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The Higgs in large extra dimensions

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Abstract

Transverse (submillimeter) and longitudinal (TeV) extra dimensions can help in dealing with the Higgs hierarchy problem. On the one hand large transverse dimensions can lower the fundamental scale of quantum gravity from the Planck scale to the TeV range. On the other hand longitudinal dimensions can provide genuine extra-dimensional symmetries (higher dimensional gauge symmetry and/or supersymmetry) to protect the Higgs mass against ultraviolet sensitivity. In this article we review recent developments along these directions. ***To cite this article: K. Benakli, M. Quirós, C. R. Physique 4 (2003).***

Résumé

Le Higgs avec de grandes dimensions supplementaires. Des dimensions supplémentaires transverses (submillimetriques) et longitudinales (TeV) peuvent aider à adresser « le problème de hiérarchie » pour la masse du Higgs. D'une part, de grandes dimensions transverses permettent d'avoir une échelle fondamentale de la gravité quantique aux énergies du TeV. D'autre part, les dimensions longitudinales fournissent de nouvelles symétries (jauge/supersymétrie) qui protègent le potentiel du Higgs d'une sensibilité aux détails de la théorie ultraviolette. Dans cet article nous passons en revue quelques développements récents de ces idées. ***Pour citer cet article : K. Benakli, M. Quirós, C. R. Physique 4 (2003).***

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1. The problem

Despite extraordinary achievements made in understanding the physics of elementary particles, many fundamental questions still remain unanswered. For instance, we do not have a fundamental understanding of the origin of the known particles mass spectrum. In particular, no fundamental reasons are known for the observed hierarchy and values of masses although these can be reproduced (fitted) in specific models. The renormalizability of the interactions of the Standard Model (SM) of fundamental particles requires these masses to originate from the vacuum expectation value (VEV) of one or many scalar fields, the Higgs

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fields. The fermion mass problem is then formulated as the problem of understanding the structure (texture) of Yukawa couplings between the Higgs field and chiral fermions. On the other hand, the W and Z gauge boson masses originate as a result of the spontaneous breaking of the electroweak symmetry by the Higgs VEV. The latter is then required to be $v \sim 246$ GeV in order to reproduce the experimental values of the W and Z boson masses. In trying to reproduce this VEV one faces the problem that the scalar field potential is generically sensitive to the ultraviolet (UV) structure of the theory, i.e., to the physics beyond the Standard Model that governs phenomena at energies higher than the UV cutoff Λ. In the minimal scenario which consists of the SM degrees of freedom the Higgs mass is quadratically divergent and is then predicted to be of order Λ while on the other hand it was expected to be of the order of its VEV v. Three different cases can then be given:

(i) The case where there is no new physics below Λ, whose expected value is then of the order of the four dimensional Planck scale (where gravitational interactions become important). A fine-tuning is needed in order to keep the Higgs mass of order M_W. That is the situation in the pure Standard Model.
(ii) The case where new physics below Λ leads to cancellation (or absence) of the quadratic divergences. This is for instance what happens in the minimal supersymmetric extension of the standard model (MSSM), or in technicolor theories.
(iii) The case of theories with a low UV cutoff in the TeV region. This is so in the presence of large extra dimensions [1–8] as we will discuss in this article.

It is important to stress that the theoretical computation of the Higgs mass, as we discuss it here, faces two separate problems. The first problem concerns the hierarchy existing between the electroweak symmetry breaking scale and the UV cutoff scale. For example in case (ii) the two scales can be decoupled by supersymmetry. In this case this problem can be reformulated as: why is the scale of supersymmetry breaking so low as compared to the Planck scale? In case (iii) the previous hierarchy is replaced by one between the UV cutoff and the Planck scale. The second problem is that quadratic divergences introduce a sensitivity to the unknown UV physics at the scale Λ and then destroy the predictivity power of the theory for the Higgs sector. In the absence of new symmetries, lowering the UV cutoff as in case (iii) does not solve the problem. This is in contrast with the case of supersymmetric extensions, for example, where at most a small logarithmic sensitivity remains at one-loop. We will discuss these issues in some detail below.

2. New physics at the TeV scale

There have been many proposals for new physics at the TeV scale, mainly motivated by the gauge hierarchy problem. The Large Hadron Collider (LHC) experiments will be able to confirm or dismiss any of these extensions. In this section we will review how a particular kind of new physics, super-large transverse extra dimensions where non-SM fields propagate, can help in solving the hierarchy problem.

2.1. Extra dimensions

A possible way to achieve small radiative corrections is to lower the fundamental scale [7,8], for instance the string scale M_s down to energies of order TeV [6]. This can be achieved in the presence of very large extra dimensions. In fact, the four dimensional Planck mass M_P is related to the string scale M_s through the relation:

$$M_P^2 = \frac{M_s^{2+D} V_D}{g_s^2}, \tag{1}$$

where g_s is the string coupling constant and V_D the volume of an internal D-dimensional space where the known (Standard Model) light states do not propagate. It is then possible to reconcile the experimentally measured value of M_P with a low value for M_s by either making the internal volume V_D big, or making g_s small [9–11] (or combining both). Here we will focus on the case where V_D is responsible for the hierarchy between M_s and M_P.

2.2. Submillimeter size extra dimensions

Among the $4 + D$ internal dimensions, $4 + d_\parallel$ are felt by Standard Model interactions, i.e., they have excitations that propagate in these dimensions. The remaining $d_\perp$ are only felt by gravitons and other experimentaly yet unknown interactions, if any. Experimental bounds allow the $d_\perp$ dimensions to be as large as the submillimeter, the only theoretical constraint being that $V_\parallel V_\perp M_s^D \sim M_P^2/M_s^2$, where $V_\perp$ and $V_\parallel$ denote the volumes spanned by the $d_\perp$ and $d_\parallel$ dimensions respectively.

2.3. TeV^{-1} size extra dimensions

In contrast to the $d_\perp$ directions, the $d_\parallel$ are constrained by experimental data to be smaller than the electroweak length. Computing these bounds can be done in a precise scenario which involve a choice of the precise embedding of the Standard Model states, as:

(1) All the Standard Model particles feel all the $d_\parallel$ dimensions of the 'bulk'. In that case the KK modes of gauge bosons can only be pair produced and present experimental bounds are rather loose, typically $1/R \geqslant 500$ GeV.
(2) Only the gauge bosons feel the extra dimensions, while all the fermions of the Standard Model are localized on a four dimensional subspace, a 'boundary'. In that case the KK modes can be single produced and present bounds from direct production and indirect precision measurements are stronger than in the previous case, $1/R \geqslant 4$ TeV [12,13].
(3) Intermediate situations where part of the Standard Model states are localized on subspaces with one or more extra dimensions less than the other states. In this case present bounds would be model dependent and align along one of the previous situations.

Moreover, for either of these cases, one needs also to specify the precise geometry of the extra dimensions.

3. Dealing with UV sensitivity

In the previous section we have seen how one could alleviate the hierarchy problem by lowering the SM cutoff in the presence of large transverse extra dimensions. In this section we will summarize possible solutions to the problem of UV sensitivity with respect to the cutoff Λ. Since the core of the hierarchy problem resides in the Higgs mass we will start this section with some general considerations concerning the Higgs field scalar potential.

3.1. The Higgs scalar potential parameters

Let us consider the case of a single scalar field ϕ with a potential of the form:

$$V(\phi) = -\frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda\phi^4 + \cdots. \tag{2}$$

For $\mu^2 > 0$, this potential has a minimum at $\langle\phi\rangle \equiv v = \mu/\sqrt{\lambda}$ and the Higgs mass at this point is $m_\phi^2 = 2\mu^2$. The field theory computation leads to quadratically divergent contributions to the scalar mass at one-loop

$$\mu^2|_{FT} = \mu_0^2 + c_{FT}^{(1)} g^2\Lambda^2 + c_{FT}^{(2)} g^4\Lambda^2 + \cdots \tag{3}$$

and unless the details of the UV theory above Λ are known, it is not possible to make any quantitative prediction. In particular we would like to stress that a string computation does not lead to a numerical identification of M_s with Λ but instead

$$\mu^2|_{ST} = \mu_0^2 + c_{ST}^{(1)} g^2 M_s^2 + c_{ST}^{(2)} g^4 M_s^2 + \cdots, \tag{4}$$

and matching (3) and (4) to $\mathcal{O}(g^2)$ leads to $c_{FT}^{(1)} g^2\Lambda^2 = c_{ST}^{(1)} g^2 M_s^2$ with the coefficient $c_{ST}^{(1)} \neq c_{FT}^{(1)}$ containing a priori a contribution from the model dependent spectrum of massive string modes [14]. It is then interesting to look for theories where the leading scalar mass parameter is computable in the effective field theory.

Before addressing some specific examples of such theories, we would like to point out the necessity of a further suppression of the Higgs parameters. From the relation $v = \mu/\sqrt{\lambda}$ one expects

$$\frac{\mu}{M_s} = \frac{\sqrt{\lambda}v}{M_s} \sim \frac{\sqrt{\lambda}}{4}\frac{TeV}{M_s} \tag{5}$$

which for a small quartic Higgs coupling implies a hierarchy between the Higgs potential mass parameter and the string scale. For instance in many (string) models, the Higgs fields originate as spin zero components of a would-be $N = 4$ supersymmetry multiplet, some of the other components being projected away by the compactification. In such cases, the tree level quartic coupling is fixed to be the same as in the MSSM, i.e., $\lambda \sim (g^2 + g'^2)/8$ which leads to: $\mu \sim 120$ GeV $\ll M_s$. A possible solution is to consider models with a low string scale where the tree-level term is absent and the Higgs mass parameter arises at one-loop with a small coefficient [14] or at higher loops [15].

3.2. Higher dimensional gauge symmetry

In order to avoid the scalar mass parameters to be dependent on the details of the fundamental theory, they must vanish in the ultraviolet limit. In theories with extra dimensions, this is ensured by implementing the higher dimensional theory with a symmetry that forbids the Higgs mass [17–27]. Upon compactification, the symmetry is spontaneously broken, so that a Higgs mass term is allowed in the IR theory. Then the knowledge of the IR degrees of freedom (including the KK modes) is sufficient in order to evaluate the resulting Higgs mass.

The simplest example [16] is provided by a class of models where a non-vanishing VEV for a scalar (Higgs) field ϕ results in shifting the mass of each KK excitation by a constant $a(\phi)$:

$$M_{\vec{m}}^2 = \sum_{i=1}^{d} \left[\frac{m_i + a_i(\phi)}{R_i} \right]^2, \tag{6}$$

where $\vec{m} = \{m_1, \ldots, m_d\}$ with m_i integers. Such mass shifts arise for instance in the presence of Wilson lines, $a_i = q \oint \frac{dy^i}{2\pi} g A_i$, where A_i is the internal component of a gauge field with gauge coupling g and q is the charge of the given state under the corresponding generator. The effective one-loop potential is given by:

$$V_{\text{eff}}(\phi) = \frac{1}{2} \sum_I (-)^{F_I} \int \frac{d^4 p}{(2\pi)^4} \log\left[p^2 + M_I^2(\phi)\right] = -\frac{1}{32\pi^2} \sum_I (-)^{F_I} \int_0^\infty dl\, l\, e^{-M_I^2(\phi)/l}, \tag{7}$$

where the sum is over all states I with masses $M_I(\phi)$. A straightforward computation shows that the ϕ-dependent part of the one-loop effective potential is given by [16]:

$$V_{\text{eff}} = -\operatorname{Tr}(-)^F \frac{\prod_{i=1}^d R_i}{32\,\pi^{(4-d)/2}} \sum_{\vec{n}} e^{2\pi i \sum_i n_i a_i} \int_0^\infty dl\, l^{(2+d)/2} f_s(l)\, e^{-\pi^2 l \sum_i n_i^2 R_i^2}, \tag{8}$$

where $F = 0, 1$ for bosons and fermions respectively and $f_s(l)$ contains the effects of string massive modes. For instance, in the case of type I models considered in [14]:

$$f_s(l) = \left[\frac{1}{4l} \frac{\theta_2}{\eta^3} \left(il + \frac{1}{2} \right) \right]^4 \to 1 \quad \text{for } l \to 0. \tag{9}$$

In the case of $R_i > 1$ (in string units), only the $l \to 0$ region contributes to the integral, we can approximate $f_s(l) \sim 1$, dropping the sensitivity to details of the UV theory, and remain with a finite field theory result given by:

$$V_{\text{eff}}(\phi) = -\operatorname{Tr}(-)^F \frac{\Gamma((4+d)/2)}{32\pi^{(12+d)/2}} \prod_{i=1}^d R_i \sum_{\vec{n} \neq \vec{0}} \frac{e^{2\pi i \sum_i n_i a_i(\phi)}}{[\sum_i n_i^2 R_i^2]^{(4+d)/2}}. \tag{10}$$

We can easily extract the one-loop mass term for a through a Taylor expansion, it is finite and given by a loop factor times the compactification energy scale. This result was expected because at scales above $1/R$ the gauge symmetry is restaured and protects A_5 from getting a mass.

Note that if we consider instead $R_i \to 0$, which by T-duality corresponds to taking the extra dimensions as transverse and very large, the one-loop effective potential receives contributions from the whole tower of string oscillators as appearing in $f_s(l)$ leading to squared UV-sensitive masses given by a loop factor times M_s^2.

Wilson lines are associated with flat directions and do not have tree-level quartic terms. They lead to unrealistic small Higgs masses $m_H \lesssim 50$ GeV. In order to construct a realistic model based on this scenario we consider an orbifold theory. We list below the most relevant points in model building:

(1) The Higgs field should be identified with the internal component of a gauge field extending the Standard Model in higher dimensions. The minimal extension is $U(3) \times U(3)$ [16]. An orbifold projection can be used in order to break down this group to the Standard Model one.
(2) The presence of a tree-level quartic interaction term requires that $d_\| > 1$ which implies the presence of extra Higgs fields.
(3) The spectrum of the Standard Model fields splits in two sets: those living in the $4 + d_\|$ bulk and those localized on the boundaries. The latter, if any, have to come in supersymmetric multiplets.
(4) Reproducing the realistic Yukawa texture remains the biggest challenge for this construction. A possibility is to add Yukawa couplings by means of Wilson lines localized at the boundaries of the orbifold and giving mass to either the bulk or the localized fermions.

The last condition, to reproduce the Yukawa texture without spoiling the UV insensitivity properties of the Higgs mass, is the most challenging one. A model based on $T^2/\mathbb{Z}_4$ and unifying the electroweak gauge group in G_2 was presented in [27].

3.3. Higher dimensional supersymmetry

The simplest solution to deal with sensitivity with respect to the cutoff Λ is by using supersymmetry in the $4 + d_\parallel$ higher dimensional space. Since most of supersymmetric models have been constructed in five dimensions, where large extra dimensions are located, we will describe here supersymmetry in this case. In five dimensions the matter supersymmetric multiplets are vector and hypermultiplets. A vector multiplet is $\mathbb{V} = (V_M, \lambda^i, \Sigma)$ where V_M, $M = \mu, 5$ is a five-dimensional vector boson, λ^i are two Majorana spinors transforming as a doublet of $\mathrm{SU}(2)_R$ (the symmetry of the two supersymmetric charges) and Σ is a real scalar. A gauge multiplet $\mathbb{V}$ should be in the adjoint representation of the gauge group. The other kind of multiplets are hypermultiplets $\mathbb{H} = (\Psi, \phi^i)$ where Ψ is a Dirac fermion and ϕ^i complex scalars transforming as doublets under $\mathrm{SU}(2)_R$.

Five-dimensional $N = 1$ supersymmetry has, as we already indicated $N = 2$ supersymmetric charges corresponding to the degrees of freedom of a Dirac spinor. We have to break from $N = 2$ to $N = 0$ supersymmetry to get the Standard Model degrees of freedom. The normal way of breaking $N = 2$ to $N = 1$ is by compactification on an orbifold. The simplest way of doing it is by defining a $\mathbb{Z}_2$ parity and compactifying the extra dimension on the orbifold $S^1/\mathbb{Z}_2$. All fields are then assigned definite transformation rules under the $\mathbb{Z}_2$-parity. If the bulk gauge group is to remain unbroken by the orbifold action there is a unique way to do this process consistent with the symmetries of the five-dimensional Lagrangian, in particular with the residual $N = 1$ supersymmetry. For the $N = 2$ gauge multiplet, the $N = 1$ gauge multiplet (A_μ, λ^1) is even and the $N = 1$ chiral multiplet $(\Sigma + \mathrm{i}A_5, \lambda^2)$ is odd. For the hypermultiplets, one of the $N = 1$ chiral multiplets (Φ_L, ϕ^1) is even and the other one (Φ_R, ϕ^2) is odd.

The residual $N = 1$ supersymmetry can be broken by compactification, the so-called Scherk–Schwarz mechanism [28–31]. It consists in imposing to the five-dimensional fields a non-trivial periodic condition under a $2\pi R$ translation (R is the radius of the circle S^1) that is represented by an operator of a global (or local) symmetry of the five-dimensional theory with generator T as

$$\Phi(x^\mu, y + 2\pi R) = \mathrm{e}^{2\pi \mathrm{i}\omega T}\Phi\big(x^\mu, y\big). \tag{11}$$

Eq. (11) can be easily solved by means of

$$\Phi(x^\mu, y) = \mathrm{e}^{\mathrm{i}\omega T y/R}\tilde{\Phi}\big(x^\mu, y\big), \tag{12}$$

where $\tilde{\Phi}(x^\mu)$ are periodic functions, Fourier expandable, and Eq. (12) implies a non-trivial y-dependence for the $n = 0$ mode that leads to a mass term in the four-dimensional theory.

The Scherk–Schwarz mechanism is a very natural way of breaking supersymmetry by compactification. This breaking has very interesting properties: it is flavor blind and it thus solves in a natural way the flavour problem, it is a global breaking of supersymmetry and thus it provides a finite Higgs mass and finally it provides a genuine extra-dimensional solution to the μ-problem of the MSSM [32–39].

3.3.1. The MSSM like models

These models are based on compactification of the extra dimension on $S^1/\mathbb{Z}_2$ as we have seen above [32–38]. The gauge sector is propagating in the bulk of the extra dimension and the Scherk–Schwarz breaking of supersymmetry is based on the global $\mathrm{SU}(2)_R$ symmetry. In particular Eq. (12) for gauginos is,

$$\lambda = \exp\left\{\frac{\mathrm{i}\omega\sigma_2 y}{R}\right\}\tilde{\lambda}. \tag{13}$$

The n-KK mass eigenstate modes are given by two Majorana fermions $\lambda_L^{1(n)} \pm \lambda_L^{2(n)}$ with masses $|n \pm \omega|/R$. For the zero mode the supersymmetry breaking Majorana mass is given by

$$M_{1/2} = \frac{\omega}{R}. \tag{14}$$

The Higgs sector can be, either localized on the boundary or in the bulk. In the former case, MSSM-like, the μ-problem has to be solved using one of the usual solutions (extra singlet, Higgs term in the Kahler potential, ...) in the literature. However if the Higgs sector propagates in the bulk the μ-problem can be solved by compactification. Let us consider two hypermultiplets corresponding to the two Higgses of the MSSM, $\mathbb{H}_a$ where $a = 1, 2$ transforms as a doublet under a global symmetry of the

Lagrangian $\mathrm{SU}(2)_H$. We can use as global symmetry for the Scherk–Schwarz mechanism in the Higgs sector the diagonal subgroup of $\mathrm{SU}(2)_R \times \mathrm{SU}(2)_H$. The compactification for the Higgs sector can now be written as

$$\mathcal{H} = \exp\left\{\frac{\mathrm{i}\omega\sigma_2 y}{R}\right\}\widetilde{\mathcal{H}}\exp\left\{\frac{-\mathrm{i}\omega\sigma_2 y}{R}\right\}, \qquad \Psi_H = \exp\left\{\frac{\mathrm{i}\omega\sigma_2 y}{R}\right\}\widetilde{\Psi}_H, \tag{15}$$

where $\mathcal{H} = H_a^i$ and $\Psi_H = \Psi_H^a$. The KK mass eigenstates are two Dirac fermions $\Psi_H^{1(n)} \pm \Psi_H^{2(n)}$ with masses $|n \pm \omega|/R$ and four scalars $h^{(\pm n)}$ and $H^{(\pm n)}$ with masses $|n|/R$ and $|n \pm 2\omega|/R$, respectively. We can see that the Higgsinos get a mass providing an extra-dimensional solution to the μ-problem while there remains a massless SM-like Higgs doublet. The fact that Higgsinos are not degenerated with the Higgs scalars is due to the interplay between the Scherk–Schwarz along $\mathrm{SU}(2)_H$ (that provides the supersymmetric mass) and the Scherk–Schwarz with respect to $\mathrm{SU}(2)_R$ (that breaks supersymmetry) as realized by the diagonal subgroup of $\mathrm{SU}(2)_R \times \mathrm{SU}(2)_H$.

As for the matter fields they can either be localized on the boundary or propagating in the bulk of the extra dimensions. Matter multiplets localized on the boundary do not feel Scherk–Schwarz sumersymmetry breaking and the corresponding fermions and sfermions are degenerate and massless at the tree level. However they interact at one-loop with the gauge sector and they then feel supersymmetry breaking. In particular we find that boundary scalars (e.g., squarks) get a mass from the gauge sector given by [33,34]

$$\Delta m_i^2 = \frac{g^2 C(R_i)}{8\pi^4 R^2}\left[2\zeta(3) - Li_3\left(\mathrm{e}^{2\pi\mathrm{i}\omega}\right) - Li_3\left(\mathrm{e}^{-2\pi\mathrm{i}\omega}\right)\right]. \tag{16}$$

However it has been recognized that in models with extra dimensions electroweak breaking is triggered at one-loop if the top quark is propagating in the bulk. It that case one gets the additional bonus that it yields a finite mass term. This can be understood since, even if the five-dimensional theory is non-renormalizable the quadratic divergence is cancelled by supersymmetry. After subtracting the five-dimensional part it remains a finite piece that is cutoff by $1/R$, as it happens in field theory at finite temperature for thermal masses. This contribution is similar to the one a localized scalar receives in (16). Including the top Yukawa h_t and gauge weak g coupling it is given by [33,34]

$$\Delta m_h^2 = \frac{6h_t^2 - 3g^2}{32\pi^4 R^2}\left[2\zeta(3) - Li_3\left(\mathrm{e}^{2\pi\mathrm{i}\omega}\right) - Li_3\left(\mathrm{e}^{-2\pi\mathrm{i}\omega}\right)\right]. \tag{17}$$

Scalar fields in hypermultiplets are $\mathrm{SU}(2)_R$ doublets and they are given boundary conditions as for the gauginos in (13). Consequently the mass spectrum is n-KK mass eigenstates with masses $|n \pm \omega|/R$ and supersymmetry breaking mass for the zero mode given by

$$M_0 = \frac{\omega}{R}. \tag{18}$$

The typical spectrum for this kind of models consists in heavy gauginos and scalars belonging to hypermultiplets propagating in the bulk, Eqs. (14) and (18) and not so heavy scalars belonging to chiral multiplets localized on the boundary, Eq. (16). The detailed predictions for the Higgs mass spectrum are of course model dependent but can always be described from the point of view of the MSSM parameter space. In particular for the models described in [40] with the Higgs doublets localized on the boundary and the top-quark propagating in the bulk the typical predictions are large $\tan\beta$, more precisely $\tan\beta \sim m_t/m_b$, and light pseudoscalar m_A. The rest of the Higgs mass spectrum is deduced from these values. The compactification radius deduced by imposing correct electroweak breaking is somewhat model dependent but typically $1/R \sim$ few TeV.

3.3.2. The single Higgs models

There is also the possibility of constructing models where the Higgs sector is SM-like, i.e., only containing a single hypermultiplet propagating in the bulk [39]. In this case, using $\mathrm{SU}(2)_R$ as the global symmetry for the Scherk–Schwarz breaking it is not possible to end up with a massless state as can be deduced from (15). In the previous section the way out was to introduce a second Higgs hypermultiplet and a global symmetry $\mathrm{SU}(2)_H$ rotating both hypermultiplets. The identification of the Scherk–Schwarz parameters for both $\mathrm{SU}(2)_R$ and $\mathrm{SU}(2)_H$ left a massless SM-like Higgs doublet. A different possibility was depicted in [39] where the Scherk–Schwarz breaking of supersymmetry was not performed using the global $\mathrm{SU}(2)_R$ but the discrete R_p (R-parity). In particular starting from the orbifold $S^1/\mathbb{Z}_2$ with radius R' one imposes the boundary conditions

$$\phi(y + 2\pi R') = R_p\phi(y). \tag{19}$$

Since $R_p = +1$ for all SM-particles and $R_p = -1$ for all supersymmetric partners, Eq. (19) translates into n-KK modes for superpartners mass eigenstates given by $|n \pm \frac{1}{2}|/R'$ while the SM-states have masses $|n|/R'$. In this way the Higgs scalar remains massless at the tree level. In this model the supersymmetry breaking masses are those in (14) and (18) with $\omega = 1/2$.

This model is dubbed $S^1/\mathbb{Z}_2 \times \mathbb{Z}_2'$ orbifold because it can be formulated as an $S^1/\mathbb{Z}_2$ orbifold with radius $R = 2R'$ and imposing the further parity $\mathbb{Z}_2'$ which corresponds to inversion with respect to $y = \pi R/2$ and $R_p = \mathbb{Z}_2 \times \mathbb{Z}_2'$. The alternative interpretation (equivalent to the Scherk–Schwarz breaking) of supersymmetry breaking in this model is by considering the two $N = 1$ supersymmetries S and S' contained in $N = 2$. The projection $\mathbb{Z}_2$ preserves one of them, S, while the projection $\mathbb{Z}_2'$ preserves the other of them, S'. The combined projection then breaks $N = 2$ to $N = 0$. At the fixed points of $\mathbb{Z}_2$, $y = 0, \pi R$ one can write a superpotential corresponding to the S supersymmetry while at the fixed points of $\mathbb{Z}_2'$, $y = \pm\pi R/2$ one can write the superpotential corresponding to the other supersymmetry S'. These interactions contain Yukawa couplings that can trigger electroweak breaking as in Eq. (17) with a finite Higgs mass. The predictions of this model are those corresponding to one with all matter particles propagating in the bulk and with Scherk–Schwarz breaking parameter $\omega = 1/2$. The compactification radius fixed by imposing correct electroweak minimum is fixed to rather low values $1/R \sim 2m_t$ while the Higgs mass stemming from the quartic coupling turns out to be $M_h \sim 127$ GeV.

However this model presents a quadratic sensitivity to the UV cutoff due to the fact that there is only a single Higgs hypermultiplet [41]. It turns out that the auxiliary field of the $U(1)$ hypercharge multiplet, D_Y receives a quadratic divergence due to the tadpole generated by the Higgs scalars localized on the boundary as

$$\frac{\xi}{\sqrt{2}}\big(\delta(y) + \delta(y - \pi R)\big) D_Y, \quad \xi \simeq \frac{g' \Lambda^2}{32\pi^2}, \tag{20}$$

where ξ is the FI term, that triggers a quadratically divergent contribution to the Higgs mass as [42]

$$\Delta m_h^2 = \frac{g'}{2}\xi. \tag{21}$$

In this model the Yukawa couplings become strong for scales $\Lambda \sim 5/R$. For such low values of the cutoff the previous quadratic divergences are not numerically relevant although they present a conceptual problem for models with a single Higgs hyperscalar. Of course models with two Higgs hyperscalars are free from FI divergences.

4. Conclusion

The Standard Model is an effective theory with a cutoff at a scale Λ that is usually supposed to be at the four dimensional Planck (quantum gravity) scale. The Higgs mass is sensitive to the cutoff through quadratic divergences that destabilize the electroweak vacuum: this is the hierarchy problem.

If the underlying theory beyond the cutoff is a string theory (the only known consistent theory of quantum gravity) there exists the possibility that the string (quantum gravity) scale M_s be in the TeV region, thus alleviating the hierarchy problem, if there are large (at most in the submillimeter range) transverse dimensions where gravity propagates. In this case there are unambiguous experimental signatures at present (Tevatron) and future (LHC) colliders corresponding to missing energy, i.e., inclusive production of extra-dimensional gravitons.

On the other hand if there are longitudinal (TeV) extra dimensions where the Standard Model matter propagates, there can be extra dimensional symmetries (gauge symmetry or supersymmetry) that protect the Higgs mass against quadratic divergences. In this case there are also unambiguous experimental signatures of longitudinal dimensions corresponding to the direct production of Kaluza–Klein modes of Standard Model gauge bosons.

In this paper we have reviewed the actual situation concerning the different higher dimensional symmetries that can protect the Higgs mass from quadratic divergences at least at the one-loop level. Unlike in four dimensions, where the only symmetry that can prevent quadratic divergences in a weakly coupled theory is supersymmetry, we have seen that there are other genuine extra-dimensional solutions. A particularly interesting solution is the case of a higher-dimensional gauge invariance.

Of course a parallel discussion can be followed if the cutoff Λ is no more due to the presence of a fundamental theory but instead of an unknown strongly coupled system. Symmetries protecting the Higgs mass from quadratic divergences should then be invoked (for instance if the Higgs originates as a pseudo-Goldstone boson in what is called little Higgs models [43]) in order to keep the UV sensitivity of the Higgs mass small.

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