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AVANCÉES EN PHYSIQUE DES PARTICULES : LA CONTRIBUTION DU LEP

ADVANCES IN PARTICLE PHYSICS: THE LEP CONTRIBUTION

Universality of electroweak couplings

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Abstract The present experimental tests of the electroweak universality in the leptonic sector are reviewed for both charged- and neutral-current electroweak interactions. ***To cite this article: A. Rougé, R. Tanaka, C. R. Physique 3 (2002) 1165–1172.***

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Universalité des couplages électrofaibles

Résumé Nous présentons l'état actuel des tests de l'universalité des interactions électrofaibles de type courant chargé et de type courant neutre dans le domaine des leptons. ***Pour citer cet article : A. Rougé, R. Tanaka, C. R. Physique 3 (2002) 1165–1172.***

universalité / lepton / interaction électrofaible

1. Introduction

The idea of universality of the weak interactions can be traced back to the period 1948–1950, when it was put forward by several authors, including Fermi himself [1–6].

In the hadronic sector, the generalization of the universality concept by Cabibbo [7] and the extension of the Cabibbo scheme by Kobayashi and Maskawa [8] opened a new chapter of physics, which is presented in the contribution by Kluit and Stocchi [9].

We describe here the precision tests of universality in the leptonic sector that have been allowed by the LEP data.

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It has been discovered at the early stage of LEP (see the contribution by Blondel [10]) that there are only three families of spin 1/2 leptons:

$$\begin{bmatrix} \nu_e \\ e^- \end{bmatrix}, \qquad \begin{bmatrix} \nu_\mu \\ \mu^- \end{bmatrix}, \qquad \begin{bmatrix} \nu_\tau \\ \tau^- \end{bmatrix}. \tag{1}$$

They interact with the massive vector bosons of the Standard Model [11–13], $W^\pm$ (charged-current) and Z (neutral-current). The universality of the electro-weak interaction implies that its strength is the same for each of the three leptonic flavours.

2. Charged-current universality

The charged-current interaction for the family of leptons (l^-, ν_l) can be written

$$\mathcal{L}_{cc}^{W-l} = \frac{g_l}{2\sqrt{2}} W_\mu^+ \bar{\nu}_l \gamma^\mu (1-\gamma_5) l + h.c., \tag{2}$$

and the universality property reads

$$g_e = g_\mu = g_\tau. \tag{3}$$

2.1. *W* decays

The first prediction of the charged-current universality is the equality of the W leptonic partial widths:

$$\Gamma(W \to e\bar{\nu}_e) = \Gamma(W \to \mu\bar{\nu}_\mu) = \Gamma(W \to \tau\bar{\nu}_\tau). \tag{4}$$

The measurements of the W branching ratios [14] can be turned into the ratios of coupling constants given in Table 1, which verify the universality at the percent level.

2.2. *τ* leptonic decays

More precise verifications can be obtained from the pure leptonic decays, $\tau^- \to \nu_\tau e^- \bar{\nu}_e$, $\tau^- \to \nu_\tau \mu^- \bar{\nu}_\mu$ and $\mu^- \to \nu_\mu e^- \bar{\nu}_e$ (Fig. 1).

The corresponding partial widths are given by

$$\Gamma(l_1 \to l_2 \nu_{l_1} \bar{\nu}_{l_2}) = \frac{G_{l_1} G_{l_2} m_{l_1}^5}{192\pi^3} f\left(\frac{m_{l_2}^2}{m_{l_1}^2}\right)(1+\delta_W)(1+\delta_\gamma), \tag{5}$$

Table 1. Constraints on charged-current lepton universality from W decays.

	$\lvert g_\mu/g_e \rvert$	$\lvert g_\tau/g_e \rvert$
W decays	0.993 ± 0.012	1.001 ± 0.014

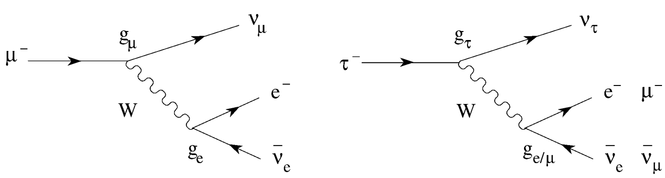

Figure 1. μ and τ decays in the Standard Model.

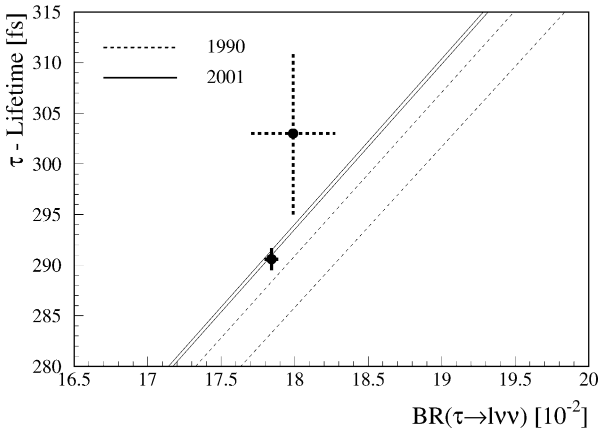

Figure 2. The relation between the lifetime and leptonic branching ratio of the τ lepton [14,20]. The lines are the predictions of universality (Eq. (8)), taking into account the uncertainties on the τ mass.

where

$$G_l = \frac{g_l^2}{4\sqrt{2}M_W^2},$$
$$f(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \ln x;$$

δ_W and δ_γ are correcting terms which take into account the structure of the W propagator and the radiative corrections [15]:

$$\delta_W = \frac{3}{5}\frac{m_{l_1}^2}{m_W^2} - 2\frac{m_{l_2}^2}{m_W^2}, \qquad \delta_\gamma = \left(\frac{25}{4} - \pi^2\right)\frac{\alpha}{2\pi}.$$

The ratio g_μ/g_e is determined by the branching fractions for τ decay into the electron and muon final states, $B_{\tau\to e}$ and $B_{\tau\to\mu}$:

$$\left(\frac{g_\mu}{g_e}\right)^2 = \frac{B_{\tau\to\mu}}{B_{\tau\to e}}\frac{f(m_e^2/m_\tau^2)}{f(m_\mu^2/m_\tau^2)}. \tag{6}$$

The knowledge of the parent lepton lifetime τ_{l_1} allows the absolute determination of the partial widths:

$$\Gamma(l_1 \to l_2 \nu_{l_1} \bar{\nu}_{l_2}) = \frac{B_{l_1\to l_2}}{\tau_{l_1}}, \tag{7}$$

hence a comparison of the τ and μ coupling constants,

$$\left(\frac{g_\tau}{g_\mu}\right)^2 = B_{\tau\to e}\frac{\tau_\mu}{\tau_\tau}\frac{f(m_e^2/m_\mu^2)}{f(m_e^2/m_\tau^2)}\left(\frac{m_\mu}{m_\tau}\right)^5(1+\delta), \tag{8}$$

where the correction δ is of the order of 10^{-4}. Taking advantage of the observed e/μ universality to combine $B_{\tau\to\mu}$ and $B_{\tau\to e}$ in a single $B_{\tau\to l}$ branching fraction, we get from the measurements [14] the value of g_τ/g_l shown in Table 2 (as $|g_\tau/g_\mu|$). Fig. 2 displays the relation between $B_{\tau\to l}$ and τ_τ expected from Eq. (8) and universality. It shows the large improvement due to LEP measurements.

2.3. τ semileptonic decays

The semileptonic decays $\tau^- \to \nu_\tau h^-$ ($h = \pi, K$) can be related in a similar way to the decays $h^- \to l^- \bar{\nu}_l$ (Fig. 3).

Table 2. Constraints on charged-current lepton universality from τ leptonic and semileptonic decays.

	$\lvert g_\mu/g_e \rvert$	$\lvert g_\tau/g_\mu \rvert$
$\tau \to l\nu_\tau\bar{\nu}_l$, $\mu \to e\nu_\mu\bar{\nu}_e$	1.0008 ± 0.0026	1.0010 ± 0.0023
$\pi \to \mu\bar{\nu}_\mu$, $\pi \to e\bar{\nu}_e$	1.0017 ± 0.0015	—
$\tau \to h\nu_\tau$, $h \to \mu\bar{\nu}_\mu$	—	1.0071 ± 0.0055

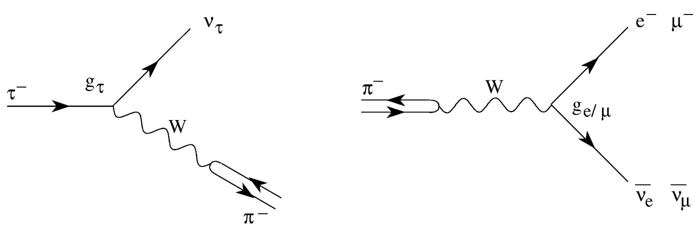

Figure 3. The decays $\tau^- \to \pi^-\nu_\tau$ and $\mu^- \to \pi^-\bar{\nu}_\mu$.

The ratios of the coupling constants can be deduced from the partial widths by using the relations (9) and (10),

$$\left(\frac{g_\mu}{g_e}\right)^2 = \frac{B_{\pi\to\mu}}{B_{\pi\to e}} \frac{m_e^2(m_\pi^2 - m_e^2)^2}{m_\mu^2(m_\pi^2 - m_\mu^2)^2}(1 + \delta_{\mu/e}), \tag{9}$$

$$\left(\frac{g_\tau}{g_\mu}\right)^2 = \frac{B_{\tau\to h}}{B_{h\to\mu}} \frac{\tau_h}{\tau_\tau} \frac{2m_\tau m_\mu^2}{m_h^3} \left(\frac{m_h^2 - m_\mu^2}{m_\tau^2 - m_h^2}\right)^2 (1 + \delta_{\tau/h}), \tag{10}$$

where the radiative corrections [16–19] amount to $\delta_{\mu/e} = -(3.76 \pm 0.04)\%$, $\delta_{\tau/\pi} = (0.16 \pm 0.14)\%$ and $\delta_{\tau/K} = (0.90 \pm 0.22)\%$.

The ratios of coupling constants computed from the measurements [14] are shown in Table 2. They are also in excellent agreement with the universality hypothesis.

3. Neutral-current universality

The interaction of a charged lepton with the Z boson can be written

$$\mathcal{L}_{nc}^{Z-l} = \frac{g}{2\cos\theta_W} Z_\mu \bar{l}\gamma^\mu(v_l - a_l\gamma_5)l, \tag{11}$$

where, in the Standard Model,

$$v_l = I_3^l - 2Q_l \sin^2\theta_W, \qquad a_l = I_3^l. \tag{12}$$

In order to test the universality property, the vector (v_l) and axial-vector (a_l) couplings will be handled here as free parameters to be determined from the data.

3.1. Observables in $e^+e^- \to l^+l^-$

The reaction $e^+e^- \to l^+l^-$ proceeds at the lowest order via s-channel photon and Z boson exchange and its properties can be computed according to the Feynman diagrams shown in Fig. 4. For e^+e^- final states, the same exchanges occur also in t-channel. At centre-of-mass energies near 91 GeV, the Z exchange is dominant in the s-channel.

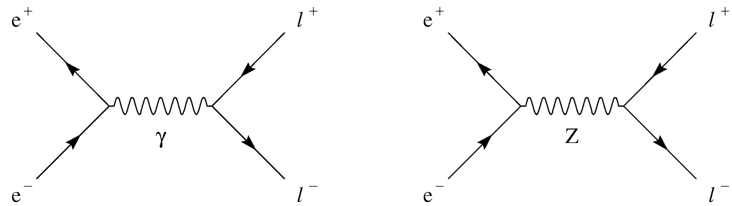

Figure 4. The lowest-order s-channel Feynman diagrams for the production of a lepton-pair in e^+e^- annihilation.

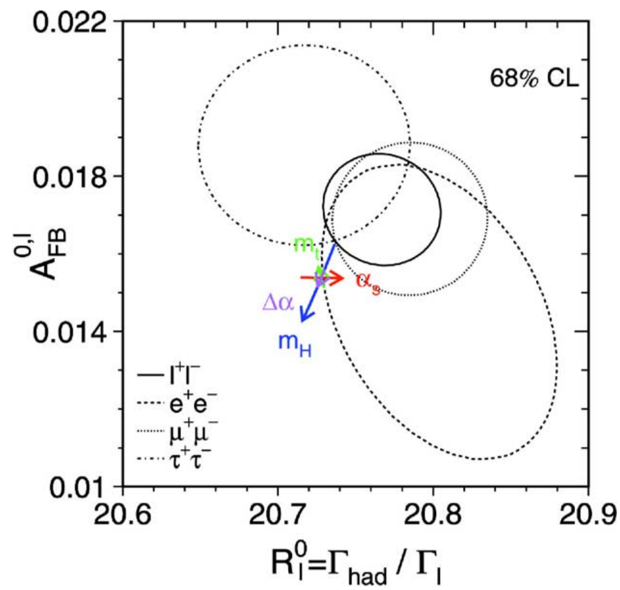

Figure 5. Contour lines (68%) in the R_l^0–$A_{FB}^{0,l}$ plane for e^+e^-, $\mu^+\mu^-$ and $\tau^+\tau^-$ final states and for all leptons combined. The data is the combination of the four LEP experiment results. The lines with arrows correspond to the Standard Model prediction when m_t, m_H and $\alpha_S(m_Z^2)$ are varied in the intervals $m_t = 174.3 \pm 5.1$ GeV/c^2 and $m_H = 300^{+700}_{-200}$ GeV/c^2, and $\alpha_S(m_Z^2) = 0.119 \pm 0.002$, respectively.

3.1.1. *Unpolarized $e^\pm$ and l polarization not observed*

The only measurable quantities are the cross-section and the angular distribution

$$\frac{1}{\sigma}\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{3}{16\pi}\left[\left(1+\cos^2\theta\right) + \frac{8}{3}\mathcal{A}_{FB}\cos\theta\right], \tag{13}$$

where θ is the scattering angle between e^- and l^-. The information carried by this angular distribution reduces itself to the forward–backward asymmetry $\mathcal{A}_{FB}$.

At the Z peak ($s = M_Z^2$) the photon exchange contribution can be neglected. The peak cross-sections $\sigma^{0,l}$ and the asymmetries are then given by:

$$\sigma^{0,l} = \frac{12\pi}{M_Z^2}\frac{\Gamma_e\Gamma_l}{\Gamma_Z^2}, \qquad A_{FB}^{0,l} = \frac{3}{4}\mathcal{A}_e\mathcal{A}_l, \tag{14}$$

$$\mathcal{A}_l = \frac{2a_l v_l}{a_l^2 + v_l^2}, \qquad \Gamma_l = \frac{G_F M_Z^3}{6\pi\sqrt{2}}\left(a_l^2 + v_l^2\right)\left(1 + \frac{3\alpha}{4\pi}\right), \tag{15}$$

where Γ_l is the Z partial decay width to the l^+l^- final state.

Subtracting the t-channel contribution to the e^+e^- channel and correcting for the τ lepton mass, the universality hypothesis can be directly checked on the measurements of the partial widths and forward–backward asymmetries.

The data of the LEP experiments at the Z peak [21] shown in Fig. 5 are in good agreement with universality.

3.1.2. *Unpolarized $e^{\pm}$ and l polarization observed*

The polarization of the final state lepton is in practice only measurable in the case of the τ lepton [22]. Since helicity is conserved at high energy for vector and axial-vector interactions, the longitudinal polarizations of the lepton and the antilepton are opposite

$$P^l \equiv P^{l^-} = 2\langle \lambda^{l^-} \rangle = -P^{l^+}. \tag{16}$$

Furthermore, the only allowed spin-observables beside the longitudinal polarization are spin-correlations. For $s = M_Z^2$, the expression of the polarization is

$$P^l(\cos\theta) = -\frac{\mathcal{A}_l(1+\cos^2\theta) + 2\mathcal{A}_e\cos\theta}{(1+\cos^2\theta) + \frac{8}{3}\mathcal{A}_{FB}\cos\theta}. \tag{17}$$

The observables are

$$\langle P^{0,l} \rangle = -A^{0,l}_{\text{pol}} = -\mathcal{A}_l, \qquad A^{0,l}_{FB,\text{pol}} = \frac{3}{4}\mathcal{A}_e, \tag{18}$$

$$C^l_{LL} = -1, \qquad C^{0,l}_{TT} = \frac{a_l^2 - v_l^2}{a_l^2 + v_l^2}. \tag{19}$$

The T-odd transverse-normal correlation receives a contribution from the $\gamma - Z$ interference proportional to $v_e a_l \Gamma_Z / M_Z$.

The measurement of $P^\tau(\cos\theta)$ at the Z peak (Fig. 6) gives a new verification of the universality prediction $\mathcal{A}_\tau = \mathcal{A}_e$: the fits of (17) to the data with and without the universality constraint are hardly distinguishable.

3.1.3. *Polarized $e^{\pm}$*

With a polarized e^- beam [1] of polarization P^e, one can also measure the left–right asymmetry between the cross sections for left- and right-handed electrons:

$$A_{LR} = \frac{\sigma(\lambda^e = -1/2) - \sigma(\lambda^e = +1/2)}{\sigma(\lambda^e = -1/2) + \sigma(\lambda^e = +1/2)} = \frac{1}{P^e}\frac{\sigma(-P^e) - \sigma(P^e)}{\sigma(-P^e) + \sigma(P^e)}, \tag{20}$$

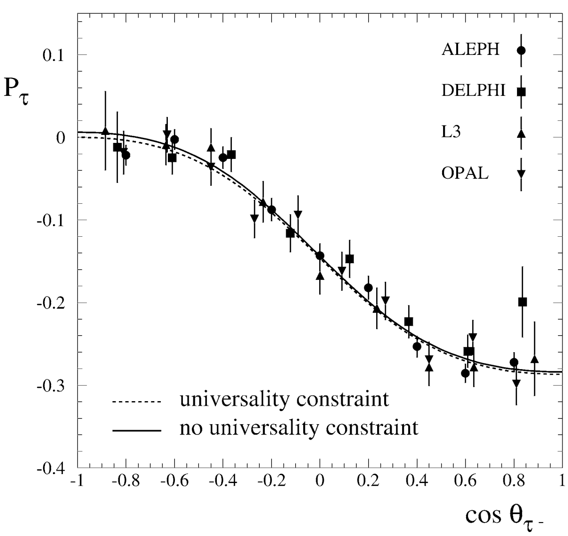

Figure 6. Test of the e/τ universality in the τ-polarization measurement.

Table 3. Results for the effective vector and axial-vector neutral-current couplings.

	Couplings
v_e	-0.03816 ± 0.00047
v_μ	-0.0367 ± 0.0023
v_τ	-0.0366 ± 0.0010
a_e	-0.50111 ± 0.00035
a_μ	-0.50120 ± 0.00054
a_τ	-0.50204 ± 0.00064
	Ratios of couplings
v_μ/v_e	0.962 ± 0.063
v_τ/v_e	0.958 ± 0.029
a_μ/a_e	1.0002 ± 0.0014
a_τ/a_e	1.0019 ± 0.0015

and double: forward–backward, left–right asymmetries for each final state. Since the roles of initial and final states are exchanged, the values of the asymmetries at the Z peak are

$$A^0_{LR} = \mathcal{A}_e, \qquad A^{0,l}_{FB,LR} = \frac{3}{4}\mathcal{A}_l. \tag{21}$$

The availability of beam polarization at SLC [21] allowed a very precise determination of the parameter $\mathcal{A}_e$, owing to the fact that no final state selection is needed to measure the left–right asymmetry.

3.2. Determination of the neutral-current leptonic couplings

The measurements of cross-sections and asymmetries at the Z peak determine the sum $a_l^2 + v_l^2$ and the product $a_l v_l$ for each charged lepton. The v/a ambiguity is solved for the τ lepton by the measurement [23, 24] of the transverse-transverse correlation ($C_{TT} = 1.01 \pm 0.12$) at LEP, which requires $|v_\tau/a_\tau| \ll 1$, and for the three families, by the pre-LEP measurements of the forward–backward asymmetries, dominated at low energy by the Z–γ interference term proportional to $a_e a_l$. By convention $a_e < 0$.

The results of the combined analysis of the data from LEP and SLC given in Table 3 show a good agreement with universality for both vector and axial-vector couplings.

For comparison, the pre-LEP values of the same parameters were $a_e = -0.513 \pm 0.025$, $v_e = -0.045 \pm 0.036$, for the electron [20] and $a_\tau = -0.484 \pm 0.034$, $v_\tau = -0.09^{+0.25}_{-0.28}$ for the τ lepton [25].

4. Conclusion

Electroweak universality has been tested in the leptonic sector for both charged and neutral currents. The analysis of LEP and SLC data has resulted in a huge improvement of the precision.

The universality of the W couplings is verified at the 2×10^{-3} level for the three families of leptons.

For the Z, the universality of the axial-vector couplings is verified at the 10^{-3} level. The precision is only of a few % for the small Z vector couplings.

[1] Because of the helicity conservation, it is not necessary to polarize both electrons and positrons.

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