, sufficient to establish FQHE correlations. The advantage of this technique is that the coupling between edges occurs on a shorter scale and the controllable QPC potential is larger than the potential fluctuations inherent in sample fabrication. In the two samples measured, the combination of QPC and random potential lead to two dominant paths for backscattering. The coherent interference between paths gives rise to nearly perfect resonant tunneling peaks in the conductance. Careful measurements of the conductance resonance showed that tunneling was coherent. This was an important check for the quasiparticle charge measurement because this ruled out the possibility of noise suppression due to multiple uncorrelated steps, similar to the $1/3$ noise reduction factor in zero field diffusive conductors. Also the resonant conductance showed nonlinear dependence on bias voltage consistent with Luttinger liquid model provided the filling factor of the bulk is used. The other group [59] used a high frequency technique in order to increase the signal bandwidth and measured the autocorrelation of the transmitted current. Here the magnetic field corresponded to a filling factor $1/3$ throughout the sample. They found few nonlinearities in the conductance, in contrast with the Luttinger liquid predictions, and this allowed them to define a bias voltage independent transmission.

In the Poissonian limit $I_B \ll I_0$, the two experiments give the same conclusion (see Fig. 9) that near filling factor $1/3$, shot noise is threefold suppressed. This is the most direct evidence that the current can be carried by a quasiparticle with a fraction of e and that the Laughlin conjecture was correct. In addition, the data showed a cross-over from thermal noise to shot noise when the applied voltage satisfies the inequality $eV/3 > 2k\theta$ (rather than $eV > 2k\theta$), indicating that the potential energy of the quasiparticles is threefold smaller as well as predicted in Eq. (11).This experiment has been now reproduced many times with different sample and measurement conditions in both laboratories.

Is it possible to go further and probe different fractional charges for less simple filling factor? Recently measurements close to $\nu = 2/5$ have given indications that the $e/5$ quasiparticles are the relevant excitations in this regime [60]. This last result has been analyzed in a model of noninteracting composite Fermions where Luttinger effects are neglected [61]. More experiments and a better theoretical understanding of the noise for Jain's filing factors are certainly needed and will be done in the future.

Acnowledgements. The authors would like to thank Yong Jin and Bernard Etienne who provide high quality samples and Laurent Saminadayar for his early contribution to noise experiments.

References

[1] K. von Klitzing, G. Dorda, M. Pepper, Phys. Rev. Lett. 45 (1980) 494.
[2] D. Tsui, H. Stromer, A. Gossard, Phys. Rev. Lett. 48 (1982) 1599.
[3] R.B. Laughlin, Phys. Rev. Lett. 50 (1983) 1395.
[4] J.M. Leynaas, L. Myrheim, Nuovo Cimento B 37 (1977) 1.
[5] S.R. Renn, D.P. Arovas, Phys. Rev. B 51 (1995) 16832.
[6] S. Ouvry, Phys. Rev. D 50 (1994) 5296.
[7] S. Wu, Phys. Rev. Lett. 73 (1994) 922.
[8] J. Jain, Phys. Rev. Lett. 63 (1989) 199.
[9] S.C. Zhang, H. Hansson, S. Kivelson, Phys. Rev. Lett. 62 (1989) 82.
[10] D.-H. Lee, S.-C. Zhang, Phys. Rev. Lett. 66 (1991) 1220.
[11] S.C. Zhang, Int. J. Mod. Phys. B 6 (1992) 25.
[12] S.L. Sondhi, A. Karlhede, S.A. Kivelson, E.H. Rezayi, Phys. Rev. B 47 (1993) 16419.
[13] S.M. Girvin, in: A. Comtet, T. Jolicoeur, S. Ouvry, F. David (Eds.), Topological Aspects of Low Dimensional Systems, NATO ASI, Les Houches Summer School, Springer, 1999, pp. 551–570.
[14] R.E. Prange, S.M. Girvin (Eds.), The Quantum Hall Effect, Springer-Verlag, New York, 1987.
[15] A.H. Mac Donald, in: E. Akkermans, G. Montambaux, J.L. Pichard, J. Zinn-Justin (Eds.), Mesoscopic Quantum Physics, Elsevier, Amsterdam, 1994.
[16] S.D. Sarma, A. Pinczuk (Eds.), Perspectives in Quantum Hall Effects, Wiley, New York, 1997.
[17] A. Comtet, T. Jolicoeur, S. Ouvry, F. David (Eds.), Topological Aspects of Low Dimensional Systems, NATO ASI, Les Houches Summer School, Springer, 1999, pp. 551–570.
[18] T. Ando, A.B. Fowler, F. Stern, Rev. Mod. Phys. 54 (1982) 437.
[19] A. Cho (Ed.), Molecular Beam Epitaxy, AIP Press, Woodbury, New York, 1994.
[20] T. Ando, Y. Matsumoto, Y. Uemura, J. Phys. Soc. Japan 39 (1975) 279.
[21] P. Streda, L. Smrcka, Phys. Stat. Solidi B 70 (1975) 537.
[22] M. Buettiker, Phys. Rev. B 38 (1988) 9375.
[23] R.J. Haug, et al., Phys. Rev. Lett. 61 (1988) 2797.
[24] D.B. Chklovskii, B.I. Shklovskii, L.I. Glazman, Phys. Rev. B 46 (1992) 4026.
[25] C.W.J. Beenakker, Phys. Rev Lett. 64 (1990) 216.
[26] A.H. MacDonald, Phys. Rev. Lett. 64 (1990) 220.
[27] H. Schulz, Fermi liquids and non-Fermi liquids, in: E. Akkermans, G. Montambaux, J.L. Pichard, J. Zinn-Justin (Eds.), Mesoscopic Quantum Physics, Proceedings of the Les Houches Summer School of Theoretical Physics, Session LXI, Springer-Verlag, 1994.
[28] X.G. Wen, Phys. Rev. B 43 (1991) 11025.

[29] X.G. Wen, Phys. Rev. Lett. 64 (1990) 2206.
[30] X.G. Wen, Phys. Rev. B 44 (1991) 5708.
[31] X.G. Wen, Int. J. Mod. Phys. B 6 (1992) 1711.
[32] J.M. Luttinger, J. Math. Phys. 4 (1963) 1154.
[33] S. Tomonaga, Prog. Theor. Phys. (Kyoto) 5 (1950) 544.
[34] C.L. Kane, M.P.A. Fisher, Phys. Rev. B 46 (1992) 15233.
[35] C.L. Kane, M.P.A. Fisher, Phys. Rev. Lett. 68 (1992) 1220.
[36] C.L. Kane, M.P.A. Fisher, Phys. Rev. B 51 (1995) 13449.
[37] A.V. Shytov, L.S. Levitov, B.I. Halperin, Phys. Rev. Lett. 80 (1998) 141.
[38] L.S. Levitov, A.V. Shytov, B.I. Halperin, Phys. Rev. B 64 (2001) 075322.
[39] A.M. Chang, L.N. Pfeiffer, K.W. West, Phys. Rev. Lett. 77 (1996) 2538.
[40] M. Grayson, D.C. Tsui, L.N. Pfeiffer, K.W. West, A.M. Chang, Phys. Rev. Lett. 80 (1998) 1062.
[41] A.M. Chang, M.K. Wu, J.C.C. Chi, L.N. Pfeiffer, K.W. West, Phys. Rev. Lett. 87 (2001) 2538.
[42] A.M. Chang, C.R. Physique 3 (2002) 677–684.
[43] F.P. Milliken, C.P. Umbach, R.A. Webb, Solid State Commun. 97 (1996) 309.
[44] P. Fendley, A.W.W. Ludwig, H. Saleur, Phys. Rev. Lett. 74 (1995) 3005.
[45] P. Fendley, H. Saleur, Phys. Rev. B 54 (1996) 10845.
[46] H. Saleur, C.R. Physique 3 (2002) 685–695.
[47] K. Moon, S.M. Girvin, Phys. Rev B 54 (1996) 4448.
[48] K. Imura, N. Nagaosa, Solid State Commun. 103 (1997) 663.
[49] L.P. Pryadko, E. Shimshoni, A. Auerbach, Phys. Rev. B 61 (2000) 10929.
[50] G.B. Lesovik, Pis'ma Zh. Eksp. Teor. Fiz. 49 (1989) 513 [JETP Lett. 49 (1989) 592].
[51] M. Reznikov, et al., Phys. Rev. Lett. 18 (1995) 3340.
[52] A. Kumar, et al., Phys. Rev. Lett. 76 (1996) 2778.
[53] W. Schottky, Ann. Phys. (Leipzig) 57 (1918) 541.
[54] C.L. Kane, M.P.A. Fisher, Phys. Rev. Lett. 72 (1994) 724.
[55] C. de C. Chamon, D.E. Freed, X.G. Wen, Phys. Rev. B 51 (1995) 2363.
[56] Th. Martin, R. Landauer, Phys. Rev. B 45 (1992) 1742.
[57] Th. Martin, R. Landauer, Physica B 175 (1991) 167.
[58] L. Saminadayar, D.C. Glattli, Y. Jin, B. Etienne, Phys. Rev. Let. 79 (1997) 2526, cond-mat/9706307.
[59] R. de-Picciotto, et al., Nature 389 (1997) 162, cond-mat/9707289.
[60] M. Reznikov, R. de-Picciotto, T.G. Griffiths, M. Heiblum, Nature 399 (1999) 238.
[61] R. de-Picciotto, cond-mat/980221.