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DOSSIER

AVANCÉES EN PHYSIQUE DES PARTICULES : LA CONTRIBUTION DU LEP

ADVANCES IN PARTICLE PHYSICS: THE LEP CONTRIBUTION

The τ lepton as a laboratory for quantum chromodynamics

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Abstract Decays of the τ lepton provide a clean environment to study hadron dynamics in an energy regime dominated by resonances. Inclusive spectral functions are the basis for quantum chromodynamics (QCD) analyses, providing a most accurate determination of the strong coupling constant and quantitative information on nonperturbative contributions. The τ vector spectral function is used together with e^+e^- data in order to compute vacuum polarization integrals arising in the calculations of the anomalous magnetic moment of the muon and the running of the electromagnetic coupling constant. ***To cite this article: M. Davier, A. Höcker, C. R. Physique 3 (2002) 1223–1233.***

τ lepton / hadrons / spectral functions / chromodynamics / quarks / vacuum polarization

Le lepton τ comme laboratoire pour l'étude de la chromodynamique quantique

Résumé Les désintégrations du lepton τ permettent une étude très propre de la dynamique des hadrons dans un domaine d'énergie dominé par les résonances. Les fonctions spectrales inclusives fournissent la base d'une étude quantitative de la chromodynamique quantique (QCD), permettant une détermination précise du couplage fort et des contributions non perturbatives. Les fonctions spectrales sont nécessaires pour le calcul des effets de polarisation du vide intervenant dans la prédiction du moment magnétique anormal du muon et dans l'évolution avec l'énergie de la constante de couplage électromagnétique. ***Pour citer cet article : M. Davier, A. Höcker, C. R. Physique 3 (2002) 1223–1233.***

lepton τ / hadrons / fonctions spectrales / chromodynamique / quarks / polarisation du vide

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1. Introduction

Unlike its lighter partners – the electron and the muon – the τ lepton is heavy enough (1777 MeV/c^2) to decay into final states with hadrons (always accompanied by a ν_τ neutrino). This provides the particle physicist with a rare opportunity to study hadron dynamics in a very clean environment, a situation only shared with e^+e^- annihilation into hadrons.

Hadrons produced in τ decays are born out of the charged weak current, i.e., out of the QCD vacuum. This property guarantees that hadronic physics factorizes in these processes which are then completely characterized for each decay channel by spectral functions as far as the total production rate is concerned. Furthermore, the produced hadronic systems have well-defined isospin and spin-parity: $I = 1$, as well as $J^P = 1^-$ for the vector current (V), and $J^P = 0^-, 1^+$ for the axial-vector current (A). The spectral functions are directly related to the invariant mass ($\sqrt{s}$) spectra of the hadronic final states, normalized to their respective branching ratios and corrected for the τ decay kinematics. For a given spin-1 vector decay, one has

$$v(s) \equiv \frac{m_\tau^2}{6|V_{ud}|^2 S_{\mathrm{EW}}} \frac{B_V}{B_e} \frac{\mathrm{d}N_V}{N_V \mathrm{d}s} \left[\left(1 - \frac{s}{m_\tau^2}\right)^2 \left(1 + \frac{2s}{M_\tau^2}\right) \right]^{-1}, \tag{1}$$

where V_{ud} denotes the Cabibbo–Kobayashi–Maskawa (CKM) weak mixing matrix element, $S_{\mathrm{EW}} = 1.0194 \pm 0.0040$ accounts for electroweak radiative corrections, and $B_V = B(\tau^- \to V^- \nu_\tau)$ and $B_e = B(\tau^- \to e^- \bar{\nu}_e \nu_\tau)$ are branching fractions for the corresponding τ decays. Isospin symmetry, often called in this context *Conserved Vector Current* (CVC), connects the τ and e^+e^- annihilation spectral functions, the latter being proportional to R, defined by the ratio of the hadronic to the point-like particle cross sections.

Hadronic τ decays are a clean probe of hadron dynamics in an interesting energy region dominated by resonances. It turns out however, that owing to the phenomenological concept of quark-hadron duality, perturbative QCD can be seriously considered due to the relatively large τ mass. Many hadronic modes have been measured and studied, while some earlier discrepancies (before 1990) have been resolved with the high-statistics and low-systematics experiments at LEP, each with samples of $\sim 3 \times 10^5$ decays. Because the τ's are produced at high energy, through the decay $Z \to \tau^+\tau^-$, conditions for low systematic uncertainties are uniquely met at LEP: the selected samples have small non-τ backgrounds ($<$1%) and large efficiency ($>$90%).

The important results obtained at LEP are reviewed in this article.

2. Overview of τ hadronic decays

Decays of the τ lepton proceed through the charged weak interaction, with the exchange of the gauge boson W between the $\tau - \nu_\tau$ leptonic current and the final state current, which can be leptonic ($e - \nu_e$ or $\mu - \nu_\mu$) or hadronic ($u - d'$, where $d' = V_{ud} d + V_{us} s$ (Fig. 1). For the first time a rather complete description of τ decays emerges and is found to follow the predictions of the Standard Model. An extensive analysis has been performed by the ALEPH experiment [1–3], taking advantage of its excellent particle identification and fine-grain electromagnetic calorimeter enabling the reconstruction of close-by photons from π^0 decays. This latter point is very important as the quantum numbers of the produced hadronic system are derived from counting the number of pions: up to small electromagnetic corrections, final states

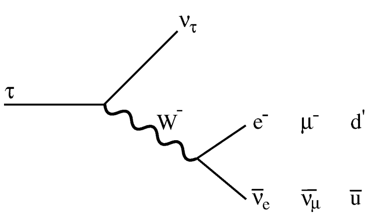

Figure 1. Feynman diagram for τ decays. The W boson connects the τ leptonic current and the final state current, which can be either leptonic or hadronic (initially a pair of quarks).

Table 1. Branching fractions for τ decays measured by ALEPH: the nonstrange vector (V), nonstrange axial-vector (A), and strange (S) modes, comprising each about 10 distinct channels, have been summed up.

Decay mode	Branching fraction ($\times 10^{-2}$)
$\tau \to \nu_\tau e \overline{\nu}_e$	17.837 ± 0.081
$\tau \to \nu_\tau \mu \overline{\nu}_\mu$	17.319 ± 0.077
$\tau \to \nu_\tau V$	31.657 ± 0.168
$\tau \to \nu_\tau A$	30.287 ± 0.188
$\tau \to \nu_\tau S$	2.895 ± 0.117

with an even number of pions originate from the vector current, while an odd number tags the axial-vector part. The ALEPH results for the decay fractions of the measured modes are given in Table 1, separately for the leptonic channels, the nonstrange vector and axial-vector states, and the modes with nonzero strange quantum number. Several observations can be made:

- the relative leptonic and hadronic rates are close to the rough prediction just counting the number of quark colours: $N_c = 3$ would yield branching fractions of 20% for each leptonic decay. The negative correction to this value is expected from QCD and provides the basis for a measurement of the strong coupling at the scale of the τ mass;
- the inclusive nonstrange vector and axial-vector rates are approximately equal: perturbative QCD with maximal parity violation in the weak current predicts an exact equality so that the small difference observed has to originate from nonperturbative contributions;
- the magnitude of the inclusive strange rate is consistent with the known ratio of the CKM matrix elements, $|V_{us}/V_{ud}|^2 \sim 0.05$.

3. Inclusive spectral functions

The τ nonstrange spectral functions have been measured by ALEPH [4,5] and OPAL [6]. The procedure requires a careful separation of vector and axial-vector states involving the reconstruction of multi-π^0 decays and the proper treatment of final states with a $K\overline{K}$ pair. The V and A spectral functions are given in Fig. 2. They are dominated by the lowest π, ρ and a_1 states (the δ distribution of the π is not drawn), with a tendency to converge at large mass towards a value close to the naive parton model expectation. Yet, the vector part stays significantly above while the axial-vector one lies below. Hence the two spectral functions are not *asymptotic* at the τ mass scale. The $V + A$ spectral function (see Fig. 3) converges towards a value above the parton level as expected in QCD. One observes that the resonance-dominated low-mass region shows an oscillatory pattern around the asymptotic smooth QCD expectation, which is given by the quark pair contribution including gluon radiation. This behaviour confirms the phenomenological concept of global quark–hadron duality [7–9]: in a global sense, i.e., suitably averaging (*smearing*) over the resonances, the hadronic physics can be quantitatively assessed using the simplifying quark and gluon picture of QCD. Fig. 2 indeed displays a textbook example of global duality, the properties of which are discussed in the next section.

4. QCD analysis of nonstrange τ decays

The total hadronic τ width, properly normalized to the known leptonic width,

$$R_\tau = \frac{\Gamma(\tau^- \to \text{hadrons}^- \nu_\tau)}{\Gamma(\tau^- \to e^- \bar{\nu}_e \nu_\tau)} = \frac{1 - B_e - B_\mu}{B_e}, \tag{2}$$

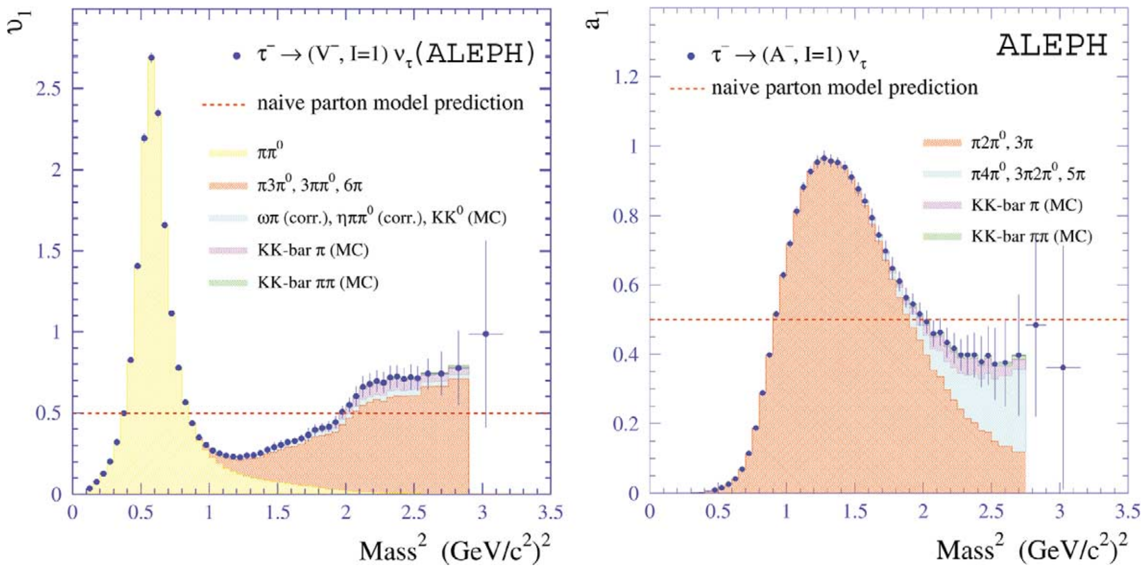

Figure 2. Inclusive nonstrange vector (left) and axial-vector (right) spectral functions from ALEPH. The contributions from the main exclusive modes are highlighted, while the dashed line is the expectation from the naive parton model.

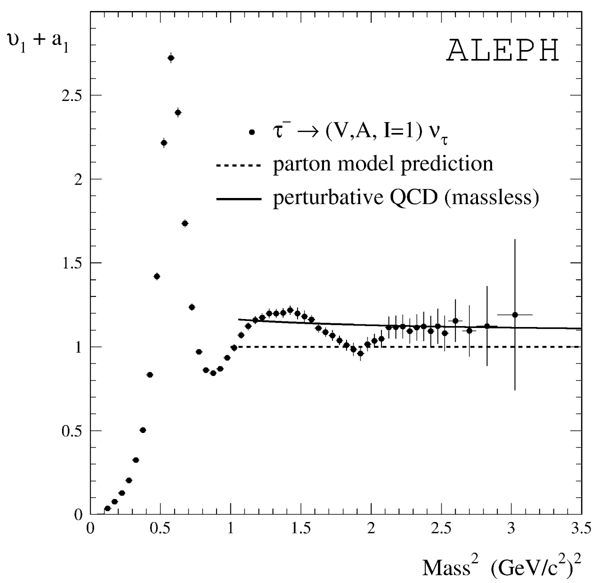

Figure 3. Inclusive $V + A$ nonstrange spectral function from ALEPH. The solid line is the prediction from massless perturbative QCD using $\alpha_s(M_Z^2) = 0.120$.

should be well predicted by QCD as it is an inclusive observable: it is the sum over the hadronic final states integrated over the accessible invariant mass spectrum from m_π to m_τ. This favourable situation could be spoiled by the fact that the energy scale is rather small, so that questions about the validity of a perturbative approach can be raised. At least two levels are to be considered: the convergence of the perturbative expansion itself and the control of the nonperturbative contributions.

4.1. Theoretical prediction for R_τ

Theoretically the ratio R_τ can be written as an integral of the spectral functions over the invariant mass-squared s of the final state hadrons, which, owing to Cauchy's theorem, can be turned into a contour integral in the complex s plane [10]. The energy scale $s_0 = m_\tau^2$ of the contour is large enough so that one can treat perturbative and nonperturbative contributions within an approach known as the *Operator Product Expansion* (OPE) [11]. The prediction for the vector and axial-vector ratio $R_{\tau,V/A}$ can then be written as:

$$R_{\tau,V/A} = \frac{3}{2}|V_{ud}|^2 S_{\mathrm{EW}}\left(1 + \delta^{(0)} + \delta^{(2-\mathrm{mass})}_{ud,V/A} + \sum_{D=4,6,8} \delta^{(D)}_{ud,V/A}\right). \qquad (3)$$

The term $\delta^{(0)}$ is the massless perturbative contribution whose expansion is known to third order α_s^3. The $\delta^{(2-\mathrm{mass})}_{ud,V/A}$ term is of dimension $D = 2$ and contains the perturbative mass correction to $\delta^{(0)}$, which is lower than 0.1% for u, d quarks. Finally, the $\delta^{(D)}$ are of nonperturbative origin and scale with the powers of $s_0^{-D/2}$.

4.2. Measurements

The ratio R_τ is obtained from measurements of the leptonic branching ratios:

$$R_\tau = 3.647 \pm 0.014, \qquad (4)$$

using the average value from the four LEP experiments including the improvement in accuracy provided by the universality assumption of leptonic currents (see the contribution by Rougé et al. [12]). The nonstrange part of R_τ is obtained by subtracting from it the measured strange contribution given in Table 1.

Two complete QCD analyses of the V and A components have been performed by ALEPH [4,5] and OPAL [6]. Both are based on the world-average leptonic branching ratios, but their own measured spectral functions which are injected into the fit through the calculations of mass-weighted moments, mainly determining the size of the nonperturbative power corrections.

4.3. Results of the fits

The results of the ALEPH fits are given in Table 2. Consistent results are found by OPAL. It is worth emphasizing that the nonperturbative contributions are found to be very small. The remarkable agreement between the $\alpha_s(m_\tau^2)$ values using vector and axial-vector data is a strong consistency check of the theoretical procedure since the V and A spectral functions have very different shapes (Fig. 2). The total nonperturbative contribution to $R_{\tau,V+A}$ is compatible with zero within an uncertainty of 0.4%, which is much smaller than the error arising from the perturbative term. This cancellation of the nonperturbative terms increases the confidence in the $\alpha_s(m_\tau^2)$ determination from the inclusive $(V + A)$ observables. The final result from

Table 2. Fit results of $\alpha_s(m_\tau^2)$ and the OPE nonperturbative contributions from vector, axial-vector and $V + A$ combined fits using the corresponding ratios R_τ and the spectral moments as input parameters. The second error is given for theoretical uncertainty.

ALEPH	$\alpha_s(m_\tau^2)$	δ_{NP}
V	$0.330 \pm 0.014 \pm 0.018$	0.020 ± 0.004
A	$0.339 \pm 0.013 \pm 0.018$	-0.027 ± 0.004
$V + A$	$0.334 \pm 0.007 \pm 0.021$	-0.003 ± 0.004

ALEPH can be written

$$\alpha_s\left(m_\tau^2\right) = 0.334 \pm 0.007_{\text{exp}} \pm 0.021_{\text{theo}}, \tag{5}$$

where the first error is experimental (statistical and systematic) and the second is theoretical. The dominant contribution to the theoretical uncertainty comes from the truncation of the perturbative QCD series after the third order.

4.4. Test of the running of $\alpha_s(s)$

A dramatic prediction of QCD is the fact that the value of the fundamental coupling of the theory depends on the energy scale through vacuum polarization effects in the gluon propagator. Indeed, the electromagnetic coupling $\alpha(s)$ increases its value with the energy (starting from the well-known $\alpha(0) \simeq 1/137$), since the gauge group $U(1)$ of electrodynamics (QED) is Abelian with a chargeless photon. On the contrary, the strong coupling $\alpha_s(s)$ is a decreasing function of energy because the gauge group $SU(3)$ of chromodynamics is non-Abelian, with the consequence that the gluon itself carries a strong charge. The evolution of the QCD coupling from very large values at low energy (long distance), responsible for the confinement of quarks into hadrons, to smaller and smaller values at high energy (asymptotically free quarks at short distance) is the most striking aspect of the theory of strong interactions. Its experimental confirmation is therefore very important.

In a first step one can compare the values of the strong coupling measured at the τ and the Z mass scales. In order to do that, the result obtained above in τ decays can be evolved to M_Z using the $SU(3)$ renormalization group to fourth order in α_s. One obtains from Eq. (5) $\alpha_s(M_Z^2) = 0.1202 \pm 0.0008_{\text{exp}} \pm 0.0024_{\text{theo}} \pm 0.0010_{\text{evol}}$, where the last error accounts for possible ambiguities in the evolution when crossing the heavy quark thresholds. This result can be compared to the best determination of the strong coupling at M_Z from the measurement of the Z width or better from the global electroweak fit (see the contribution by Olchevski et al. [13]). The variable R_Z, given by the rate of Z decays to hadrons divided by the leptonic rate for $Z \to e^+e^-$, has similar advantages to R_τ. Because of the much larger scale it has a weaker sensitivity to $\alpha_s(s)$, paired with a better converging perturbative series. It turns out that this determination is dominated by experimental errors with very small theoretical uncertainties, i.e. the reverse of the situation encountered in τ decays. The electroweak fit yields $\alpha_s(M_Z^2) = 0.1183 \pm 0.0027$, in excellent

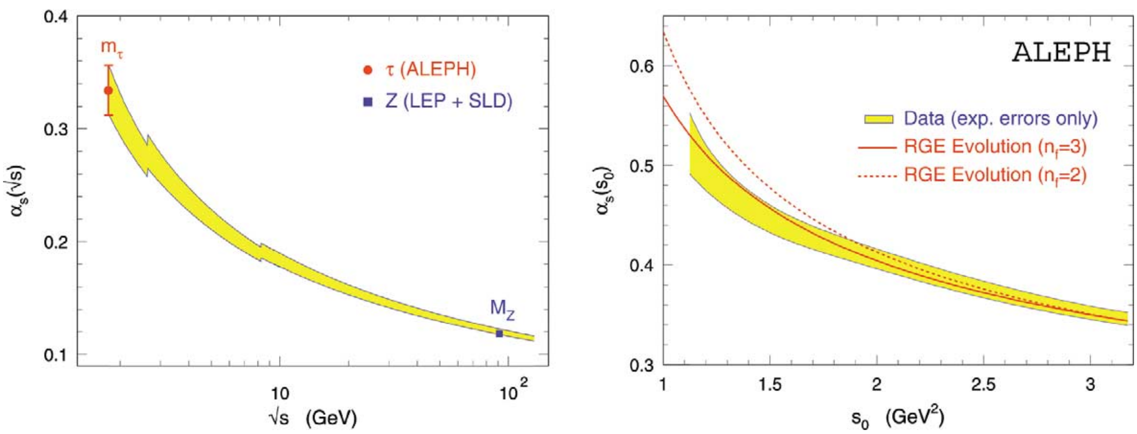

Figure 4. Left: high-energy evolution of the strong coupling (measured at m_τ^2) to M_Z^2 predicted by QCD compared to the direct measurement. The evolution is carried out at 4 loops, while the flavour matching is accomplished at 3 loops using the matching scales $2m_c$ and $2m_b$, respectively. Right: the low-energy running of $\alpha_s(s_0)$ obtained from the fit of the theoretical prediction to $R_{\tau,V+A}(s_0)$. The shaded band shows the data within their experimental errors. The curves give the evolutions from the fourth-order renormalization group equations for two and three flavours.

agreement with the result from the τ analysis. Fig. 4 illustrates the agreement between the evolution of $\alpha_s(m_\tau^2)$, predicted by QCD, and the measured $\alpha_s(M_Z^2)$.

A second, more adventurous test of the running can be performed going from the τ mass scale down to lower energies. One uses for this the measured τ spectral functions allowing one to simulate the physics of a hypothetical τ lepton with a mass $\sqrt{s_0}$ smaller than m_τ. Assuming quark–hadron duality, the evolution of $R_\tau(s_0)$ provides a direct test of the running of $\alpha_s(s_0)$, as well as a check of the validity of the OPE approach in τ decays. Fig. 4 shows the experimental determination of $\alpha_s(s_0)$, obtained at every s_0 value from the comparison of data and theory. Good agreement is observed with the four-loop renormalization group evolution given the expected number of quark flavours (u, d, s), indicating that the α_s determination from the inclusive $V + A$ data is robust.

Summarizing this part, the strong coupling has been measured using τ and Z decays at LEP: it runs according to the $SU(3)$ gauge group with a fall off by about a factor of five when the energy scale increases from 1 GeV to 100 GeV.

5. Applications to hadronic vacuum polarization

5.1. Running of the QED fine structure constant and muon magnetic anomaly

The running of the QED fine structure constant $\alpha(s)$ and the anomalous magnetic moment of the muon are prominent observables the theoretical precisions of which are limited by second order loop effects from hadronic vacuum polarization. The precision tests of the electroweak theory performed at LEP require an accurate knowledge of the QED coupling at the Z mass scale, $\alpha(M_Z^2)$.

While the quantum fluctuations from pairs of virtual particles are readily computed in QED for leptons, the loops involving hadrons cannot be inferred from first principles, because QCD is an infrared-divergent theory, as discussed above. However, by virtue of the analyticity of the vacuum polarization function and unitarity, the hadronic contribution can be calculated via a dispersion integral involving the cross section for e^+e^- annihilation into hadrons, $\sigma_{\rm had}(s)$. Resumming the contributions from multiple loops, the running electromagnetic coupling is obtained by

$$\alpha(s) = \frac{\alpha(0)}{1 - \Delta\alpha(s)}, \tag{6}$$

where $4\pi\alpha(0)$ is the square of the electron charge in the long-wavelength Thomson limit. The leading order leptonic contribution at the Z scale amounts to 314.2×10^{-4}. The dispersion integral for the contribution of hadronic vacuum polarization from the five lighter quarks can be written [14,15]

$$\Delta\alpha_{\rm had}^{(5)}\big(M_{\rm Z}^2\big) = -\frac{M_{\rm Z}^2}{4\pi^2\alpha}{\rm Re}\int_{4m_\pi^2}^{\infty} {\rm d}s\,\frac{\sigma_{\rm had}(s)}{s - M_{\rm Z}^2 - {\rm i}\varepsilon}, \tag{7}$$

while the top quark contribution, which is outside experimental reach for the moment, can be calculated from perturbative QCD.

Similarly, the contribution of the hadronic vacuum polarization to a_μ can be calculated via the dispersion integral [16,17], as sketched in Fig. 5

$$a_\mu^{\rm had} = \frac{1}{4\pi^3}\int_{4m_\pi^2}^{\infty} {\rm d}s\,\sigma_{\rm had}(s)K(s). \tag{8}$$

The known QED function $K(s)$ decreases monotonically with increasing s. It gives a strong weight to the low energy part of the integral (8). About 91% of the total contribution to $a_\mu^{\rm had}$ is accumulated at c.m.

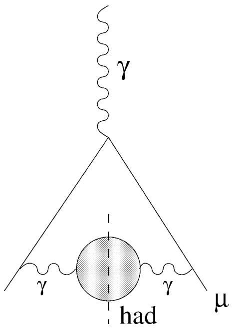

Figure 5. Feynman diagram for the lowest order vacuum polarization contribution from hadronic fluctuations in the photon propagator. As indicated by the vertical dashed line such a contribution can be evaluated using data for the cross section $e^+e^- \to$ *hadrons* through a dispersion relation.

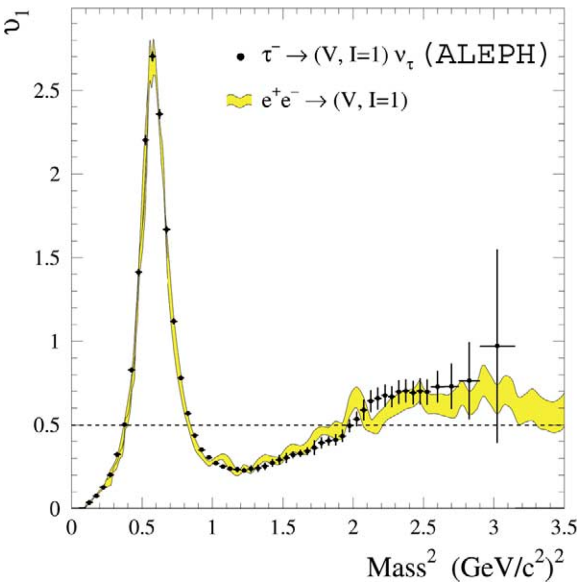

Figure 6. Global test of CVC using isospin-rotated τ and e^+e^- vector spectral functions.

energies $\sqrt{s}$ below 2.1 GeV, and 72% of $a_\mu^{\rm had}$ is covered by the two-pion final state which is dominated by the $\rho(770)$ resonance.

The calculation of the hadronic vacuum polarization effects can therefore proceed, given reliable and precise data on $\sigma_{\rm had}(s)$, or equivalently on the vector spectral functions using the relation

$$\sigma_{\rm had}(s) = \frac{4\pi\alpha^2}{s} v(s). \tag{9}$$

5.2. Improvements to the standard calculations from τ data

The studies on τ decays have shown that:

- The $I = 1$ vector spectral function from τ decays agrees with that from e^+e^- annihilation, while it is more precise for masses less than 1.6 GeV$\cdot c^{-2}$ as can be seen on Fig. 6. Isospin-violating effects are predicted at a few 10^{-3} level.
- The description of R_τ by perturbative QCD works down to a scale of 1 GeV. Nonperturbative contributions at 1.8 GeV are well below 1% in this case. They are larger ($\sim$2%) for the vector

part alone, but reasonably well described by OPE. The complete (perturbative and nonperturbative) description is accurate at the 1% level at 1.8 GeV for integrals over the vector spectral function such as $R_{\tau,V}$.

This has direct applications to calculations of hadronic vacuum polarisation which involve the knowledge of the vector spectral function. For both, the running of α and a_μ, the standard method involves the computation of the corresponding dispersion integrals ((7), (8)) over the vector spectral function taken from the $e^+e^- \to$ hadrons data. Hence the precision of the calculation is given by the accuracy of the data, which is poor above 1.5 GeV. It has been shown that at low energies the precision on the integrals can be significantly improved by using τ data [18].

Another breakthrough comes about when one applies the prediction of perturbative QCD far above quark thresholds, but at sufficiently low energies (compatible with the remarks above) in place of poor and noncompetitive experimental data [19]. Finally, it is possible to obtain theoretical constraints on the data-dominated low-energy domain by using analyticity and QCD sum rules, basically without any additional assumption. This idea [20] has been used within the procedure described above to further improve the calculations [21].

The various measurements of $R(s)$ and the QCD prediction are shown in Fig. 7.

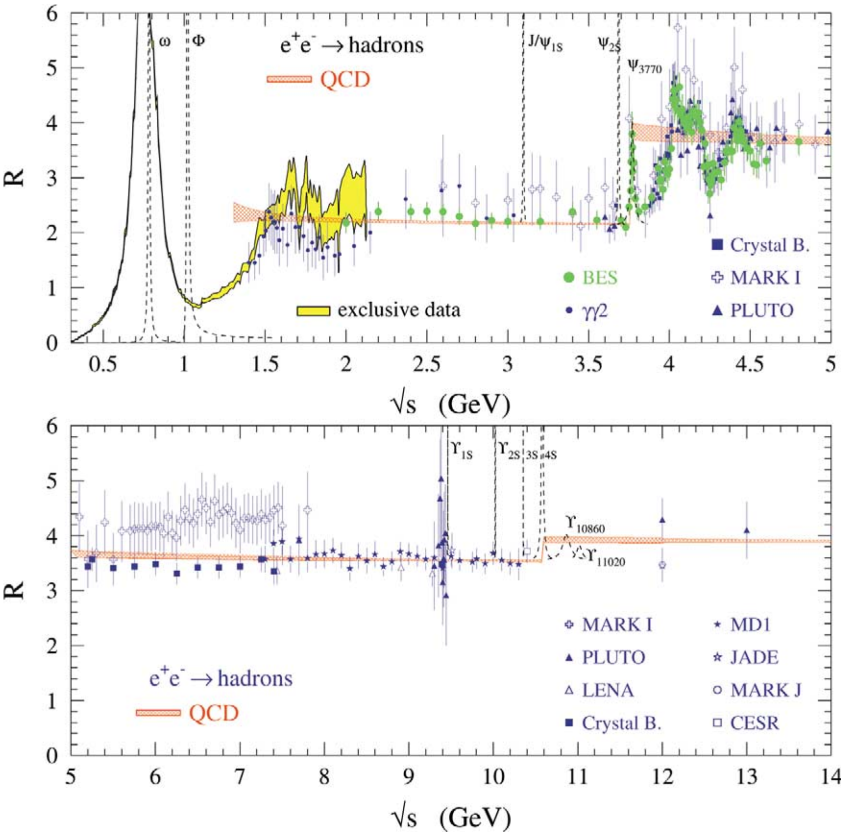

Figure 7. Inclusive hadronic cross section ratio in e^+e^- annihilation versus the centre-of-mass energy $\sqrt{s}$. Additionally shown is the QCD prediction of the continuum contribution from reference [19] as explained in the text. The shaded areas depict regions where experimental data are used for the evaluation of $\Delta\alpha_{\rm had}(M_Z^2)$ and $a_\mu^{\rm had}$. References can be found in [18]. The new data from BES [22] in the 2–5 GeV range agree with the QCD prediction.

5.3. Results

The evaluation of the integrals (7) and (8) leads to the results

$$\begin{aligned}\Delta\alpha_{\text{had}}\left(M_Z^2\right) &= (276.3 \pm 1.1_{\text{exp}} \pm 1.1_{\text{theo}}) \times 10^{-4},\\ \alpha^{-1}\left(M_Z^2\right) &= 128.933 \pm 0.015_{\text{exp}} \pm 0.015_{\text{theo}},\\ a_\mu^{\text{had}} &= (692.4 \pm 5.6_{\text{exp}} \pm 2.6_{\text{theo}}) \times 10^{-10},\\ a_\mu^{\text{SM}} &= (11659176.6 \pm 5.6_{\text{exp}} \pm 4.4_{\text{theo}}) \times 10^{-10},\end{aligned} \tag{10}$$

and $a_e^{\text{had}} = (187.5 \pm 1.7_{\text{exp}} \pm 0.7_{\text{theo}}) \times 10^{-14}$ for the leading order hadronic contribution to a_e. The total a_μ^{SM} value includes additional contributions from non-leading order hadronic vacuum polarization and light-by-light scattering contributions. For this latter part, important progress has been recently achieved [23,24], correcting a sign mistake in the previous calculations. The given results provide a significant improvement in precision over the previous determinations by factors of 2.5 and 5 for a_μ^{had} and $\Delta\alpha_{\text{had}}(M_Z^2)$, respectively. New measurements of R have been obtained in Beijing [22]. As seen in Fig. 7 they are in perfect agreement with the QCD prediction in the 2–3.75 GeV range considered above as 'globally' asymptotic for the u, d and s quarks. Since most of the changes between previous data and QCD occurred in this range it is reassuring that the new and more precise data do indeed agree with the prediction.

5.4. Comparison with experiment and outlook

These results have direct implications for phenomenology and on-going experimental programs. Most of the sensitivity to the Higgs boson mass originates from the measurements of asymmetries in the process $e^+e^- \to Z \to$ fermion pairs. In order to extract M_H it is necessary to know the value of α at the Z energy scale with precision. Using the determination quoted in Eq. (10) the Higgs mass from the global electroweak fit turns out to be

$$M_H = \left(80^{+44}_{-28}\right)\ \text{GeV}, \tag{11}$$

representing some improvement over the value obtained using the more traditional estimate for $\alpha(M_Z^2)$ (contribution by Olchevski et al. [13]).

The interest in reducing the uncertainty in the hadronic contribution to a_μ^{had} is directly linked to the possibility of measuring the weak contribution, $a_\mu^{\text{weak}} = (15.2 \pm 0.1) \times 10^{-10}$ [25], included in Eq. (10). The present value from the BNL experiment [26] marks a significant achievement in accuracy over the previous determinations at CERN and BNL,

$$a_\mu = (11659202 \pm 15) \times 10^{-10}. \tag{12}$$

It is in fair agreement with the theoretical prediction given in Eq. (10). This was not the case when it was published since, at that time, the small 'light-by-light' contribution had been computed with the wrong sign. The resulting 2.6 σ discrepancy generated a flood of theoretical papers attempting to interpret the effect in terms of new physics, among which Supersymmetry appeared to be the natural 'explanation'. The excitement may not be over as the precision of the Brookhaven experiment should be further improved to a level of 4×10^{-10}, well below the expected size of the weak contribution. Such a program makes sense only if the uncertainty on the hadronic contribution can be made sufficiently small. The improvements described above represent a significant step in this direction.

6. Conclusions

The decays $\tau \to \nu_\tau +$ hadrons constitute a clean and powerful way to study hadronic physics up to $\sqrt{s} \sim 1.8$ GeV. Probably the major surprise has been the fact that inclusive hadron production is well described by perturbative QCD with very small nonperturbative components at the τ mass. Since the low-energy region is dominated by hadron resonances, this is a success for the concept of global quark-hadron duality. The measurement of the vector and axial-vector spectral functions has opened the way to quantitative analyses. Precise determinations of α_s agree for both spectral functions and they also agree with all the other determinations from the Z width, the rate of Z to jets and deep inelastic lepton scattering. The value from τ decays, $\alpha_s(M_Z^2)_\tau = 0.1202 \pm 0.0027$, is in excellent agreement with the average from all other determinations [27], $\alpha_s(M_Z^2)_{\text{non}-\tau} = 0.1187 \pm 0.0020$.

The use of the τ vector spectral function and the theoretical approach (QCD) tested in τ decays considerably improve the calculations of hadronic vacuum polarization. Significantly improved results have been obtained for the running of α to the Z mass and for the muon anomalous magnetic moment. Both of these quantities must be calculated with high precision to be compared to accurate measurements, thus providing an opportunity to reveal new physics.

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