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DOSSIER

AVANCÉES EN PHYSIQUE DES PARTICULES : LA CONTRIBUTION DU LEP

ADVANCES IN PARTICLE PHYSICS: THE LEP CONTRIBUTION

High precision tests of the Standard Model and determination of the top quark and Higgs boson masses

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Abstract Global precision tests of the Standard Model are presented. They demonstrate its validity at the per mille level. This precision, combined with the level of agreement between measured and predicted values of the observables, allowed to determine the top quark mass with ±5% accuracy and to constrain the Higgs mass within a narrow kinematical domain. ***To cite this article: A. Olchevski, M. Winter, C. R. Physique 3 (2002) 1183–1191.***

precision tests / Standard Model tests / electro-weak corrections / bosonic corrections / top mass / Higgs mass

Tests de précision du Modèle Standard et détermination des masses du quark top et du boson de Higgs

Résumé Les tests de précision globaux du Modèle Standard sont exposés. Ils ont établi sa validité au pour-mille près. Grâce à cette précision, et sur la base de l'accord constaté entre mesures expérimentales et prédictions théoriques, la masse du quark top a pu être déterminée à ±5% près et celle du boson de Higgs a pu être confinée dans un domaine cinématique restreint. ***Pour citer cet article : A. Olchevski, M. Winter, C. R. Physique 3 (2002) 1183–1191.***

tests de précision / tests du Modèle Standard / corrections électro-faibles / corrections bosoniques / masse du top / masse du Higgs

1. Introduction

Precision tests of the validity of the Standard Model (SM) were a major issue of the LEP programme. They were, in particular, motivated by a general conviction that the model does not provide a fully

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satisfactory and complete description of nature, and that signs of physics not included in the SM would manifest themselves.

The tests consisted in comparing the measured value of various physics observables to their value predicted by the Standard Model. The latter rely, however, on the knowledge of more than 20 parameters which are not predicted by the SM, and need therefore to be measured experimentally. Typical examples of these *external* parameters are the masses of the 12 elementary fermions and the couplings characterising the intensity of each of the 3 subatomic forces.

All external parameters do not have the same influence on the SM predictions. Moreover, this influence depends on the energy range for which the predictions are calculated. At LEP energies, only a handful of parameters have a significant influence. These are:

- α, the fine structure constant, which expresses the intensity of the electromagnetic force. Its value is known with a precision of the order of 10^{-8};
- G_F, the Fermi constant, which was extracted from the muon life time with a precision of the order of 10^{-5};
- M_Z, the mass of the Z^0 boson, which LEP allowed us to determine with a precision close to 2×10^{-5} (see [1]).

These three fundamental parameters allow us to compute the value of a large variety of observables within the framework of the SM. Expressions (1) and (2) illustrate this feature. They show how the W boson mass (M_W) and the weak mixing angle (θ_W) are related to α, G_F and M_Z, and can thus be derived from the knowledge of these three external parameters:

$$M_W^2\left(1-\frac{M_W^2}{M_Z^2}\right)=\frac{\pi}{\sqrt{2}}\frac{\alpha}{G_F}\,\frac{1}{1-\Delta r}\longrightarrow M_W, \tag{1}$$

$$\cos^2\theta_W\sin^2\theta_W=\frac{\pi}{\sqrt{2}}\frac{\alpha}{G_F M_Z^2}\,\frac{1}{1-\Delta r}\longrightarrow \theta_W. \tag{2}$$

Besides α, G_F and M_Z, these expressions also involve a function Δr which accounts for quantum corrections. The latter are discussed in the next section.

2. Quantum corrections

The quantum corrections contained in Δr are due to fluctuations of the physical vacuum. The latter are expressed by the Heisenberg relations, which tell that the energy available in a physical system may fluctuate by amounts (ΔE) all the bigger the fluctuation (Δt) is shorter, i.e.:

$$\Delta E \cdot \Delta t \leqslant \frac{\hbar}{2}.$$

These fluctuations modify dominantly the propagation of the Z^0 boson and originate mainly from virtual pairs of top-antitop quarks and W^+W^- bosons as well as from virtual Higgs bosons, as shown in Fig. 1.

Each of these quantum fluctuations (more information on the Standard Model and vacuum quantum corrections may be found in [2–4]) influences the values of the observables computed in the SM framework in a specific way: the correction due to top-antitop pairs is a quadratic function of the top quark mass (m_t), whereas the correction associated to Higgs boson fluctuations is essentially a logarithmic function of its mass (M_H). As for the W^+W^- pairs, their coupling to the Z^0 boson is a triple gauge coupling (see the theoretical introduction by Boudjema and Zeppenfeld [5]), expressing the dynamical properties of the weak force. Thus the physical observables measured at LEP-1 are (though indirectly) sensitive to the top quark, to the Higgs boson and to the gauge couplings, well below the kinematical threshold of the production of final states made of top-antitop or W^+W^- pairs, or containing a Higgs boson.

The function Δr, which accounts for these quantum fluctuations, is made of two contributions, i.e., $\Delta r = \Delta r_W + \Delta\alpha$. $\Delta\alpha$ dominates the value of Δr but it accounts only for (purely electromagnetic) quantum

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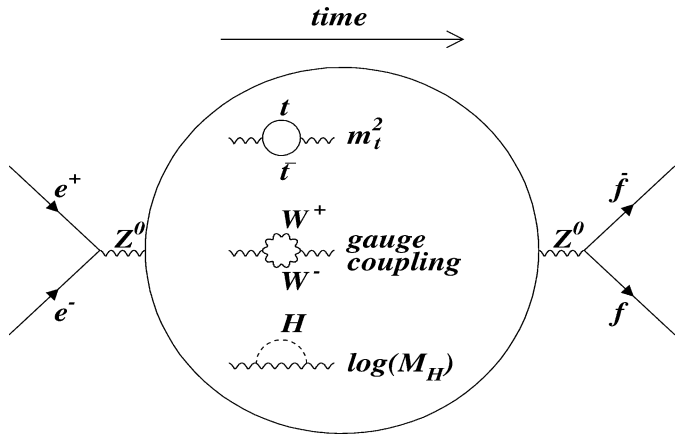

Figure 1. Diagram representing the most frequent process occuring in e^+e^- annihilations at LEP, corrected for the dominant quantum fluctuations affecting the propagation of the Z^0 boson: virtual pair of top-antitop ($t\bar{t}$) quarks or of W^+W^- bosons, and virtual Higgs boson.

corrections to α. All quantum fluctuations of interest are actually concentrated in Δr_W, which amounts to a few per cent. It originates mainly from the top quark, the contribution of the terms depending on M_H amounting to less than one per cent.

Since the magnitude of the vacuum quantum corrections depends on the values of m_t and M_H, the latter may be determined by comparing the SM prediction for a given observable to its experimental value, and chosing the values of m_t and M_H which bring the SM prediction closest to the experimental value. This approach requires an experimental accuracy on the relevant observables well below one per cent. Experiments at LEP (and at the Stanford Linear Collider) were first to provide such a precision. To make use of it, the accuracy of the theoretical calculations deserved substantial improvements. A constant effort was made in the last decade to bring this accuracy to a fraction of a per mille, thus matching the experimental precision well. Moreover, the extraction of Δr_W from the measured value of Δr required an accurate knowledge of $\Delta\alpha$, which motivated numerous extensive studies.

3. Validity tests of the Standard Model

About twenty observables were determined at LEP in order to test the validity of the SM. Their definitions and measurement techniques are exposed in [1,6]. Except for the Z^0 mass (M_Z), the value of each observable is computable within the SM framework. Fig. 2 shows how the theoretical predictions compare to the observed values. The difference between both sets of values is expressed in numbers of standard deviations.

Some of the observables considered on the figure were determined at other places than LEP: A_l(SLD) at the Stanford Linear Collider, $M_W^{(\mathrm{TEV})}$ and m_t at the Tevatron proton–antiproton collider of Fermilab (Chicago), $\sin^2\theta_W(\nu N)$ in interactions of neutrinos with nucleons and $Q_W(Cs)$ in experiments measuring parity violation in atomic transitions.

The probability describing the overall compatibility between the measurements and the predictions amounts to 8.5%, thus confirming the validity of the SM at the per mille level. Such a high level of agreement was far from obvious before LEP started, and it sets stringent limits on fundamental aspects of theoretical models imagined to overcome some of the intrinsic difficulties of the SM (see the contribution by Binètruy and Grivaz [7]).

Since the SM predictions are confirmed with such a high precision, the existence of the vacuum quantum corrections due to the top quark, to the Higgs boson and to the gauge couplings (shown in Fig. 1) was investigated. Moreover, an attempt was made to make evidence of the sole contribution from the Higgs boson and W^+W^- pairs (called *bosonic corrections*), i.e., to isolate the contribution to the function Δr_W in expression (1) and (2) which cannot be attributed to the top quark.

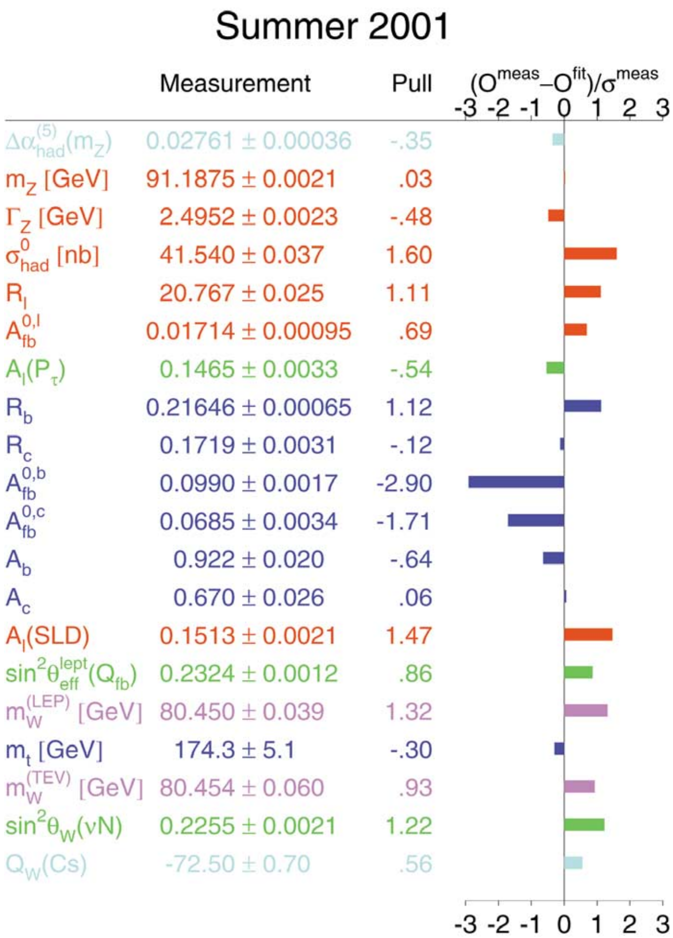

Figure 2. Difference, expressed in number of standard deviations (i.e., pulls), between the measured values and the SM predictions for electroweak observables measured at LEP and elsewhere [11].

This test is achieved by comparing the measured value of $\sin^2\theta_W$ to its value predicted within the SM, expressed as a function of the top quark mass. Fig. 3 shows how the measured value of $\sin^2\theta_W$ [1] compares to the SM prediction. The latter is shown before and after including the quantum corrections from the Higgs boson and W^+W^- pairs (i.e., without and with bosonic corrections). The experimental value of $\sin^2\theta_W$ is clearly incompatible with the SM prediction without bosonic corrections, while it is well reproduced by the complete prediction, computed with the value of m_t measured at the Tevatron proton-antiproton collider (represented by the vertical band in Fig. 3).

4. Determination of the top quark mass

Once the manifestation of quantum corrections due to vacuum fluctuations were established as predicted by the SM, it was possible to search for the value of the top quark mass which is prefered by the data in order to compare it with the value extracted from the direct measurement, and check whether both values agree or not.

The top quark was discovered in 1994, i.e., several years after the start of LEP and SLC (more information on precision measurements at LEP-1 and SLC may be found in [8]). The sensitivity of the electroweak observables to m_t could thus be used to predict its value before its discovery.

The derivation of m_t from the measured vacuum quantum corrections (i.e., from Δr_W) is however ambiguous since each electroweak observable provides a value of Δr_W which combines the corrections

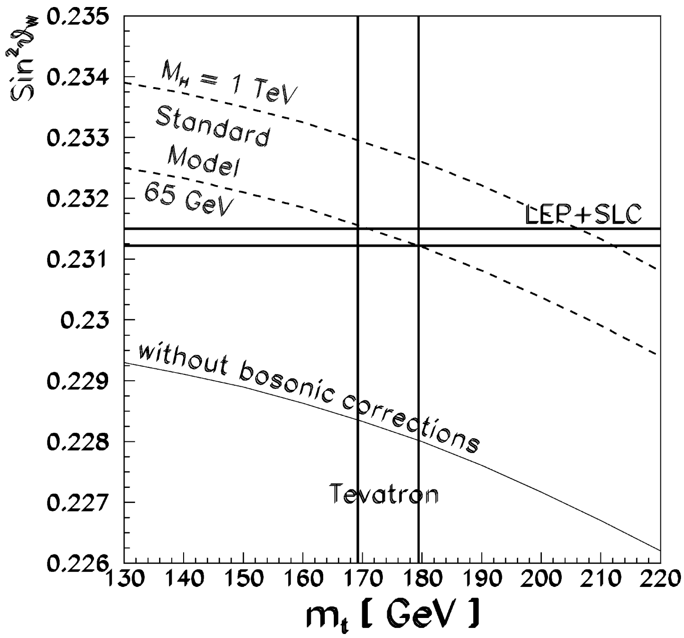

Figure 3. Comparison of the measured values of $\sin^2\theta_W$ and m_t to the SM predictions expressed as a function of m_t. The measurements performed at LEP-1 and SLC (resp. at the Tevatron) are represented with horizontal (resp. vertical) bands centered on the measurements and ±1 standard deviation wide. The SM predictions are shown in two cases: before (continuous line) and after (dashed lines) inclusion of bosonic quantum corrections reflecting physical vacuum fluctuations due to virtual pairs of W^+W^- bosons and to virtual Higgs bosons, as shown in Fig. 1. The complete SM prediction is displayed for two extreme values of the Higgs mass.

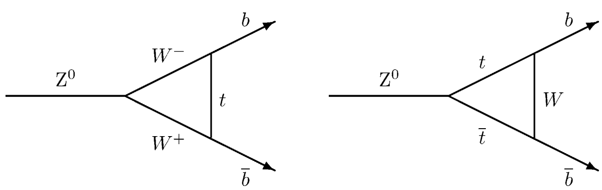

Figure 4. Quantum corrections due to physical vacuum fluctuations sensitive to the top quark mass, occuring only when the Z^0 decays into a $b\overline{b}$ pair.

due to m_t with those originating from the Higgs mass. The relative weight of each of both contributions varying from one observable to another, the combination of all observables is essential to have a chance of distinguishing both contributions. A particularly powerful observable is the branching fraction of the Z^0 into $b\overline{b}$ pairs (i.e., R_b), which is much more sensitive to m_t than to M_H since the coupling of the Z^0 to $b\overline{b}$ pairs is affected by specific quantum corrections involving a top–antitop pair, as shown in Fig. 4.

The sensitivity of R_b to m_t is shown on Fig. 5, together with that of the Z^0 total width (Γ_Z). The latter is among the most sensitive observables to m_t but – as shown by the figure – its SM prediction is significantly affected by the experimental uncertainties on the external parameters used in the computation.

The domains where the measured observables coincide with their SM predictions determine the value of m_t. The accuracy on m_t was for long mainly limited by statistics. During the whole running period of LEP-1, the extraction of m_t was regularly repeated as the collected data sample was growing. In spring 1994, shortly before the discovery of the top quark at the Tevatron, the electroweak observables measured

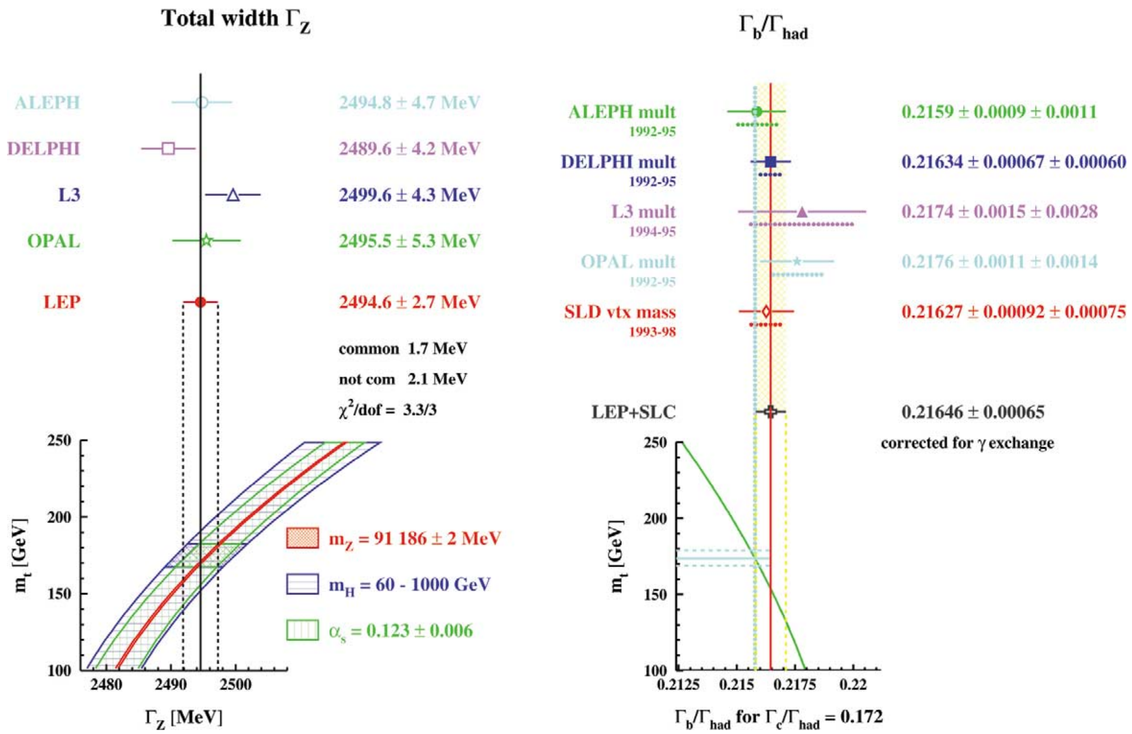

Figure 5. Experimental and theoretical values of the total width of the Z^0 (left) and of its branching fraction into $b\overline{b}$ pairs (right). The measurements are shown for each contributing experiment (including SLD), as well as for the combined results (vertical band, ± 1 standard deviation wide). The SM prediction is shown as a curved band function of m_t. The band width reflects the experimental uncertainty on the main external parameters on which the SM calculations are based: M_H, the intensity of the strong interaction (α_S) and the Z^0 mass (M_Z).

at LEP-1 and SLC translated into a value of m_t of $(177 \pm 10 \pm 20)$ GeV, where the second uncertainty reflected the interval in which the Higgs mass was allowed to be. The first measurement performed at the Tevatron in 1994 ($m_t = (174 \pm 16)$ GeV) confirmed the prediction remarkably.

Since then, the data collected at LEP-1 and SLC allowed to refine this prediction (i.e., $m_t = (181^{+11}_{-9})$ GeV). Its compatibility with the present result of the direct measurement (i.e., $m_t = (174.3 \pm 5.1)$ GeV) illustrates how predictive the method is, both to estimate parameters not directly calculable within the SM (like fermion masses) and to make evidence of phenomena not explained by the SM which would destroy this internal consistency.

5. Determination of the Higgs mass

The weak mixing angle is by far the parameter most sensitive to M_H since it concentrates the sensitivities of all electroweak observables expressing the vectorial couplings of the Z^0 to fermions. As for the top quark corrections, these sensitivities vary from one observable to another.

Fig. 6 illustrates the compatibility of the different measurements of $\sin^2\theta_W$ extracted from various asymmetries, and shows how their combined value compares to the SM prediction. The latter is displayed as a function of M_H, showing that relatively low masses are favoured (a χ^2 fit leads to $M_H = (81^{+109}_{-40})$ GeV). The accuracy on M_H improves substantially when the values of m_t and M_W measured directly are taken into account. These direct measurements help disentangling the vacuum corrections embedded in $\sin^2\theta_W$ due to the Higgs boson from those originating from the top quark (as suggested by relation (1) and (2)).

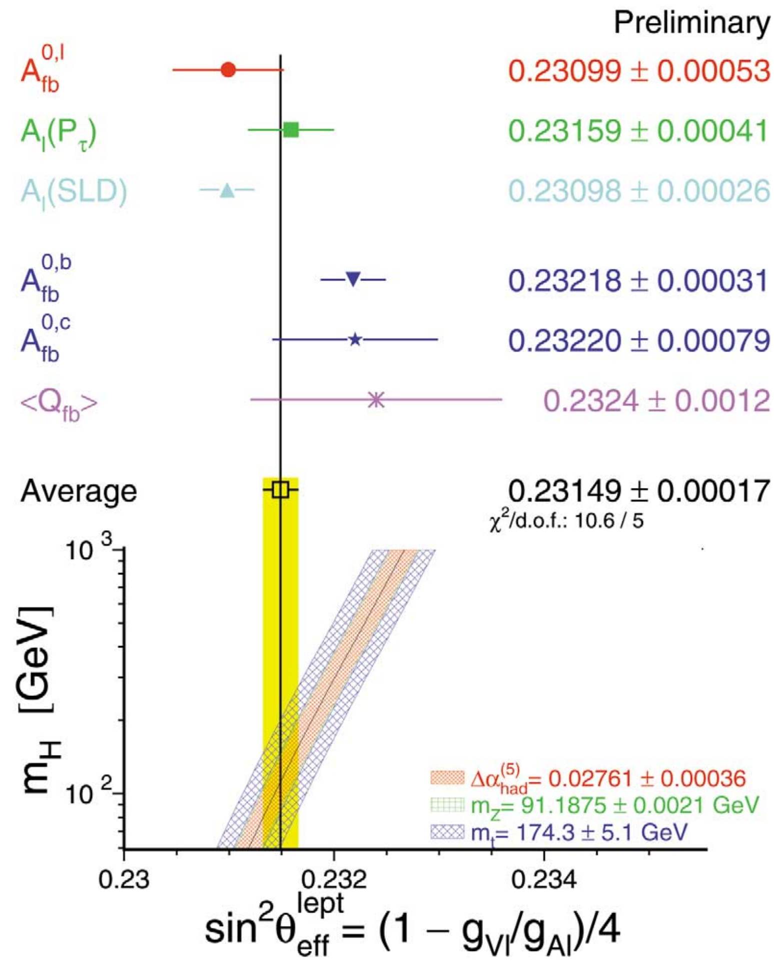

Figure 6. Values of $\sin^2\theta_W$ (called $\sin^2\theta_{eff}^{lept}$ here) extracted from various asymmetries (i.e., observables expressing the non-conservation of parity in weak interactions). The average of all measurements is compared to the SM prediction, represented by the oblique band varying with M_H. The band width reflects the main uncertainties on the prediction: the knowledge of m_t, of $\Delta\alpha$ and of M_Z.

Fig. 7 (left) illustrates how the direct measurements of M_W and m_t (dashed contour) allow to derive a range of M_H values from their comparison to the SM prediction expressed as a function of M_H. The figure shows also the domain (continuous contour) derived from the vacuum fluctuations evaluated with the Z^0 observables (e.g., with $\sin^2\theta_W$ measured at LEP and SLC).

The two contours are well compatible, thus corroborating the validity of the SM (a discrepancy between the contours would have hinted to the breakdown of the SM). Combining both contours allows us to predict M_H with the highest accuracy presently available. In order to find the value of M_H which makes the SM prediction match the combined contour, a fit was performed. Its χ^2 distribution is displayed in Fig. 7 (right), its minimum leading to:

$$M_H = \left(88^{+53}_{-35}\right)\ \text{GeV}.$$

This value of M_H will be confronted to the result of direct searches of the Higgs boson performed at LEP in the contribution by Janot and Kado [9].

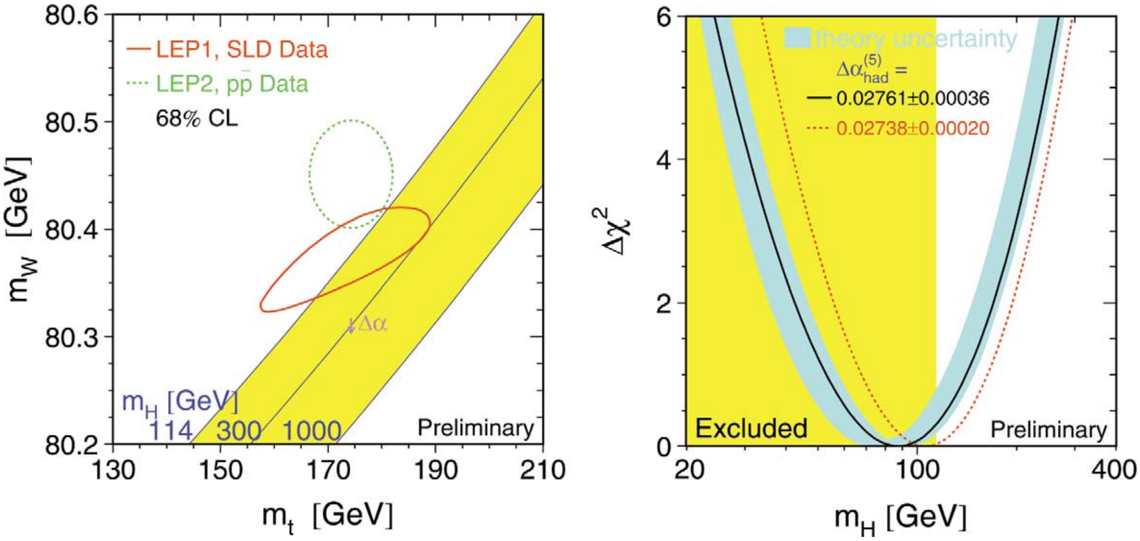

Figure 7. Left: Experimental correlations of the direct measurements of M_W and m_t and of their indirect determination (based on the Z^0 observables), compared to the SM predictions. The experimental measurements are represented by their 68% confidence level contours (dashed line for the direct measurements and continuous line for the indirect determination). The SM predictions are shown as a band reflecting values of M_H ranging from 114 to 1000 GeV.
Right: χ^2 distribution of the fit of M_H to all experimental data. The domain in grey stopping at $M_H = 114$ GeV stands for the kinematical domain excluded by the direct searches of the Higgs boson (see the contribution by Janot and Kado [9]). Two fits are represented, which differ by the value of the quantum correction $\Delta\alpha$ used in the theoretical calculation: the full line corresponds to a value based on low energy data alone, while the dotted line was obtained with a value including additional theoretical assumptions. The band around the left parabola reflects the uncertainty on the SM predictions of pure theoretical origin.

Though indirect, this prediction has important consequences, vacuum quantum corrections having already shown their prediction power for the top quark mass. The fit result sets in particular upper bounds on M_H: the SM Higgs boson is lighter than 300 GeV with a confidence level of about 99% and has 95% probability to be lighter than about 200 GeV.

Fig. 7 shows that the uncertainty on $\Delta\alpha$, which reflects our poor knowledge of light quark masses, has a significant influence on the value of M_H. The fit result is displayed for two alternative values of $\Delta\alpha$: one which is extracted from data collected at low energies only and one which relies on additionnal theoretical assumptions. The latter, which leads to the most precise value of $\Delta\alpha$, was pioneered several years ago (see, for instance, [10]); the value used in Fig. 7 comes from one the most recent determinations of this kind.

6. Summary

The SM predictions were tested up to the per mille level with about twenty different observables. Overall, the predictions match the observations, and show that purely electroweak quantum corrections manifest themselves at LEP energies as predicted by the SM. This observation sets stringent limits on theories more general than the SM which have been imagined to cure (at least part of) its weaknesses.

Based on virtual effects caused by quantum fluctuations of the physical vacuum, a value of the top quark mass was extracted from various electroweak observables. It allowed us to predict m_t before the discovery of the top quark, and was well confirmed by measurements performed in proton-antiproton collisions.

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The manifestations of vacuum fluctuations were also converted in a value of the Higgs mass, which came out to be (88^{+53}_{-35}) GeV. This result is of prominent importance for future particle physics experiments since it settles an upper limit on M_H below 300 GeV and indicates that M_H is likely to be less than 200 GeV.

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1 Derived from the final states made of charged lepton-pairs.

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