C. R. Physique 3 (2002) 1203–1210

Physique subatomique/*Subatomic physics*

AVANCÉES EN PHYSIQUE DES PARTICULES :
LA CONTRIBUTION DU LEP

ADVANCES IN PARTICLE PHYSICS: THE LEP CONTRIBUTION

DOSSIER

Heavy quarks and the CKM matrix

Peter Kluit [a], **Achille Stocchi** [b*]

[a] **NIKHEF, Postbus 41882, NL-1009 DB Amsterdam, The Netherlands**

[b] **Laboratoire de l'accélérateur linéaire, IN2P3-CNRS et Université de Paris-Sud, 91405 Orsay, France**

Received 15 May 2002; accepted 1 July 2002

Note presented by Guy Laval.

Abstract In the last decade, the LEP experiments played a central role in the study of B hadrons (hadrons containing a b quark). New B hadrons have been observed (B_s^0, Λ_B, Ξ_b and B^{**}) and their production and decay properties have been measured. In this paper we will focus on measurements of the CKM matrix elements: $|V_{cb}|$, $|V_{ub}|$, $|V_{td}|$ and $|V_{ts}|$. We will show how all these measurements, together with theoretical developments, have significantly improved our knowledge on the flavour sector of the Standard Model. ***To cite this article: P. Kluit, A. Stocchi, C. R. Physique 3 (2002) 1203–1210.***

beauty (B) hadrons / B decays / $B^0 - \overline{B^0}$ **oscillations / CKM matrix / CP violation**

Les quarks lourds et la matrice CKM

Résumé Dans les dernières dix années, les expériences LEP ont joué un rôle primordial dans les études des hadrons beaux (hadrons contenant un quark b). Des nouveaux hadrons beaux ont été observés (B_s^0, Λ_B, Ξ_b et B^{**}) et leurs taux de production ainsi que les caractéristiques liées à leurs processus de désintégration ont été mesurés. Dans ce papier nous expliquerons les mesures des éléments de la matrice CKM : $|V_{cb}|$, $|V_{ub}|$ ainsi que $|V_{td}|$ et $|V_{ts}|$. Nous montrerons la manière dont ces mesures ainsi que les développements théoriques qui leur sont reliés ont amélioré de façon significative notre connaissance du secteur des saveurs tel qu'il est décrit dans le Modèle Standard. ***Pour citer cet article : P. Kluit, A. Stocchi, C. R. Physique 3 (2002) 1203–1210.***

hadrons beaux (B) / désintégrations des hadrons B / oscillations $B^0 - \overline{B^0}$ **/ matrice CKM / violation de CP**

1. Introduction

In the last decade, the LEP experiments played an important role in the study of B hadrons. At the start of the LEP accelerator in 1989, only the B_d and the B^+ hadrons were known and their properties were

* Corresponding author.
E-mail address: stocchi@lal.in2p3.fr (A. Stocchi).

S1631-0705(02)01396-8/REV

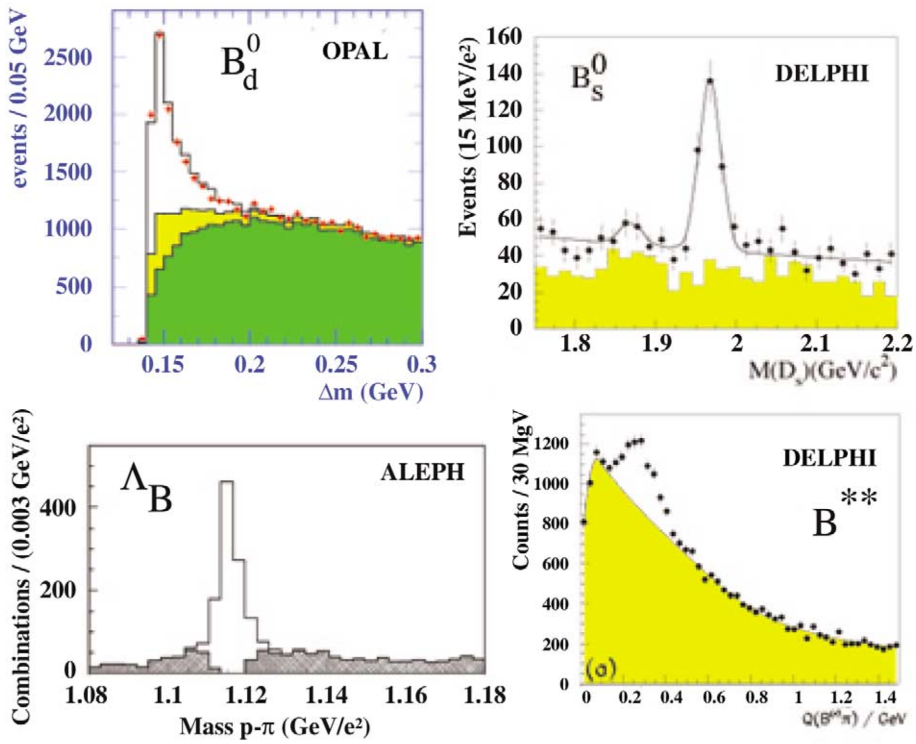

Figure 1. B hadrons at LEP. The three plots from top left to bottom left show the invariant mass spectra of $((D^0\pi) - D^0)$, D_s, Λ which are obtained in correlation with an opposite sign lepton. These events are attributed mainly to the semileptonic decays of B_d^0, B_s^0 and Λ_B hadrons, respectively. The bottom right figure shows the mass difference between an orbitally excited B meson (B^{**}) and a $B^{(*)}$.

under study. New weakly decaying B hadrons have been observed (B_s^0, Λ_B, Ξ_b) for the first time and their production and decay properties have been measured. New strongly decaying hadrons, the orbitally ($L = 1$) excited B (B^{**}) mesons have been also observed and their mass and production rates measured. An overview of the signals used to study these new states is given in Fig. 1.

Precise measurements of the production and decay properties of B hadrons have been performed [1]. For an overview of the B hadron lifetime measurements, the reader is referred to the dedicated article in this volume [2]. In this paper we will further focus on the measurements of the CKM matrix elements. Section 2 introduces the CKM matrix and in Sections 3, 4 the measurements of the matrix elements $|V_{cb}|$ and $|V_{ub}|$ are described. The next section discusses the results for $B^0 - \overline{B^0}$ oscillations from which the elements $|V_{td}|$ and $|V_{ts}|$ have been determined. Finally in Section 6 the impact of these measurements on the determination of the CKM parameters is presented.

2. The CKM matrix

The CKM matrix V_{CKM} appears [3,4] when one writes the most general $V - A$ current between up $U \equiv (u, c, t)$ and down $D \equiv (d, s, b)$ quark states

$$J^\mu \sim \overline{U}\gamma^\mu\left(\frac{1-\gamma^5}{2}\right)V_{\mathrm{CKM}}D. \tag{1}$$

It is known that for at least 3 quark doublets (what is indeed observed), the unitary CKM matrix becomes in general complex. For 3 doublets its elements

$$V_{\mathrm{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \tag{2}$$

depend in general on four parameters: three Euler angles and one complex phase. Several parametrizations exist, but it is convenient to use the improved Wolfenstein [5] parametrization, expressed in terms of the parameters λ, A, ρ and η

$$V_{\mathrm{CKM}} = \begin{pmatrix} 1-\frac{\lambda^2}{2}-\frac{\lambda^4}{8} & \lambda & A\lambda^3(\rho-\mathrm{i}\eta) \\ -\lambda+\frac{A^2\lambda^5}{2}(1-2\rho)-\mathrm{i}A^2\lambda^5\eta & 1-\frac{\lambda^2}{2}-\lambda^4(\frac{1}{8}+\frac{A^2}{2}) & A\lambda^2 \\ A\lambda^3[1-(1-\frac{\lambda^2}{2})(\rho+\mathrm{i}\eta)] & -A\lambda^2(1-\frac{\lambda^2}{2})(1+\lambda^2(\rho+\mathrm{i}\eta)) & 1-\frac{A^2\lambda^4}{2} \end{pmatrix} + \mathrm{O}(\lambda^6). \tag{3}$$

The CKM matrix elements can be expressed as:

$$V_{ub} = A\lambda^3(\overline{\rho}-\mathrm{i}\overline{\eta})/(1-\lambda^2/2), \quad V_{cb} = A\lambda^2, \quad V_{td} = A\lambda^3(1-\overline{\rho}+\mathrm{i}\overline{\eta}), \tag{4}$$

where the parameters $\overline{\rho}$ and $\overline{\eta}$ have been introduced [6]. [1]

The parameter λ is precisely determined to be 0.2237 ± 0.0033 using semileptonic kaon decays [7]. The other parameters: A, $\overline{\rho}$ and $\overline{\eta}$ were rather unprecisely known.

The LEP experiments contributed to the determination of the CKM matrix elements moduli $|V_{cb}|$, $|V_{ub}|$, $|V_{td}|$ and $|V_{ts}|$. An overview of the measurements discussed below as well as a description of the procedure to combine the results of the different experiments can be found in [1].

The parameter η makes the elements of the CKM matrix complex, with the important consequence that weak processes involving quarks do not preserve CP symmetry: thus $\eta \neq 0$ accounts for the violation of the CP symmetry (observed since 1964 in the $K^0\overline{K^0}$ system) in the Standard Model.

3. Measurement of $|V_{cb}|$

The $|V_{cb}|$ element of the CKM matrix can be accessed at LEP by studying the decay rates of inclusive and exclusive semileptonic b-decays. The first method is based on the measured inclusive semileptonic width Γ_{sl}:

$$\Gamma_{\mathrm{sl}} = \frac{BR(b \to cl\nu)}{\tau_b} = \gamma_{\mathrm{theory}}|V_{cb}|^2. \tag{5}$$

Using the OPE (Operator Product Expansion) for heavy quarks, the theoretical factor γ_{theory} can be determined and $|V_{cb}|$ can be extracted. The combined inclusive result is [8]:

$$|V_{cb}| = (40.7 \pm 0.7_{\mathrm{exp}} \pm 0.8_{\mathrm{th}}) \times 10^{-3}.$$

The second method is based on exclusive $\overline{\mathrm{B}^0}_{\mathrm{d}} \to \mathrm{D}^{*+}\ell^-\overline{\nu_l}$ decays. Using HQET (Heavy Quark Effective Theory) an expression for the differential rate – proportional to $|V_{cb}|^2$ – can be derived. The exclusive result is:

$$|V_{cb}| = (41.5 \pm 1.2_{\mathrm{stat}} \pm 1.8_{\mathrm{sys}}) \times 10^{-3}.$$

The combined inclusive and exclusive result is: [2]

$$|V_{cb}| = (41.8 \pm 1.0) \times 10^{-3},$$

which corresponds to a value for A of 0.835 ± 0.020.

4. Measurement of $|V_{ub}|$

The CKM matrix element $|V_{ub}|$ has been measured at LEP using semileptonic b to u decays. This measurement is rather difficult because one has to suppress the large background from the more abundant semileptonic b to c quark transitions. By using kinematical and topological variables, the LEP experiments have succeeded in measuring the semileptonic b to u branching ratio [8], and obtain:

$$BR(b \to l^- \bar{\nu} X_u) = (1.71 \pm 0.53) \times 10^{-3}.$$

Using models based on the Operator Product Expansion, a value for $|V_{ub}|$ is obtained:

$$|V_{ub}| = (40.9 \pm 6.1 \pm 3.1) \times 10^{-4} \text{ LEP}. \tag{6}$$

The CLEO collaboration [9] has measured the branching fraction for the decay $\overline{\mathrm{B}^0_\mathrm{d}} \to \rho^+ \ell^- \overline{\nu_\ell}$ and deduced a value for $|V_{ub}|$ using several models to describe the decay form factors:

$$|V_{ub}| = (32.5 \pm 2.9 \pm 5.5) \times 10^{-4} \text{ CLEO}.$$

These results are compatible within their respective errors.

5. Study of $\mathrm{B}^0 - \overline{\mathrm{B}^0}$ oscillations

The probability that a B^0 meson oscillates into a $\overline{\mathrm{B}^0}$ or remains a B^0 is given by:

$$P_{\mathrm{B}^0_\mathrm{q} \to \mathrm{B}^0_\mathrm{q}(\overline{\mathrm{B}^0_\mathrm{q}})} = \frac{1}{2}\, \mathrm{e}^{-\mathrm{t}/\tau_\mathrm{q}} (1 \pm \cos \Delta \mathrm{m_q t}), \tag{7}$$

where t is the proper time, τ_q the lifetime of the B^0_q meson, and $\Delta \mathrm{m_q} = \mathrm{m}_{\mathrm{B}^0_1} - \mathrm{m}_{\mathrm{B}^0_2}$ the mass difference between the two physical mass eigenstates. [3] To derive this formula the effects of CP violation and lifetime differences for the two states have been neglected.

The Standard Model predicts:

$$\Delta \mathrm{m_d} \propto |V_{td}|^2 \mathrm{f}^2_{\mathrm{B_d}} \mathrm{B}_{\mathrm{B_d}}; \qquad \Delta \mathrm{m_s} \propto |V_{ts}|^2 \mathrm{f}^2_{\mathrm{B_s}} \mathrm{B}_{\mathrm{B_s}}, \tag{8}$$

where $f_{\mathrm{B_{q=s,d}}}$ is the B decay constant and $\mathrm{B}_{\mathrm{B_{q=s,d}}}$ the bag factor, both depending on large distance hadronic physics which controls the B wave function. A measurement of the mass difference gives thus access to the CKM matrix elements V_{td} and V_{ts}.

A time dependent study of $\mathrm{B}^0 - \overline{\mathrm{B}^0}$ oscillations requires:

- the measurement of the proper time t;
- to know if a B^0 or a $\overline{\mathrm{B}^0}$ decays at time t (decay tag);
- to know if a b or a $\bar{b}$ quark has been produced at $\mathrm{t} = 0$ (production tag).

5.1. $\Delta \mathrm{m_d}$ measurements

Analyses using different samples have been performed at LEP. A typical time distribution is shown in Fig. 2. $\mathrm{B}^0_\mathrm{d} - \overline{\mathrm{B}^0_\mathrm{d}}$ oscillations with a frequency $\Delta \mathrm{m_d} \sim 0.530\ \mathrm{ps}^{-1}$ are clearly visible. As with many other results from LEP, this will be a textbook plot! The present summary of the results on $\Delta \mathrm{m_d}$ is shown in Fig. 2. Combining LEP, CDF and SLD measurements it follows that [10]:

$$\Delta \mathrm{m_d} = (0.498 \pm 0.013)\ \mathrm{ps}^{-1}. \tag{9}$$

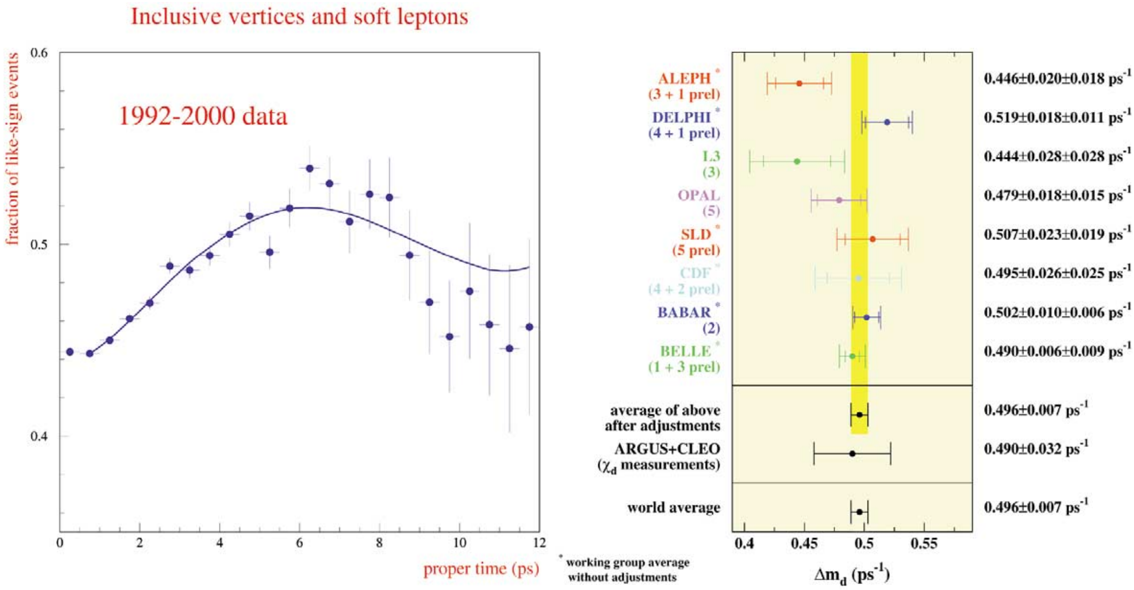

Figure 2. The plot on the left shows the $B_d^0 - \overline{B_d^0}$ oscillations. The points with error bars are the data. The curve shows the result of the fit using $\Delta m_d = 0.53\ ps^{-1}$. On the right, the summary of the Δm_d results from LEP, SLD, CDF, BABAR and BELLE is given.

Table 1. Characteristics of the different analyses are given in terms of statistics (N), B_s^0 purity ($P(B_s)$, tagging purities — i.e. the fraction of correctly tagged events — at the production and decay time ($\varepsilon_1, \varepsilon_2$) and average time resolution within the first picosecond.

Analysis	N (events)	P (B_s) (%)	ε_1 (%)	ε_2 (%)	σ_t ($t < 1$ ps) (ps)
Dipole	~ 700000	~ 10	~ 70	~ 60	~ 0.25
Inclusive lepton	~ 50000	~ 10	~ 70	~ 90	~ 0.25
$D_s^{\pm} h^{\mp}$	~ 3000	~ 15	~ 72	~ 90	~ 0.22
$D_s^{\pm} \ell^{\mp}$	~ 400	~ 60	~ 78	~ 90	~ 0.18
Exclusive B_s^0	~ 25	~ 70	~ 78	~ 100	~ 0.08

Δm_d has been first measured with high precision by the LEP/SLD/CDF experiments. The new and precise measurements performed at the B-factories confirmed these measurements and improved the precision by a factor 2. the combined result is: $\Delta m_d = (0.496 \pm 0.007)\ ps^{-1}$.

5.2. Analyses on Δm_s

The search for $B_s^0 - \overline{B_s^0}$ oscillations is more difficult because the oscillation frequency is much higher. In the Standard Model one expects $\Delta m_s \sim 20 \Delta m_d$. The proper time resolution will therefore play an essential role. Five different types of analyses have been performed at LEP/SLD. An overview is given in Table 1.

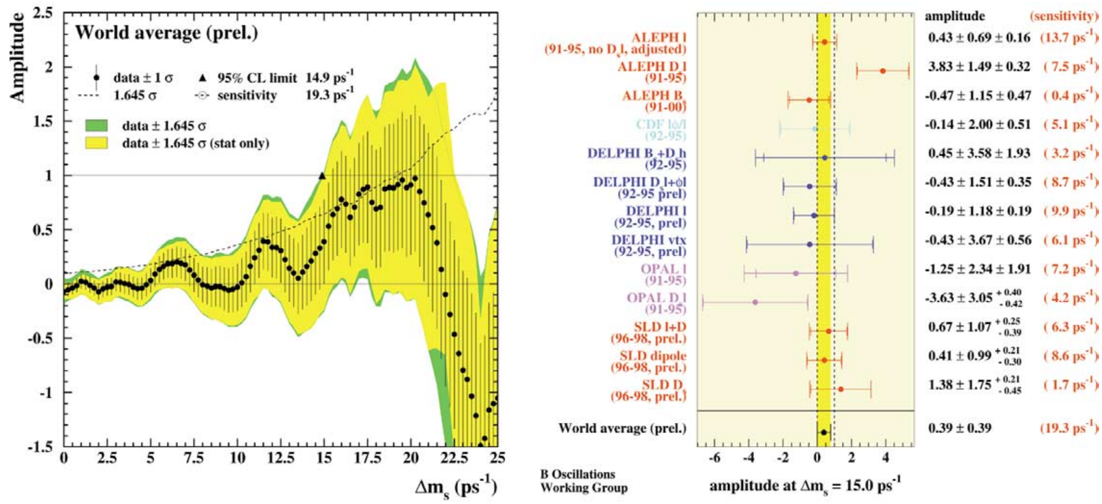

Figure 3. The plot on the left shows the combined Δm_s results from LEP/SLD/CDF analyses for the amplitude versus Δm_s. The points with error bars are the data; the lines show the 95% C.L. curves (the systematics have been included in black) [10]. The dotted curve shows the sensitivity. The plot on the right shows the summary of the Δm_s results per experiment. The errors are given at $\Delta m_s = 15\ \mathrm{ps}^{-1}$ (the sensitivity is also given).

The so-called amplitude method [11] has been developed to combine the data of different experiments and equation 7 is modified in the following way:

$$1 \pm \cos \Delta m_s t \to 1 \pm A \cos \Delta m_s t,$$

A and σ_A are measured at fixed values of Δm_s. In case of a clear oscillation signal, the measured amplitude is compatible with $A = 1$ at the corresponding value of Δm_s. With this method it is also easy to set an exclusion limit. The values of Δm_s excluded at 95% C.L. are those satisfying the condition $A(\Delta m_s) + 1.645\sigma_A(\Delta m_s) < 1$. Furthermore, the sensitivity of the experiment can be defined as the value of Δm_s corresponding to $1.645\sigma_A(\Delta m_s) = 1$ (for $A(\Delta m_s) = 0$, namely supposing that the 'true' value of Δm_s is well above the measurable value of Δm_s).

The combined result of the LEP/SLD/CDF [10] analyses is shown in Fig. 3 and is:

$$\Delta m_s > 14.9\ \mathrm{ps}^{-1} \quad \text{at 95\% C.L.}$$

for an expected sensitivity of $19.3\ \mathrm{ps}^{-1}$.

6. CKM fits

The different measurements allow for a determination of the parameters of the CKM matrix.

- from the measurements of $|V_{cb}|$, the A parameter is determined;
- the measurement of $|V_{ub}|$ gives a circular constraint of the form: $\overline{\rho}^2 + \overline{\eta}^2$;
- the measurement of Δm_d gives a constraint of the form: $(1 - \overline{\rho})^2 + \overline{\eta}^2$;
- an upper limit on the ratio $\Delta m_d/\Delta m_s$ gives the same type of constraint in the $\overline{\rho}$–$\overline{\eta}$ plane, as a measurement of Δm_d, but this ratio is expected to have a smaller theoretical uncertainty since the ratio $f^2_{B_d} B_{B_d} / f^2_{B_s} B_{B_s}$ is better known than the absolute value of $f^2_{B_d} B_{B_d}$;
- the measurement of ϵ_K defines an hyperbola in the $\overline{\rho}$–$\overline{\eta}$ plane.

These constraints are shown in Fig. 4.

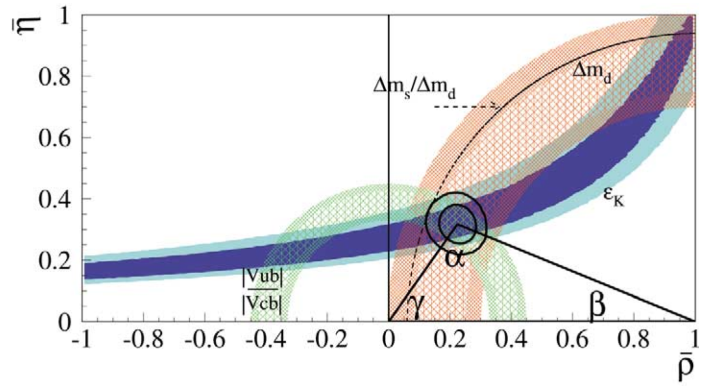

Figure 4. The allowed regions for $\overline{\rho}$ and $\overline{\eta}$ (contours at 68%, 95%) are compared with the uncertainty bands for $|V_{ub}|/|V_{cb}|$, ε_K, Δm_d and the limit on $\Delta m_s/\Delta m_d$ (dotted curve).

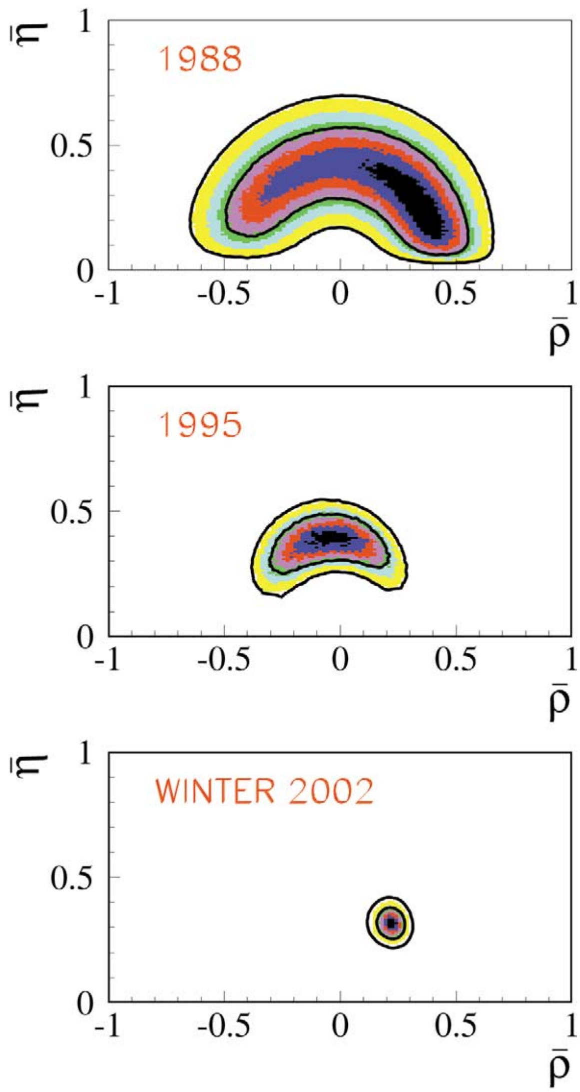

Figure 5. Evolution during the years of the allowed regions for $\overline{\rho}$ and $\overline{\eta}$ (contours at 68%, 95%).

All these quantities must – if the Standard Model is correct – be compatible, inside measurement errors and theoretical uncertainties, with one point in the $\overline{\rho}$–$\overline{\eta}$ plane. As shown in Fig. 4, this point defines a triangle where sides have been measured at LEP/CDF/SLD and CLEO and whose angles can be measured also at B-factories.

Using the available and most recent measurements and also up to date theoretical calculations [7,8,10] the allowed region in the $\overline{\rho}-\overline{\eta}$ is determined, as shown in Fig. 4. It corresponds to:

$$\overline{\rho} = 0.224 \pm 0.038; \qquad \overline{\eta} = 0.317 \pm 0.040.$$

The value of $\sin 2\beta$ can be also determined and compared with those measured recently at B-factories using $B^0 \to J/\psi K^0$ decays (which are based on CP-violating asymmetries)

$$\sin 2\beta = 0.698 \pm 0.066 \quad \text{from sides measurements and } \varepsilon_K \ (+\,\text{lattice QCD}), \tag{10}$$

$$\sin 2\beta = 0.78 \pm 0.08 \quad \text{from } B^0 \to J/\psi K^0 \ (\text{B} - \text{factories}). \tag{11}$$

The agreement of these values shows the consistency of the Standard Model in describing the CP violation phenomena in terms of one single parameter η. It is of interest to give predictions for the following quantities that will be measured in the future:

$$\gamma = (54.8 \pm 6.2)^0, \tag{12}$$

$$\Delta m_s = (16.3 \pm 3.4)\ \text{ps}^{-1}. \tag{13}$$

The evolution of our knowledge concerning the allowed region in the $\overline{\rho}$–$\overline{\eta}$ plane is shown in Fig. 5. The reduction of the error bands from the year 1995 to 2002 is essentially due to the analyses described in this paper which have been performed after the end of the data taking at the Z^0 pole and to the progress in lattice QCD determinations.

Thus, at the moment, the complex nature of the CKM matrix seems to be fully responsible for the observed CP violation in weak processes involving quarks.

---

[1] $\overline{\rho} = \rho(1 - \lambda^2/2)$; $\overline{\eta} = \eta(1 - \lambda^2/2)$.

[2] In this average the results from CLEO and Belle Collaborations are included. LEP still has a predominant weight.

[3] Δm_q is usually given in ps^{-1}: 1 ps^{-1} corresponds to 6.58×10^{-4} eV.

References

[1] ALEPH, CDF, DELPHI, L3, OPAL, SLD Collaborations, CERN-EP/2001-050.
[2] P. Coyle, O. Schneider, C. R. Physique 3 (2002) 1143.
[3] N. Cabibbo, Phys. Rev. Lett. 10 (1963) 531.
[4] M. Kobayashi, M. Maskawa, Prog. Theor. Phys. 49 (1973) 652.
[5] L. Wolfenstein, Phys. Rev. Lett. 51 (1983) 1945.
[6] A.J. Buras, M.E. Lautenbacher, G. Ostermaier, Phys. Rev. D 50 (1994) 3433.
[7] M. Ciuchini, et al., JHEP 0107:013, 2001 hep-ph/0012308.
[8] Results presented at the CKM Unitarity Triangle Workshop, CERN Feb 2002. http://ckm-workshop.web.cern.ch/ckm-workshop/.
LEP Working group on $|V_{cb}|$ http://lepvcb.web.cern.ch/LEPVCB/ Winter 2002 averages.
LEP Working group on $|V_{ub}|$ http://battagl.home.cern.ch/battagl/vub/vub.html.
[9] B.H. Behrens, et al., CLEO Collaboration, Phys. Rev. D 61 (2000) 052001.
[10] Results presented at the Workshop on the CKM Unitarity Triangle, CERN Feb 2002. http://ckm-workshop.web.cern.ch/ckm-workshop/,
http://lepbosc.web.cern.ch/LEPBOSC/combined_results/CKM_workshop_2002/.
[11] H.G. Moser, A. Roussarie, Nucl. Instr. and Meth. A 384 (1997) 491.